Title: String Theory Lecture

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Collection: String Theory 2023/24

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$$\boxed{S(X) = \frac{T}{2}\int \frac{dX^\mu}{dw^a}\frac{dX_\mu}{dw_a}\, dw^1 dw^2}$$

$$T^{ab} = 0$$

$$\parallel$$

$$\frac{\partial X^\mu}{\partial u^a}\frac{\partial X^\nu}{\partial u^b} - \delta_{ab} \cdots$$

$$T^{a}{}_{a} = 0$$

$$\boxed{\frac{\partial^2 X^\lambda}{\partial u^a \partial u_a} = 0}$$

$$\boxed{S(X) = \frac{T}{2}\int \frac{dX^\mu}{dw^a}\frac{dX_\mu}{dw_a}\, dw^1 dw^2}$$

$$T^{ab} = 0$$

$$\| $$

$$\frac{\partial X^\mu}{\partial u^a}\frac{\partial X^\nu}{\partial u^b} -$$

$$T^{aa} = 0$$

$$\langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2) \cdots \rangle = \sum \prod \langle X^{\mu_i}(u_i)\, X^{\mu_j}(u_j)\rangle$$

$$\Box_1^2 \langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2)\rangle = \delta^{(2)}(u_1 - u_2)\,\eta^{\mu_1\mu_2}$$

$$\frac{\delta S}{\delta h_{ab}} \equiv T^{ab}$$

$$\boxed{S(X) = \frac{T}{2}\int \frac{dX^{\mu}}{dw^{a}} \frac{dX_{\mu}}{dw_{a}} dw^{1} dw^{2}}$$

$$\langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2) \cdots \rangle = \sum \prod \langle X^{\mu_i}(u_i)\, X^{\mu_j}(u_j) \rangle$$

$$\Box_1^2 \langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2) \rangle = \delta^{(2)}(u_1 - u_2)\, \eta^{\mu_1 \mu_2}$$

