Title: Quantum Gravity Lecture

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Collection: Quantum Gravity 2023/24

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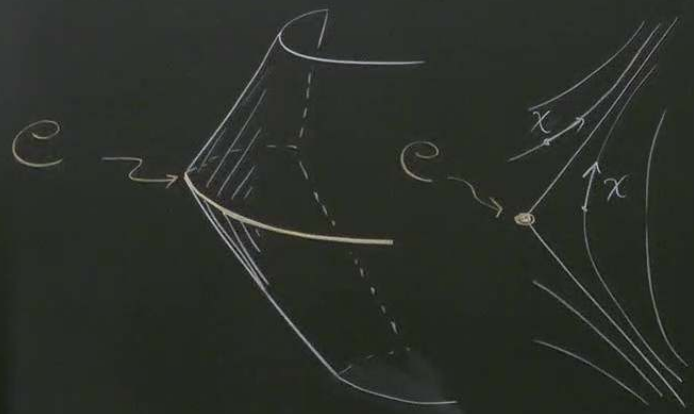
BH ENTROPY IS THE NOETHER CHARGE

[Wald, Wald + Iyer]

- Bifurcate killing horizon



ZEROTH-LAW

- $\nabla_a(\chi^b\chi_b)\big|_{\mathcal{H}} = -2\kappa\chi_a$

κ ↑ SURFACE GRAVITY

κ constant on the horizon

- THM (Rigidity)

Stationary BH $\Rightarrow$ axisym

$\Rightarrow$ The event horizon is a killing horizon for

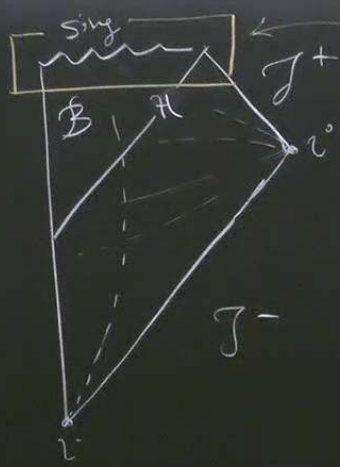
$$\chi^a = t^a + \omega_H \phi^a$$

$\omega_H \in \mathbb{R}$ "angular vel."

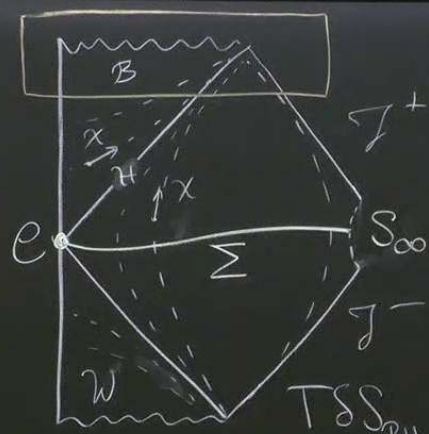
- THM [Wald & Racz]

Stationary BH, $\kappa \neq 0$

$\Rightarrow$ In the maximally extended spt of the BH the event horizon $\mathcal{H}$ is a branch of a bifurcate killing hor.



max ext



$\partial\Sigma = e \cup S_\infty$ ↑ sphere at ∞

ADM mass, ang. mom.

$$T\delta S_{BH} \equiv \kappa\, \delta A = \delta M - \omega_H \delta J$$

(T, κ: ↑)

Lemma

At e : $\nabla_a \chi_b = \kappa\, \varepsilon_{ab}$

deduce from ?

$$\dot{i}_{\xi}\Omega_\Sigma = -\delta Q_\Sigma(\xi) - \int_\Sigma i_\xi \underline{E} + \oint_{\partial\Sigma} i_\xi \underline{\Theta}$$

$\underline{E} = 0$

idea: $\xi = \chi = t^a + \omega_H \phi^a$ (↑ M, ↑ J)

$$Q_\Sigma(\xi) = \int_\Sigma \xi^a \underline{\tilde{C}_{\xi a}} + \oint_{\partial\Sigma} \underline{\tilde{J}}(\xi)$$

$= 0$

Subtlety

χ^a is a Killing vf

$\left(\mathcal{L}_\chi g_{ab} = 0 \right)$

$\Rightarrow d\chi \neq 0$

How does the Hamiltonian flow eq generalize?

$$\dot{\phi}(\chi)\,\Omega_\Sigma = -(d\mathcal{Q}_\xi)(\chi) - \int_\Sigma i_\chi \underline{E} + \oint_{\partial\Sigma} i_\chi \underline{\Theta}$$

$$\approx -(d q_e)(\chi) - (d q_\infty)(\chi) + \oint_e i_\chi \underline{\Theta} + \oint_{S_\infty} i_\chi \underline{\Theta}$$

① $\delta^{(1)}\mathcal{H} = \int_{\Sigma} (\mathcal{L}_X \underline{\omega}) \overset{\text{eqn}}{=} 0$ $\leftarrow$ Killing

② X at $\mathcal{C}$ vanishes.

