Title: Quantum Gravity Lecture

Speakers: Aldo Riello

Collection: Quantum Gravity 2023/24

Date: April 19, 2024 - 9:00 AM

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RECAP

Lagn. sym $\quad \mathbb{L}_{\rho(\xi)} \underline{L} = d\underline{R}(\xi)$

$\underline{J}(\xi) := \mathring{\mathbb{I}}_{\rho(\xi)} \underline{\Theta} - \underline{R}(\xi)$

- NOETHER 1

$d\underline{J}(\xi) = \delta_\xi \varphi^I \underline{E}_I \approx 0$

$\mathring{\mathbb{I}}_{\rho(\xi)} \underline{\Omega} = -\mathbb{d}\,\underline{J}(\xi) + \underline{S}(\xi)$

$\underline{\Omega}$ ↳ top-1

$\underline{S}(\xi)$ ↳ $\approx d\underline{r}(\xi)$

$\underline{r}$ ↳ top-2

- "Best case scenario" $\underline{R}(\xi) = 0$, $\mathbb{L}_{\rho(\xi)} \underline{\Theta} = 0$

$\to \mathring{\mathbb{I}}_{\rho(\xi)} \underline{\Omega} = -\mathbb{d}\,\underline{J}(\xi)$, $\underline{J}(\xi) = \mathring{\mathbb{I}}_{\rho(\xi)} \underline{\Theta}$

Local sym

using N1

⇝ NOETHER

$(\mathbb{D}^\dagger)$

↳ n

b.c.

→ u

→ t

$d\underline{R}(\xi)$

$i_{\rho(\xi)}\underline{\Theta} - \underline{R}(\xi)$

≈ 0

$) + \underline{S}(\xi)$

$\approx d\underline{r}(\xi)$

top-2

$\underline{R}(\xi) = 0 \quad , \quad \mathbb{L}_{\rho(\xi)}\underline{\Theta} = 0$

$(\xi) \quad , \quad \underline{J}(\xi) = \mathring{i}_{\rho(\xi)}\underline{\Theta}$

<u>Local</u> sym $\quad \xi(x) \in \mathcal{G} = \begin{cases} C^\infty(M, \mathfrak{g}) \\ \mathfrak{X}^1(M) \end{cases}$

using N1

$\leadsto$ <u>NOETHER 2</u> $\quad \delta_\xi \varphi^I =: \mathbb{D}^I_\alpha \xi^\alpha$

$$(\mathbb{D}^\dagger)^I_\alpha E_I \equiv 0$$

$\hookrightarrow$ not all eom are indep.

b.c. $\#\alpha$ relations among $\#I$ eom.

$\to$ underdetermined dynamics

$\to$ to save determinism, need to identify sols

<u>Ex</u>

YM

Ex

YM: $\varphi^I = A_a^\alpha$, $I = (\alpha, a)$

$E_I = D^b F_{ba}^\alpha$ $\quad D = d + A$

$\delta_\xi \varphi^I = (D\xi)_a^\alpha$

$\mathbb{D}_\alpha^I = \mathbb{D}_{a\alpha}{}^\beta = \delta_\alpha^\beta \partial_a + f^\beta{}_{\alpha\gamma} A_a^\gamma = D_a$

$(\mathbb{D}^\dagger)_\alpha^I = \mathrm{div}_A$

$(\mathbb{D}^\dagger)_\alpha^I E_I = D^a D^b F_{ab}^\alpha \equiv 0$

GR $\quad \varphi^I = g_{ab}$ $\quad I = (a, b)$

$E_I = G_{ab} \equiv R_{ab} - \frac{1}{2} R g_{ab}$

$\delta_\xi g_{ab} = \mathcal{L}_\xi g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a$

$\vdots$

$(\mathbb{D}^\dagger)_\alpha^I E_I = \nabla^a G_{ab} \equiv 0$

"Bianchi"

$Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi) \approx 0$

$\partial\Sigma = \emptyset$

Ex

YM:

$$\varphi^I = A^\alpha_a \,, \quad I = (\alpha, a)$$

$$E_I = D^b F^\alpha_{ba} \qquad D = d + A$$

$$\delta_\xi \varphi^I = (\mathbb{D}\,\xi)^\alpha_a$$

$$\mathbb{D}^I_\alpha = \mathbb{D}_{a\alpha}{}^\beta = \delta^\beta_\alpha \partial_a + f^\beta{}_{\alpha\gamma} A^\gamma_a = D_a$$

$$(\mathbb{D}^\dagger)^I_\alpha = \mathrm{div}_A$$

$$(\mathbb{D}^\dagger)^I_\alpha E_I = D^a D^b F^\alpha_{ab} \equiv 0$$

GR

$$\varphi^I = g_{ab} \qquad I = (a, b)$$

$$E_I = G_{ab} \equiv R_{ab} - \tfrac{1}{2} R\, g_{ab}$$

$$\delta_\xi g_{ab} = L_\xi g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$\vdots$

$$(\mathbb{D}^\dagger)^I_\alpha E_I = \nabla^a G_{ab} \equiv 0$$

"Bianchi"

- $$Q_\Sigma(\xi) = \int_\Sigma \underline{J}(\xi) \approx 0 \qquad \partial\Sigma = \emptyset$$

holds $\forall \xi, \ \forall \Sigma$

THM

$\hookrightarrow \underline{J}(\xi) \approx dj(\xi)$

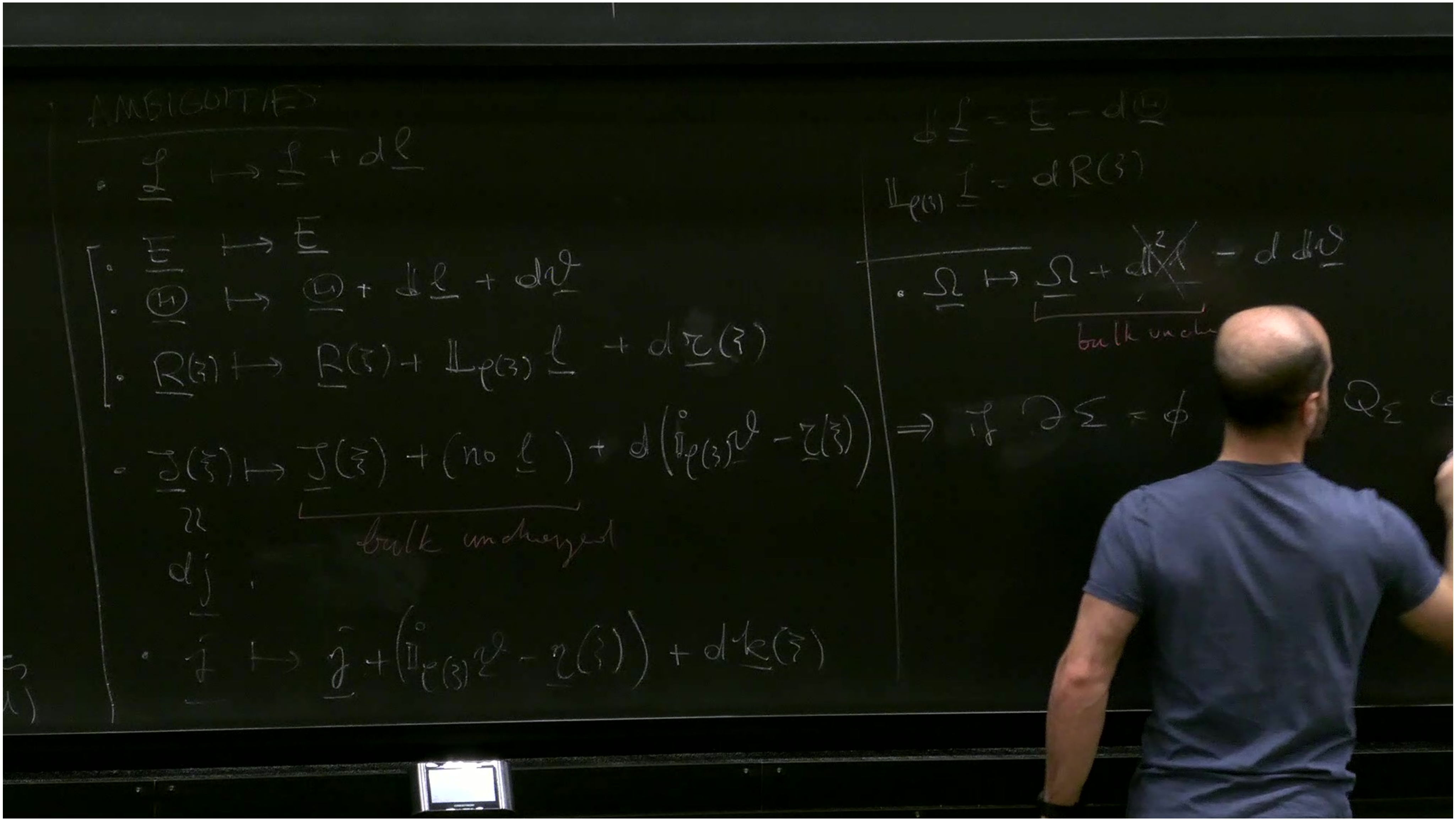
Generally: $\underline{J}(\xi) = \xi^\alpha \boxed{\underset{\underset{0}{\approx}}{\underline{C}_\alpha}} + dj(\xi)$

