Title: Quantum Gravity Lecture

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Collection: Quantum Gravity 2023/24

Date: April 22, 2024 - 2:00 PM

URL: https://pirsa.org/24040029

GENERAL RELATIVITY

$$\mathcal{F} = \{ g_{ab} \text{ on } M \}$$

$$S = \int \sqrt{g}\left(\tfrac{1}{2}R - \Lambda\right) d^4x$$

$$\underline{L}\ (\Lambda = 0)$$

$$\underline{E}_{ab} = -\left(G_{ab} + \Lambda g_{ab}\right)\underline{\epsilon}$$

$$\underline{\Theta} = \tfrac{1}{2}\left(\nabla_b\, \delta g^{ab} - \nabla^a\, \delta g\right)\underline{\epsilon}_a$$

$$\underline{\epsilon} = \sqrt{g}\, d^4x$$

$$\mathcal{G} = \mathrm{diff}(M) \simeq \mathfrak{X}'(M)$$

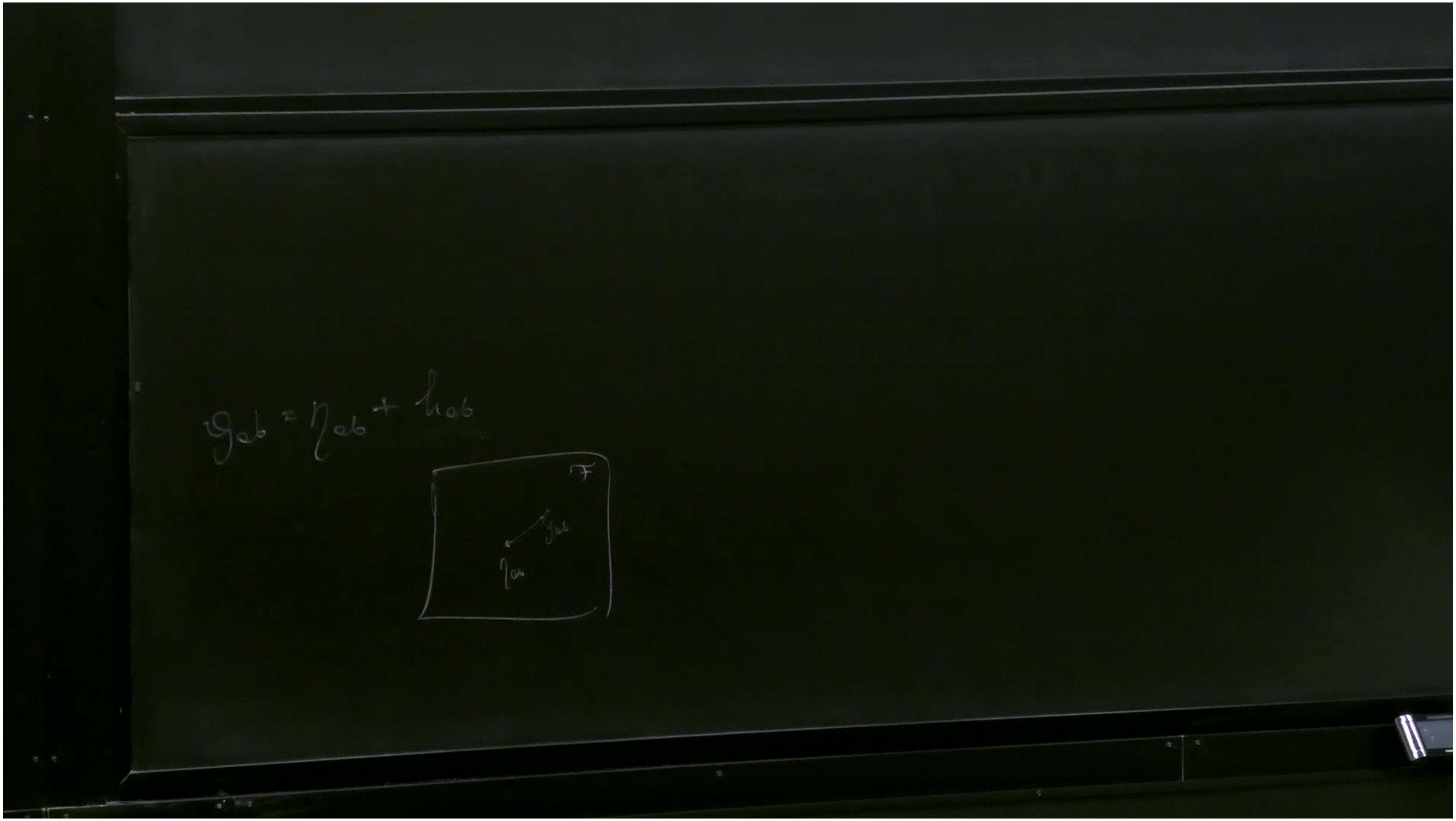
$$\delta_\xi g_{ab} = L_\xi g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

