Title: Quantum Gravity Lecture

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Collection: Quantum Gravity 2023/24

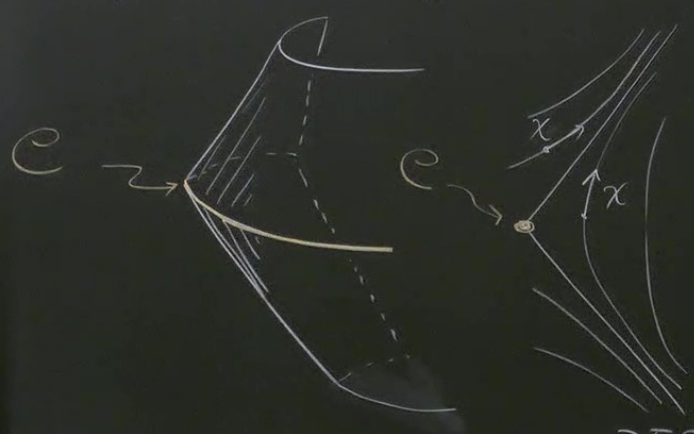
Date: April 24, 2024 - 9:00 AM

URL: https://pirsa.org/24040030

BH ENTROPY IS THE NOETHER CHARGE

[Wald, Wald + Iyer]

• Bifurcate Killing horizon



• $\nabla_a(\chi^b\chi_b) = -2\kappa\,\chi_a$

κ 'SURFACE GRAVITY'

ZEROTH-LAW

$\underline{\kappa}$ constant on the horizon

• THM (Rigidity)

