Title: Strong Gravity Lecture

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Collection: Strong Gravity 2023/24

Date: April 15, 2024 - 2:00 PM

URL: https://pirsa.org/24040019

Bad time slicing, an example (see Carroll, A

Gaussian normal coördinates: $\alpha = 1$, $\beta^i = 0$ $(\Longleftrightarrow$

$n^a = (1, \vec{0})$

$n^a \nabla_a n^b = -\Gamma^b_{tt} = 0$, geodesic an example (see Carroll, Appendix F)

linates: $\alpha = 1$, $\beta^i = 0$ $\left(\Longleftrightarrow g_{++} = -1, \; g_{+i} = 0 \right)$

$n^a = (1, \vec{0})$

$= 0$,

geodesic

$$K = g_{ab}K^{ab} = -g^{ab}(g^c_a + n_a n^c)\nabla_c n_b = -\nabla_a n^a = -\Theta$$

expansion

Raychaudhuri's Eqn:

$$\frac{d\theta}{d\tau} + \frac{1}{3}\Theta^2 + \sigma_{ab}\sigma^{ab} - \omega_{ab}\omega^{ab} = -R_{ab}u^a u^b$$

$u^a \to$ geodesics,

$\Theta = \nabla_a u^a$ σ_{ab} is shear

$\omega_{ab} = \frac{1}{2}(\nabla_a u_b - \nabla_b u_a)$ rotation tensor expansion

U^b

$$\sigma_{ab}\sigma^{ab} \geq 0 \quad \text{(spatial tensor)}$$

$$R_{ab}U^aU^b = 8\pi\left(T_{ab} - \tfrac{1}{2}T g_{ab}\right)U^aU^b \geq 0$$

Strong energy condition

$$\frac{d\theta}{d\tau} + \frac{1}{3}\theta^2 - \omega_{ab}\omega^{ab} \leq 0$$

For $u^a = n^a$, $\Theta = -K$, $\omega_{ab} = \frac{1}{2}(\partial_a n_b - \partial_b n_a) = 0$

