Title: String Theory Lecture

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$$\partial X = \sum a_n e^{-ns}$$

$$T = -\frac{1}{2} \lim \left(\partial X(s)\, \partial X(s') - \frac{1}{(s-s')^2} \right)$$

$$T = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$L_n = \frac{1}{2} \sum_m : a_{n-m} a_m :$$

$$[L_n, a_m] = m\, a_{n+m}$$

$$[L_n, X(s)] = e^{ns}\partial_s X(s)$$

$$[L_n, \partial X(s)] = \partial(e^{ns}\partial_s X) = e^{ns}\partial(\partial X) + n e^{ns}\partial_s X$$

$$[L_n, \partial X(s)\partial X(s')] = \cdots$$

$$[L_n, T(s)] = e^{ns}\partial_s T(s) + 2n e^{ns} T(s) + \frac{n^3}{12} e^{ns}$$

$$T^{ab}=0$$

$$\langle PHYS | T | PHYS \rangle = 0$$

$$\langle PHYS | L_n | PHYS \rangle = 0$$

$$L_n | PHYS \rangle = 0 \qquad n > 0$$

$$\langle PHYS | L_{-n} = 0$$

$$(L_0 - \boxed{1}) | PHYS \rangle$$

$$\boxed{D=26}$$

$$|NULL\rangle = L_{-n} | \cdots \rangle$$

$$\frac{KER(L_n,\ L_0 - 1)}{KER(L_n, L_0 - 1) \cap IM(L_{-n})}$$

X^{μ}

$T^{ab} = 0$

$|NULL\rangle = L_{-n}|\cdots\rangle$

$\langle PHYS | T | PHYS \rangle = 0$

$\langle PHYS | L_n | PHYS \rangle = 0$

$$\frac{KER(L_n, L_0 - 1)}{KER(L_n, L_0 - 1) \cap IM(L_{-n})}$$

$L_n |PHYS\rangle = 0 \qquad n > 0$

$\langle PHYS | L_{-n} = 0$

$(L_0 - \boxed{1}) |PHYS\rangle$

A^{μ}

$= 26$

$a_n \quad \bar{a}_n$

X^μ

$L_{-n} | \cdots \rangle$

$(L_n , L_0 - 1)$

$M(L_{-n})$

