Title: Quantum Gravity Lecture

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Collection: Quantum Gravity 2023/24

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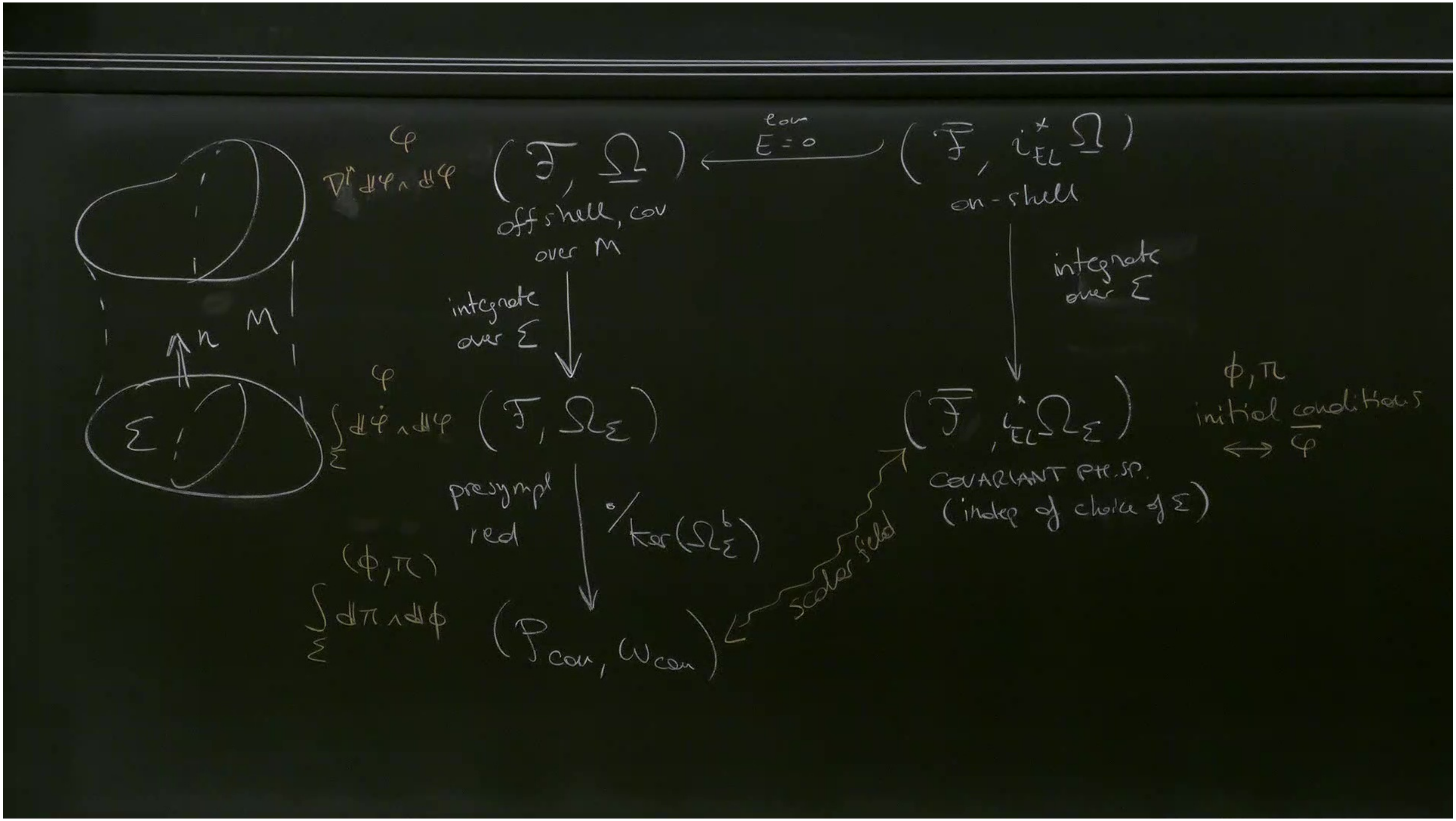