T

$\|$

$\frac{\partial X}{\partial u}$

$\frac{T}{2}\int \frac{dX^\mu}{dw^a}\frac{dX_\mu}{dw_a}\, dw^1 dw^2$

$T^{ab}=0$

$\|$

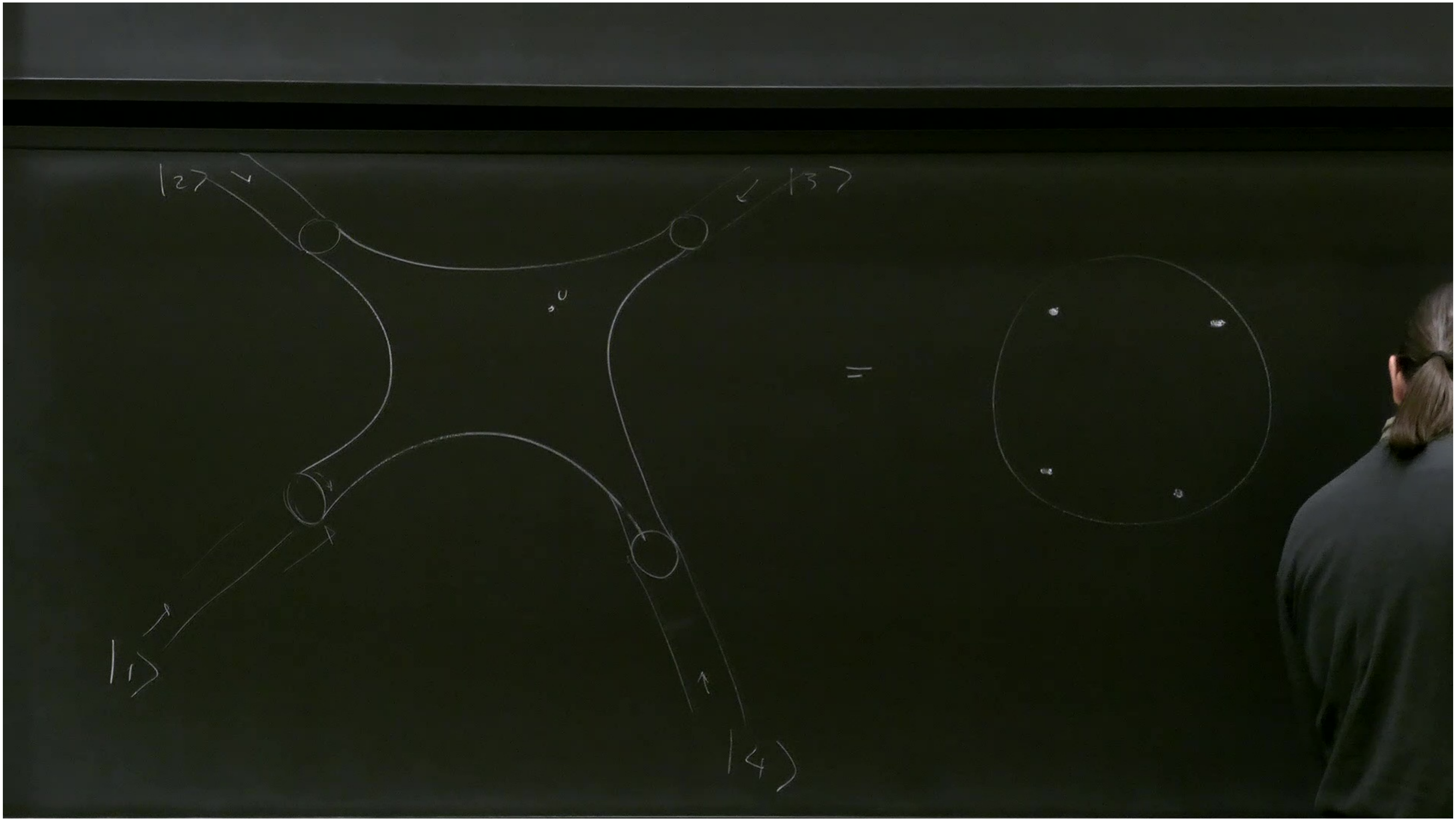
$\frac{\partial X^\mu}{\partial u^a}\frac{\partial X^\nu}{\partial u^b} - \delta_{ab}\cdots$

$T^{aa}=0$

$\boxed{\frac{\partial^2 X^\lambda}{\partial u^a \partial u_a} = 0}$

$\cdots\rangle = \sum \prod \langle X^{\mu_i}(u_i) X^{\mu_j}(u_j)\rangle$

$^{\mu_2}(u_2)\rangle = \delta^{(2)}(u_1 - u_2)\,\eta^{\mu_1\mu_2}$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$$\boxed{S(X) = \frac{T}{2}\int \frac{dX^\mu}{dw^a}\frac{dX_\mu}{dw_a}\, dw^1 dw^2}$$

$$T^{ab} = 0$$

$$\| $$

$$\frac{\partial X^\mu}{\partial u^a}\frac{\partial X^\nu}{\partial u^b} - \delta$$

$$\langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2) \cdots \rangle = \sum \prod \langle X^{\mu_i}(u_i)\, X^{\mu_j}(u_j) \rangle$$

$$T^{a}{}_{a} = 0$$

$$\Box_1^2 \langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2) \rangle = \delta^{(2)}(u_1 - u_2)\, \eta^{\mu_1\mu_2}$$

$$\langle X^{\mu_1}(u) X^{\mu_2}(u) \quad \cdots\cdots \rangle$$

$$\frac{T}{2}\int \frac{dX^\mu}{dw^a}\frac{dX_\mu}{dw_a}\, dw^1 dw^2$$

$$T^{ab}=0$$
$$\parallel$$
$$\frac{\partial X^\mu}{\partial u^a}\frac{\partial X^\nu}{\partial u^b}-\delta_{ab}\cdots$$

$$\boxed{\frac{\partial^2 X^\lambda}{\partial u^a \partial u_a}=0}$$

$$\cdots \rangle = \sum \prod \langle X^{\mu_i}(u_i)\, X^{\mu_j}(u_j)\rangle \quad T^{aa}=0$$

$$\cdots{}^{\mu_2}(u_2)\rangle = \delta^{(2)}(u_1-u_2)\,\eta^{\mu_1\mu_2}$$

$$\cdots\cdots\rangle$$

ab

$$\boxed{S(X) = \frac{T}{2} \int \frac{dX^{\mu}}{dw^{a}} \frac{dX_{\mu}}{dw_{a}} \, dw^{1} dw^{2}}$$