Background indep. (general cov.)

$$\mathbb{L}_{\rho(\xi)} \underline{L} = L_\xi \underline{L}$$

Recall, $\underline{J}(\xi) = \mathbb{i}_{\rho(\xi)} \underline{\Theta} - \underline{R}(\xi)$

$$d\underline{R}(\xi) := \mathbb{L}_{\rho(\xi)} \underline{L} \underset{B.T.}{=}$$


$g_{ab} = \eta_{ab} + h_{ab}$
F
g_{ab}
η_{ab}

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Recall, $\underline{J}(\xi) := \mathbb{i}_{\rho(\xi)} \underline{\Theta} - \underline{R}(\xi)$

$$d\underline{R}(\xi) := \mathbb{L}_{\rho(\xi)} \underline{\mathcal{L}} \underset{B.I.}{=} L_\xi \underline{\mathcal{L}} = d i_\xi \underline{\mathcal{L}} + i_\xi (\cancel{d \underline{\mathcal{L}}})$$

$$\underline{R}(\xi) - i_\xi \underline{\mathcal{L}} \neq 0$$

$(\xi) = \xi^b \tilde{C}^a_b + \nabla_b \tilde{j}^{ba}(\xi)$

$\tilde{C}^{ab} = -2E^{ab} \approx 0$

$\tilde{j}^{ba} = \frac{1}{2}(\nabla^b \xi^a - \nabla^a \xi^b) = \tilde{j}^{[ba]}$ Komar

$\underline{J} = \xi^b \underline{C}_b + d\underline{j} \quad , \quad \underline{j} = \frac{1}{2} \tilde{j}^{ba} \underline{\epsilon}_{ba} = \frac{1}{2} * d\xi^b$

$n_b \tilde{C}^{ba}\big|_\Sigma$ constraints of GR at Σ.

"Hamiltonian" flow eq.

$\mathbb{I}_{\rho(\xi)} \underline{\Omega} = -\mathbb{d}\underline{J}(\xi) + \underline{S}(\xi)$

the obstruction $\underline{S}$ can be rewritten as

$\underline{S}(\xi) = \mathbb{I}_{\rho(\xi)} \underline{\Theta} - \mathbb{d}\underline{B}(\xi) \in \Omega^{top-1,1}(M \times F)$

$\underset{B.I.}{=} L_\xi \underline{\Theta} - d\, i_\xi \underline{\mathcal{L}}$

$= i_\xi \cancel{d\underline{\Theta}} + d i_\xi \underline{\Theta}$

$+ i_\xi(\underline{E} - \cancel{d\underline{\Theta}}) = i_\xi \underline{E} + d i_\xi \underline{\Theta}$

$$\underline{S}(\xi) = i_\xi \underline{E} + d\, i_\xi \underline{\omega} \approx d\, i_\xi \underline{\omega}$$

$\uparrow$ eom

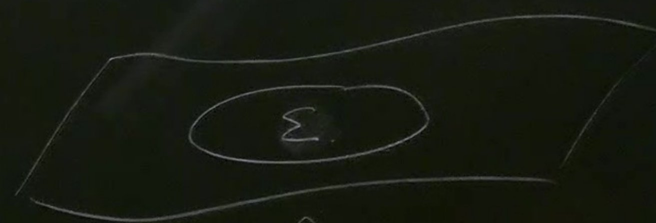
Integrating over Σ:

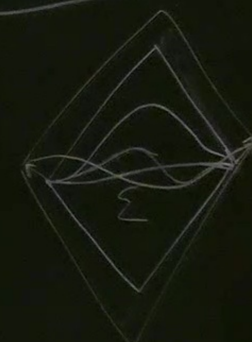
$$\mathring{i}_{\rho(\xi)} \Omega_\Sigma = -\, \mathrm{d}\!\!\mathrm{d}\, Q_\Sigma(\xi) + \int_\Sigma \underbrace{i_\xi \underline{E}}_{} + \oint_{\partial\Sigma} \underbrace{i_\xi \underline{\omega}}_{\text{corner}}$$

$\sim E\ \xi^a \epsilon_a$

$\sim E\,(\xi^a n_a)\, \epsilon_\Sigma$

$\neq 0$ if ξ transverse to $\partial\Sigma$





$$\underline{S}(\xi) = i_\xi \underset{\uparrow \text{eom}}{\underline{E}} + d\, i_\xi \underline{\Theta} \approx d\, i_\xi \underline{\Theta}$$