$$-\frac{dK}{d\tau} + \frac{1}{3}K^2 \le 0$$

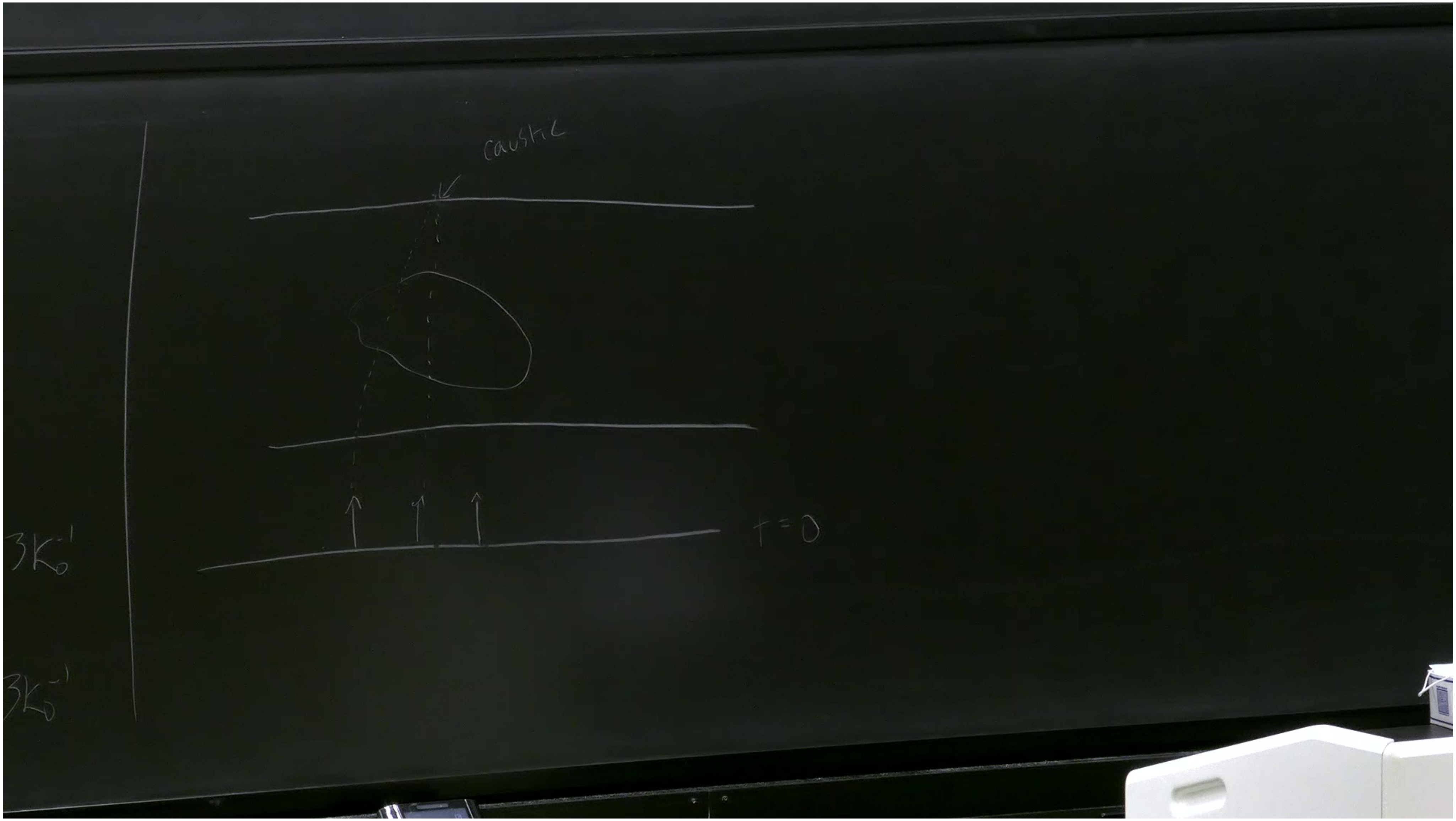
$K(\tau=0) = K_0 > 0$

$$\int_{K_0}^{K} \left(-\frac{1}{K'^2}\right) dK' \le \int_0^{\tau} -\frac{1}{3}\, d\tau'$$

$\tau > 3K_0^{-1}$

$$K^{-1} \le K_0^{-1} - \tau/3$$

$K^{-1} > 0$ at $\tau = 0$, $K^{-1} < 0$ at $\tau = 3K_0^{-1}$


caustic
t=0
$3K_0^{-1}$
$3K_0^{-1}$

3+1 Decomposition of the Einstein Eqn.

$$^{(3)}R_{abc}{}^{d}\,v_d = D_a D_b v_c - D_b D_a v_c$$

$$D_a D_b v_c = \gamma^d_a \gamma^e_b \gamma^f_c \nabla_d \left(\gamma^g_e \gamma^h_f \nabla_g v_h \right)$$

$$\gamma_{ab} = g_{ab} + n_a n_b \quad , \quad \nabla_a n^c = -K^c_a - \left(n^d \nabla_d n^c \right) n_a$$

$$^{(3)}R^c{}_{dab} = \gamma^e_a \gamma^f_b \gamma^c_h \gamma^g_d R^h{}_{gef} - K^c_a K_{db} + K^c_b K_{ad}$$

$$^{(3)}R_{db} + K K_{db} - K^a_b K_{ad} = \gamma^f_b \gamma^e_h \gamma^g_d R^h{}_{gef} \quad (\Diamond)$$

$$ {}^{(3)}R + K^2 - K_{ab}K^{ab} = R + 2R_{ab}n^a n^b \quad (*) $$

$$ R^c{}_{dab}\, n^d = (\nabla_a \nabla_b - \nabla_b \nabla_a) n^c $$

$$ \gamma^c_d\, n^e\, \gamma^f_a\, \gamma^g_b\, R^d{}_{efg} = D_b K^c_a - D_a K^c_b $$

