Title: Quantum Gravity Lecture

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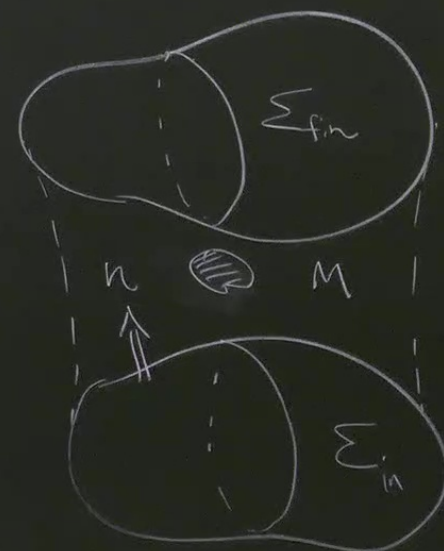
COVARIANT vs CANONICAL ph. sp. ($\partial\Sigma = \phi$)

I argued that

covariant

$$\left(\bar{\mathcal{F}},\ \iota_{EL}^{*}\,\Omega_{\Sigma}\right) = \text{phase space of th.}$$

on-shell $\Longrightarrow$ Σ-indep



Question

- is $(\bar{\mathcal{F}},$
- how does

Quick rev

Recipe t

$S = \int$

(π, φ)

Rmk

$(\partial\Sigma = \emptyset)$

t.

of th..

Question

• is $(\mathcal{F}, \iota_{EL}^{*}\Omega_\Sigma)$ symplectic?

• how does it compare to canonical ph. sp.?

Quick reminder:

Recipe to construct canonical ph. sp.:

$$\int_\Sigma \ominus$$

$$S = \int_M \underline{L} \qquad \delta S = \int_M E\,\delta m + \int_{\partial M} \underbrace{\frac{\partial L}{\partial \dot\varphi}}_{\to\, \pi} \delta\varphi$$

(π, φ) canonically conjugate, $\{\pi(x), \varphi(y)\} = \delta(x,y)$

$$\iff \omega = \int_\Sigma \delta\pi \wedge \delta\varphi$$

Rmk: we never went on-shell!

Canonic

covarian

$\mathcal{F}$

$\underline{\Omega} =$

Σ :

Rmk :

• d

• b

Idea :

...al ph.sp?

...ph.sp.

$$\int_{\Sigma} \underline{\Theta}$$

$$\ldots n \to \int_{\partial M} \underbrace{\frac{\partial L}{\partial \dot\varphi}}_{\to \pi} \delta\varphi$$

...e, $\{\pi(x), \varphi(y)\} = \delta(x,y)$

$\Longleftrightarrow \omega = \int_{\Sigma} \delta\pi \wedge \delta\varphi$

...l!

Canonical ph. sp. from covariant picture.

$\mathcal{F}$ = off-shell field sp.

$\underline{\Omega} = d\,\underline{\Theta}$ sympl. current

Σ : $\Omega_\Sigma = \int_\Sigma \underline{\Omega}$

Rmk: $(\mathcal{F}, \Omega_\Sigma)$ is not sympl!

- $d\Omega_\Sigma = 0$ because d exact
- but highly degenerate!

Idea: "quotient out the kernel" to define the canonical ph. sp.

Example: Sc...

$\underline{L} = -\frac{1}{2}(\nabla\varphi)^2 \ldots$

$\underline{\Theta} = d\varphi\, \nabla \ldots$

$\Omega_\Sigma = \int_\Sigma (n^a \ldots$

Degenerate b...

$\mathbb{X} = \int_M \delta_{\mathbb{X}}\varphi \frac{\delta}{\delta \ldots}$

is s.t. $\mathbb{I}_{\mathbb{X}} \Omega_\Sigma \ldots$

from
ne -
shell field sp.

sympl. current

$$\int_\Sigma \underline{\Omega}$$

Ω_Σ) is not sympl!

because $\mathbb{d}$ exact

ly degenerate!
t out the kernel" to define
nonical ph. sp.