$$[a^\mu_n , a^\nu_m] = n\,\delta_{n+m,0}\,\eta^{\mu\nu}$$

$$\langle p | a^\mu_{-1} \; a^\nu_1 | p \rangle = \eta^{\mu\nu}\,\delta(p-p')$$

$|p\rangle$

$a^\mu_{-1}|p\rangle \, , \; \bar{a}^\mu_{-1}|p\rangle$

$a^\mu_{-1}\bar{a}^\nu_{-1}|p\rangle + \cdots$

$\partial X \quad \bar{\partial} X$

$a_n \quad \bar{a}_n$

X^μ

$|NULL\rangle = L_{-n} | \cdots \rangle$

$[\, a^\mu_n \,,\, a^\nu_m \,] =$

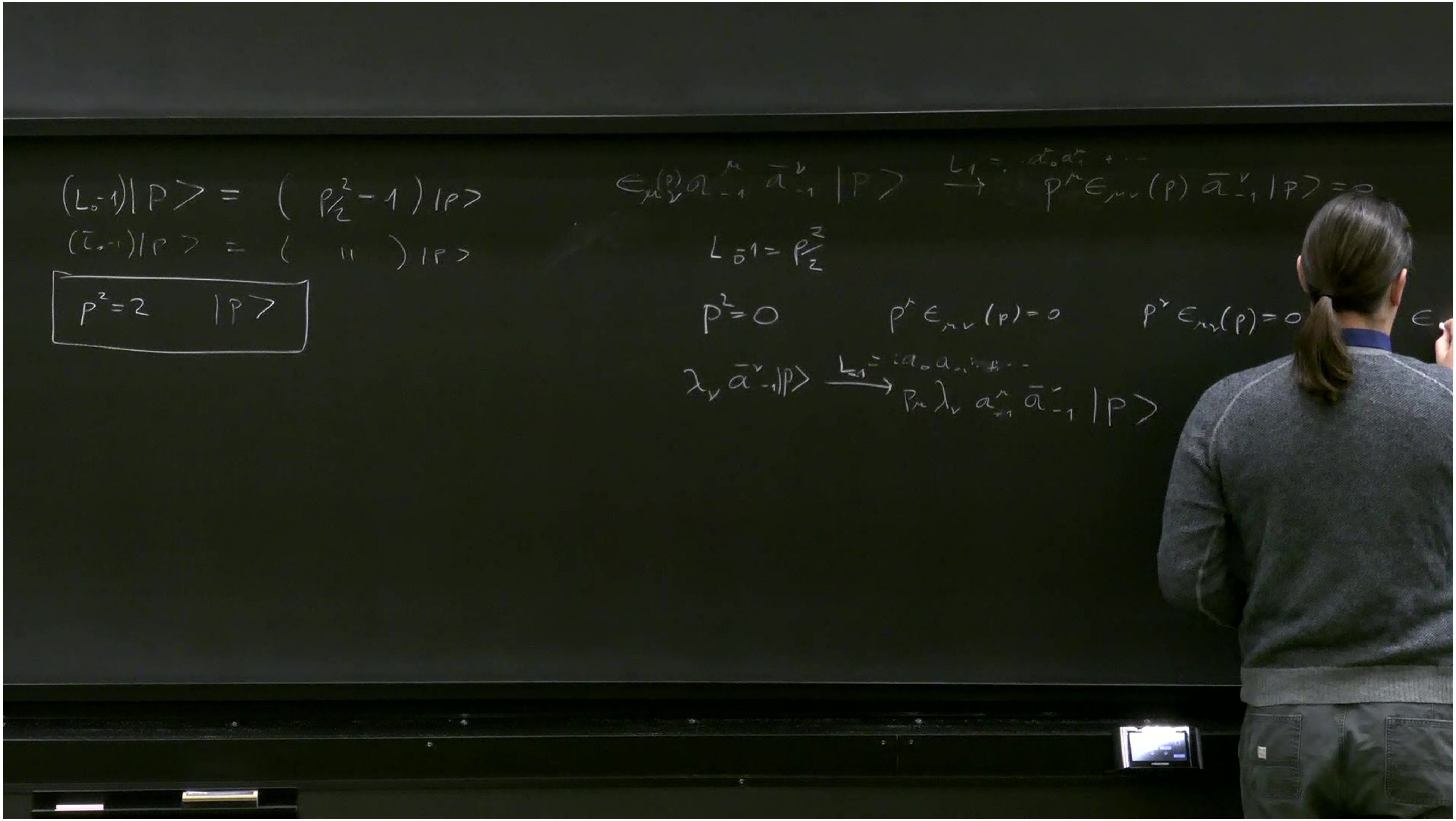
$$\frac{KER\left(\overset{\bar{L}_n}{L_n},\ \overset{\bar{L}_0 - 1}{L_0 - 1}\right)}{KER(L_n, L_0 - 1) \cap IM\left(\underset{\bar{L}_{-n}}{L_{-n}}\right)}$$

$\langle p | a^\mu_{-1} \; a^\nu_1 | p \rangle = \eta^{\mu\nu}$

$|p\rangle$

$a^\mu_{-1} |p\rangle \,,\ \bar{a}^\mu_{-1} |p\rangle$

$a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle + \cdots$


$(L_0 - 1)|p\rangle = \left(\frac{p^2}{2} - 1\right)|p\rangle$
$(\bar{L}_0 - 1)|p\rangle = (\quad " \quad)|p\rangle$
$\boxed{p^2 = 2 \qquad |p\rangle}$
$\epsilon_{\mu\nu}(p)\, a^{\mu}_{-1}\, \bar{a}^{\nu}_{-1}|p\rangle \xrightarrow{L_1 = \alpha_0 \alpha_1 + \cdots} p^{\mu}\epsilon_{\mu\nu}(p)\, \bar{a}^{\nu}_{-1}|p\rangle = 0$
$L_0 - 1 = \frac{p^2}{2}$
$p^2 = 0 \qquad p^{\mu}\epsilon_{\mu\nu}(p) = 0 \qquad p^{\nu}\epsilon_{\mu\nu}(p) = 0 \qquad \epsilon$
$\lambda_\nu \bar{a}^{\nu}_{-1}|p\rangle \xrightarrow{L_1 = \alpha_0 \alpha_1 + \cdots} p_\mu \lambda_\nu a^{\mu}_{+1} \bar{a}^{\nu}_{-1}|p\rangle$

