Title: String Theory Lecture

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Collection: String Theory 2023/24

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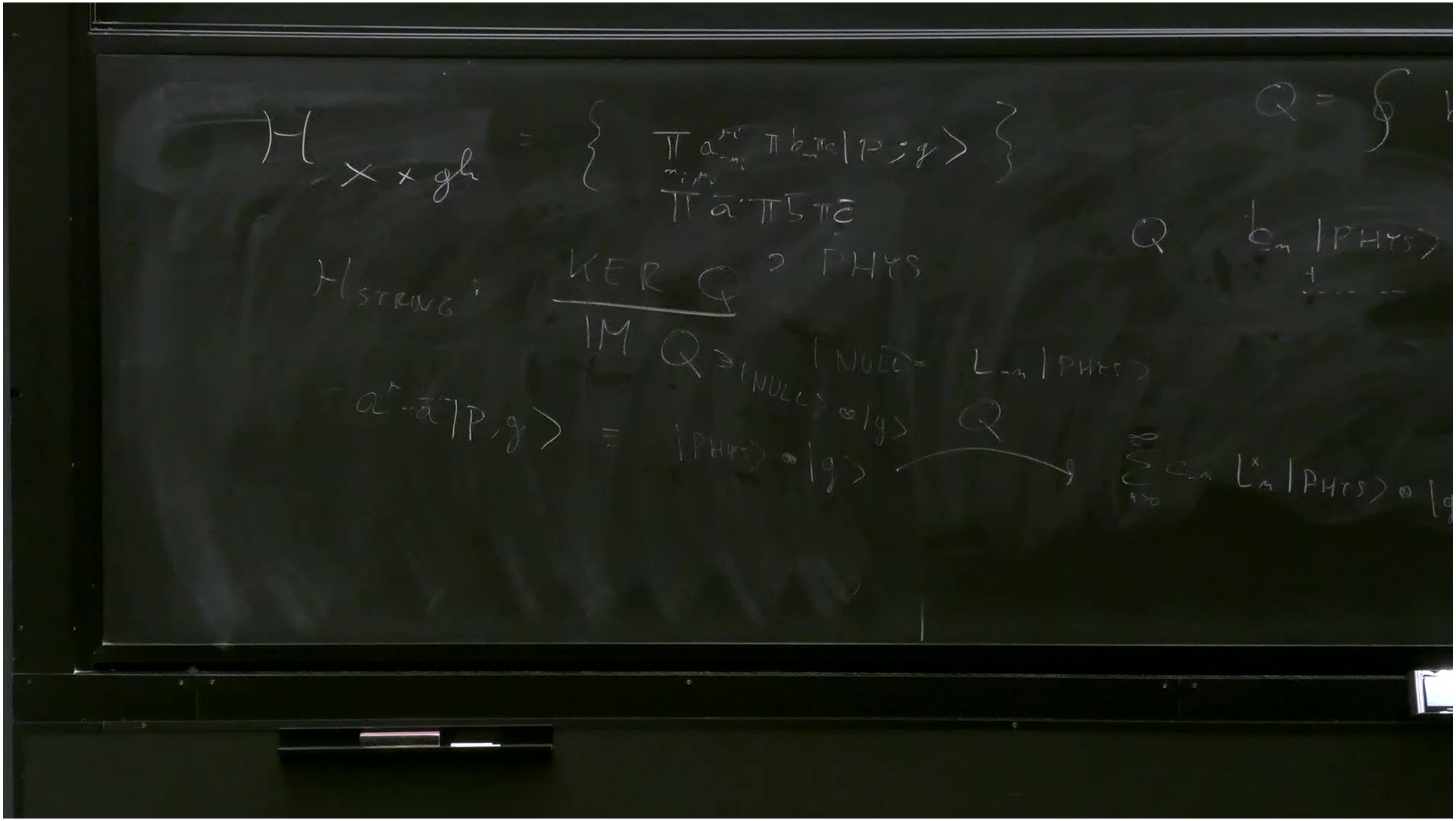
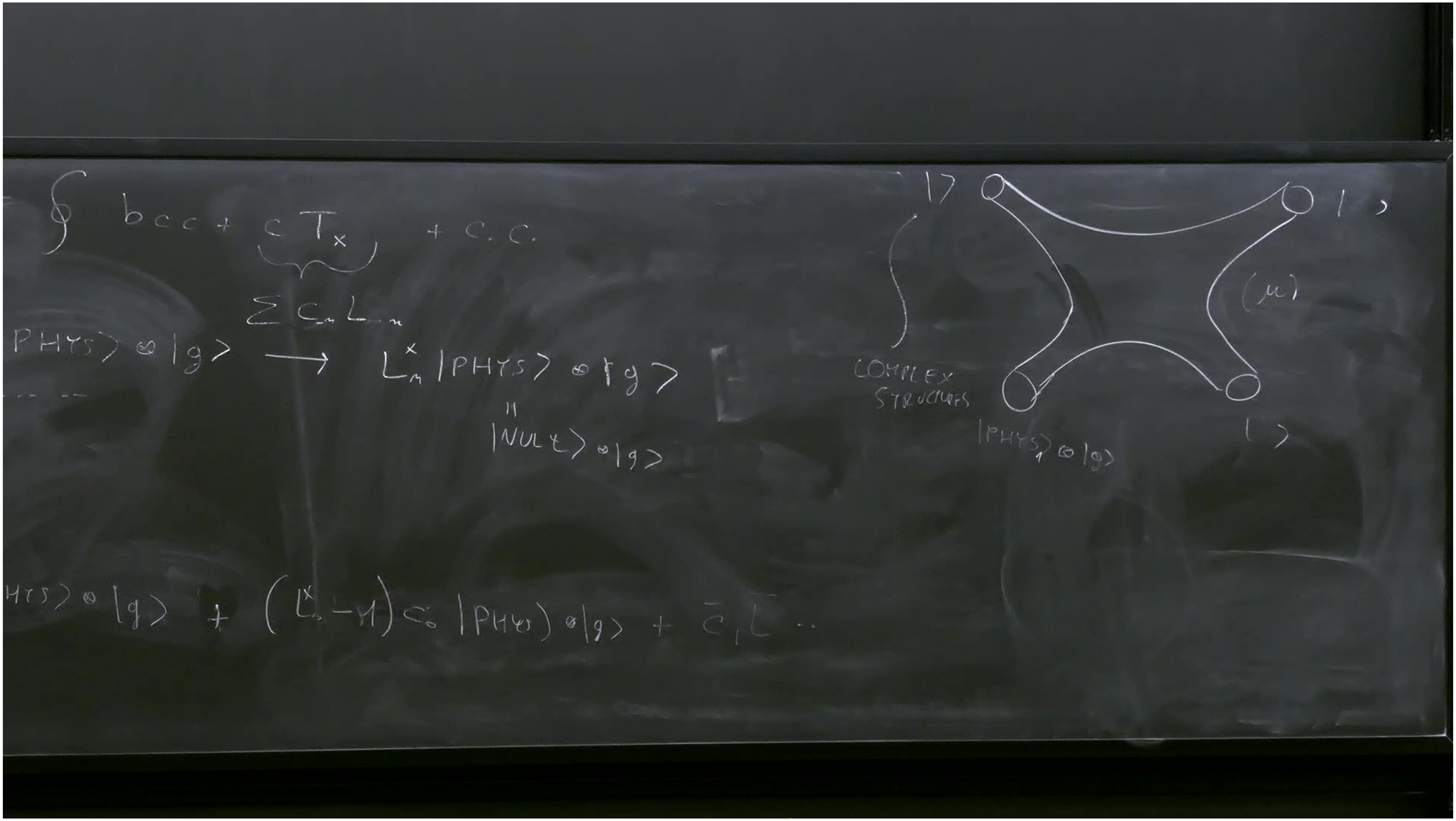
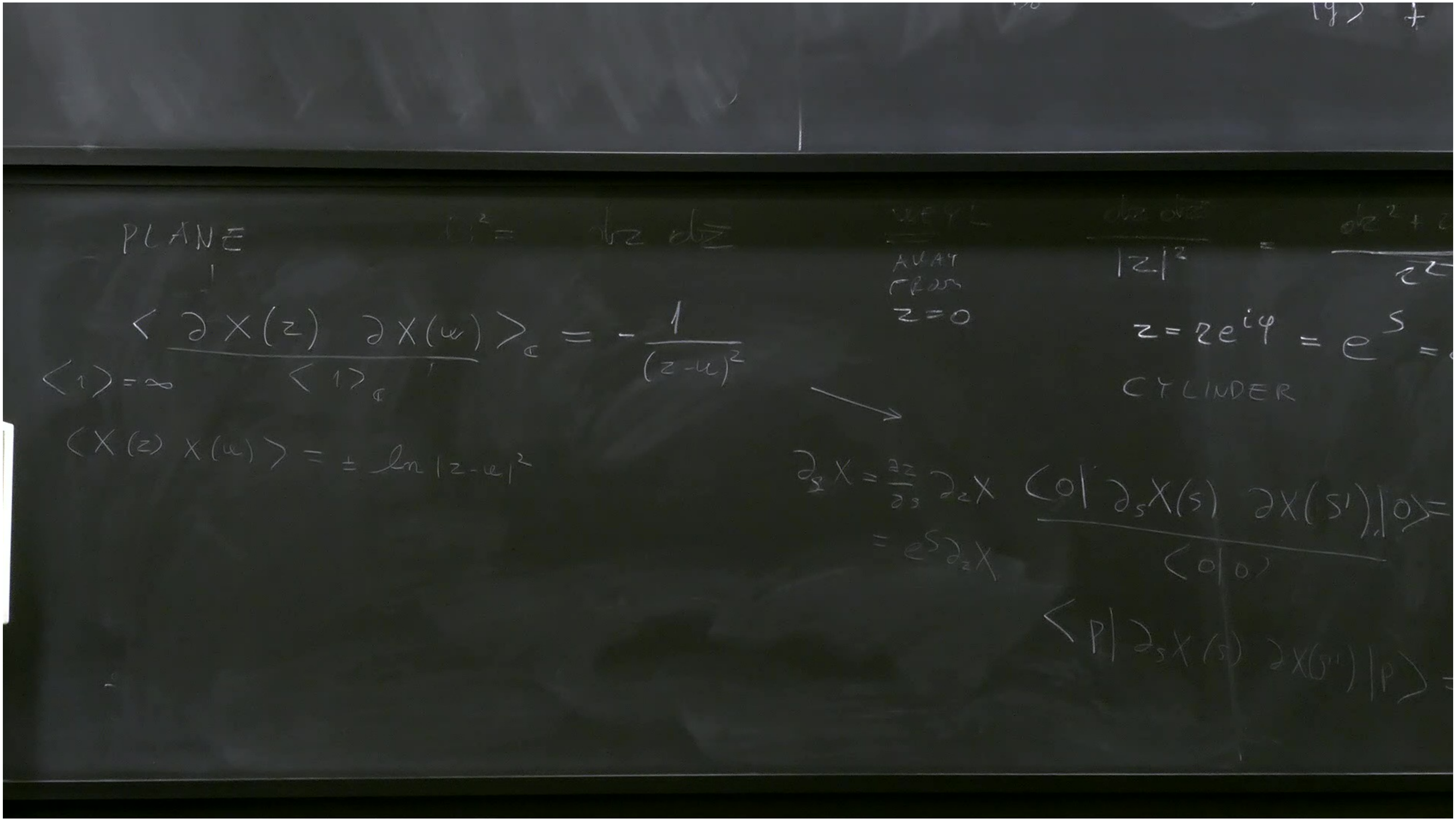