Stationary BH $\Rightarrow$ axisym

$\Rightarrow$ The event horizon is a Killing horizon for

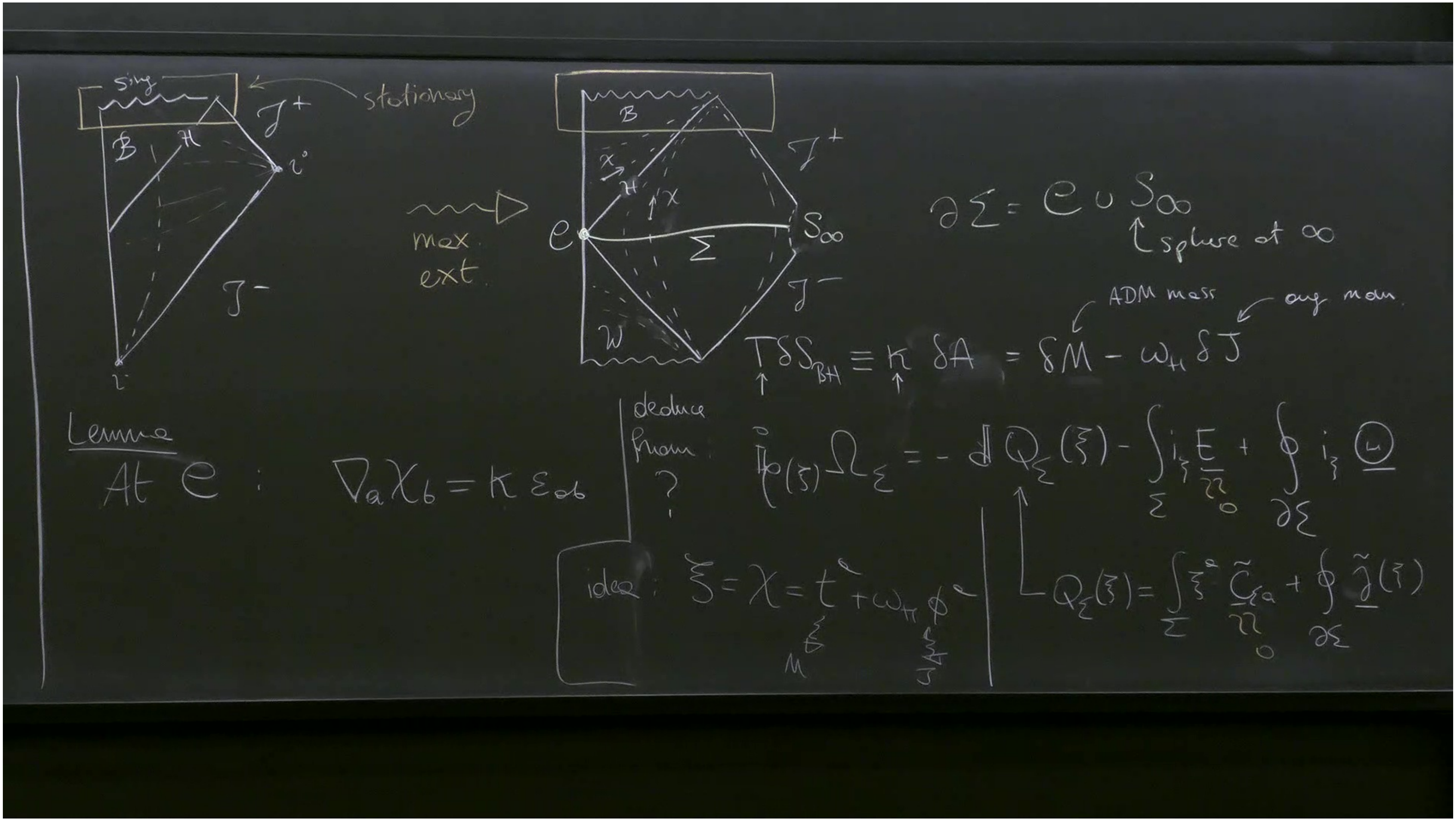
$$\chi^a = t^a + \omega_H \phi^a$$

$\omega_H \in \mathbb{R}$ "angular vel"

• THM [Wald & Racz]

Stationary BH, $\kappa \neq 0$

$\Rightarrow$ In the maximally extended spt of the BH the event horizon $\mathcal{H}$ is a branch of a bifurcate Killing hor.


sing
stationary
$\mathcal{J}^+$
$\mathcal{B}$
$\mathcal{H}$
i^0
$\mathcal{J}^-$
i^-
max. ext.
$\mathcal{B}$
χ
$\mathcal{H}$
$\mathcal{C}$
Σ
S_∞
$\mathcal{J}^+$
$\mathcal{J}^-$
$\mathcal{W}$
$\partial\Sigma = \mathcal{C} \cup S_\infty$
sphere at ∞
ADM mass
ang mom.
$T\delta S_{BH} \equiv \kappa\, \delta A = \delta M - \omega_H \delta J$
Lemma
At $\mathcal{C}$: $\nabla_a \chi_b = \kappa\, \varepsilon_{ab}$
deduce from ?
$I_{\delta}(\xi)\Omega_\Sigma = -\, \delta Q_\Sigma(\xi) - \int_\Sigma i_\xi E + \oint_{\partial\Sigma} i_\xi \omega$
idea: $\xi = \chi = t^a + \omega_H \phi^a$
M
J
$Q_\Sigma(\xi) = \int_\Sigma \xi^a \tilde{C}_a + \oint_{\partial\Sigma} \tilde{j}(\xi)$

Subtlety

χ^a is a Killing vf

$\delta \left(\mathcal{L}_\chi g_{ab} = 0 \right)$

$\Rightarrow \delta \chi \neq 0$

How does the Ham flow eq generalize?

$$\mathring{I}_{\rho(\chi)} \Omega_\Sigma = -\left(\delta Q_\Sigma\right)(\chi) - \int_\Sigma i_\chi E + \oint_{\partial\Sigma} i_\chi \underline{\Theta}$$

$$\approx -\left(\delta q_e\right)(\chi) - \left(\delta q_\infty\right)(\chi) + \oint_{e} i_\chi \underline{\Theta} + \oint_{S_\infty} i_\chi \underline{\Theta}$$

1. $C^{(X)} = \int (\mathcal{L}_X g_{ab}) \frac{\delta}{\delta g_{ab}} = 0$ ← Killing

2. χ at $\mathcal{C}$ vanishes.

$$\oint_{\mathcal{C}} i_\chi \underline{\omega} = 0$$

because $\phi^a \in TS_\infty$

$$\Longrightarrow 0 \approx -(\mathbb{d}q_{\mathcal{C}})(\chi) - (\mathbb{d}q_\infty)(S_\infty) + \oint_{S_\infty} i_\chi \underline{\omega}$$

$$\approx -(\mathbb{d}q_{\mathcal{C}})(\chi) - \omega_H (\mathbb{d}q_\infty)(\phi^a) - (\mathbb{d}q_\infty)(t^a) + \oint_\infty i_t \underline{\omega} + \omega_H \underbrace{\oint_\infty i_\phi \underline{\omega}}_{=0}$$

<u>Rmk</u>: since $(M, g_{ab}) \xrightarrow[\infty]{} $ Minkowski

$t^a, \phi^a \xrightarrow[\infty]{}$ "absolute" time transl. & rotations , $\mathbb{d}\phi^a|_\infty = \mathbb{d}t^a|_\infty = 0$ $\Longrightarrow (\mathbb{d}q_\infty)(t) = \mathbb{d}(q_\infty(t))$

Compute now Komar charges at ∞!

