Title: Strong Gravity Lecture

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Collection: Strong Gravity 2023/24

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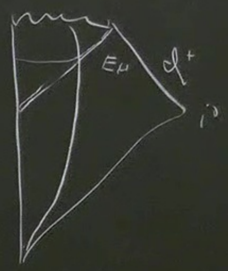
URL: https://pirsa.org/24040016

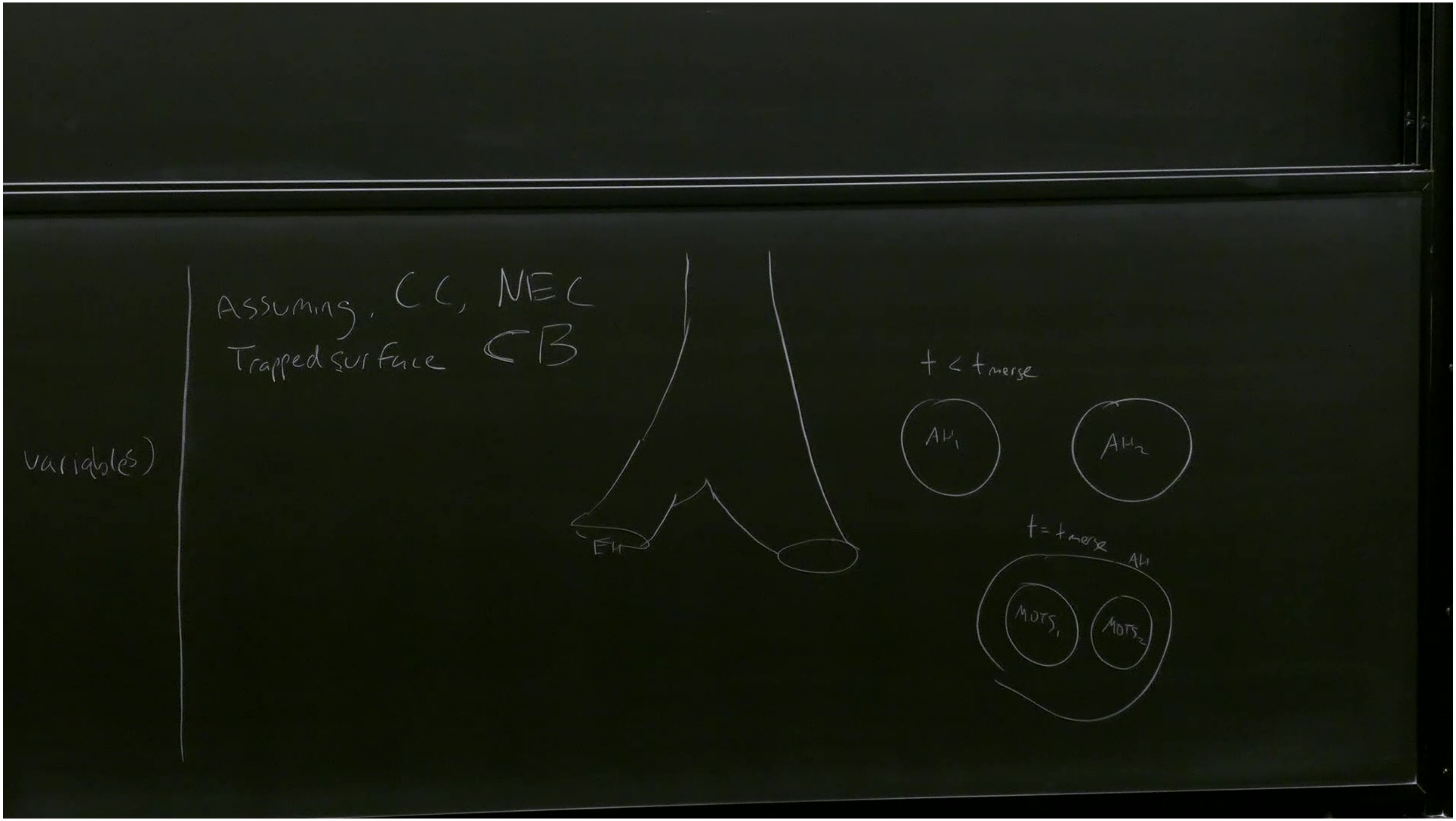
Apparent horizon : Outer most MOTS

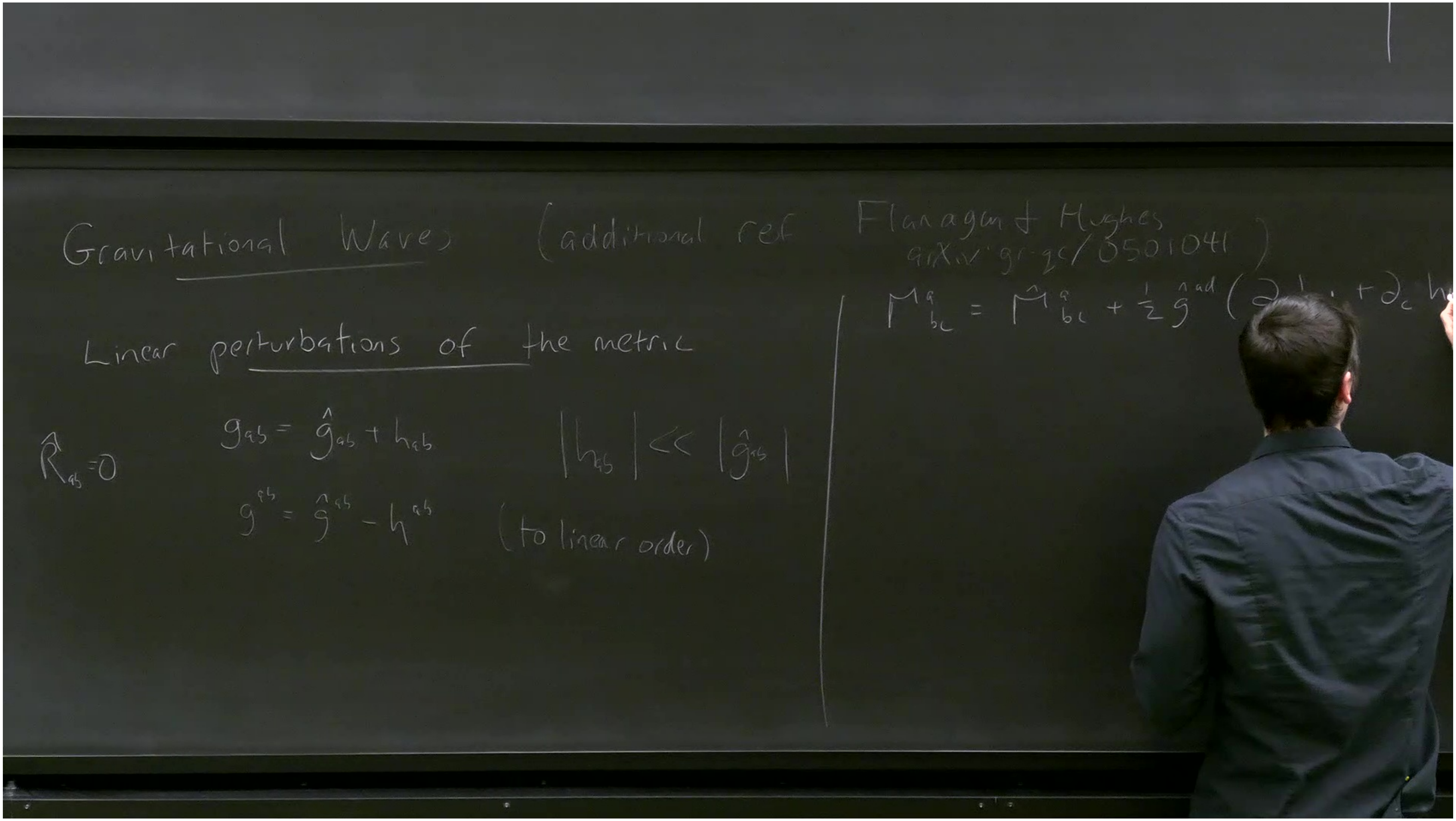
MOTS : $\theta_{\ell^+} = q^{ab} \nabla_a \ell^+_b = 0$

$= D_i s^i + K_{ij} s^i s^j - K$ (in 3+1 variables)