Σ
n
M
φ
$\int_\Sigma d\dot\varphi \wedge d\varphi$
$(\mathcal{F}, \Omega)$
off-shell, cov
over M
integrate
over Σ
$(\mathcal{F}, \Omega_\Sigma)$
presympl
red
$/\ker(\Omega_\Sigma^\flat)$
(ϕ, π)
$\int_\Sigma d\pi \wedge d\phi$
(P_{cov}, ω_{cov})
eom
$E=0$
$(\bar{\mathcal{F}}, i^*_{EL}\Omega)$
on-shell
integrate
over Σ
$(\bar{\mathcal{F}}, i^*_{EL}\Omega_\Sigma)$
COVARIANT PH.SP.
(indep of choice of Σ)
scalar field
ϕ, π
initial conditions
$\longleftrightarrow \bar\varphi$

Maxwell

$$\left(A_\mu,\ dF^{\mu\nu}\wedge dA_\nu\right) \longleftarrow \text{(Sol. of Maxwell eqs)}$$

$$\Sigma = \{t=0\} \qquad \left(A_\mu,\ \int_\Sigma dF^{0i}\wedge dA_i\right) \qquad \left(\mathcal{J},\ i^*_{EL}\,\Omega_\Sigma\right)$$

?

$$a_i = A_i\big|_\Sigma \qquad E^i = F^{0i}\big|_\Sigma \qquad \left((a_i, E^i),\ \int_\Sigma dE^i\wedge da_i\right)$$

off shell canonical ph. space.

Not the same b.c.

there leftover

com constraining

E^i (at "fixed time")

i.e. Gauss constraint

$$\nabla_i E^i \approx 0$$

(Sol. of Maxwell eq's)

$$\left(\bar{\mathcal{J}},\ i^*_{EL}\,\Omega_\Sigma\right)$$

?

Not the same b.c.
there left over eom constraining E^i (at "fixed time")

