Title: Strong Gravity Lecture

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Collection: Strong Gravity 2023/24

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Prelude, example of Maxwell Eqns.

$$0 = \partial^a F_{ab} = \partial^a \partial_a A_b - \partial^a \partial_b A_a \qquad (A_a, \partial_t A_a)\ t=0$$

$b=t$

$$-\cancel{\partial_t^2 A_t} + \partial^i \partial_i A_b + \cancel{\partial_t^2 A_t} - \partial^i \partial_t A_i = 0$$

$$\partial_i \partial^i A_t - \partial_t \partial^i A_i = 0 \qquad \text{Constraint}$$

$$\nabla \cdot \vec{E} = 0$$

With Lorenz gauge: $\partial_a A^a = 0$, $\partial^a \partial_a A_b = 0$, $\partial^a \partial_a (\partial_b A^b) = 0$

Generalized harmonic formulation of Einstein Eqns

$$R_{ab} = 4\pi\left(2T_{ab} - g_{ab}T\right)$$

$$= -\frac{1}{2} g^{cd}\partial_c\partial_d g_{ab} - \partial_c g_{d(a}\partial_{b)} g^{cd} + \nabla_{(a}\Gamma_{b)} - \Gamma^c{}_{da}\Gamma^d{}_{cb} - \Gamma$$

$$\Gamma_a = g^{bc}\Gamma_{abc} = -\Box x_a \quad , \quad \Box x^a = \nabla_b \nabla^b x^a$$

Fix gauge $\Box x^a = H^a \leftarrow$ specify

H^a independent variables

nic formulation of Einstein Eqns

$2T_{ab} - g_{ab}T)$

$\partial_c g_{d(a}\partial_{b)} g^{cd} + \nabla_{(a}\Gamma_{b)} - \Gamma^c_{da}\Gamma^d_{cb} - \Gamma_c \Gamma^c_{ab}$

$_{abc} = -\Box x_a \quad , \quad \Box x^a = \nabla_b \nabla^b x^a = \frac{1}{\sqrt{-g}}\partial_b\left(\sqrt{-g}\, g^{ba}\right) = -\Gamma^a$

$H^a \leftarrow$ specify

iables

Generalized harmonic Eqns

$H_a = -\Gamma_a$

$$g^{cd}\partial_c\partial_d g_{ab} + \partial_b g^{cd}\partial_c g_{ad} + \partial_a g^{cd}\partial_c g_{bd}$$

$$+\partial_a H_b + \partial_b H_a - 2H_d\Gamma^d_{\ ab} + 2\Gamma^c_{\ db}\Gamma^d_{\ ca}$$

$$= -8\pi\left(2T_{ab} - g_{ab}T\right)$$

$L(H^a) = 0$

some evolution operator

Simplest choice:

$H^a = 0$ (harmonic gauge)

Simple choice

$H^a = F^a(g_{bc})$

Evolve

$g_{ab}, \partial_t g_{ab}, H^a, \partial_t H^a$

$$C^a = H^a - \Box x^a$$

$C^a = 0$ for all time?