Q =
KER Q
IM Q
PHYS
STRING
|NULL〉
|PHYS〉
|PHYS〉 ⊗ |g〉
|P,g〉


∮ bcc + cT_x + c.c.
Σ c_m L_m
PHYS> ⊗ |g> ⟶ L^x_m |PHYS> ⊗ |g>
||
|NULL> ⊗ |g>
HYS> ⊗ |g> + (L^x_0 − M) c_0 |PHYS> ⊗ |g> + c̄_1 L̄ ..
COMPLEX
STRUCTURES
|PHYS>_1 ⊗ |g>
(μ)


PLANE
⟨∂X(z) ∂X(w)⟩_C / ⟨1⟩_C = −1/(z−w)^2
⟨1⟩=∞
⟨X(z) X(w)⟩ = ± ln|z−w|^2
AWAY FROM z=0
|z|^2
z = re^{iφ} = e^s
CYLINDER
∂_s X = ∂z/∂s ∂_z X = e^s ∂_z X
⟨0|∂_s X(s) ∂X(s')|0⟩ / ⟨0|0⟩
⟨p|∂_s X(s) ∂X(s'')|p⟩

$$|g\rangle + (L_0^X - 1)\, c_0\, |p(\psi)\rangle \otimes |g\rangle + \bar{c}_1 \bar{L} \ldots$$

$$\frac{dz\, d\bar{z}}{|z|^2} = \frac{dr^2 + r^2 d\varphi^2}{r^2} = \frac{dr^2}{r^2} + d\varphi^2 \qquad r = e^{\tau}$$

$$z = re^{i\varphi} = e^{s} = e^{\tau} e^{-i\sigma} = d\tau^2 + d\varphi^2 \qquad \langle p | p' \rangle = \delta(p - p')$$

CYLINDER

$$\frac{\langle 0 | \partial_s X(s)\, \partial X(s') | 0 \rangle}{\langle 0 | 0 \rangle} = \frac{e^{s+s'}}{(e^{s} - e^{s'})^2} \qquad \delta(0)$$

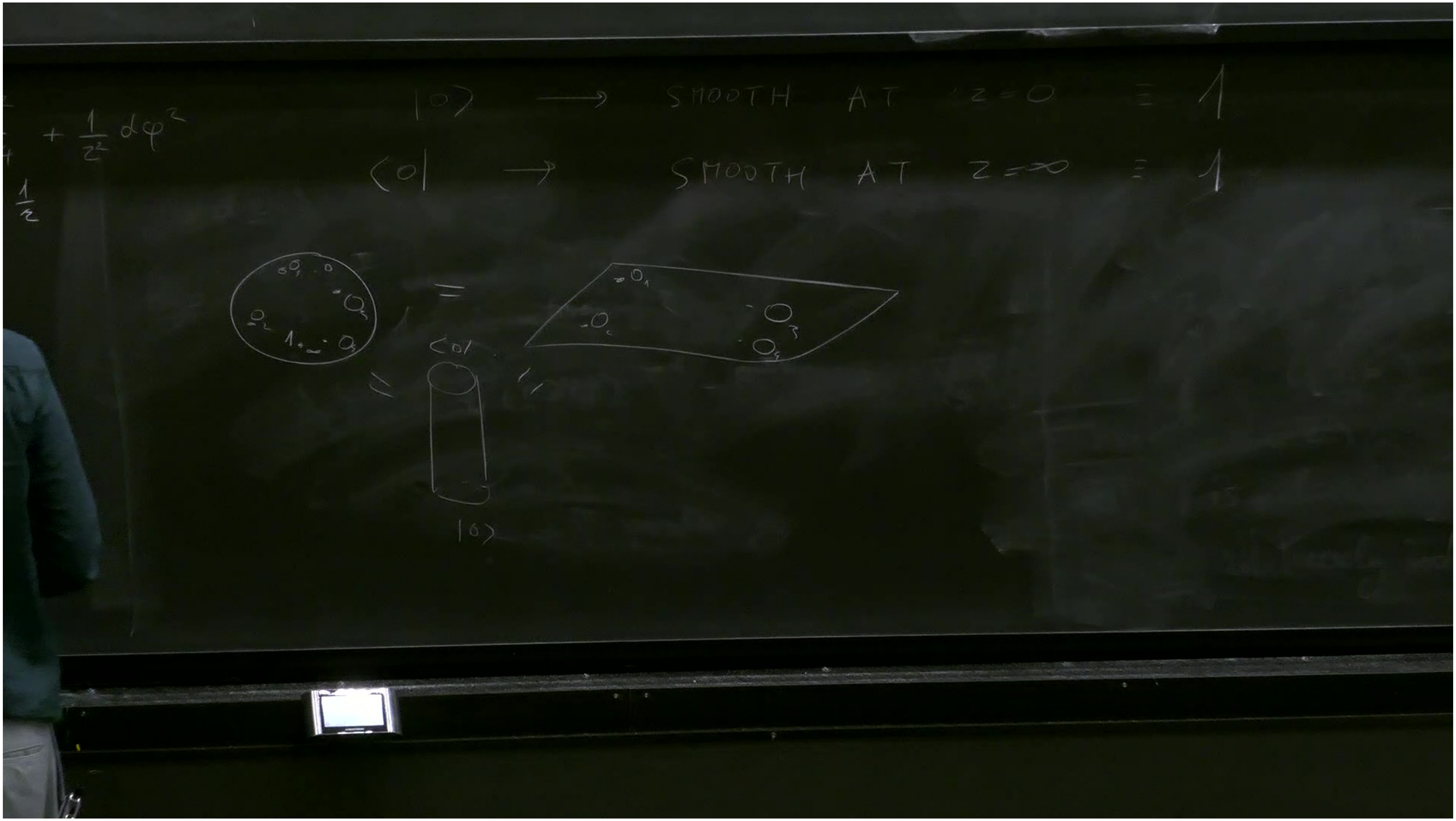
$$\langle p | \partial_s X(s)\, \partial X(s') | p \rangle = \frac{p^2}{2} + \text{''}$$

