

Title: String Theory Lecture

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$$\begin{array}{ccc}
 h^{zz} & h^{z\bar{z}} & h^{\bar{z}\bar{z}} \\
 \parallel & & \parallel \\
 0 & & 0 \\
 & h_{z\bar{z}} \circ h_{\bar{z}z} & 
 \end{array}$$

$$\begin{array}{ccc}
 b_{zz} & & b_{\bar{z}\bar{z}} \\
 \parallel & & \parallel \\
 b & & \bar{b}
 \end{array}$$

$$\boxed{\{Q, b\} = T} \quad \boxed{\{Q, \bar{b}\} = \bar{T}}$$

$$v^z \frac{\partial}{\partial z} + v^{\bar{z}} \frac{\partial}{\partial \bar{z}}$$

$$\begin{array}{ccc}
 c^z & & c^{\bar{z}} \\
 \parallel & & \parallel \\
 0 & & 0
 \end{array}$$

$$\boxed{\{Q, X^\mu\} = c \partial X^\mu + \bar{c} \bar{\partial} X^\mu}$$

$$h_{z\bar{z}} \\ \parallel \\ 0$$

dz

$$b_{z\bar{z}} \\ \parallel \\ b$$

$$v^{-z} \frac{\partial}{\partial z} + v^{-\bar{z}} \frac{\partial}{\partial \bar{z}}$$

$$c = 0$$

$$\bar{c} = 0$$

$$\begin{pmatrix} \partial v & \partial \bar{v} \\ \bar{\partial} v & \bar{\partial} \bar{v} \end{pmatrix}$$

$$\{v, v'\} = v \partial v' - v' \partial v$$

$$S_{gh} = \int [ \dots ]$$

$$S_x = T$$

$$\{Q, X^\mu\} = c \partial X^\mu + \bar{c} \bar{\partial} X^\mu$$

$$\{Q, \bar{b}\} = \bar{T}$$

$$\{Q, c\} = c \partial c \\ Q^2 = 0$$

$$Q \Rightarrow J_{BRST}$$

$$v^{\mu} = \begin{pmatrix} \partial v & \partial \bar{v} \\ \bar{\partial} v & \bar{\partial} \bar{v} \end{pmatrix}$$

$$v^{\mu} = v \partial v^{\mu} - v^{\mu} \partial v$$

$$\delta h^{zz} = h^{z\bar{z}} \partial_{\bar{z}} v^z + \dots$$

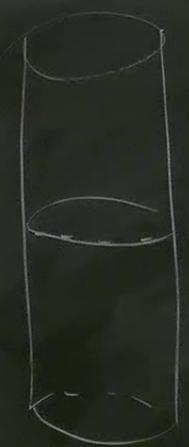
$$\delta h^{\bar{z}\bar{z}} = h^{z\bar{z}} \partial_z v^{\bar{z}}$$

$$T_{z\bar{z}} = 0$$

$$S_{gh} = \int [b_{zz} \bar{\partial} c^z + b_{\bar{z}\bar{z}} \partial c^{\bar{z}}] dz d\bar{z}$$

$$S_x = T \int \partial X^{\mu} \bar{\partial} X^{\mu} dz d\bar{z}$$

$$Q \Rightarrow T_{BRST} = \frac{1}{2} c \partial X \partial X + \dots b c \partial c$$



$$T_{BRST} = \lim_{s' \rightarrow s} \left[ \frac{1}{2} c(s) \partial X(s) \partial X(s') + \dots \right]$$
$$Q = \int_0^{2\pi} J_{BRST} ds + \overline{J}_{BRST} d\bar{s}$$

$\{Q, b\}$

?  $Q^2 = 0$  ?

$$[Q, J_{BRST}] = \# c \partial c \quad (D-26)$$



$$\partial X(s') + \cdot b(s) \mid c(s') \partial c(s') - ? ]$$

$$\{Q, b\} = T = T^x + T^{gl} \quad [L, L] = (1L + \frac{\#}{12} n^2(n-1))$$

$$\{Q, b_n\} = L_n$$

$$[L_m, \{Q, b_n\}] = \{L_m, Q\} b_n + \{Q, L_m, b_n\} = (m-n)L_{m+n}$$

$$S_{gh} = \int b \bar{\partial} c \, dz d\bar{z} + \int \bar{b} \partial \bar{c} \, dz d\bar{z} \quad \bar{\partial} c = 0 \quad \partial \bar{c} = 0$$

$$\partial b = 0 \quad \bar{\partial} \bar{b} = 0$$

$b_{zz} \quad c^z$

$$b' = b \left( \frac{dz}{dz'} \right)^2$$

$$c' = c \left( \frac{dz}{dz'} \right)^{-1}$$

$$T_{gh} = c \partial_s b + 2 \partial_s c b$$

$$- \partial_s \bar{b} c$$

$$\mathcal{T}^{gh} = cb$$

$$\langle b(s) | c(s') \rangle$$

$$J^{gh} = \lim_{s \rightarrow s'} \left( b(s) c(s') - \frac{1}{s-s'} \right)$$

$$s = s(z)$$

$$J_s = \left( b(s) c(s') - \frac{1}{s-s'} \right) =$$

$$J_z = \left( b(z) c(z') - \frac{1}{z-z'} \right)$$

$J^g h$  is "ANOMALOUS"  $\Rightarrow \langle O_1 \dots O_m \rangle_{\Sigma} = \sum_i g^h(O_i) = \text{NUMBER}(\Sigma)$

$$= \left( \left( \frac{dz}{ds} \right)^2 \left( \frac{ds'}{dz'} \right) b(z) c(z') - \frac{1}{s-s'} \right)$$

$$\left( \frac{dz}{ds} \right)^2 \left( \frac{ds'}{dz'} \right) \left( \dots \right) + \left( \frac{dz}{ds} \right)^2 \left( \frac{ds'}{dz'} \right) \frac{1}{z-z'} - \frac{1}{s(z)-s(z')}$$