$T^{ab} = 0$

$\parallel$

$\frac{\partial X^{\mu}}{\partial u^{a}} \frac{\partial X}{\partial u}$

$T^{aa} = 0$

$$\langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2) \cdots \rangle = \sum \prod \langle X^{\mu_i}(u_i)\, X^{\mu_j}(u_j) \rangle$$

$$\Box_1^2 \langle X^{\mu_1}(u_1)\, X^{\mu_2}(u_2) \rangle = \delta^{(2)}(u_1 - u_2)\, \eta^{\mu_1 \mu_2}$$

$$\langle :X^{\mu_1}(u)\, X^{\mu_2}(u): \quad - - - - \rangle$$

NORMAL ORDERING =

DROP $\sqcap$ WITHIN OPERATOR

$$\boxed{\frac{T}{2}\int \frac{dX^{\mu}}{dw^{a}}\frac{dX_{\mu}}{dw_{a}}\,dw^{1}dw^{2}}$$

$$T^{ab}=0$$

$$\frac{\partial X^{\mu}}{\partial u^{a}}\frac{\partial X^{\nu}}{\partial u^{b}}-\delta_{ab}\cdots$$

$$T^{aa}=0$$

$$\boxed{\frac{\partial^{2}X^{\lambda}}{\partial u^{a}\partial u_{a}}=0}$$

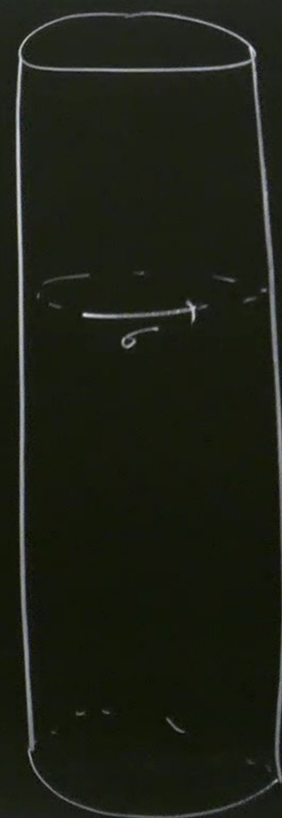
$$\ldots_{2})\;\cdots\;\rangle=\sum\prod\langle X^{\mu_i}(u_i)\,X^{\mu_j}(u_j)\rangle$$

$$X^{\mu_2}(u_2)\rangle=\delta^{(2)}(u_1-u_2)\,\eta^{\mu_1\mu_2}$$

$$\ldots:\;\;----\;\rangle$$

...ERING :

... WITHIN OPERATOR



$\uparrow \tau = it$

$\sigma \to \sigma + 2\pi L$

$s = \tau - i\sigma$

$\partial_s \partial_{\bar{s}} X = 0$

$X = f(s) + g(\bar{s})$

$X(\sigma,\tau) =$

$$\partial_s \partial_{\bar{s}} X = 0$$

$$X = f(s) + g(\bar{s})$$

$$\left[X(\sigma,0),\ i\partial_\tau X(\sigma',0)\right] = 4\pi i\, \delta(\sigma-\sigma')$$

$$X(\sigma,\tau) = x - \frac{2i}{L} p\tau + \sum_{n\neq 0} \left[\frac{i}{n} a_n e^{i\frac{n}{L}(\sigma+i\tau)} - \frac{i}{n} \bar{a}_n e^{-i\frac{n}{L}(\sigma-i\tau)} \right]$$

$$[x,p] = i \qquad [a_n, a_{-n}] = n \qquad [\bar{a}_n, \bar{a}_{-n}] = n$$

$|p\rangle \qquad a_n|p\rangle = \bar{a}_n|p\rangle = 0 \qquad n>0$

$a_{-1}|p\rangle \;,\; \bar{a}_{-1}|p\rangle$

$a_{-2}|p\rangle \,,\, a_{-1}^2|p\rangle \,,\, \bar{a}_{-2}|p\rangle \,,\, \bar{a}_{-1}^2|p\rangle$