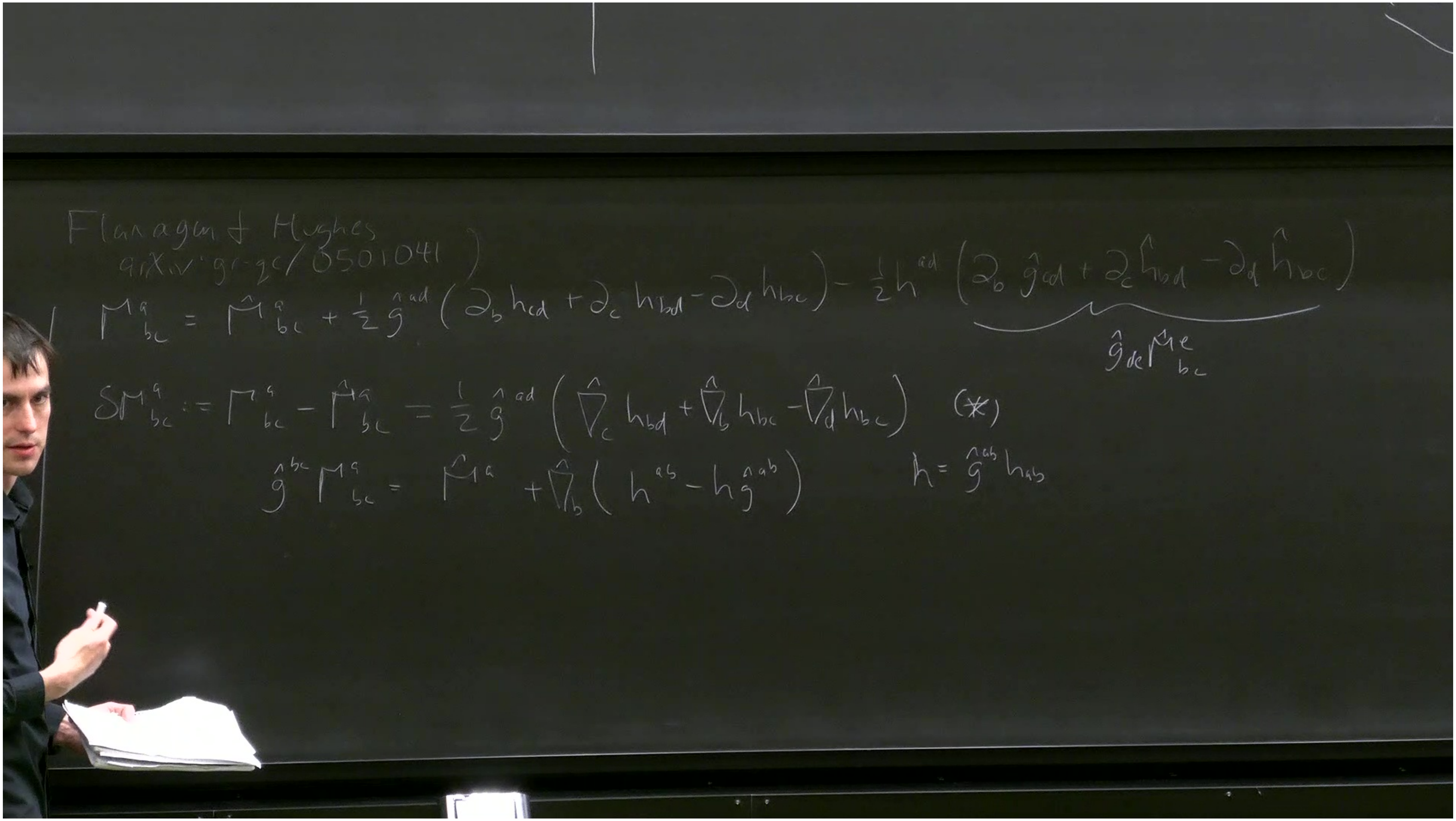
$\theta_{\ell^-} < 0$

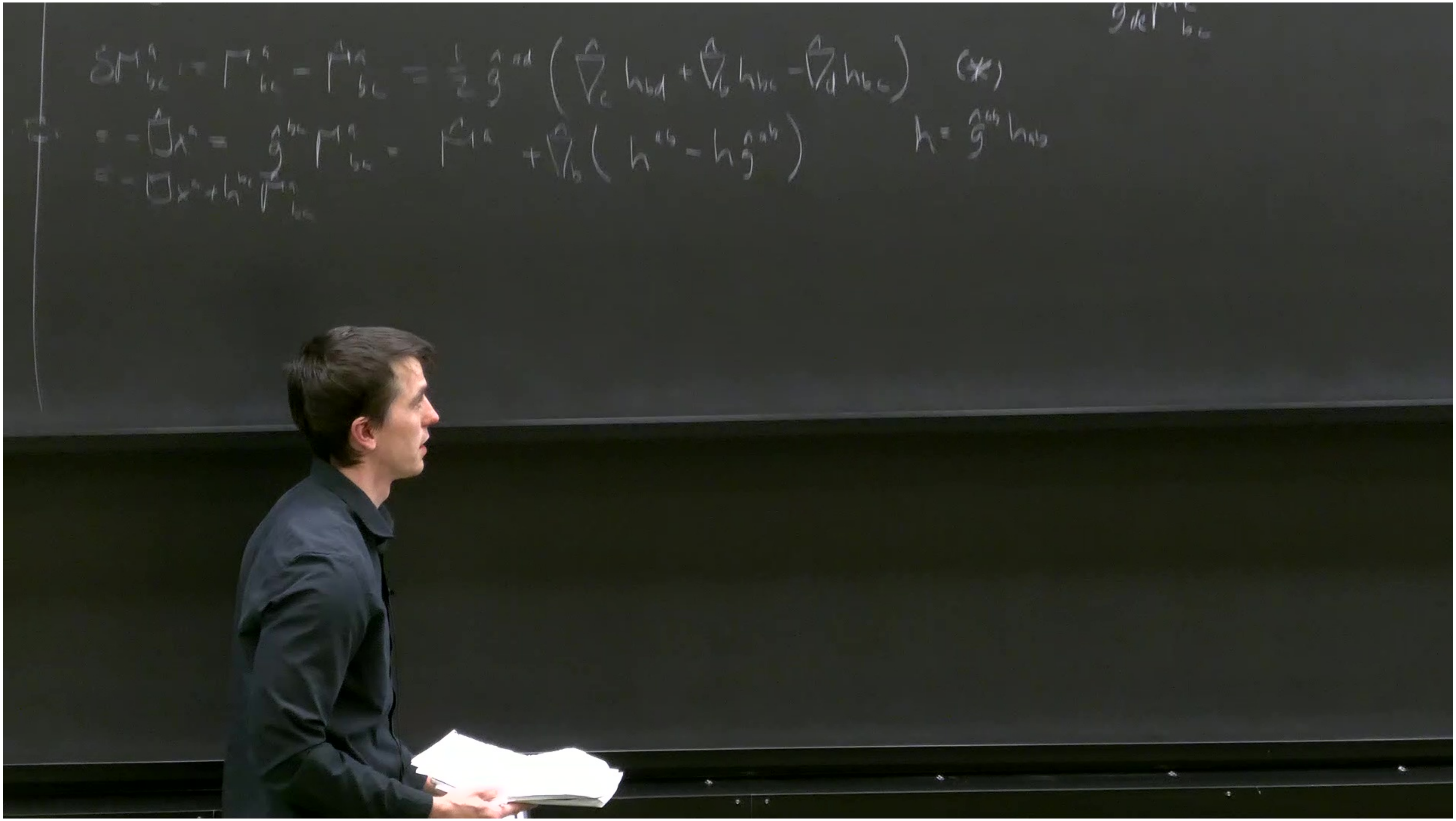



variables)
Assuming, CC, NEC
Trapped surface ⊂ B
EH
t < t_merse
AH_1
AH_2
t = t_merse
AH
MOTS_1
MOTS_2



Gravitational Waves (additional ref Flanagan + Hughes arXiv: gr-qc/0501041)
Linear perturbations of the metric
$\hat{R}_{ab}=0$
$g_{ab} = \hat{g}_{ab} + h_{ab}$
$|h_{ab}| \ll |\hat{g}_{ab}|$
$g^{ab} = \hat{g}^{ab} - h^{ab}$
(to linear order)
$\Gamma^{a}_{bc} = \hat{\Gamma}^{a}_{bc} + \frac{1}{2}\hat{g}^{ad}(\partial$
$+ \partial_c h$


Flanagan & Hughes
arXiv:gr-qc/0501041
$\Gamma^a_{bc} = \hat{\Gamma}^a_{bc} + \frac{1}{2}\hat{g}^{ad}\left(\partial_b h_{cd} + \partial_c h_{bd} - \partial_d h_{bc}\right) - \frac{1}{2}h^{ad}\underbrace{\left(\partial_b \hat{g}_{cd} + \partial_c \hat{g}_{bd} - \partial_d \hat{g}_{bc}\right)}_{\hat{g}_{de}\hat{\Gamma}^e_{bc}}$