ALOUS"  $\Rightarrow \langle O_1 \dots O_n \rangle_{\Sigma}$   $\sum_i g^h(O_i) = \text{NUMBER}(\Sigma)$

$\langle \dots \rangle_{S^2}$   $\#c - \#b = 3$

$$b(z)c(z') - \frac{1}{s-s'}$$

$$\dots + \left(\frac{dz}{ds}\right)^2 \left(\frac{ds'}{dz'}\right) \frac{1}{z-z'} - \frac{1}{s(z)-s(z')}$$



$$\bar{\partial} c = 0$$
$$\bar{\partial} b = 0$$

$$\partial \bar{c} = 0$$
$$\partial \bar{b} = 0$$

$$\bar{\partial}_s \langle b(s) | c(s') \rangle \rightarrow = - \int^{(2)} (s-s')$$

$$\langle b(s) | c(s') \rangle \sim \frac{1}{s-s'} + \dots$$

$$\partial_s c b = [L_m^{gh}, L_m^{g^2}] = (n-m) L_{m+n}^{gh} + \frac{-26}{12} m(m^2-1) \delta_{m+n,0}$$

$T^X + T^{gh}$  # D-26

$$S_{gh} = \int b \bar{\partial} c \, d\zeta d\bar{\zeta} + \int \bar{b} \partial \bar{c} \, d\zeta d\bar{\zeta} \quad \bar{\partial} c = 0 \quad \partial \bar{c} = 0$$

$$\bar{\partial} b = 0 \quad \partial \bar{b} = 0$$

$$b_{zz} \quad c^z$$

$$b' = b \left( \frac{dz}{dz'} \right)^2$$

$$c' = c \left( \frac{dz}{dz'} \right)^{-1}$$

$$T_{gh} = c \partial_s b + 2 \partial_s c b$$

$$- \partial_s \bar{b} c$$

$$T^{gh} = cb$$

$$T^x + T^{gh}$$

$$b(s) = \sum_n b_n e^{-ns}$$

$$c(s) = \sum_n c_n e^{-ns}$$

$$\{b, b\} = 0$$

$$\{c, c\} = 0$$

$$\{b_m, c_m\} = \delta_{m+m, 0}$$

$$|g\rangle$$

$$c_n |g\rangle = 0$$

$$b_n |g\rangle = 0$$

$$n > 0$$

$$|g\rangle$$

$$\bar{b} = \sum \bar{b}_n e^{-n\bar{s}}$$

$$\bar{c} = \sum \bar{c}_n e^{-n\bar{s}}$$

$$b(s) = \sum_n b_n e^{-ns}$$

$$c(s) = \sum_n c_n e^{-ns}$$

$$\bar{b} = \sum \bar{b}_n e^{-n\bar{s}}$$

$$\bar{c} = \sum \bar{c}_n e^{-n\bar{s}}$$

$$\{b, b\} = 0$$

$$\{c, c\} = 0$$

$$\{b_m, c_n\} = \delta_{m+n, 0}$$

$$\begin{array}{c}
 b_1 |g\rangle \\
 \uparrow \\
 c_0 |g\rangle \quad |g\rangle \\
 \downarrow \\
 c_{-1} |g\rangle
 \end{array}$$

$$c_n |g\rangle = 0 \quad b_n |g\rangle = 0 \quad n > 0$$

$$b_0 |g\rangle = 0$$

$$\begin{array}{c}
 c_{-1} |g\rangle \\
 b_{-1} |g\rangle
 \end{array}$$

$$b_n e^{-n\bar{s}}$$

$$E_n e^{-n\bar{s}}$$

$$\{b_n, c_n\} = 1 \quad b_n^2 = 0 \quad c_n^2 = 0$$

$$\delta_{m+n, 0}$$

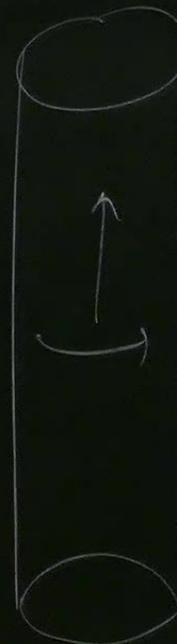
$$\langle \tilde{g} | g \rangle = 1$$

$$\langle \tilde{g} | c_n = 0$$

$$\langle g | b_n = 0$$

$$\langle g | c_n \bar{c}_n | g \rangle = 1$$

$$\langle \tilde{g} | b_n \bar{b}_n | \tilde{g} \rangle = 1$$

 $\partial \bar{s}$  $\partial s$