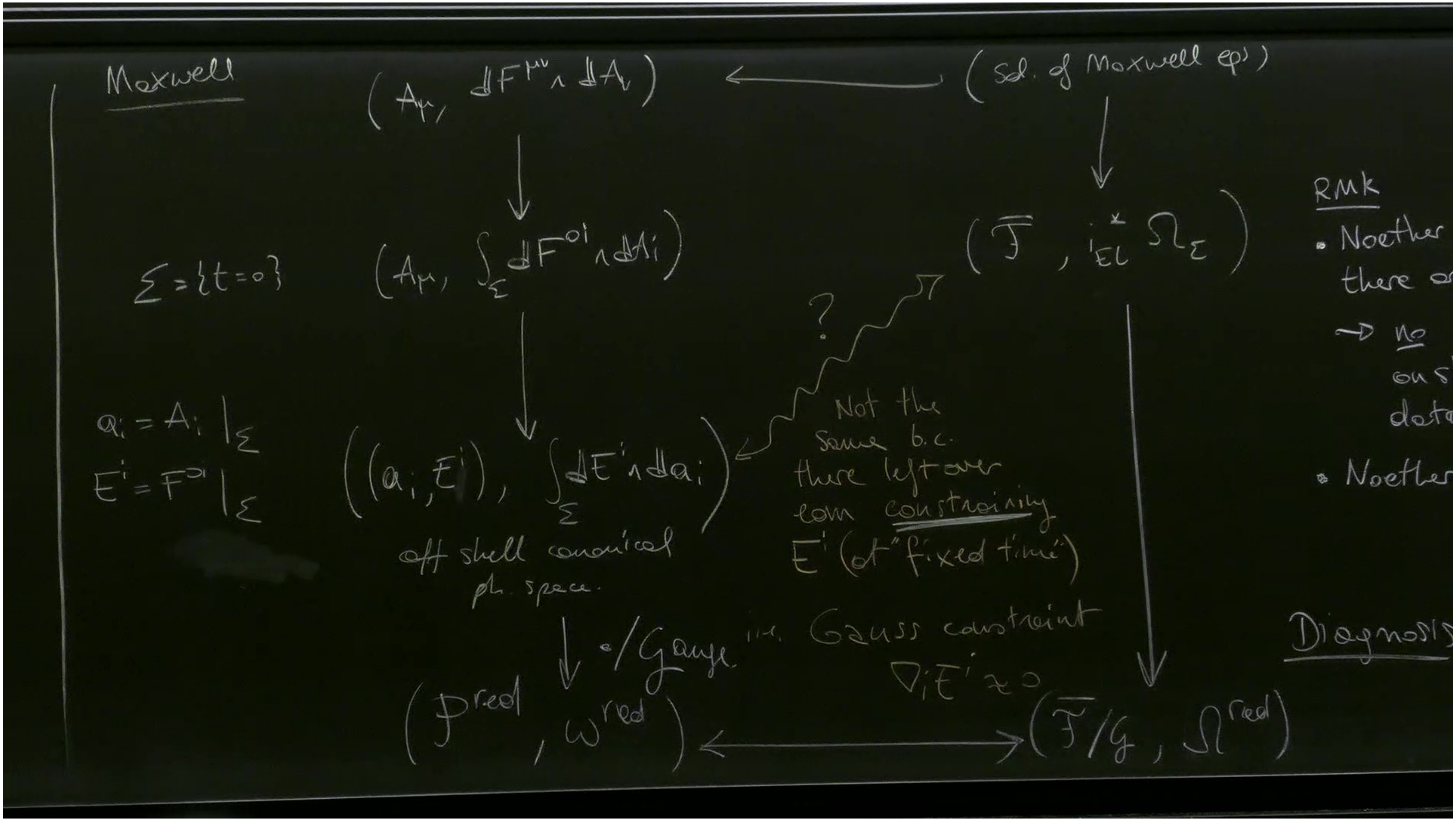
i.e. Gauss constraint

$\nabla_i E^i \approx 0$

RMK

- Noether 2 $\left((D^I_\alpha)^\dagger E_I \equiv 0\right)$
there are less eom than fields
→ no 1-to-1 corresp. between on shell histories and initial data
- Noether 2′ $\left(i^*_\Sigma \underline{J} = \underline{C}_\alpha \xi^\alpha + d\underline{j}\right)$
$Q_\Sigma(\xi) \approx 0$
Gauss constraint

Diagnosis: need to quotient out gauge sym!


Maxwell
(A_μ, dF^μν ∧ dA_ν)
(Sol. of Maxwell eq's)
Σ = {t=0}
(A_μ, ∫_Σ dF^0i ∧ dA_i)
(F̄, i*_EL Ω_Σ)
?
a_i = A_i |_Σ
E^i = F^0i |_Σ
((a_i, E^i), ∫_Σ dE^i ∧ da_i)
off shell canonical
ph. space
Not the
same b.c.
there left over
com constraining
E^i (at "fixed time")
/Gauge i.e. Gauss constraint
∇_i E^i ≈ 0
(P^red, ω^red)
(F̄/G, Ω^red)
RMK
• Noether
there
→ no
• Noether
Diagnosis

Symplectic reduction

$(\mathcal{P}, \omega)$: (a_i, E^i) $\quad \omega = \int_\Sigma \mathbb{d}E^i \wedge \mathbb{d}A_i$

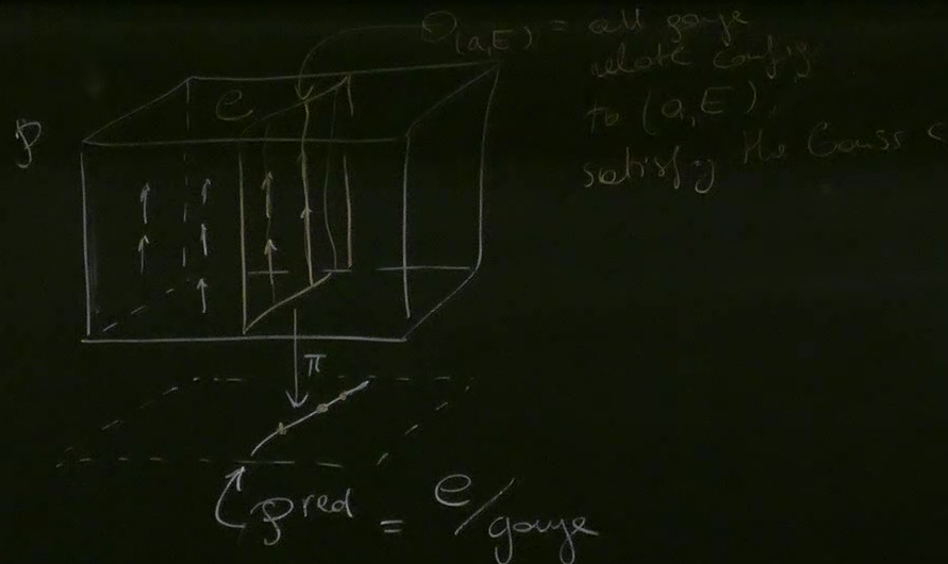
constraint : $C = \nabla_i E^i \approx 0$