Integrating over Σ:

$$\mathring{i}_{\rho(\xi)} \Omega_\Sigma = -\mathbb{d}\, Q_\Sigma(\xi) + \int_\Sigma \underbrace{i_\xi \underline{E}} + \oint_{\partial\Sigma} \underbrace{i_\xi \underline{\Theta}}$$

$$\sim E\ \xi^a \epsilon_a$$

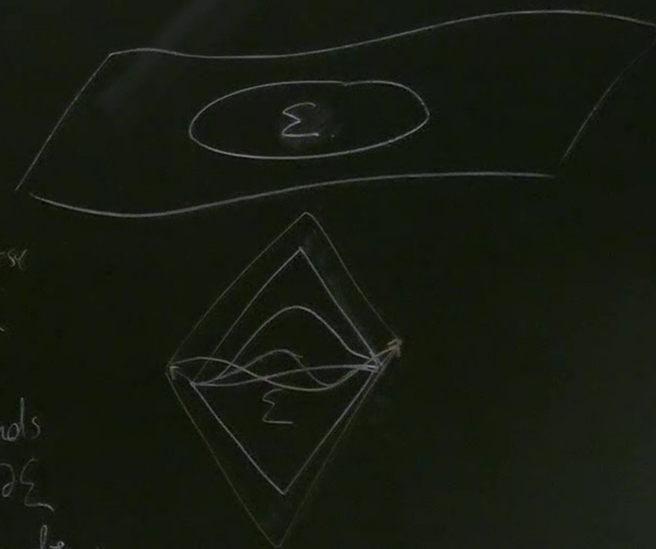
$$\sim E\,(\xi^a n_a)\, \epsilon_\Sigma$$

corner

$\neq 0$ if ξ transverse to $\partial\Sigma$

$\Downarrow$

if boundary conds are imposed at $\partial\Sigma$ maybe this term can be reabsorbed.



$$\chi^a = t^a + \omega_H \phi^a$$

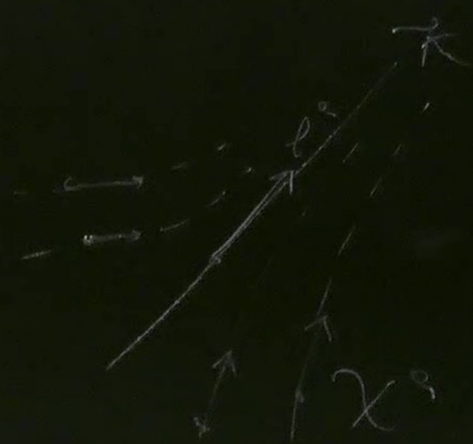
$$\delta Q \sim \delta E + \omega_H \delta J = T \delta S$$