$\delta\Gamma^a_{bc} := \Gamma^a_{bc} - \hat{\Gamma}^a_{bc} = \frac{1}{2}\hat{g}^{ad}\left(\hat{\nabla}_c h_{bd} + \hat{\nabla}_b h_{bc} - \hat{\nabla}_d h_{bc}\right)$ (*)
$\hat{g}^{bc}\Gamma^a_{bc} = \hat{\Gamma}^a + \hat{\nabla}_b\left(h^{ab} - h\hat{g}^{ab}\right)$
$h = \hat{g}^{ab}h_{ab}$


$\delta\Gamma^a_{bc} := \Gamma^a_{bc} - \hat{\Gamma}^a_{bc} = \frac{1}{2}\hat{g}^{ad}\left(\hat{\nabla}_c h_{bd} + \hat{\nabla}_b h_{dc} - \hat{\nabla}_d h_{bc}\right)$ (*)
$= -\hat{\Box}x^a = g^{bc}\Gamma^a_{bc} - \hat{\Gamma}^a + \hat{\nabla}_b\left(h^{ab} - h\hat{g}^{ab}\right)$
$h = \hat{g}^{ab}h_{ab}$
$= -\Box x^a + h^{bc}\hat{\Gamma}^a_{bc}$

$$R^a{}_{bcd} = \partial_c \Gamma^a{}_{bd} - \partial_d \Gamma^a{}_{bc} + \Gamma^a{}_{ce}\Gamma^e{}_{bd} - \Gamma^a{}_{de}\Gamma^e{}_{bc}$$