$$R_{ab} - 4\pi(2T_{ab} - g_{ab}T) = \nabla_{(a}C_{b)}$$

$n^a n^b$

$\gamma^a{}_c n^b$

$n^a n^b(\nabla_{(a}C_{b)}) =$ Hamiltonian

$\gamma^a_c n^b (\nabla_{(a}C_{b)}) = 0$

At $t=0$, $C^a = 0$, Hamiltonian, momentum constraints

$\Rightarrow \partial_t C_a = 0$ at $t=0$

$(R_{ab} -$

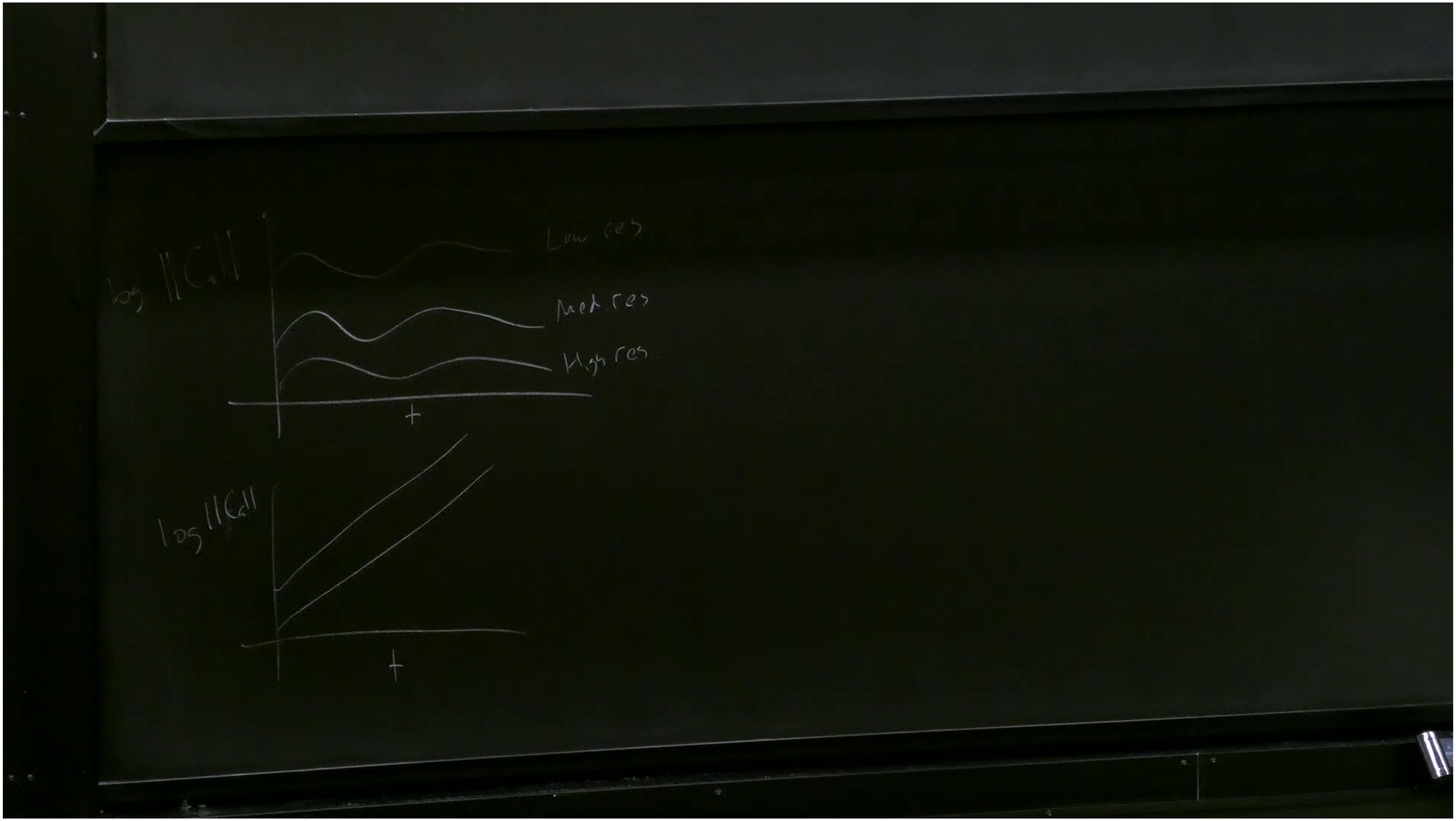
$\Rightarrow$

$$\left(R_{ab} - \tfrac{1}{2} R g_{ab}\right) - 8\pi T_{ab} = \nabla_{(a} C_{b)} - \tfrac{1}{2} g_{ab} g^{cd} \nabla_{(c} C_{d)}$$

$$\Rightarrow \nabla^a \underbrace{\left(R_{ab} - \tfrac{1}{2} R g_{ab}\right)}_{\text{Bianch. Id}\ \to 0} - 8\pi \underbrace{\nabla^a T_{ab}}_{\text{S.E. cons}\ \to 0} = 0 = \nabla^a \nabla_a C_b + \nabla^a \nabla_b C_a - \nabla_b \nabla^a C_a$$