$\cdots\cdots$

$\langle p|p'\rangle = \delta(p-p') \qquad (a_{-1}|p\rangle)^\dagger = \langle p|a_1$

$\langle p|a_n a_{-n}|p'\rangle \overset{?}{=} n\,\delta(p-p')$

SYMMETRY: $X \to X + c$

$X \to X + f(s) + g(\bar{s})$

$\int \frac{\partial X}{\partial v^a} \frac{\partial X}{\partial v_a} d^2 v \to$

$J_a = \partial_a X$

$\partial_a J^a = \Box^2 X = 0$

$J = \partial_s X \qquad \bar{J} = \partial_{\bar{s}} X$

$\bar{\partial} J = \bar{\partial} \partial X = 0$

$\bar{\partial}\Big(f(s) J(s) \Big) = 0$

$$\frac{\partial X}{\partial v_a} d^2v \rightarrow \int \frac{\partial X}{\partial v^a} \frac{\partial c}{\partial v_a} dw^2$$

$$\frac{\partial X}{\partial \bar{\zeta}} d\zeta d\bar{\zeta}$$

$$\zeta \rightarrow f(\zeta')$$

$$\bar{\partial} O = 0$$

$$\bar{\partial} f(\zeta) O = 0$$

$$\int \frac{\partial X}{\partial v^a}\frac{\partial X}{\partial v_a}\, d^2v \to \int \frac{\partial X}{\partial v^a}\frac{\partial c}{\partial v_a}\, dw^2$$

$$\int \frac{\partial X}{\partial z}\frac{\partial X}{\partial \bar z}\, dz\, d\bar z$$

$$z \to f(z')$$

$$\bar\partial O = 0$$

$$\bar\partial f(z)\, O = 0$$

$$\langle p | a_n a_m | p' \rangle = \delta_{n+m,0} \cdots \qquad i\partial X = \frac{\ }{L} + \sum_{n\neq 0}$$

$$\langle p | \partial X(s) \ \partial X(s') | p' \rangle = \left(-\frac{p^2}{L^2} - \frac{1}{L^2} \sum_{n>0} n \, e^{\frac{n}{L}(s'-s)} \right) \delta(p-p') =$$

$$\langle p | T(s) | p' \rangle$$

$$T(s) = \lim_{s' \to s} \left[-\frac{1}{2} \partial X(s) \, \partial X(s') \right.$$

$\langle p | a_n a_m | p' \rangle = \delta_{n+m,0} \cdots$ $\quad i\partial X = \frac{}{L} + \frac{}{L} \sum_{n\neq 0}$

$$\langle p | \, \partial X(s) \, \partial X(s') \, | p' \rangle = \left(-\frac{p^2}{L^2} - \frac{1}{L^2} \sum_{n>0} n \, e^{\frac{n}{L}(s'-s)} \right) \delta(p-p') = \Big($$

$\langle p | T(s) | p' \rangle$

$$T(s) = -\frac{1}{2} \lim_{s' \to s} \left[\partial X(s) \, \partial X(s') + \frac{1}{(s-s')^2} \right]$$

$\langle p | T(s) | p' \rangle =$

$$i\partial X = \frac{p}{L} + \frac{1}{L}\sum_{n\neq 0} a_n e$$

$$= \left(-\frac{p^2}{L^2} - \frac{1}{L^2}\sum_{n>0} n\, e^{\frac{n}{L}(s'-s)}\right)\delta(p-p') = \left(-\frac{p^2}{L^2} - \frac{1}{L^2}\frac{e^{\frac{s+s'}{L}}}{\left(e^{s/L}-e^{s'/L}\right)^2}\right)\delta(p-p')$$

$$\sum_{n=1}^{\infty} n = $$

$$\langle p|T(s)|p'\rangle = \left(\frac{p^2}{2L^2}\boxed{-\frac{1}{24L^2}}\right)\delta(p-p')$$

$$X(s') + \frac{1}{(s-s')^2}\Bigg]$$

$\langle p | a_n a_m | p' \rangle = \delta_{n+m,0} \cdots \qquad i\partial X = \frac{}{L} + \frac{}{L}\sum_{n\neq 0}$

$$\langle p | \partial X(s) \, \partial X(s') | p' \rangle = \left(-\frac{p^2}{L^2} - \frac{1}{L^2} \sum_{n>0} n e^{\frac{n}{L}(s'-s)} \right) \delta(p-p') =$$