$s \to -s$ $\qquad z \to \frac{1}{z}$ $\qquad \frac{dz\,d\bar{z}}{|z|^4} = \frac{dr^2}{r^4} + \frac{1}{r^2}\,d\varphi^2$

$z \to \frac{1}{z}$

$$S^2 = \frac{dz\,d\bar{z}}{(1+|z|^2)^2} \xrightarrow{z \to \frac{1}{z}} \frac{\frac{dz\,d\bar{z}}{|z|^4}}{\left(1+\frac{1}{|z|^4}\right)^2} = \frac{dz\,d\bar{z}}{(1+|z|^4)^2}$$

$$\frac{dz\,d\bar{z}}{(1+|z|^2)^2} = \frac{ds\,d\bar{s}\,e^{s+\bar{s}}}{\left(1+e^{s+\bar{s}}\right)^2} = \frac{ds\,d\bar{s}}{\left(2\cosh\frac{s+\bar{s}}{2}\right)^2}$$

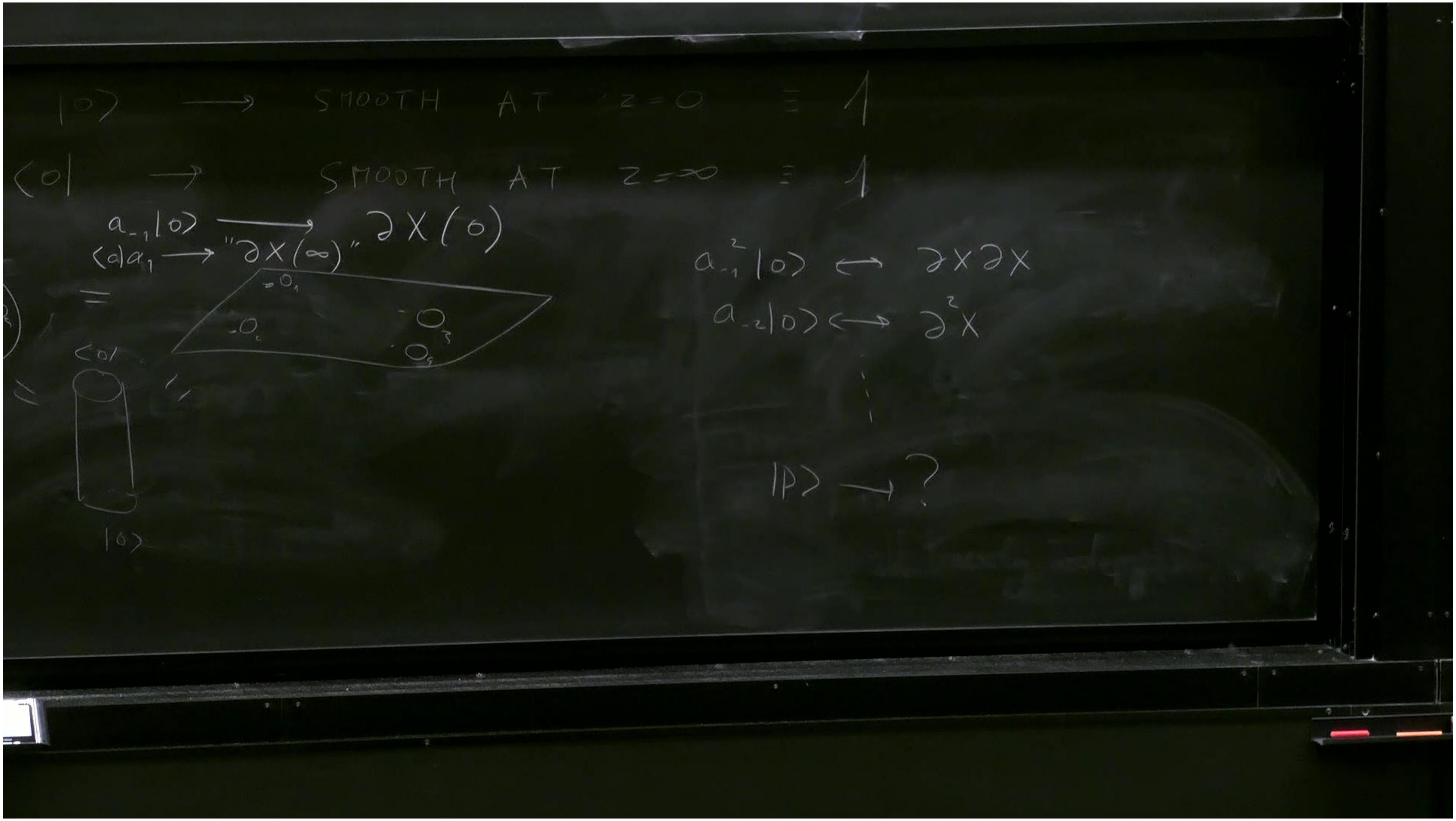

|0⟩ ⟶ SMOOTH AT z = 0 ≡ 1
⟨0| ⟶ SMOOTH AT z = ∞ ≡ 1
⟨0|
|0⟩

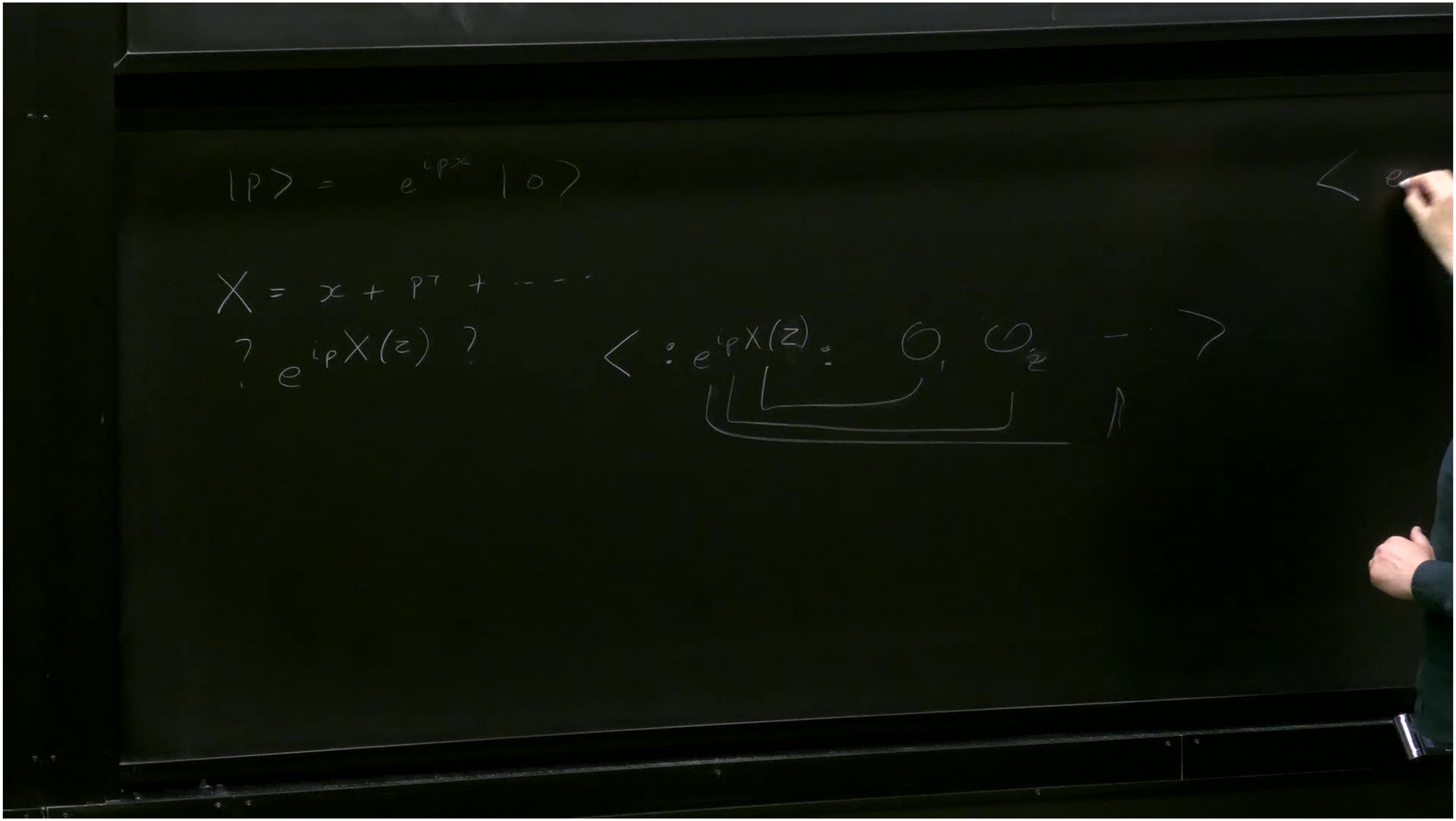
$$\partial_z X(z) \longrightarrow \partial_z X(0)$$

$$e^{-s}\,\partial_s X(s)\,|0\rangle \xrightarrow{s\to-\infty} a_{-1}|0\rangle$$

$$e^{-s}\sum a_n e^{-ns}|0\rangle = a_{-1}|0\rangle + e^{s} a_{-2}|0\rangle + e^{2s} a_{-3}|0\rangle \; \ldots$$