Example: Scalar field

$$\underline{L} = -\frac{1}{2}(\nabla\varphi)^2 \, \underline{\epsilon}$$

$$\underline{\theta} = \mathbb{d}\varphi \, \nabla^a\varphi \, \underline{\epsilon}_a \qquad \imath_\Sigma^* \underline{\epsilon}_a = n_a \underline{\epsilon}_\Sigma$$

$$\Omega_\Sigma = \int_\Sigma \underbrace{(n^a \nabla_a \mathbb{d}\varphi)}_{\equiv \mathbb{d}\dot{\varphi}} \wedge \mathbb{d}\varphi \, \underline{\epsilon}_\Sigma$$

Degenerate because $\Omega_\Sigma \in \Omega^2(\mathcal{F})$

space of fields over M

$\mathbb{X} = \int_M \delta_{\mathbb{X}}\varphi \frac{\delta}{\delta\varphi}$, $\delta_{\mathbb{X}}\varphi$ of compact support disjoint from Σ

is s.t. $\mathbb{I}_{\mathbb{X}} \Omega_\Sigma = 0$

More generally

$X \in \ker(\Omega^\flat_\Sigma)$ iff

$$0 = i_X \Omega_\Sigma = \int_\Sigma (\nabla_n \delta_X \varphi)\, d\varphi - (\underbrace{\nabla_n d\varphi}_{d\dot{\varphi}}) \cdot \delta_X \varphi$$

indep. functions on Σ

iff $\begin{cases} \delta_X \varphi = 0 \\ \nabla_n \delta_X \varphi = 0 \end{cases}$

That is: X is in the kernel iff it leaves φ and $\dot{\varphi}$ unchanged at Σ

(rmk: e.g. it could still change $\ddot{\varphi}$)

We wo

"kernel

$\Rightarrow$ we

th

(φ

We d

by d

Whi

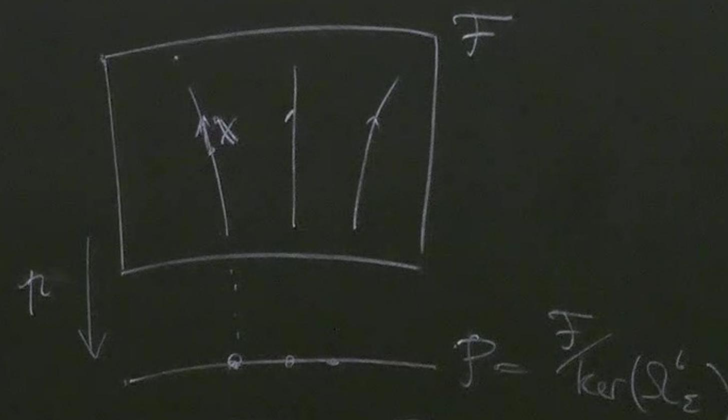
We want to get rid of kernel

$\Rightarrow$ we "identify" all config that have the same

$$\begin{cases} \varphi|_\Sigma = \phi \\ \dot{\varphi}|_\Sigma = \pi \end{cases}$$

$$(\phi, \pi) = [\varphi] \in \mathcal{P}_{can} = \mathcal{F} / \ker(\Omega_\Sigma^\flat)$$

$\mathbb{X} \in \ker(\Omega_\Sigma^\flat)$



$$p : \mathcal{F} \longrightarrow \mathcal{P}$$
$$\varphi \longmapsto [\varphi] \quad \text{projection}$$

We define $\omega \in \Omega^2(\mathcal{P})$

by demanding: $p^* \omega = \Omega_\Sigma$

Which is non-deg by construction

$\mathcal{F}$

$\mathcal{P} = \mathcal{F} / \ker(\Omega'_\Sigma)$

projection

This procedure

$(\mathcal{F}, \Omega_\Sigma) \xrightarrow[\text{by kernel}]{\text{quotient}} (\mathcal{P}, \omega)$

Ω_Σ with a kernel; ω non-deg.

is called "presymplectic reduction".

For the scalar field: $\omega = \int_\Sigma d\pi \wedge d\phi$,

$\boxed{(\phi, \pi)} := p(\varphi) = (\varphi|_\Sigma, \dot{\varphi}|_\Sigma) \qquad p^* \omega = \Omega_\Sigma$

$(\mathcal{P}, \omega)$ is the expected can ph sp.

we used no eq's of motion!