$$ n^e \gamma^g_b R_{eg} = D_b K - D_a K^a_b \quad (☺) $$

$$ \gamma^4 R,\ \gamma^3 n R,\ \gamma^2 n^2 R,\ n^3 R = 0 $$

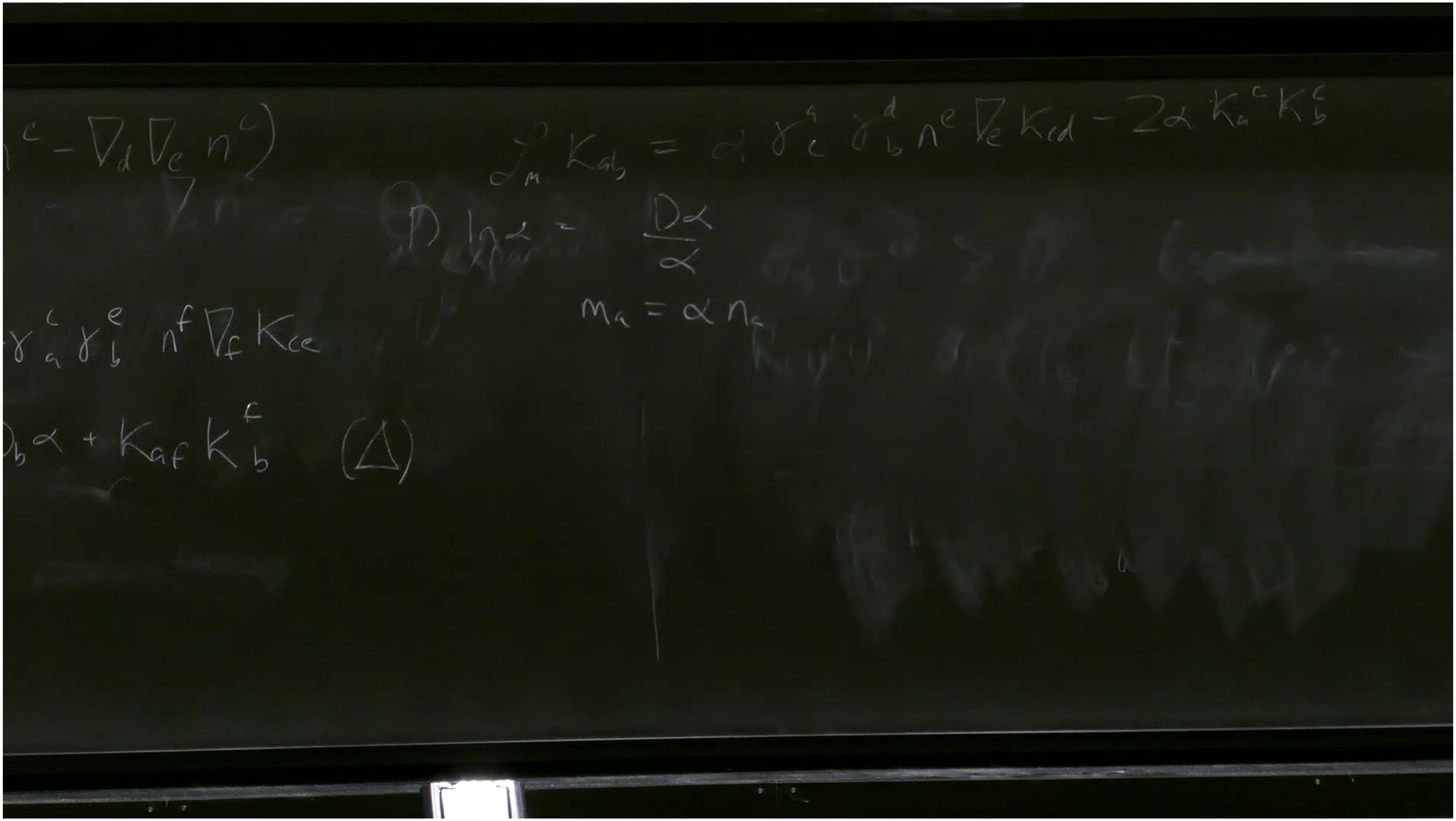
n_a

$$\gamma_{ac} n^d \gamma^e_b R^c{}_{fed} n^f = \gamma_{ac} n^d \gamma^e_b \left(\nabla_e \nabla_d n^c - \nabla_d \nabla_e n^c\right)$$

