

Title: Quantum Gravity Lecture

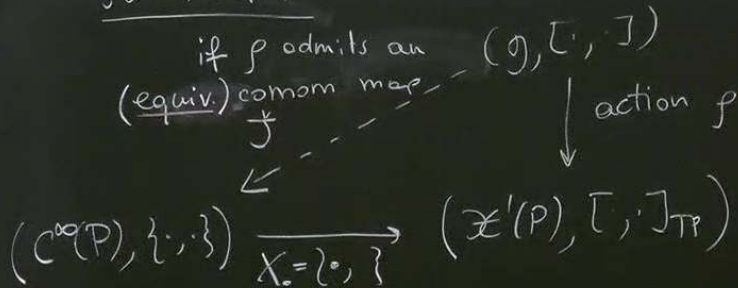
Speakers: Aldo Riello

Collection: Quantum Gravity 2023/24

Date: April 11, 2024 - 11:30 AM

URL: <https://pirsa.org/24040023>

SUMMARY



$\rho(\xi) \omega = -d\check{J}(\xi)$

$\rho(\xi) = \{ \check{J}(\xi), \cdot \}$

See lecture notes for SYMPL REDUCTION

CHAPTER 2

In field theory, two relations

• A mom. map \exists iff $\check{J} \cdot \mathfrak{g} / [\mathfrak{g}, \mathfrak{g}] \rightarrow \mathfrak{H}'(P)$ vanishes identically.

• Equiv: $\{ \check{J}(\xi), \check{J}(\eta) \} = \check{J}([\xi, \eta])$

|||

$L_{\rho(\xi)} \check{J}(\eta)$

See lecture notes
for SYMPL. REDUCTION

CHAPTER 2

In field theory, two relevant manifolds:

(i) Spacetime M , $\dim < \infty$

(ii) Field Space F , $\dim = \infty$

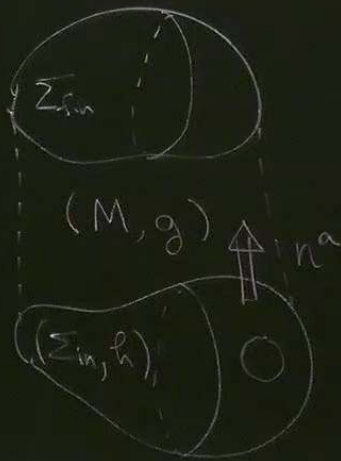
↑ plays the role of P (morally)

Need tools to do geom. on F .

(i) Spacetime
Globally Hyp.

$$M = \Sigma \times \mathbb{R}, \quad \partial \Sigma = \emptyset$$

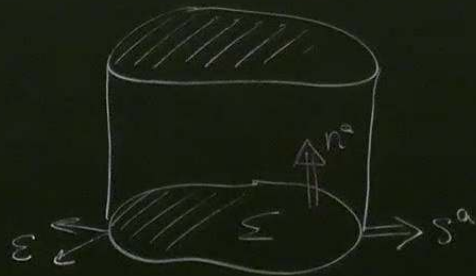
$$M = \Sigma \times [t_{in}, t_{fin}]$$



$$\Sigma, \partial \Sigma = \emptyset$$

In some cases
we will consider Σ with boundary
 $\partial \Sigma \neq \emptyset$

In some cases
we will consider Σ with boundaries
 $\partial\Sigma + \phi$



$$\partial\Sigma = C$$

$$\partial M = \Sigma_{in} \times \Sigma_{fin} \times \underbrace{(C \times [t_{in}, t_{fin}])}_B$$

$$\xi^{ab} = \frac{n^a s^b - n^b s^a}{\sqrt{1 + (ns)^2}}$$

$N \subset M$ submanifold of dim n .

$$\Omega^k(N)$$

$$\Omega^{\text{top-}k}(N) = \Omega^{n-k}(N)$$

$$* \Sigma_{\text{fin}} * \underbrace{(C \times [t_{\text{in}}, t_{\text{fin}}])}_B$$

(ii) Field space

$$\mathcal{F} = \Gamma(M, F)$$

$\pi: \mathcal{F} \rightarrow M$ fiber bundle over M

Ex:

1) real scalar field

$$F = M \times \mathbb{R} \rightarrow M, \Gamma(M, F) = C^\infty(M, \mathbb{R})$$

(non-deg)

2) GR

$$F = T^*M \otimes T^*M \rightarrow M, \Gamma(M, F) = \text{symmetric covariant 2-tensors}$$

2) GR

$$F = T^*M \underset{M}{\otimes} T^*M, \quad F(M, F) = \begin{matrix} \text{symmetric} \\ \text{covariant} \\ \text{2-tensors} \end{matrix}$$

3) YM : $A \in \mathcal{A}(M, \mathfrak{g})$ $F = \text{Conn}$ (it can be done)

4) Particles as 0+1d field theory
 $M = \text{time} = \mathbb{R}$ $F = M \times \mathbb{R}^{3N}$, $\vec{q}(t) \in \mathcal{F} = C(M, \mathbb{R}^3)$