$$\partial X \partial X \xrightarrow{z\to 0}$$

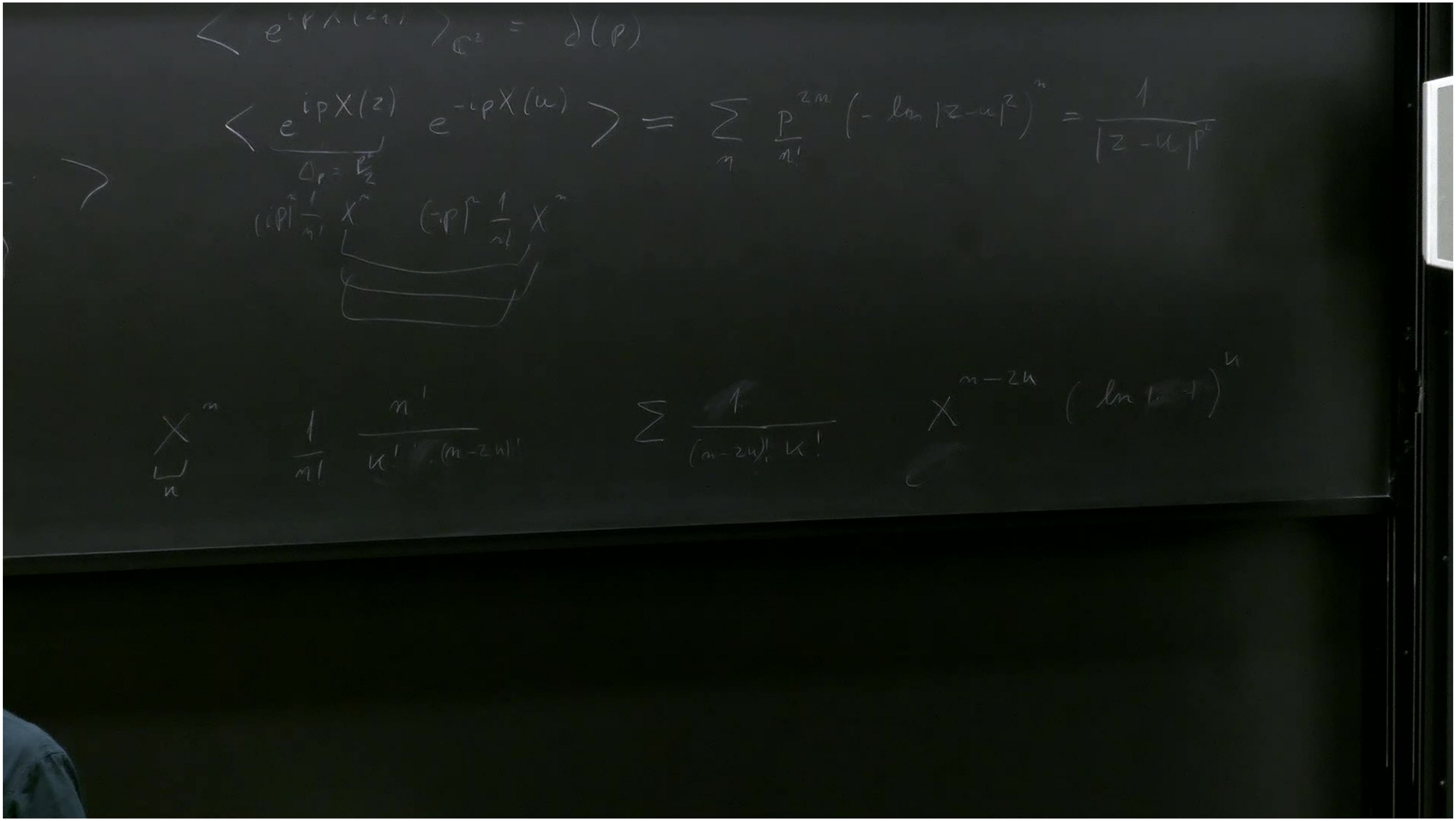

$|0\rangle \longrightarrow$ SMOOTH AT $z=0$ $\equiv 1$
$\langle 0| \longrightarrow$ SMOOTH AT $z=\infty$ $\equiv 1$
$a_{-1}|0\rangle \longrightarrow \partial X(0)$
$\langle 0|a_1 \longrightarrow$ "$\partial X(\infty)$"
O_1
O_2
O_3
O_4
$\langle 0|$
$|0\rangle$
$a_{-1}^2|0\rangle \longleftrightarrow \partial X \partial X$
$a_{-2}|0\rangle \longleftrightarrow \partial^2 X$
$|p\rangle \longrightarrow ?$