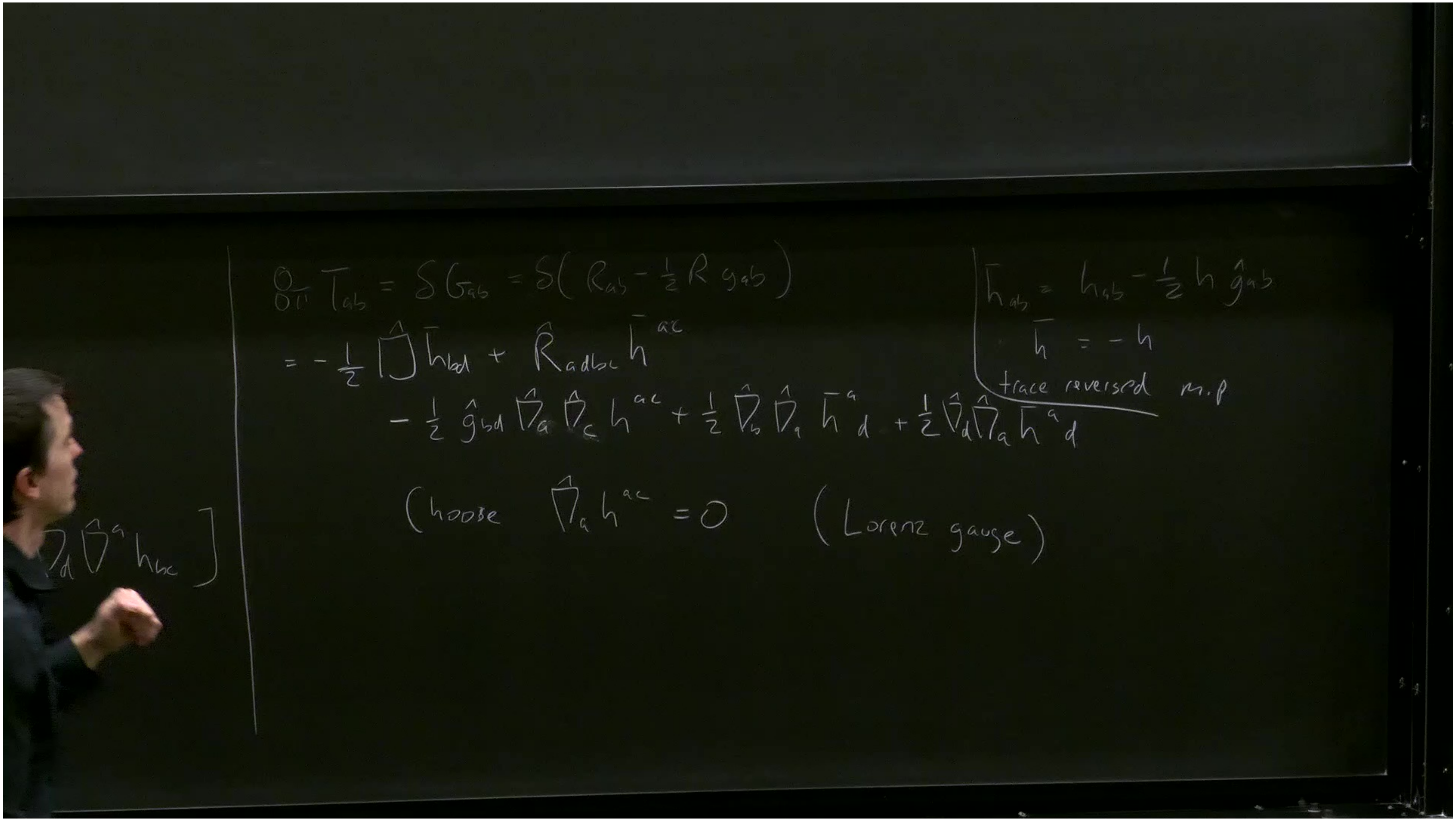
in locally flat coordinates

$$R^a{}_{bcd} = \partial_c \hat{\Gamma}^a{}_{bd} - \partial_d \hat{\Gamma}^a{}_{bc} + \partial_c \delta\Gamma^a{}_{bd} - \partial_d \delta\Gamma^a{}_{bc}$$

$$= \hat{R}^a{}_{bcd} + \hat{\nabla}_c \delta\Gamma^a{}_{bd} - \hat{\nabla}_d \delta\Gamma^a{}_{bc}$$

$$\delta R^a{}_{bcd} = \frac{1}{2}\left[\hat{\nabla}_c \hat{\nabla}_b h^a{}_d + \hat{\nabla}_c \hat{\nabla}_d h^a{}_d - \hat{\nabla}_c \hat{\nabla}^a h_{bd} - \hat{\nabla}_d \hat{\nabla}_b h^a{}_c - \hat{\nabla}_d \hat{\nabla}_c h^a{}_b + \hat{\nabla}_d \hat{\nabla}^a h_{bc} \right]$$

$$\delta R_{bd} = \hat{\nabla}_a \hat{\nabla}_{(b} h^a{}_{d)} - \frac{1}{2}\hat{\nabla}_a \hat{\nabla}^a h_{bd} - \frac{1}{2}\hat{\nabla}_d \hat{\nabla}_b h$$


$\frac{\partial}{\partial \epsilon} T_{ab} = \delta G_{ab} = \delta\left(R_{ab} - \frac{1}{2} R g_{ab} \right)$
$= -\frac{1}{2} \hat{\Box} \bar{h}_{bd} + \hat{R}_{adbc} \bar{h}^{ac}$
$- \frac{1}{2} \hat{g}_{bd} \hat{\nabla}_a \hat{\nabla}_c h^{ac} + \frac{1}{2} \hat{\nabla}_b \hat{\nabla}_a \bar{h}^a{}_d + \frac{1}{2} \hat{\nabla}_d \hat{\nabla}_a \bar{h}^a{}_d$
Choose $\hat{\nabla}_a h^{ac} = 0$ (Lorenz gauge)
$\bar{h}_{ab} = h_{ab} - \frac{1}{2} h \hat{g}_{ab}$
$\bar{h} = -h$
trace reversed m.p
$\nabla_d \hat{\nabla}^a h_{bc}]$

$g_{de}\Gamma^{e}{}_{bc}$

$\frac{1}{2}\hat{g}^{ad}\left(\hat{\nabla}_c h_{bd} + \hat{\nabla}_b h_{bc} - \hat{\nabla}_d h_{bc}\right) \quad (*)$

$\hat{\Gamma}^{a} \quad + \hat{\nabla}_b\underbrace{\left(h^{ab} - h\hat{g}^{ab}\right)}_{\bar{h}^{ab}} \qquad h = \hat{g}^{ab}h_{ab}$

$-\frac{1}{2}g_{bd}\nabla_a\nabla_c h \quad + \frac{1}{2}\nabla_b\nabla_a h^{a}{}_{d} + \frac{1}{2}\nabla_d\nabla_a h^{a}{}_{d}$