$\epsilon_{\mu\nu}(p)\, a^{\mu}_{-1}\, \bar{a}^{\nu}_{-1} |p\rangle \xrightarrow{L_1 = :a_0^\mu a_1^\mu: + \cdots} p^\mu \epsilon_{\mu\nu}(p)\, \bar{a}^{\nu}_{-1} |p\rangle = 0$

$L_0 - 1 = \frac{p^2}{2}$

$p^2 = 0 \qquad p^\mu \epsilon_{\mu\nu}(p) = 0 \qquad p^\nu \epsilon_{\mu\nu}(p) = 0 \qquad \epsilon_{\mu\nu}(p) \to \epsilon_{\mu\nu}(p) + p_\mu \lambda_\nu + \bar{\lambda}$

$\lambda_\nu \bar{a}^\nu_{-1}|p\rangle \xrightarrow{L_{-1} = :a_0 a_{-1}: + \cdots} p_\mu \lambda_\nu a^\mu_{-1} \bar{a}^\nu_{-1} |p\rangle$

SPIN 2 MASSLESS

$\epsilon_{\mu\nu}(p) = \epsilon^{S}_{\mu\nu}(p) + \epsilon^{A}_{\mu\nu}(p) + \eta_{\mu\nu}\epsilon(p)$

$p_\mu \lambda_\nu + p_\nu \lambda_\mu - p\lambda\, \eta_{\mu\nu} \qquad p_\mu \tilde{\lambda}_\nu - p_\nu \tilde{\lambda}_\mu$

PARTICLE $\int ds + \int A_\mu \dot{x}^\mu$

$$H_{\mu\nu\sigma} = \epsilon_{\mu\nu\sigma\tau}\,\partial_\tau \phi$$

STRING $\int dA + \int B_{[\mu\nu]} \frac{\partial x^\mu}{\partial \sigma^1} \frac{\partial x^\nu}{\partial \sigma^2}$

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

$1)\,|p\rangle$

$)\,|p\rangle$

$$\epsilon_{\mu\nu}(p)\,\alpha^{\mu}_{-1}\,\bar{\alpha}^{\nu}_{-1}\,|p\rangle \xrightarrow{L_1 = \alpha_0 \alpha_1 + \cdots} p^{\mu}\epsilon_{\mu\nu}(p)\,\bar{\alpha}^{\nu}_{-1}\,|p\rangle = 0$$

$$L_0 - 1 = \frac{p^2}{2}$$

$$p^2 = 0 \qquad p^{\mu}\epsilon_{\mu\nu}(p) = 0 \qquad p^{\nu}\epsilon_{\mu\nu}(p) = 0 \qquad \epsilon_{\mu\nu}(p) \to \epsilon_{\mu\nu}(p) + p_{\mu}$$

$$\lambda_{\nu}\,\bar{\alpha}^{\nu}_{-1}|p\rangle \xrightarrow{L_{-1} = :\alpha_0 \alpha_{-1}: + \cdots} p_{\mu}\lambda_{\nu}\,\alpha^{\mu}_{-1}\,\bar{\alpha}^{\nu}_{-1}\,|p\rangle$$

SPIN 2 MASSLESS   B-FIELD   DILATON

$$\epsilon_{\mu\nu}(p) = \epsilon^{S}_{\mu\nu}(p) + \epsilon^{A}_{\mu\nu}(p) + \eta_{\mu\nu}\,\epsilon(p)$$

$$p_{\mu}\lambda_{\nu} + p_{\nu}\lambda_{\mu} - p\cdot\lambda\,\eta_{\mu\nu} \qquad p_{\mu}\tilde{\lambda}_{\nu} - p_{\nu}\tilde{\lambda}_{\mu}$$

