Title: Strong Gravity Lecture

Speakers: William East

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Scalar (test) field

$$\Box \varphi = g^{ab} \nabla_a \nabla_b \varphi = 0$$

Kerr solution in B.L. coordinates

Ansatz: $\varphi = R(r)\, \Theta(\theta)\, e^{im\phi}\, e^{-i\omega t}$

(*)

$$(*) \quad \frac{d}{dr}\left(\Delta \frac{dR}{dr}\right) + \left[ \frac{\omega^2(r^2+a^2)^2 - 4Mam\omega r + m^2a^2}{\Delta} - (\omega^2a^2+\Lambda) \right] R = 0$$

$$(**) \quad \frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) + \left[ a^2\omega^2\cos^2\theta - \frac{m^2}{\sin^2\theta} + \Lambda \right]\Theta = 0$$

$\Delta = (r-r_+)(r-r_-)$

$\uparrow$ separation constant

$a=0$ (**) doesn't depend to $\omega$

$\Theta \to P_\ell(\cos\theta)$ $\Theta e^{im\phi} = Y_{\ell m}(\theta,\phi)$ spherical harmonics, $\Lambda = \ell(\ell+1)$

$a \neq 0$ $\Theta = S_{\ell m}(\cos\theta, c)$ $c = a\omega$

$a = 0$, $(*) \Rightarrow \dfrac{d^2 R}{dr_*^2} + (\omega^2 - V) R = 0$, $V = \left(1 - \dfrac{2M}{r}\right)\left[\dfrac{\ell(\ell+1)}{r^2} + 2\right.$

$r_* = r + 2M \log\left(\dfrac{r}{2M} - 1\right)$

tortise coördinate

General Solution: $\psi = \sum_{\ell,m} \int dt\, e^{-i\omega t} R_{\ell m \omega}(r)\, S_{\ell m}(\theta, c)\, e^{im\phi}$

$0 \quad , \quad V = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M\sigma}{r^3}\right] \qquad \sigma = 1$

$$\int dt\, e^{-i\omega t} R_{\ell m\omega}(r)\, S_{\ell m}(\theta, c)\, e^{im\phi}$$

$$\hat{\Box} h_{ab} - 2 \hat{R}_{acbd} h^{cd} = 0$$

$$h_{ab} = \tilde{h}_{ab}(r,\theta)\, e^{im\phi}\, e^{-i\omega t}$$

Specialize to $a=0$

Odd (Regge-Wheeler)

$$\tilde{h}_{ab} = \begin{pmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ h_0(r) & h_1(r) & 0 & 0 \end{pmatrix} \left(\sin\theta \frac{\partial}{\partial\theta}\right) Y_{\ell 0}(\theta)$$

$O$

$e^{-i\omega t}$

$$h_1(r) = \frac{r^2}{r-2M}\Psi^-_{s=2}\,, \quad h_0(r) = \frac{i}{\omega}\frac{d}{dr_*}\left(r\Psi^-_{s=-2}\right)$$

$\Psi^-_{s=2}$ is a solution to $(\langle\rangle)$

$\sigma = 1 - s^2 = -3$

Even (Zerrilli)

$$\tilde{h}_{ab} = A(r)\, Y_{\ell 0}(\theta)$$

$$\begin{pmatrix} 0 & 0 & h_0(r) \\ & 0 & h_1(r) \\ 0 & 0 & 0 \\ 0 & & \\ h_1(r) & 0 & 0 \end{pmatrix}\left(\sin\theta\frac{\partial}{\partial\theta}\right)Y_{\ell 0}(\theta)$$