$$\mathcal{C} = \{ (a, E) : \nabla_i E^i = 0 \} \hookrightarrow \mathcal{P}$$

$i^*_{\mathcal{C}}\,\omega$ will be degenerate, in fact:

$$\mathbb{I}_{\rho(\xi)}\,\omega = -\mathbb{d}\,Q_\Sigma(\xi) = -\mathbb{d}\int_\Sigma \nabla_i E^i\, \xi + \underset{\partial\Sigma = \emptyset}{(0)}$$

pullback to $\mathcal{C}$ $= 0 \Rightarrow \rho(\xi)$ is in the kernel of $i^*_{\mathcal{C}}\,\omega$

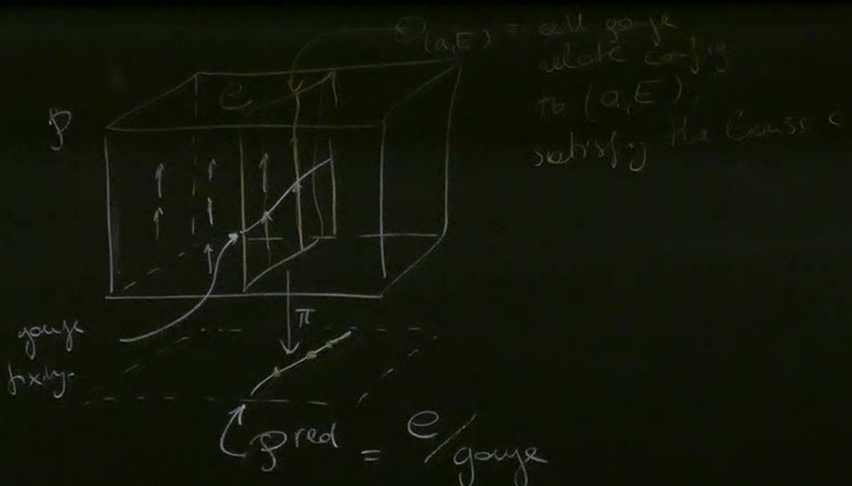


$$\begin{tikzcd}
(\mathcal{P}, \omega) \arrow[dr, squiggly, "\text{symp'. reduction at } \mathcal{C}"'] & (\mathcal{C}, \ \imath_\mathcal{C}^*\omega = \pi^*\omega^{red}) \arrow[l, "\imath_\mathcal{C}"'] \arrow[d, "\pi"] \\
 & (\mathcal{P}^{red} = \mathcal{C}/\text{gauge}, \ \omega^{red})
\end{tikzcd}$$

ω^{red} defined by demand:

$$\imath_\mathcal{C}^*\omega = \pi^*\omega^{red}$$

$$(\mathcal{P}^{red}, \omega^{red}) \longleftrightarrow (\mathcal{F}/\mathcal{G}, \Omega^{red})$$



$$\begin{tikzcd}
(\mathcal{P}, \omega) \arrow[dr, squiggly, "\text{symp. reduction at } \mathcal{C}"'] & (\mathcal{C},\ \iota_{\mathcal{C}}^{*}\omega = \pi^{*}\omega^{red}) \arrow[l, "\iota_{\mathcal{C}}"'] \arrow[d, "\pi"] \\
 & (\mathcal{P}^{red} = \mathcal{C}/gauge,\ \omega^{red})
\end{tikzcd}$$