$$\nabla_b n_a = -K_{ab} - \underbrace{(D_a \ln \alpha)}_{\text{acceleration}} n_b$$

$$= -K_{af} K^f_b + \frac{1}{\alpha} D_b D_a \alpha + \gamma^c_a \gamma^e_b n^f \nabla_f K_{ce}$$

$$\gamma_{ac} n^d \gamma^e_b R^c{}_{fed} n^f = \frac{1}{\alpha} \mathcal{L}_m K_{ab} + \frac{1}{\alpha} D_a D_b \alpha + K_{af} K^f_b$$


$n^c - \nabla_d \nabla_e n^c)$
$\mathcal{L}_m K_{ab} = \alpha \gamma^a_c \gamma^d_b n^e \nabla_e K_{cd} - 2\alpha K_a{}^c K^c_b$
$D \ln\alpha = \frac{D\alpha}{\alpha}$
$m_a = \alpha n_a$
$\gamma^c_a \gamma^e_b n^f \nabla_f K_{ce}$
$D_b \alpha + K_{af} K^f_b$ (Δ)

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}$$

$$E := n^a n^b T_{ab}$$ energy density

$$p^a := -\gamma^a{}_c n^b T_{ab}$$ momentum density

$$S_{ab} := \gamma_a{}^c \gamma_b{}^d T_{cd}$$: matter stress tensor

Trace: $T = S - E$

Fluid: $T_{ab} = (\rho + P) u_a u_b + P g_{ab}$

density

tum density

stress tensor

$$E := (\rho + P) W^2 - P \qquad W = -n_a U^a$$

$$p^a = (E + P) U^a$$

$$U^a = \frac{1}{W} \gamma^a_b u^b$$

$$S_{ab} = (E + P) U_a U_b + P \gamma_{ab}$$

$$R_{ab} n^a n^b + \frac{1}{2} R = 8\pi E$$

$$(*) \Rightarrow {}^{(3)}R + K^2 - K_{ab}K^{ab} = 16\pi E$$

Hamiltonian

$$R_{ab}\gamma^a_c n^b - \frac{1}{2} R \cancelto{0}{n_a \gamma^a_c} = -8\pi p_c$$

: Momentum constraint

$$☺ \Rightarrow D_a K^a_b - D_b K = 8\pi p_c$$

$$R_{ab} = 8\pi \left(T_{ab} - \frac{1}{2} T g_{ab} \right)$$

$$\gamma^a{}_c \gamma^b{}_d R_{ab} = 8\pi \left[S_{cd} - \frac{1}{2}(S-E)\gamma_{cd} \right]$$

$$-\frac{1}{2}\mathcal{L}_m K_{cd} - \frac{1}{\alpha} D_c D_d \alpha + {}^{(3)}R_{cd} + K K_{cd} - 2 K_{ca} K^a{}_d = 8\pi [\dots]$$

$$\mathcal{L}_m \gamma_{ab} = -2\alpha K_{ab}$$

$$\gamma^d_b n^e \nabla_e K_{cd} - 2\alpha K_a^c K_b^c$$

$$E^{ab} = G^{ab} - 8\pi T^{ab}$$

Constraints: $E^{ab} n_b = 0$

$$\nabla_a E^{ab} = 0$$

$\nabla_t E^{ta}$ = terms involve E_{ab}, $\partial_i E_{ab}$

Maxwell's Eqn s

$\nabla\cdot\vec{E} = \rho$, $\nabla\cdot\vec{B} = 0$

constraints

$\partial_t \vec{E} = \nabla\times\vec{B} - \vec{J}$, $\partial_t \vec{B} = -\nabla\times\vec{E}$

evolution eqns.

$$\partial_t(\nabla\cdot\vec{B}) = 0$$

$$\partial_t(\nabla\cdot\vec{E}) = -\nabla\cdot\vec{J} = 0$$

$$\partial_t \rho + \nabla\cdot\vec{J} = 0$$