$$\nabla_a \nabla^a C_b = -R^a{}_b C_a$$

Hence $C_a = \partial_t C_a = 0$ at $t=0 \Rightarrow$ $C_a = 0$ for all time

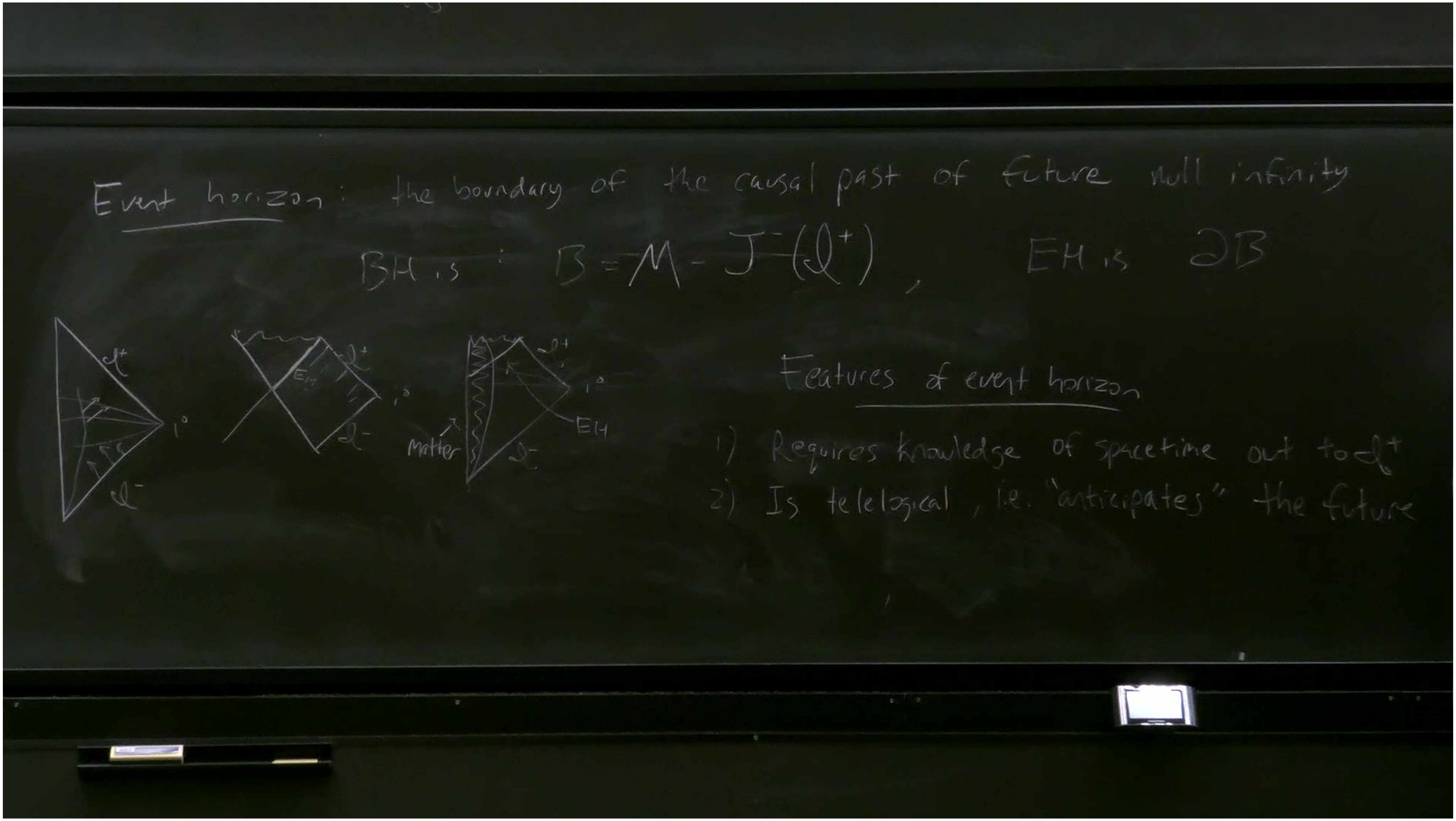

log ||C_a||
Low res
Med res
High res
t
log ||C_a||
t

Solution to exponentially growing $||C_a||$

Add $\kappa\left[n_{(a}C_{b)} - \frac{1}{2} g_{ab} n^d C_d\right]$ to (*)

$$\Box C^a = -R^a{}_b C^b + \kappa \nabla_b \left[n^{(b} C^{a)}\right]$$

$\kappa \sim \frac{1}{[L]}$ → damps constraint violation on κ^{-1} length/timescale


Event horizon: the boundary of the causal past of future null infinity
BH is: $B = M - J^-(\mathscr{I}^+)$, EH is ∂B
$\mathscr{I}^+$
i^0
$\mathscr{I}^-$
EH
Matter
Features of event horizon
1) Requires knowledge of spacetime out to $\mathscr{I}^+$
2) Is teleological, i.e. "anticipates" the future

ty

Shell of mass M at radius r

$r = 2M$

to $\mathcal{I}^+$

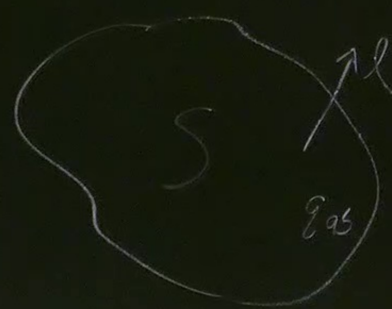
future

Apparent Horizon

Let Σ_t be some 3-dim timeslice

Let S be a closed 2-dim surface in Σ_t

with induced metric q_{ab}, ℓ normal vector field



Define expansion

$$\theta := \mathcal{L}_\ell \left(\ln \left(\sqrt{q} \right) \right) = q^{ab} \nabla_a \ell_b$$

Case 1: the null

Outward/inward $\ell_\pm^a = n^a \pm s^a$

unit normal to Σ

unit spacelike vector normal to S

$n_a n^a = -1$, $s_a s^a = 1$, $\ell_\pm^a \ell_{\pm a} = 0$, $\ell_+ \cdot \ell_- = -2$

Outward null expansion: $\Theta_+ = q^{ab} \nabla_a \ell^+_b$

$= D_a s^a + K_{ab} s^a s^b - K$ (on 3-d variable)

...celike vector
...ormal to S.

$l_+^a l_{-a} = -2$

... $-K$ (in 3+1 variables)

If $\theta_{l_+} \leq 0$ everywhere on S,
S is outer trapped surface

$\theta_{l_+} = 0$ marginally outer trapped surface (MOTS)

Apparent horizon: outermost MOTS