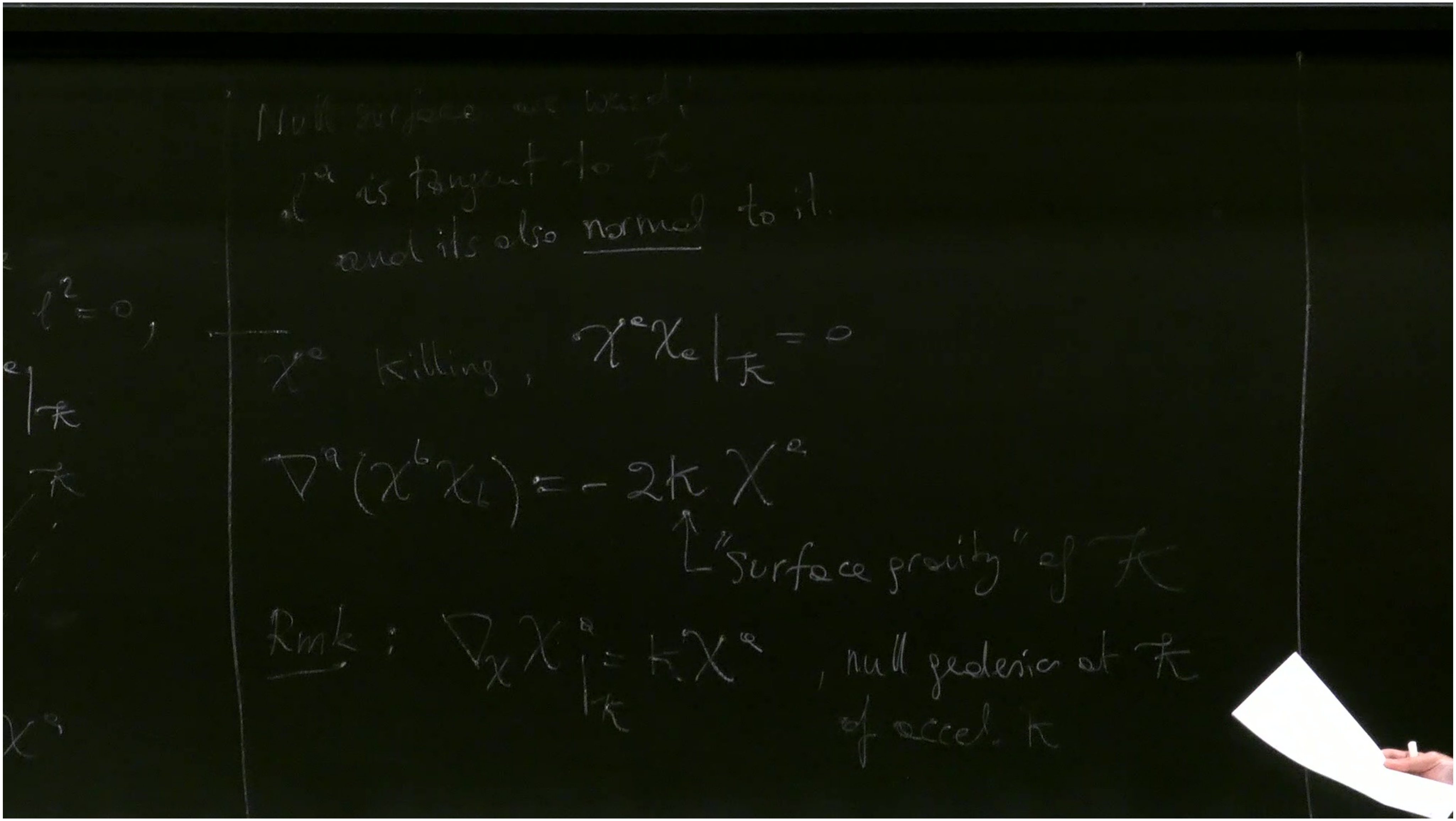
$$\kappa \, \delta A$$

Wald 1999 (review, CQG)

DEF [Killing horizon]

$\mathcal{K}$ = null surface whose null generator ℓ^a, $\ell^2 = 0$, is Killing v.f. $\chi^a|_{\mathcal{K}}$




Null surfaces are weird;
ℓ^a is tangent to $\mathcal{K}$
and it's also normal to it.
$\ell^2 = 0$,
χ^a Killing, $\chi^a \chi_a |_{\mathcal{K}} = 0$
$\nabla^a(\chi^b \chi_b) = -2\kappa \chi^a$
"surface gravity" of $\mathcal{K}$
Rmk: $\nabla_\chi \chi^a |_{\mathcal{K}} = \kappa \chi^a$, null geodesic at $\mathcal{K}$ of accel. κ

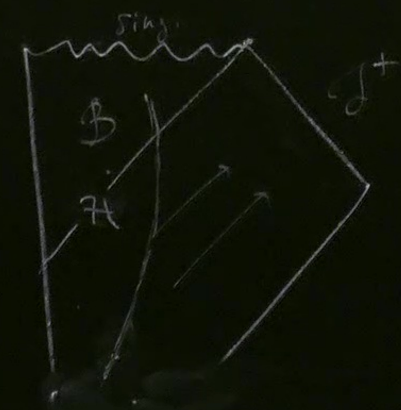
$\hbar$

$\hbar$

"BLACK HOLE"

$(\Lambda = 0)$

- asymptotically flat spacetimes
  $\longrightarrow$ $\mathscr{J}^+$ future as. null infinity
  "the place light rays go in the asympt future"



- $\mathcal{B} = M \setminus I^{-1}(\mathscr{J}^+)$

BLACK HOLE interior / region

- $\mathcal{H} = \partial \mathcal{B}$ null surface

EVENT HORIZ

(Rigidity)

THM (Carter & Hawking)

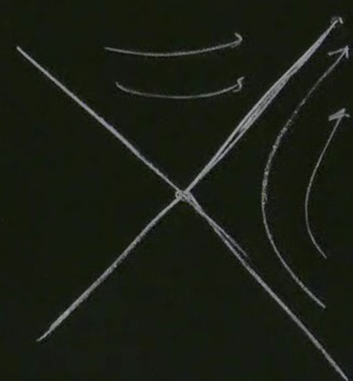
· Stationary BH (timelike KVF, t^a)

→ axisym (ϕ^a)

→ $\exists\ \omega_H$ const: $\chi^a = t^a + \omega_H \phi^a$

such that $\mathcal{H}$ is a Killing horiz. for χ^a.

Rindler.



DEF [Bifurcate Killing Hor.]

- $\mathcal{K}_1, \mathcal{K}_2$ 2 Killing horiz for same χ^a

- $\mathcal{C} := \mathcal{K}_1 \cap \mathcal{K}_2$ - 2 sphere
$\neq \phi$ bifurcation surface.

PROP $\chi^a = 0$ @ $\mathcal{C}$
& converse is also true.

THM [Carter & Bardeen]
Zeroth law: κ is const. on $\mathcal{H}$.

$k = 0 \to$ unstable B.H.

(Dafermos)