constraints
(Gauss in YM)

$\partial\Sigma = \emptyset,\ \xi = 0$ (YM)

$$\mathbb{I}_{\rho(\xi)} \Omega_\Sigma = -\mathbb{d}\, Q_\Sigma(\xi) = -\int_\Sigma \xi^\alpha\, \mathbb{d}\,\underline{C}_\alpha$$

→ if all goes for the "best", the constraints generate gauge transf. (off shell)


AMBIGUITIES
bulk unchanged
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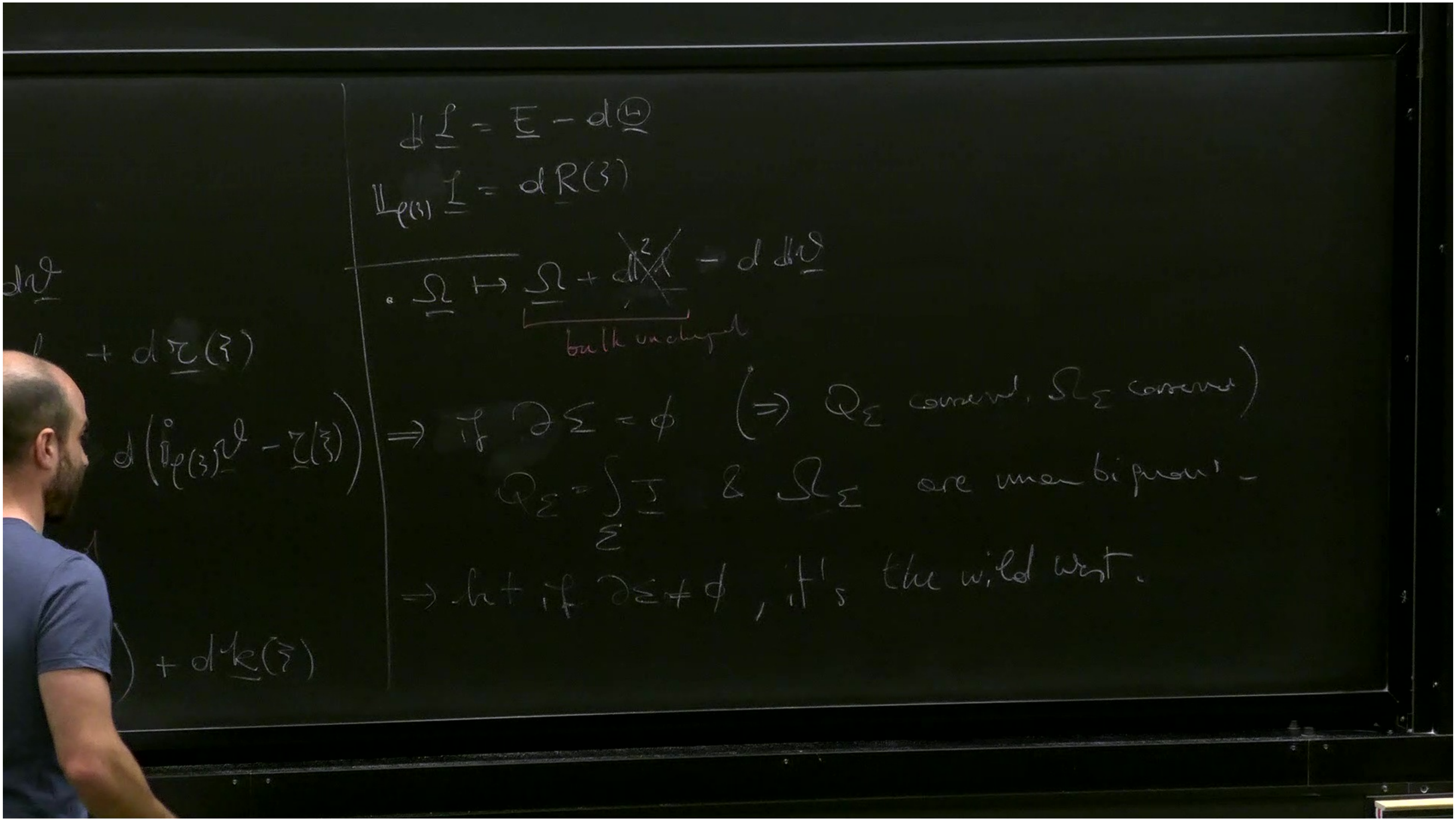
dϑ
+ dτ(ξ)
d(I_{ρ(ξ)}ϑ − τ(ξ))
+ dk(ξ)
dL = E − dΘ
I_{ρ(ξ)} L = dR(ξ)
Ω ↦ Ω + dd²λ − d dϑ
bulk unchanged
⇒ if ∂Σ = ϕ (⇒ Q_Σ conserved, Ω_Σ conserved)
Q_Σ = ∫_Σ J & Ω_Σ are unambiguous.
→ but if ∂Σ ≠ ϕ, it's the wild west.