- $q_\infty(\phi) = -J$

- $q_\infty(t) = \frac{1}{2} M_{ADM}$

- miracle is that, thanks to the asymptotic properties of BH spt ("boundary cond" = asympt. flat)

$$\oint_\infty i_t \underline{\Theta} = d \oint_\infty \underline{B} \quad , \quad \oint_\infty \underline{B} = \frac{1}{2} M_{ADM}$$

$$\Rightarrow (\delta q_e)(X) \simeq \delta M - \omega_H \, \delta J$$

outside the variation!

Left hand side?

warm up: $q_e(X) = \frac{1}{4} \oint_e (\nabla_a \chi_b - \nabla_b \chi_a) \underline{\epsilon}^{ab}$

lemma

$$= \frac{1}{2} \oint_e \kappa \, \underbrace{\varepsilon_{ab} \underline{\epsilon}^{ab}}_{2 \underline{\epsilon}_e = 2\sqrt{\gamma}\, d^2x}$$

zeroth law

$$= \kappa \, \text{Area}(e)$$

M

& rotations , $d\varphi|_\infty = dC|_\infty = 0$

$$q(\xi) = \frac{1}{4}\int (\nabla_a \xi_b - \nabla_b \xi_a)\underline{\epsilon}^{ab}$$
$$= \frac{1}{4}\int (\partial_a \xi_b - \partial_b \xi_a)\underline{\epsilon}^{ab}$$
$$= \frac{1}{4}\int \left[\partial_a (g_{bc}\xi^c) - \dots\right]\underline{\epsilon}^{ab}$$

$$\underline{\epsilon}_{ab} = \varepsilon_{ab}\sqrt{\gamma}\, d^2x$$
$$\varepsilon_a{}^b \varepsilon_b{}^a \equiv 2$$
$$0 = d\!\!\!d(\varepsilon_a{}^b \varepsilon_b{}^a) = 2\varepsilon_a{}^b\, d\!\!\!d\, \varepsilon_b{}^a$$

$$(d\!\!\!d q)_e(\chi) = \frac{1}{4}\int_e \left[\partial_a (d\!\!\!d g_{bc}\chi^c) - \dots\right]\underline{\epsilon}^{ab} + \frac{1}{4}\int_e (\nabla_a \chi_b - \dots)\, d\!\!\!d\,\underline{\epsilon}^{ab}$$

$\chi|_e = 0$

0^{th} law

$$= \frac{1}{4}\int_e \Big(d\!\!\!d g_{bc}\underbrace{\nabla_a\chi^c}_{\kappa\,\varepsilon_a{}^c} - \dots\Big)\underline{\epsilon}^{ab} + \frac{1}{2}\int \kappa\,\varepsilon_{ab}\, d\!\!\!d\,\underline{\epsilon}^{ab}$$

$$= \frac{\kappa}{2}\int_C \varepsilon^c{}_a\, d\!\!\!d\,\underline{\epsilon}^a{}_c = \kappa\int_C d\!\!\!d\sqrt{\gamma} = \kappa\, d\!\!\!d\,\mathrm{Area}(C)$$

Conclusion

$\kappa \, dA \approx dM - \omega_H \, dJ$

1st law of BH mech.

Quantum?

$\kappa \overset{?}{\longleftrightarrow}$ Temperature

$T = 2\pi\kappa$

$S_{BH} = \frac{A}{2\pi}$, $\ell_{pl}^2 = 8\pi G\hbar$

Rmk

$\underline{J}$ is a momentum map

Mom maps are equivariant but not invariant!

$\mathbb{L}_{\rho(\xi)} \underline{J}(\eta) = -\underline{J}([\xi,\eta]) \neq 0$

but for χ a killing (field dep! $\mathbb{L}_{\rho(\xi)}\chi \neq 0$)

$\mathbb{L}_{\rho(\xi)} \underline{J}(\chi) = 0$

Rmk

$\underline{J}$ is a momentum map

Mom. maps are equivariant
but not invariant!

$$\mathbb{L}_{\rho(\xi)} \underline{J}(\eta) = -\underline{J}([\xi,\eta]) \neq 0$$

but for χ a killing (field dep! $\mathbb{L}_{\rho(\xi)}\chi \neq 0$)

$$\mathbb{L}_{\rho(\xi)} \underline{J}(\chi) = 0$$ invariant! $q(\chi)$ is a [illegible]!

diffeo inv. charge!