5) Parametrized
 $M = \text{"parameter time"}$ $F = M \times \mathbb{R}^4$
 $= \mathbb{R}$

Doing geometry on M & F

$$\boxed{M} \ni x$$

• vector fields

$$X = \sum_i X^i(x) \frac{\partial}{\partial x^i}$$

$$X(f) = \lim_{\varepsilon \rightarrow 0} \frac{f(x_i + \varepsilon X^i) - f(x_i)}{\varepsilon}$$
$$= \sum_i X^i \frac{\partial f}{\partial x^i}$$

$$\boxed{F} \ni \varphi$$

• vector fields

$$X \equiv \int dx \delta_{xx} \varphi(x) \frac{\delta}{\delta \varphi(x)}$$

Interested in local function(al)s over F

$$S(\varphi) = \int dx \mathcal{L}(\varphi(x), \partial \varphi(x), x)$$

$$X(S) = \int dx \frac{\delta S}{\delta \varphi} (\delta_{xx} \varphi) = \int dx \left(\frac{\partial \mathcal{L}}{\partial \varphi} \right)$$

- 1) Scalar field $F =$
- 2) GR $F =$

$\boxed{F} \ni \varphi$ ↙ not the EL derivative

• vector fields " $\delta_{\mathbb{X}} \varphi(x) \frac{\delta}{\delta \varphi(x)}$ "

$$\mathbb{X} = \int dx \delta_{\mathbb{X}} \varphi(x) \frac{\delta}{\delta \varphi(x)}$$

Interested in local function(al)s over F

$$S(\varphi) = \int dx \mathcal{L}(\varphi(x), \partial \varphi(x), x)$$

$$\mathbb{X}(S) = \int dx \frac{\delta S}{\delta \varphi} (\delta_{\mathbb{X}} \varphi) = \int dx \left(\frac{\partial \mathcal{L}}{\partial \varphi(x)} \delta_{\mathbb{X}} \varphi(x) + \frac{\partial \mathcal{L}}{\partial \partial_i \varphi(x)} \boxed{\partial_i \delta_{\mathbb{X}} \varphi(x)} \right) = \lim_{\epsilon \rightarrow 0} \frac{S(\varphi + \epsilon \delta_{\mathbb{X}} \varphi) - S(\varphi)}{\epsilon}$$

↑ differential operator

- 3) YM : $A \in \mathcal{S}'(M, \mathfrak{g})$
- 4) Particles as odd field F
 $M = \text{time} = \mathbb{R}$
- 5) Parametrized
 $M = \text{"parameter time"} = \mathbb{R}$

Doing geometry on M & F

$$\boxed{M} \ni x$$

• vector fields

$$X = \sum_i X^i(x) \frac{\partial}{\partial x^i}$$

$$X(f) = \lim_{\epsilon \rightarrow 0} \frac{f(x_i + \epsilon X^i) - f(x_i)}{\epsilon}$$

$$= \sum_i X^i \frac{\partial f}{\partial x^i}$$

• differential

$$df = \sum_i \frac{\partial f}{\partial x^i} dx^i$$

$$\boxed{F} \ni \varphi$$

↙ not the EL derivative

• vector fields

$$X = \int dx \delta_X \varphi(x) \frac{\delta}{\delta \varphi(x)}$$

Interested in local function(al)s over F

$$S(\varphi) = \int dx \mathcal{L}(\varphi(x), \partial \varphi(x), x)$$

$$X(S) = \int dx \frac{\delta S}{\delta \varphi} (\delta_X \varphi) = \int dx \left(\frac{\partial \mathcal{L}}{\partial \varphi(x)} \delta_X \varphi \right)$$

• diff forms

$$dS = \int dx \frac{\delta S}{\delta \varphi} (d\varphi)$$

↑ differential operator

$$\boxed{F} \ni \varphi$$

not the EL derivative

• vector fields

$$X = \int dx \delta_X \varphi(x) \frac{\delta}{\delta \varphi(x)}$$

Interested in local function(al)s over F

$$S(\varphi) = \int dx \mathcal{L}(\varphi(x), \partial\varphi(x), x)$$

$$X(S) = \int dx \frac{\delta S}{\delta \varphi} (\delta_X \varphi) = \int dx \left(\frac{\partial \mathcal{L}}{\partial \varphi(x)} \delta_X \varphi(x) + \frac{\partial \mathcal{L}}{\partial \partial_i \varphi(x)} \boxed{\partial_i \delta_X \varphi(x)} \right) = \lim_{\epsilon \rightarrow 0} \frac{S(\varphi + \epsilon X \varphi) - S(\varphi)}{\epsilon}$$

↑ differential operator

• diff forms

$$dS = \int dx \frac{\delta S}{\delta \varphi} (d\varphi)$$

- 3) YM
- 4) Particles M
- 5) Paramet M

Cartan's calculus

(M)