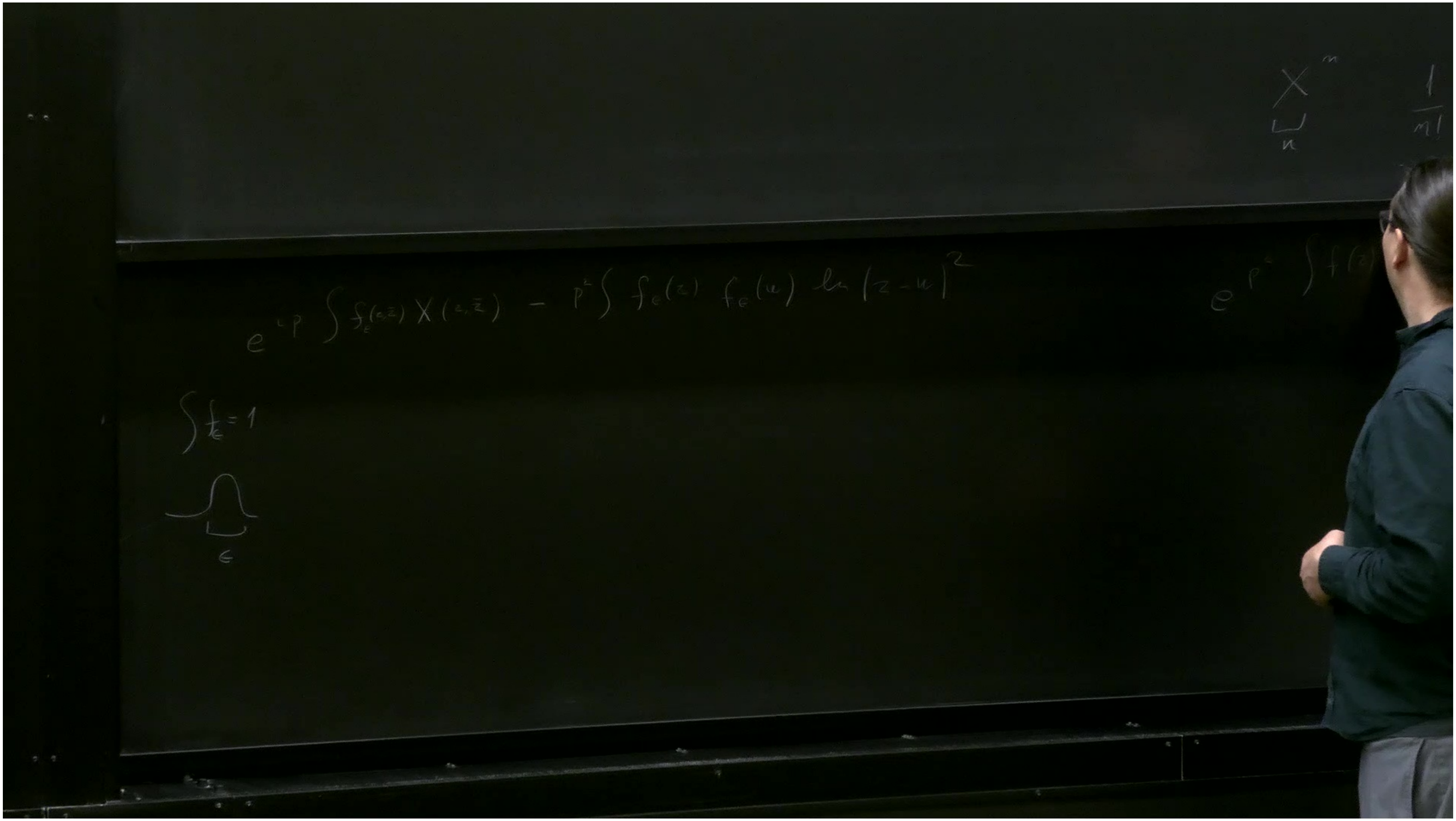

$|p\rangle = e^{ipx}|0\rangle$
$X = x + p\tau + \cdots$
$?\, e^{ipX(z)}\, ?$
$\langle :e^{ipX(z)}: \mathcal{O}_1 \mathcal{O}_2 \cdots \rangle$

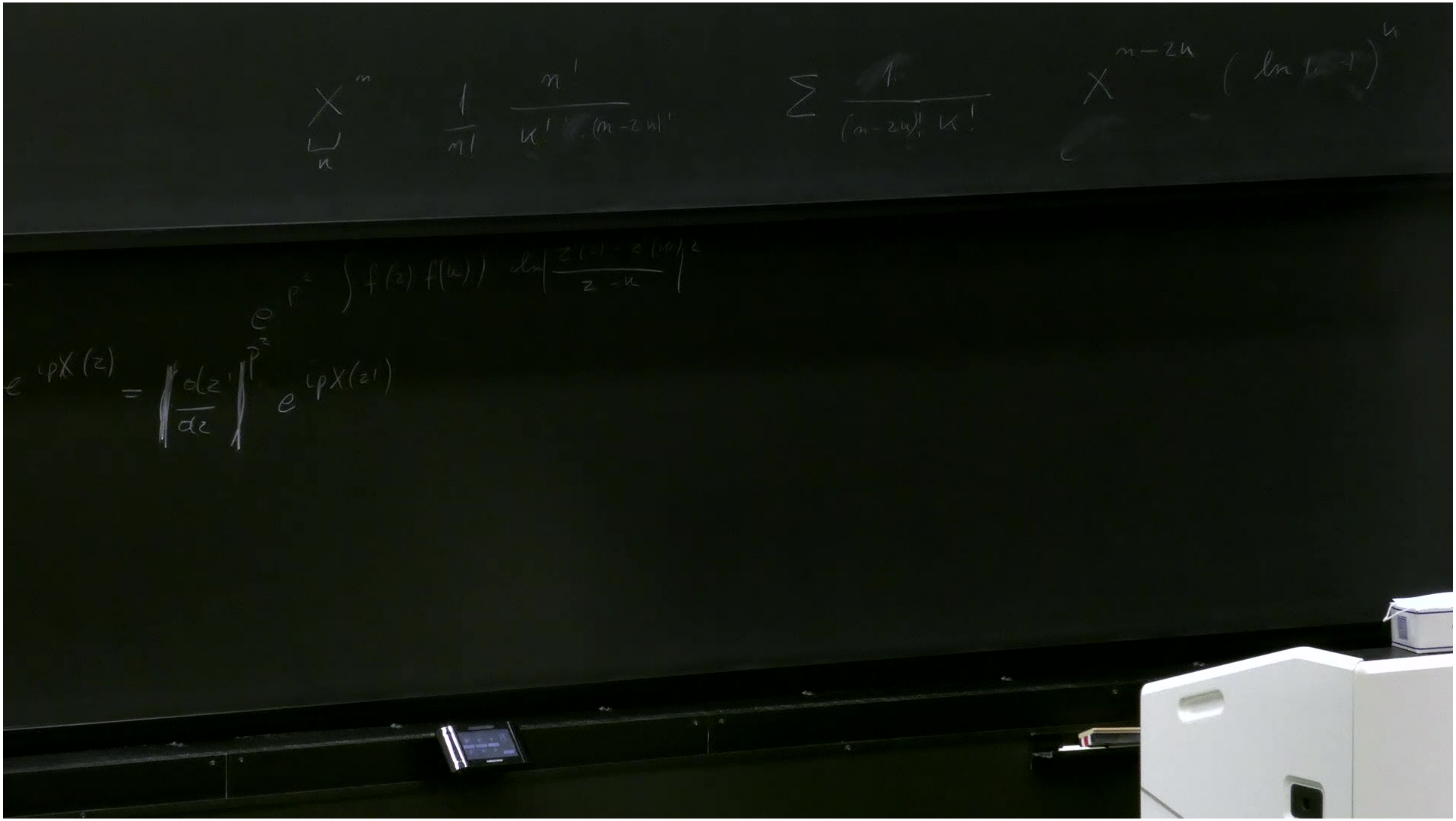
$$\langle e^{ipX(z_1)} \rangle_{\mathbb{C}^2} = \delta(p)$$

$$\langle \underbrace{e^{ipX(z)}}_{\Delta_p = \frac{p^2}{2}} e^{-ipX(w)} \rangle = \sum_n \frac{p^{2n}}{n!} \left(-\ln|z-w|^2\right)^n = \frac{1}{|z-w|^{p^2}}$$