$+\hat{\nabla}_d\hat{\nabla}^a h_{bc}\,]$

Choose $\hat{\nabla}_a h^{ac} = 0$ (Lorenz gauge)

$$x^a \to x'^a = x^a + \xi^a$$

$$h_{ab} \to h'_{ab} = h_{ab} - 2\hat{\nabla}_{(a}\xi_{b)}$$

$$\hat{\nabla}_a h^{ab} = \hat{\nabla}_a h'^{ab} - \hat{\Box}\xi^b$$

Originally gauge freedom $\xi^a(t, x^i)$

with Lorenz gauge $\xi^a(t=0, x^i)$, $\partial_t \xi^a(t=0, x^i)$

Particular cases

(i) Vacuum $(T_{ab}=0)$

Particular cases

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Can choose $\bar{h} = 0 = h$

$h_{ab} = \bar{h}_{ab}$

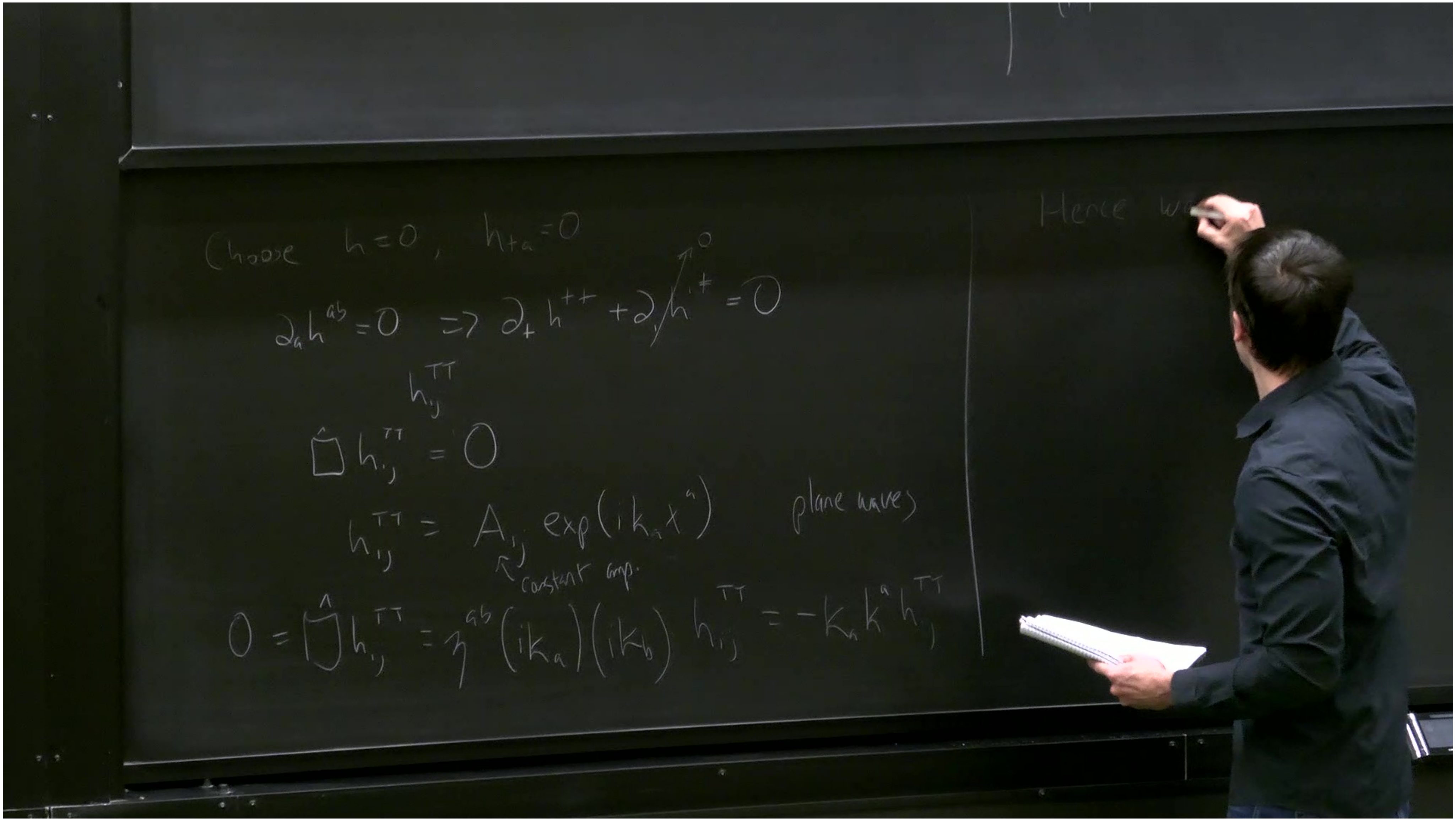
(ia) Vacuum and flat background $(\hat{g}_{ab} = \eta_{ab})$