$$\oint_{\mathcal{C}} i_X \underline{\omega} = 0$$

$$\Rightarrow 0 \approx -(d\mathcal{H}_{\mathcal{C}})(X) - (d\mathcal{H}_{\infty})(S_\infty) + \oint_{S_\infty} i_X \underline{\omega}$$

because $\phi^a \in TS_\infty$, so $0 = \| \; 0$

$$\approx -(d\mathcal{H}_{\mathcal{C}})(X) - \omega_H (d\mathcal{H}_{\infty})(\phi^a) - (d\mathcal{H}_{\infty})(t^a) + \oint_{\infty} i_t \underline{\omega} + \omega_H \oint_{\infty} i_\phi \underline{\omega}$$

Rmk : since $(M, g_{ab}) \xrightarrow[\infty]{} \text{Minkowski}$

$t^a, \phi^a \xrightarrow[\infty]{}$ "absolute" time transl. & rotations, $\; d\phi\, i_\infty = d(t^a)_\infty = 0 \Rightarrow (d\mathcal{H}_\infty)(t) = d(\mathcal{H}_\infty(t))$

Compute now Komar charges at ∞!

- $q_\infty(\phi) = -J$
- $q_\infty(t) = \frac{1}{2} M_{ADM}$
- miracle is that, thanks to the asymptotic properties of BH spt ("boundary cond" = asympt. flat)

$$\oint_{\infty} i_t \underline{\underline{L}} = d \oint_{\infty} \underline{B} \quad , \quad \oint_{\infty} \underline{B} = \frac{1}{2} M_{ADM}$$

$$\Rightarrow (d q_\ell)(X) \approx dM - \omega_H\, dJ$$

↑ outside the variation!

Left hand side?

Warm up: $q_\ell(X) = \frac{1}{4}\oint_\ell (\nabla_a X_b - \nabla_b X_a)\,\underline{\epsilon}^{ab}$

Leaves ↓

$$= \frac{1}{2}\oint_\ell \kappa\, \underbrace{\varepsilon_{ab}\underline{\epsilon}^{ab}}_{-2}$$

Zeroth law ↓

$$\underline{\epsilon}_\ell = 2\sqrt{\gamma}\, d^2x$$

$$= \kappa\, \mathrm{Area}(C)$$

M

& rotations

$$q(\xi) = \frac{1}{4}\int (\nabla_a \xi_b - \nabla_b \xi_a)\,\underline{\epsilon}^{ab}$$
$$= \frac{1}{4}\int (\partial_a \xi_b - \partial_b \xi_a)\,\underline{\epsilon}^{ab}$$
$$= \frac{1}{4}\int \left[\partial_a (g_{bc}\xi^c) - \dots\right]\underline{\epsilon}^{ab}$$

$$\underline{\epsilon}_{ab} = \varepsilon_{ab}\sqrt{\gamma}\,d^2x$$
$$\varepsilon_a{}^b\,\varepsilon_b{}^a \equiv 2$$
$$0 = d(\varepsilon_a{}^b\,\varepsilon_b{}^a) = 2\,\varepsilon_a{}^b\,d\varepsilon_b{}^a$$

$$(\delta_Q q)(\chi) = \frac{1}{4}\int_e \left[\partial_a(\delta g_{bc}\,\chi^c) - \dots\right]\underline{\epsilon}^{ab} + \frac{1}{4}\int_e \left(\nabla_a \chi_b - \dots\right)\delta\underline{\epsilon}^{ab}$$

$\chi|_{\mathcal{E}=0}$

$$= \frac{1}{4}\int_e \left(\delta g_{bc}\,\underbrace{\nabla_a \chi^c}_{\kappa\,\varepsilon_a{}^c} - \dots\right)\underline{\epsilon}^{ab} + \frac{1}{2}\int \kappa\,\varepsilon_{ab}\,\delta\underline{\epsilon}^{ab}$$

0th law

$$= \frac{\kappa}{2}\int_C \varepsilon^c{}_a\,\delta\underline{\epsilon}^a{}_c = \kappa\int_C \delta\sqrt{\gamma} = \kappa\,\delta\,\mathrm{Area}(C)$$

Conclusion

$\kappa \, d\mathcal{A} \approx d\mathcal{M} - \omega_H \, d\mathcal{J}$

1st law of BH mech.

Quantum?

$\kappa \overset{?}{\longleftrightarrow}$ Temperature

$T = \frac{\hbar \kappa}{2\pi}$

$S_{BH} = \frac{\mathcal{A}}{2\hbar}$, $l_{Pl}^2 = 8\pi G\hbar$

Rmk

$\underline{J}$ is a momentum map

Mom maps are equivariant but not invariant!

$\mathbb{L}_{\mathfrak{e}(3)} \underline{J}(\eta) = -\underline{J}([\xi, \eta]) \neq 0$

but for χ a killing (field dep! $\mathbb{L}_{\mathfrak{e}(3)}\chi \neq 0$)

$\mathbb{L}_{\mathfrak{e}(3)} \underline{J}(\chi) = 0$

Rmk

$\underline{J}$ is a momentum map

Mom. maps are equivariant but not invariant!

$\mathcal{L}_{\varrho(\xi)} \underline{J}(\eta) = -\underline{J}([\xi, \eta]) \neq 0$

but for χ a killing (field alg! $\mathcal{L}_{\varrho(\xi)}\chi \neq 0$)

$\mathcal{L}_{\varrho(\xi)} \underline{J}(\chi) = 0$ invariant! $q(\chi)$ is a diffeo inv. charge!