$\langle p | T(s) | p' \rangle$

$$\langle p | T(s) | p' \rangle = \left(\frac{p^2}{2L^2} \right.$$

$$T(s) = -\frac{1}{2} \lim_{s' \to s} \left[\partial X(s) \, \partial X(s') + \frac{1}{(s-s')^2} \right]$$

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{m,n} :a_n a_m: e^{-(n+m)s} \equiv -\frac{1}{24} + \sum_n L_n e^{-ns}$$

CLASSICALLY

$$T_{ss} = -\frac{1}{2}(\partial X)^2$$

$$T_{\bar{s}\bar{s}} = -\frac{1}{2}(\bar{\partial} X)^2$$

a_1

$$i\partial X = \frac{p}{L} + \frac{1}{L}\sum_{n\neq 0} a_n e^{-\frac{n}{L}s}$$

$$p = a_0 = \bar{a}_0$$

$$\bar{\partial} T = 0$$

$$\bar{\partial}\, v(s) T = 0$$

$$s \to s + v(s)$$

$$v(s)\partial_s$$

$$- \frac{1}{L^2}\sum_{n>0} n\, e^{\frac{n}{L}(s'-s)} \Big) \delta(p-p') = \left(-\frac{p^2}{L^2} - \frac{1}{L^2}\frac{e^{\frac{s+s'}{L}}}{(e^{s/L} - e^{s'/L})^2} \right) \delta(p-p')$$

$$\sum_{n=1}^{\infty} n = -\frac{1}{12}$$

$$\langle p | T(s) | p' \rangle = \left(\frac{p^2}{2L^2} - \boxed{\frac{1}{24L^2}} \right) \delta(p-p')$$

$\frac{1}{L}\sum_{n\neq 0} a_n e^{-\frac{ns}{L}}$

$= a_0 = \bar{a}_0$

$\bar{\partial}\, v(s)\, T = 0 \qquad s \to s + v(s)$

$v(s)\partial_s$

$[v\partial_s, v'\partial_s] = (v\partial v' - v'\partial v)\partial_s$

$) \; \delta(p-p') = \left(-\frac{p^2}{L^2} - \frac{1}{L^2}\frac{e^{\frac{s+s'}{L}}}{(e^{s/L} - e^{s'/L})^2}\right)\delta(p-p')$

$\sum_{n=1}^{\infty} n = -\frac{1}{12}$

$p|T(s)|p'\rangle = \left(\frac{p^2}{2L^2} \boxed{-\boxed{\frac{1}{24L^2}}}\right)\delta(p-p')$

$L_n \Rightarrow e^{ns}\partial_s$

$T = -\frac{1}{24} + \sum_n L_n e^{-ns}$

ns

$[L_n, L_m] = (n-m)L_{n+m} + \frac{1}{12}n(n^2-1)\delta_{n+m,0}$

SYMMETRY: $X \to X + c$

$X \to X + f(z) + g(\bar{z})$

$J_a = \partial_a X \qquad \partial_a J^a = \Box^2 X = 0$

$J = \partial_z X \qquad \bar{J} = \partial_{\bar{z}} X$

$\bar{\partial} J = \bar{\partial}\partial X = 0$

$\bar{\partial}\big(f(z) J(z)\big) = 0$

$$\int \frac{\partial X}{\partial u^a}\frac{\partial X}{\partial u_a}\, d^2u \to \int \frac{\partial X}{\partial u^a}\frac{\partial c}{\partial u_a}\, d^2u$$

$$\int \frac{\partial X}{\partial z}\frac{\partial X}{\partial \bar{z}}\, dz\, d\bar{z}$$

$\bar{\partial} f$