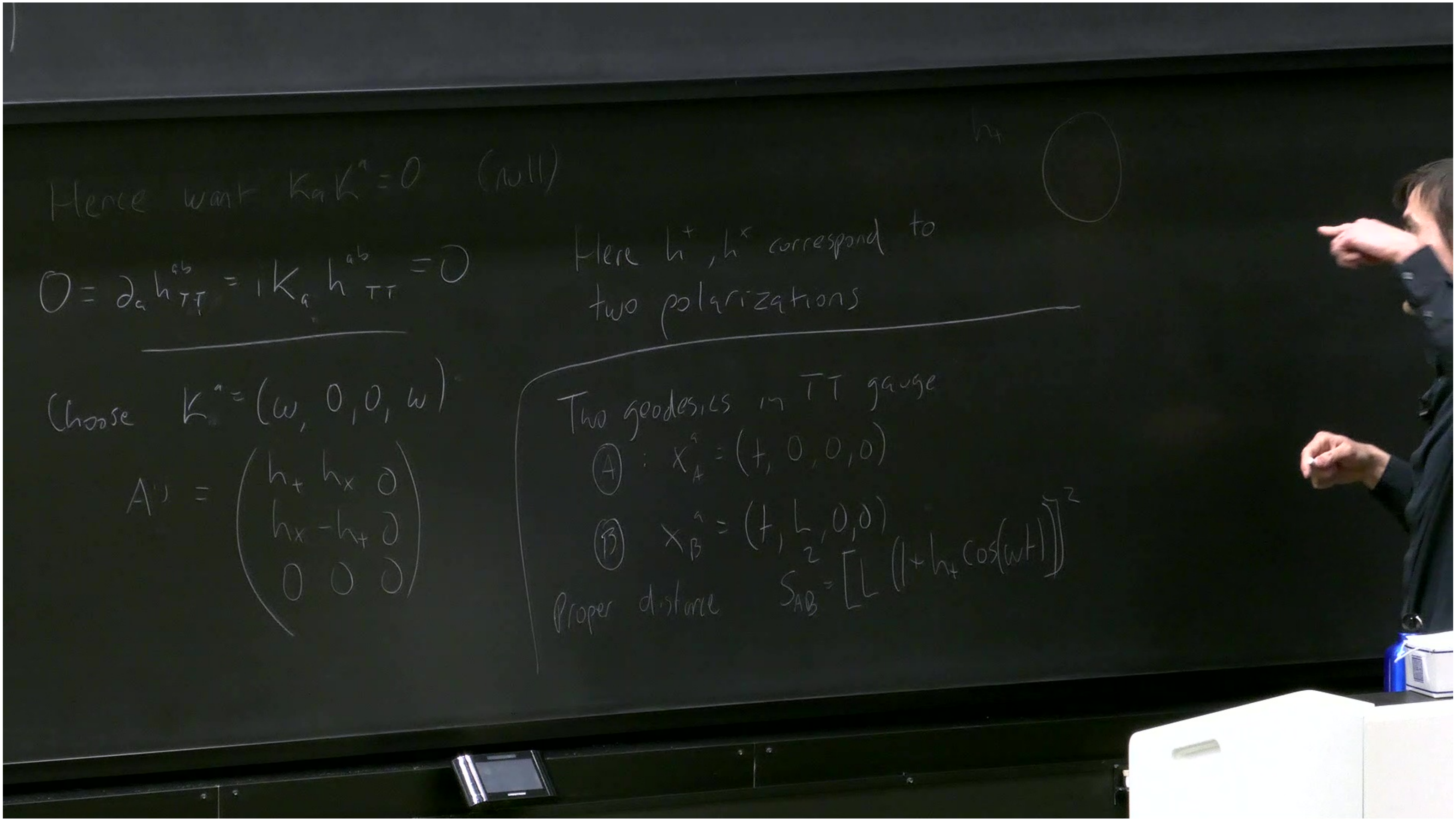
[wavezone]