Rmk in this case $(\bar{\mathcal{F}}, \iota_{EL}^* \Omega_\Sigma) \simeq (\mathcal{P}, \omega)_{can}$

EOM

No EOM

(rmk: e.g. it could still change $\ddot{\varphi}$)

Which is non-deg by construction.

Example: Maxwell

$$\underline{L} = -\frac{1}{4} F_{ab} F^{ab} \underline{\epsilon}$$

$$\mathcal{F} = \Omega^1(M) \ni A$$

$$\underline{\omega} = \mathbb{d}A_a F^{ab} \underline{\epsilon}_b$$

$$\Omega_\Sigma = \int_\Sigma \mathbb{d} \underbrace{F^{ab}}_{F(A)} \wedge \mathbb{d}A_a \underline{\epsilon}_b$$

$$= \int_\Sigma \left(n_b \, \mathbb{d}F^{ab} \wedge \mathbb{d}A_a\right) \underline{\epsilon}_\Sigma$$

If $\Sigma = \{t=0\}$ of Mink

$$\Omega_\Sigma = \int \mathbb{d}F^{0i} \wedge \mathbb{d}A_i$$

$$-F^{0i} = \partial_0 A_i - \partial_i A_0$$

Rmk: A_0 appears only in F^{0i}, no $\mathbb{d}A_0$ by itself!
(there is no $\dot{A}_0$ in $\underline{L}$)

Kernel: every $\mathbb{X}$ that leaves F^{0i} & A_i unchanged.

$$\Rightarrow \mathcal{P} = \mathcal{F}/\ker(\Omega_\Sigma^b) \ni (a_i, E^i) = (A_i|_\Sigma, F^{0i}|_\Sigma) = [A]$$

$$\omega = \int_\Sigma \mathbb{d}E^i \wedge \mathbb{d}A_i$$

Rmk in this case $(\bar{\mathcal{F}}, \iota_{E\ell}^{*}\Omega_{\Sigma}) \cong (\mathcal{P},$

Question

$(\bar{\mathcal{F}}, \iota_{E\ell}^{*}\Omega_{\Sigma}) \cong (\mathcal{P}, \omega)$ for Maxwell?

No! Because...

The shell requires $D_i E^i \approx 0$

which is not imposed anywhere in $\mathcal{P}$!

→ $\boxed{\text{Gauss constraint}}$ (on initial data)

Claim: Gauss constraint is a consequence of gauge symmetry

...changed.

$\ldots_{\Sigma}) = [A]$

Rmk in this case $(\mathcal{J}, \iota_{\xi_C}\delta\iota_\Sigma) = (\mathcal{P}, \omega)_{\text{con}}$

$(\mathcal{P}, \omega)$ for Maxwell?

se ...

ven $\quad \nabla_i E^i \approx 0$

nposed anywhere in $\mathcal{P}$!

$\boxed{\text{raint}}$ (on initial data)

constraint is a consequence of gauge

ny

1) Noether 2

$(D^I_\alpha)^+ E_I = 0$

i.e. eom are not all indep

$\Rightarrow$ to get a deterministic system we need to identify config. related by local sym.

$\rightarrow$ but we have not mod-out gauge yet in $\mathcal{P}$!

2) Noether 2 bis

$$Q_\Sigma(\xi) \approx 0 \Leftrightarrow \underline{J}(\xi) = \xi^\alpha \underset{\overset{??}{0}}{\underline{C_\alpha}} + dj(\xi)$$

For Maxwell

$$\tilde{\underline{J}}^b(\xi) = \nabla_a \xi F^{ab}$$

$$Q_\Sigma(\xi) = \int_\Sigma (\nabla_i \xi) F^{0i} = -\int_\Sigma \xi \underbrace{\nabla_i E^i}_{\text{Gauss constraint!}}$$

$\partial\Sigma = \phi$

Recall : $\mathring{i}_{\rho(\xi)} \underline{\Omega} = -\mathbb{d}\, \underline{J}(\xi)$

$\mathcal{J}$: $\mathring{i}_{\rho(\xi)} \Omega_\Sigma = -\mathbb{d}\, Q_\Sigma(\xi)$

$\mathcal{P}$: $\mathring{i}_{\rho(\xi)} \omega = -\int_\Sigma \xi \nabla_i E^i$

$\rho(\xi)(a_i, E^i) = (\partial_i \xi, 0)$