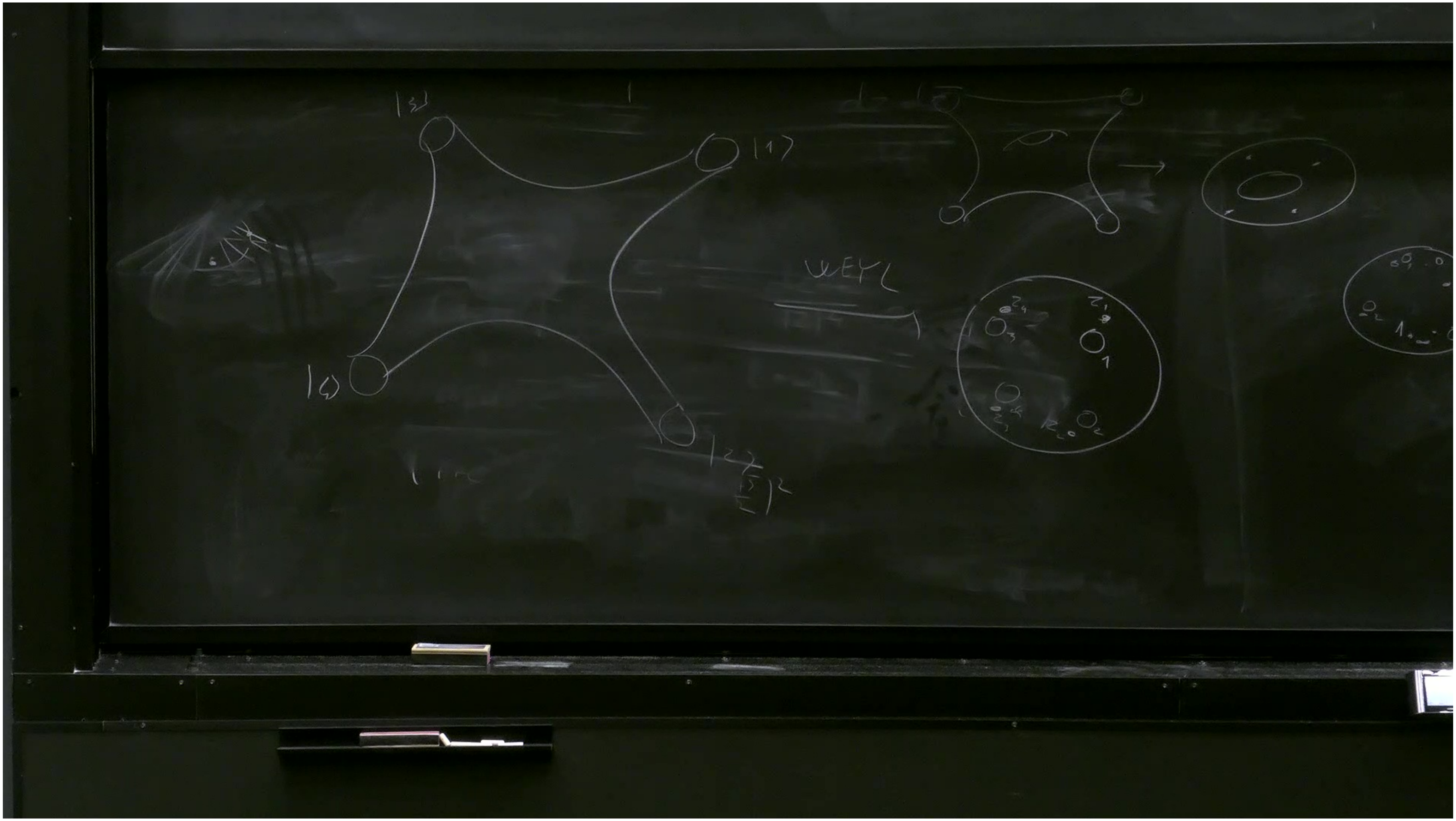
$$(ip)^n \frac{1}{n!} X^n \qquad (-ip)^n \frac{1}{n!} X^n$$


$\langle e^{ipX(z_1)} \rangle_{\mathbb{C}^2} = \delta(p)$
$\langle e^{ipX(z)} e^{-ipX(u)} \rangle = \sum_n \frac{p^{2n}}{n!} (-\ln|z-u|^2)^n = \frac{1}{|z-u|^{p^2}}$
$\Delta_p = \frac{p^2}{2}$
$(ip)^n \frac{1}{n!} X^n \quad (-ip)^n \frac{1}{n!} X^n$
$X^n \quad \frac{1}{n!} \frac{n!}{k!\,(n-2k)!} \quad \sum \frac{1}{(n-2k)!\,k!} X^{n-2k} (\ln \ldots)^k$


$e^{ip\int f_\epsilon(z,\bar z)X(z,\bar z) - p^2\int f_\epsilon(z) f_\epsilon(u) \ln|z-u|^2}$
$\int f_\epsilon = 1$
ϵ
$\frac{1}{n!}$



e ipX(z) = | dz' / dz |p² e ipX(z')
∫ f(z) f(w)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