$h_{ta} = 0$

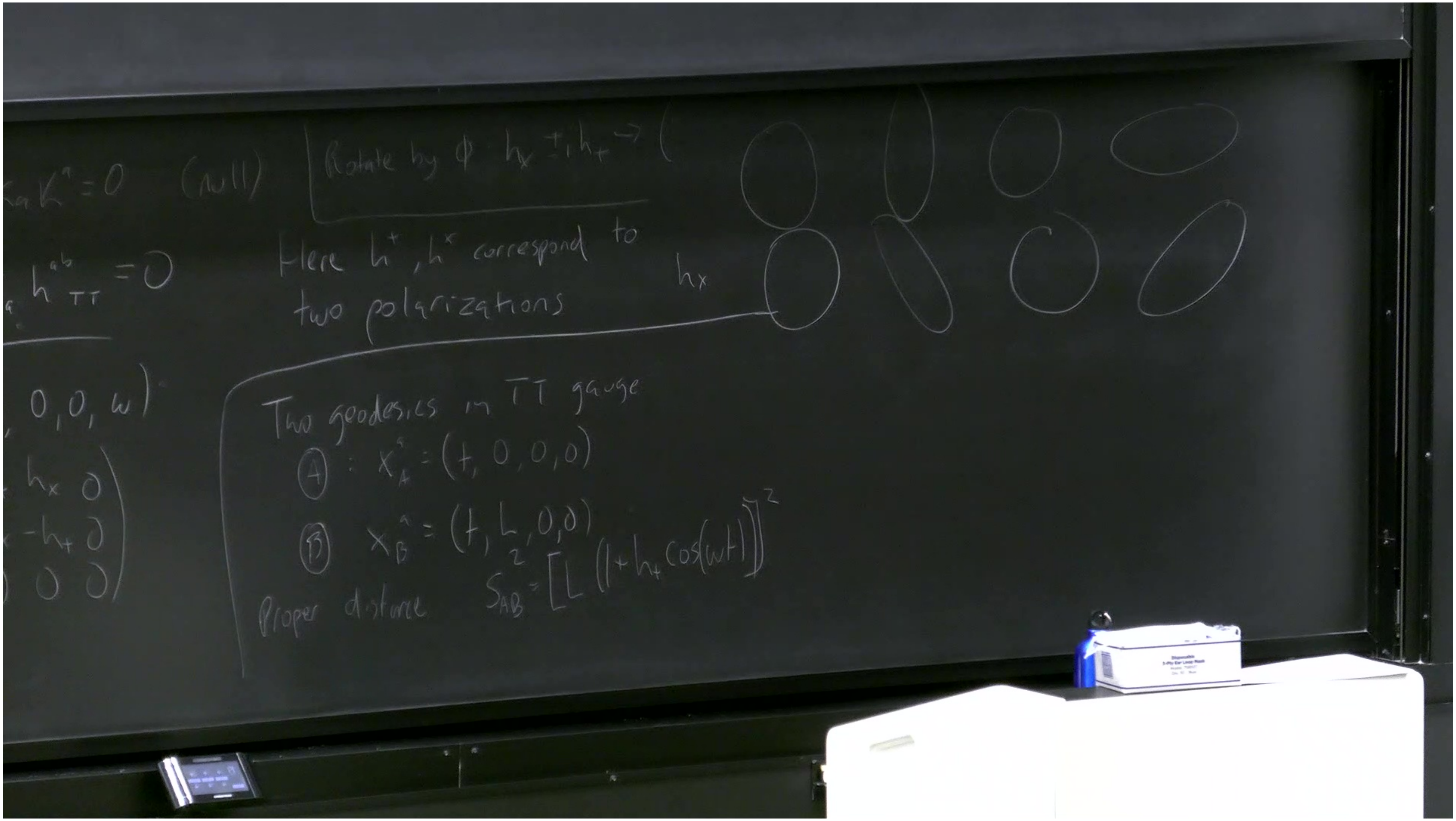
(ib) $\hat{g}_{ab} = g^{Kerr}_{ab}$ (black hole perturbation)

(ii) Non-vacuum, flat background $(T_{ab} \neq 0,\ \hat{g}_{ab} = \eta_{ab})$


Choose $h=0$, $h_{+a}=0$
$\partial_a h^{ab}=0 \Rightarrow \partial_+ h^{++} + \cancel{\partial_i h^{i+}}^{\,0} = 0$
h_{ij}^{TT}
$\hat{\Box} h_{ij}^{TT} = 0$
$h_{ij}^{TT} = A_{ij} \exp(i k_a x^a)$
plane waves
constant amp.
$0 = \hat{\Box} h_{ij}^{TT} = \eta^{ab}(ik_a)(ik_b)\, h_{ij}^{TT} = -k_a k^a h_{ij}^{TT}$
Hence w



Hence want $k_a k^a = 0$ (null)
$0 = \partial_a h^{ab}_{TT} = i k_a h^{ab}_{TT} = 0$
Choose $k^a = (\omega, 0, 0, \omega)$
$A^{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$
Here $h^+, h^\times$ correspond to two polarizations
h_+
Two geodesics in TT gauge
(A) : $x^a_A = (t, 0, 0, 0)$
(B) $x^a_B = (t, \frac{L}{2}, 0, 0)$
Proper distance $S_{AB} = [L(1 + h_+ \cos(\omega t))]^2$



Rotate by ϕ $h_\times \pm i h_+ \rightarrow$
(null)
Here h^+, $h^\times$ correspond to two polarizations
$h_\times$
Two geodesics in TT gauge
A : $x^a_A = (t, 0, 0, 0)$
B $x^a_B = (t, \frac{L}{2}, 0, 0)$
Proper distance $S_{AB} = [L(1+h_+\cos(\omega t))]^2$

$$+ \hat{\nabla}_d \hat{\nabla}^a h_{bc} \Big]$$

$$8\pi T_{ab} = \delta G_{ab} = \delta\left(R_{ab} - \tfrac{1}{2} R\, g_{ab}\right)$$

$$h_\times \pm i h_+ \to (h_\times \pm i h_+)\, e^{\mp 2i\phi}$$

$$= -\tfrac{1}{2} \hat{\Box} \bar{h}_{bd} + \hat{R}_{adbc} \bar{h}^{ac}$$

$$- \tfrac{1}{2} \hat{g}_{bd} \hat{\nabla}_a \hat{\nabla}_c h^{ac} + \tfrac{1}{2} \hat{\nabla}_b \hat{\nabla}_a$$

Choose $\hat{\nabla}_a h^{ac} = 0$

$$\hat{\Box} \bar{h}_{bd} - 2 \hat{R}_{adbc} \bar{h}^{ac} = -16\pi$$

$$\hat{\Box} \bar{h} = -16\pi T$$