- d
- i_X
- $L_X = i_X d + d i_X$

$$\left. \begin{array}{l} \cdot d \quad (\text{circled}) \\ \cdot i_X \quad (\text{crossed}) \\ \cdot L_X = i_X d + d i_X \end{array} \right\}$$

$$L_X S = i_X d S = i_X \int dx \frac{\delta S}{\delta \varphi} (d\varphi) = \int dx \frac{\delta S}{\delta \varphi} (i_X d\varphi)$$

$\delta_X \varphi$

Mixed forms

$$\alpha \in \Omega^{p,q}(\underline{M}, \underline{F})$$

$$\alpha(x) = \partial_a \varphi(x) \boxed{dx^a} \wedge \boxed{d\varphi(x)} \in \Omega^{1,1}(\underline{M}, \underline{F})$$

$$\hookrightarrow \alpha = d\varphi \wedge d\varphi$$

$$\bullet \quad \boxed{d\mathbb{d} + \mathbb{d}d = 0} \iff \begin{aligned} D &= d + \mathbb{d} \\ D^2 &= 0 \end{aligned}$$

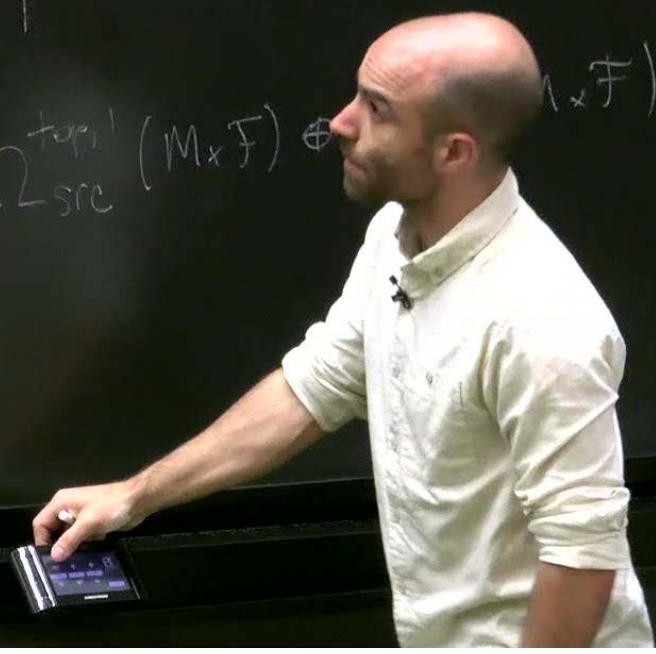
• one \wedge for all

$$\alpha_1 \wedge \alpha_2 = (-1)^{(p_1+q_1) \cdot (p_2+q_2)} \alpha_2 \wedge \alpha_1$$

$$\begin{aligned} \cdot d\hat{i}_X &= -i_X d \\ \cdot d i_X &= -i_X d + L_X \\ \cdot d L_X &= L_X d \\ \text{etc.} \end{aligned}$$

THM (Takeus)

$$\Omega^{\text{top}, 1}(M \times F) = \Omega^{\text{top}, 1}(M \times F) \oplus \Omega^{\text{top}, 1}(M \times F)$$



$\cdot dL_x = L_x d$
 etc.

THM (Takeus)

$$\Omega^{\text{top}, 1}(M \times F) = \Omega_{\text{src}}^{\text{top}, 1}(M \times F) \oplus \Omega_{\text{bdry}}^{\text{top}, 1}(M \times F)$$

$$\parallel \Omega^{\text{top}, 1}(M \times F)$$

a form \parallel that is
 "proportional" to
 $d\varphi(x)$
 without derivatives, i.e. no $\partial^k \varphi$, $k \geq 1$



without de

Lograngien $L = L(x, \varphi, \partial\varphi) \in \Omega^{\text{top}, 0}(M \times F)$, $S = \dots$

$$dL \stackrel{=}{=} E_I \lrcorner d\varphi^I + d\langle H \rangle$$

↑ Tokens ↑ Euler-Lagrange: $\frac{\partial L}{\partial \varphi} - \partial_a \frac{\partial L}{\partial \dot{\varphi}^a}$

$\langle H \rangle \in \Omega^{\text{top}-1, 1}(M \times F)$

↑ universal current
 (pre) symplectic potential current
 top-1 form on M

without dec

Lograngien $L = L(x, \varphi, \partial\varphi) \in \Omega^{\text{top}, 0}(M \times F)$, $S = \int L$

$dL \stackrel{=}{=} E_I d\varphi^I + d\Theta$
↑ Tokens
↑ Euler-Lagrange: $\frac{\partial L}{\partial \varphi} - \frac{\partial L}{\partial \partial \varphi}$

$\Theta \in \Omega^{\text{top}-1, 1}(M \times F)$

↑ universal current
(pre) symplectic potential current
↳ will give us the ph. sp. structure
top-1 form on M

$\Omega_\Sigma := \int_\Sigma d\Theta$
will be a degenerate
2-form on F^{-1}