$) \bar{a}^{\nu}_{-1} |p\rangle = 0$

$g_{\mu\nu} \to g_{\mu\nu} + \partial_\mu v_\nu + \partial_\nu v_\mu$

$\epsilon_{\mu\nu}(p) = 0$

$\epsilon_{\mu\nu}(p) \to \epsilon_{\mu\nu}(p) + p_\mu \lambda_\nu + \bar{\lambda}_\mu p_\nu$

STATES OF MASS^2 = $2n$

SPIN UP TO $2n+2$

DILATON

$+ \eta_{\mu\nu} \epsilon(p)$

$p_\nu \tilde{\lambda}_\mu$

$p\rangle = \left(p^2/2 - 1 \right) |p\rangle$

$|p\rangle = \left(\quad '' \quad \right) |p\rangle$

$\boxed{2 \quad |p\rangle}$

$\epsilon_{\mu\nu}(p)\, \alpha^{\mu}_{-1}\, \bar{\alpha}^{\nu}_{-1} |p\rangle \xrightarrow{L_1 = \alpha_0 \alpha_1 + \cdots} p^{\mu}\epsilon_{\mu\nu}(p)\, \bar{\alpha}^{\nu}_{-1} |p\rangle = 0$

$L_0 - 1 = p^2/2$

$p^2 = 0 \qquad p^{\mu}\epsilon_{\mu\nu}(p) = 0 \qquad p^{\nu}\epsilon_{\mu\nu}(p) = 0 \qquad \epsilon_{\mu\nu}($

$\lambda_{\nu}\, \bar{\alpha}^{\nu}_{-1}|p\rangle \xrightarrow{L_{-1} = \alpha_0 \alpha_{-1} + \cdots} p_{\mu}\lambda_{\nu}\, \alpha^{\mu}_{-1}\, \bar{\alpha}^{\nu}_{-1} |p\rangle$

SPIN 2 MASSLESS $\qquad$ B-FIELD $\qquad$ DILATON

$\epsilon_{\mu\nu}(p) = \epsilon^{S}_{\mu\nu}(p) + \epsilon^{A}_{\mu\nu}(p) + \eta_{\mu\nu}\epsilon(p)$

$p_{\mu}\lambda_{\nu} + p_{\nu}\lambda_{\mu} - p\cdot\lambda\, \eta_{\mu\nu} \qquad p_{\mu}\tilde{\lambda}_{\nu} - p_{\nu}\tilde{\lambda}_{\mu}$

$(L_0 - \boxed{1})\,|PHYS\rangle$

$\boxed{D=26}$

$\boxed{c=26, \quad \Delta = 1}$

$(L_0 - 1)|p\rangle = \left(\frac{p^2}{2} - 1\right)|p\rangle$

$(\bar{L}_0 - 1)|p\rangle = (\quad '' \quad)|p\rangle$

$\boxed{p^2 = 2 \qquad |p\rangle}$

$\epsilon_{\mu\nu}(p)\, a^{\mu}_{-1}\, \bar{a}^{\nu}_{-1}\,|p\rangle \xrightarrow{L_1 = \alpha_0^{\mu} a_1^{\mu} + \cdots} p^{\mu}\epsilon_{\mu\nu}(p)\,\bar{a}^{\nu}_{-1}$

$L_0 - 1 = \frac{p^2}{2}$

$p^2 = 0 \qquad p^{\mu}\epsilon_{\mu\nu}(p) = 0 \qquad p^{\nu}\epsilon_{\mu\nu}(p) =$

$\lambda_{\nu}\,\bar{a}^{\nu}_{-1}|p\rangle \xrightarrow{L_{-1} = :a_0 a_{-1}: + \cdots} p_{\mu}\lambda_{\nu}\, a^{\mu}_{-1}\,\bar{a}^{\nu}_{-1}\,|p\rangle$

SPIN 2 MASSLESS — B-FIELD — D

$\epsilon_{\mu\nu}(p) = \epsilon^{S}_{\mu\nu}(p) + \epsilon^{A}_{\mu\nu}(p) + \eta_{\mu\nu}$

$p_{\mu}\lambda_{\nu} + p_{\nu}\lambda_{\mu} - p\cdot\lambda\,\eta_{\mu\nu} \qquad p_{\mu}\tilde{\lambda}_{\nu} - p_{\nu}\tilde{\lambda}_{\mu}$