ω^{red} defined by demand:

$$\boxed{\iota_{\mathcal{C}}^{*}\omega = \pi^{*}\omega^{red}}$$

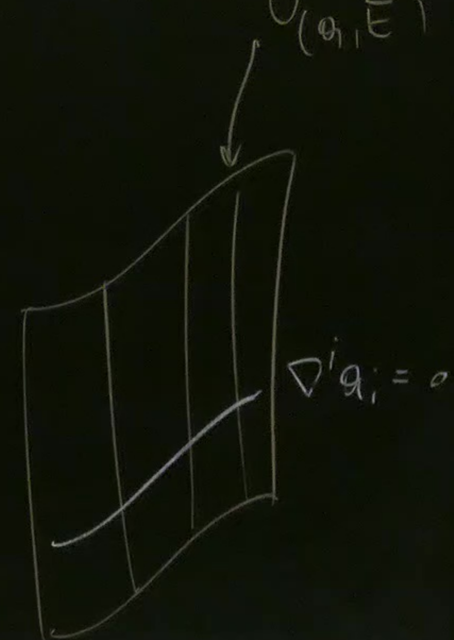
In practice, working on $\mathcal{P}_{red}$ is hard (locality), so one introduces a "gauge fixing" in $\mathcal{C}$. E.g. on each gauge orbit of (a,E) there is $1!$ conf such that $\nabla^i a_i = 0$

/gauge, ω^{red})

demand:

$\boxed{\omega^{red}}$

$\exists!$ conf such that $\nabla^i q_i = 0$



pullback to $\mathcal{C}$ $\approx 0 \Rightarrow \rho(\xi)$ is in the kernel of $i_{\mathcal{C}}^{*}\omega$ $\quad \partial\Sigma = \emptyset$ | a "gauge

In YM

$\xi(x) \in C^\infty(M) \rightsquigarrow \xi^\alpha(x) \in C^\infty(M, \mathfrak{g})$

Maxwell $\qquad$ in YM

α = basis index in $\mathfrak{g}$

$$\begin{cases} \Phi = (a_i^\alpha, E_\alpha^i) \\ \omega = \int_\Sigma \sum_\alpha dE^i_\alpha \wedge da^\alpha_i \end{cases}$$

Gauss constraint: $D_i E^i_\alpha \equiv \nabla_i E^i_\alpha + f_{\alpha\beta}{}^{\gamma} a_i^\beta E^i_\gamma \approx 0$

$$\mathring{\mathbb{I}}_{\rho(\xi)}\,\omega = -\,dQ_\Sigma(\xi) \approx 0$$

$\mathring{\mathbb{I}}_{\rho}$

a "gauge fixing" in $\mathcal{C}$. E.g. on each gauge orbit of (a, E) there is $1!$ conf ...

$$\mathring{i}_{\rho(\eta)} \mathring{i}_{\rho(\xi)} \omega = -\mathbb{L}_{\rho(\eta)} Q_\Sigma(\xi) \underset{\substack{\uparrow \\ \text{equivariance}}}{=} -Q_\Sigma([\eta, \xi])$$

$$\| $$

$$-\{Q_\Sigma(\eta), Q_\Sigma(\xi)\}$$

→ in YM the momentum map / Noether charge is equiv.

$$\{Q_\Sigma(\eta), Q_\Sigma(\xi)\} = Q_\Sigma([\xi, \eta])$$

≈ 0

i.e. the charge (Poisson) algebra is a representation of the gauge alg.

y" in $\mathcal{C}$. E.g. on each gauge orbit of (q,E) there is 1!

$\rho(\xi)\, \omega = -\mathbb{L}_{\rho(\eta)} Q_\Sigma(\xi) \underset{\substack{\uparrow \\ \text{equivariance}}}{=} -Q_\Sigma([\eta,\xi])$

$\eta), Q_\Sigma(\xi)\}$

YM the momentum map / Noether charge is equiv.

$\{Q_\Sigma(\eta), Q_\Sigma(\xi)\} = Q_\Sigma([\xi,\eta])$

the charge (Poisson) algebra is a representation of the gauge alg.

This guarantees that the reduction procedure is self-consistent

(gauge orbits are submfds of $\mathcal{C}$ by Frobenius thm)

$[\rho(\xi), \rho(\eta)] = \rho([\xi,\eta])$

Rmk: watch out in GR! this will not quite hold – [structural feature of timelike diffeo]