Boundary conditions: Ingoing at horizon

$r \to r_+$ , $r_* \to -\infty$ $\qquad R \sim \psi \sim e^{-i\omega r_*}$

Outgoing radiation

$r_*, r \to \infty$ $\qquad R, \psi \sim e^{i\omega r_*}$

Solve $\quad \dfrac{d^2\psi}{dr_*^2} + Q(r_*)\psi = 0 \qquad Q(r_*) = \omega^2 - V(r_*)$

$$Q(r_*) = Q_0 + \frac{1}{2} Q_0'' \left(r_* - \bar{r}_*\right)^2$$

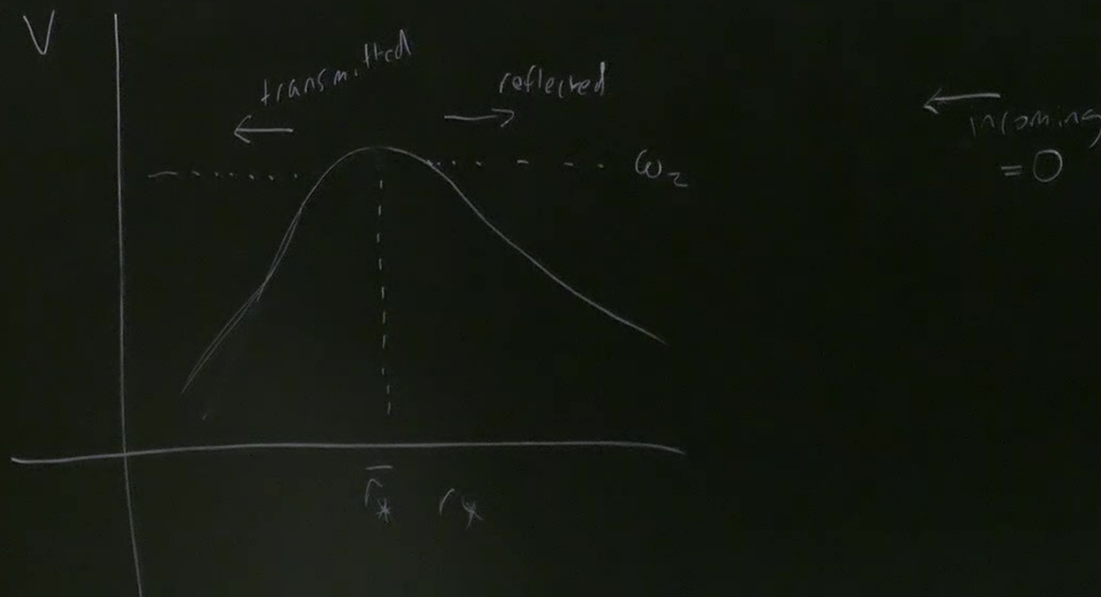
horizon

$r_* \to -\infty$ $\quad R \sim \psi \sim e^{-i\omega r_*}$

n

, $r \to \infty$ $\quad R, \psi \sim e^{i\omega r_*}$

$$Q(r_*) = \omega^2 - V(r_*)$$



$$\Psi = A\, D_\nu(z) + B\, D_{-\nu-1}(iz)$$

$$\nu = \frac{-iQ_0}{\sqrt{2Q_0''}} - \frac{1}{2}, \quad z = (2Q_0'')^{1/4} e^{i\pi/4} x$$

$$x = (r_* - \bar{r}_*)$$

Can satisfy BCs iff $\nu = 0, 1, 2, \ldots = n$

$$(M\omega_n)^2 = V(\bar{r}_*) - i\left(n + \frac{1}{2}\right)\left[-2\frac{d^2 V_\ell}{dr_*^2}\right]^{1/2}$$

$$\Psi = A\, D_\nu(z) + B\, D_{-\nu-1}(iz)$$

$$\nu = \frac{-iQ_0}{\sqrt{2Q_0''}} - \frac{1}{2}, \quad z = (2Q_0'')^{1/4} e^{i\pi/4} (r_* - \bar{r}_*)$$