→ "if all goes for the best", The constraints generate gauge transf. (off shell)

• $\dot{f} \mapsto \dot{g}$

GENERAL RELATIVITY

$$\mathcal{F} = \{ g_{ab}(x) \} = \mathrm{Riem}_{d,1}(M)$$

$$S_{EH} = \frac{\hbar}{l_{Pl}^2} \int_M \sqrt{g} \left(\frac{1}{2} R - \Lambda \right) + \underbrace{\frac{\hbar}{l_{Pl}^2} \int_{\partial M} \sqrt{h}\, K}_{YGH}$$

$$l_{Pl}^2 = 8\pi G \hbar$$

Rmks 1) non polynomial in g_{ab}
(fixable using Palatini-Cartan, Ashtekar)
= tetrad & (spin) conn as indep. fields

2) $\partial^2 g \Rightarrow$ pbs with action principle $\Theta \propto \delta g$
fixable by adding YGH term

→ if all goes for the "best", the constraints generate gauge transf. (off shell)

• $\underline{\tilde{j}} \mapsto \underline{\tilde{j}} + (\mathring{\mathbb{I}}_{\xi(3)} \mathcal{V}$

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$$\mathbb{d} R = \nabla \mathbb{d}\Gamma = \nabla(\nabla \mathbb{d} g)$$

$$\updownarrow$$

$$\mathbb{d} F = D \mathbb{d} A$$

$$[\nabla, \nabla] = \mathrm{Riem}$$

$$[D, D] = F$$

$$\Gamma = g^{-1}\partial g + \ldots$$

$$\mathbb{d}\Gamma = \nabla \mathbb{d} g$$

$(\xi) \ldots - \underline{\eta}(\xi) \Big) + d\, \underline{k}(\xi)$

$d \mathcal{L}_{EH} = d g_{ab}\, \underline{E}^{ab} - d\, \underline{\Theta}$

- $\underline{E}^{ab} = -\frac{1}{2}\left(G^{ab} + \Lambda g^{ab} \right) \underline{\epsilon}$
- $\underline{\Theta} = \frac{1}{2}\left(\nabla_b\, d g^{ab} - \nabla^a d g \right) \underline{\epsilon}_a$

<u>Sym</u> $\quad \xi(x) \in \mathrm{diff}(M) = \mathfrak{X}^1(M)$

$\mathbb{L}_{\rho(\xi)}\, g_{ab} = L_\xi\, g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a$

$[\rho(\xi), \rho(\eta)] = -\rho([\xi,\eta])$

antihomom.

Background indep
(General covariance)

$$\boxed{\mathbb{L}_{\rho(\xi)}\, \underline{\mathcal{L}} = L_\xi\, \underline{\mathcal{L}}}$$