Can satisfy BCs iff $\nu = 0, 1, 2, \ldots = n$

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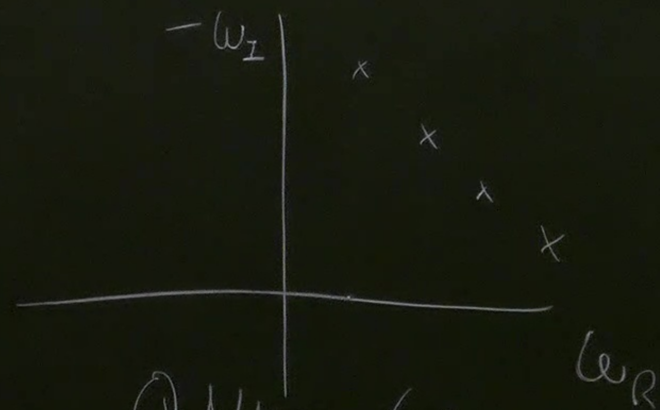
$\ell = 2,\ n = 0 \qquad M\omega = 0.37 - 0.09i$

$$\left[ \frac{\ell(\ell+1)}{r^2} + \frac{2M\sigma}{r^3} \right] \qquad \sigma = 1 \qquad \Psi = A$$

$$S_{\ell m}(\theta, c)\, e^{im\phi}$$




QNMs $(\ell, m, n)$

$$\sim e^{-i\omega t} = e^{-i\omega_R + i\omega_I t}$$

Can satisfy BCs iff

$$(M\omega_n)^2$$

$\ell =$

Weyl tensor: $R_{abcd} = C_{abcd} + \frac{1}{2}\left( g_{a[c} R_{d]b} - g_{b[c} R_{d]a} \right.$

$$\Psi_4 = C_{abcd}\, n^a m^{*b} n^c m^{*d}$$

n

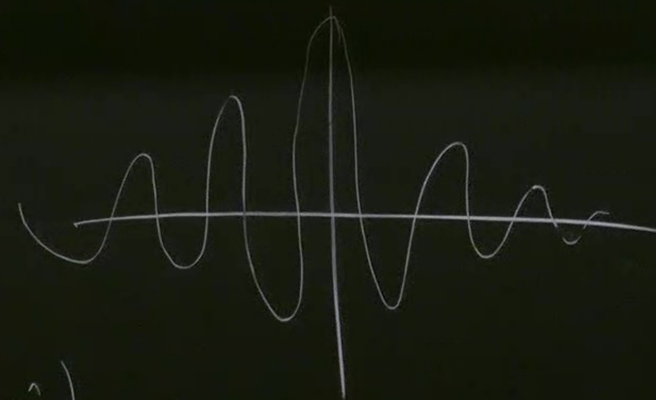
m

$$\Psi_4 = \frac{1}{2}\left( \ddot{h}_{\hat{\phi}\hat{\phi}} - \ddot{h}_{\hat{\theta}\hat{\theta}} \right) - i\, \ddot{h}_{\hat{\theta}\hat{\phi}} = \ddot{h}_+ - i\ddot{h}_\times$$

$$\Psi_4 = \rho^4 \sum \int d\omega\, e^{-i\omega t}\, {}_{-2}S_{\ell m}(\theta, c)\, e^{im\phi}\, e^{-i\omega t}\, R_{\ell m \omega}(r)$$

$g_{a[c}\, g_{d]b}$

$+a^2, -\Delta, 0, a)$ $\qquad \rho^{-1} = r - i a \sin\theta$



$ia\sin\theta, 0, 1, i\sin\theta)$

$l \simeq (\hat{t} + \hat{r})$ outgoing

$n = (\hat{t} - \hat{r})$ ingoing

$m^a = \frac{1}{\sqrt{2}}(\hat{\theta} + i\hat{\phi})^a$

st damped QNM in Kerr

$l = m = 2 \quad (n=0)$

$\simeq 1.53 - (1.16)(1 - a/M)^{0.13}$

$2$

$= \frac{\omega_R}{2\omega_I} \simeq 0.70 + 1.42(1 - a/M)^{-0.5}$

$a/M \simeq 0.7, \quad \omega_{22} \simeq 0.53, \quad \omega_I \simeq 0.081$