Title: Strong Gravity Lecture

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Collection: Strong Gravity 2023/24

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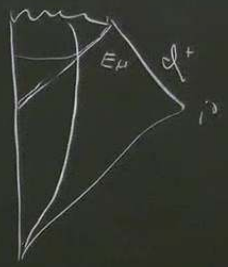
URL: https://pirsa.org/24040016

Apparent horizon : Outer most MOTS

MOTS: $\Theta_{\ell^+} = q^{ab} \nabla_a \ell^+_b = 0$

$= D_i s^i + K_{ij} s^i s^j - K$ (in 3+1 variables)

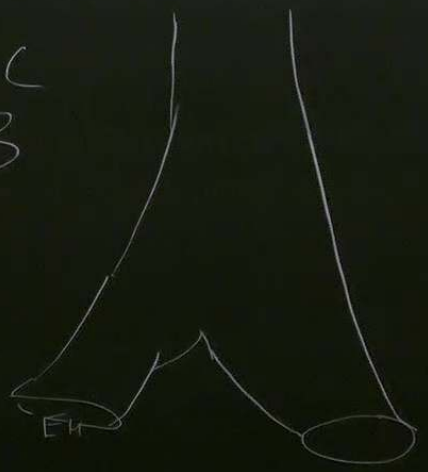
$\Theta_{\ell^-} < 0$



Assuming: CC, NEC

Trapped surface $\subset \mathcal{B}$

variables)

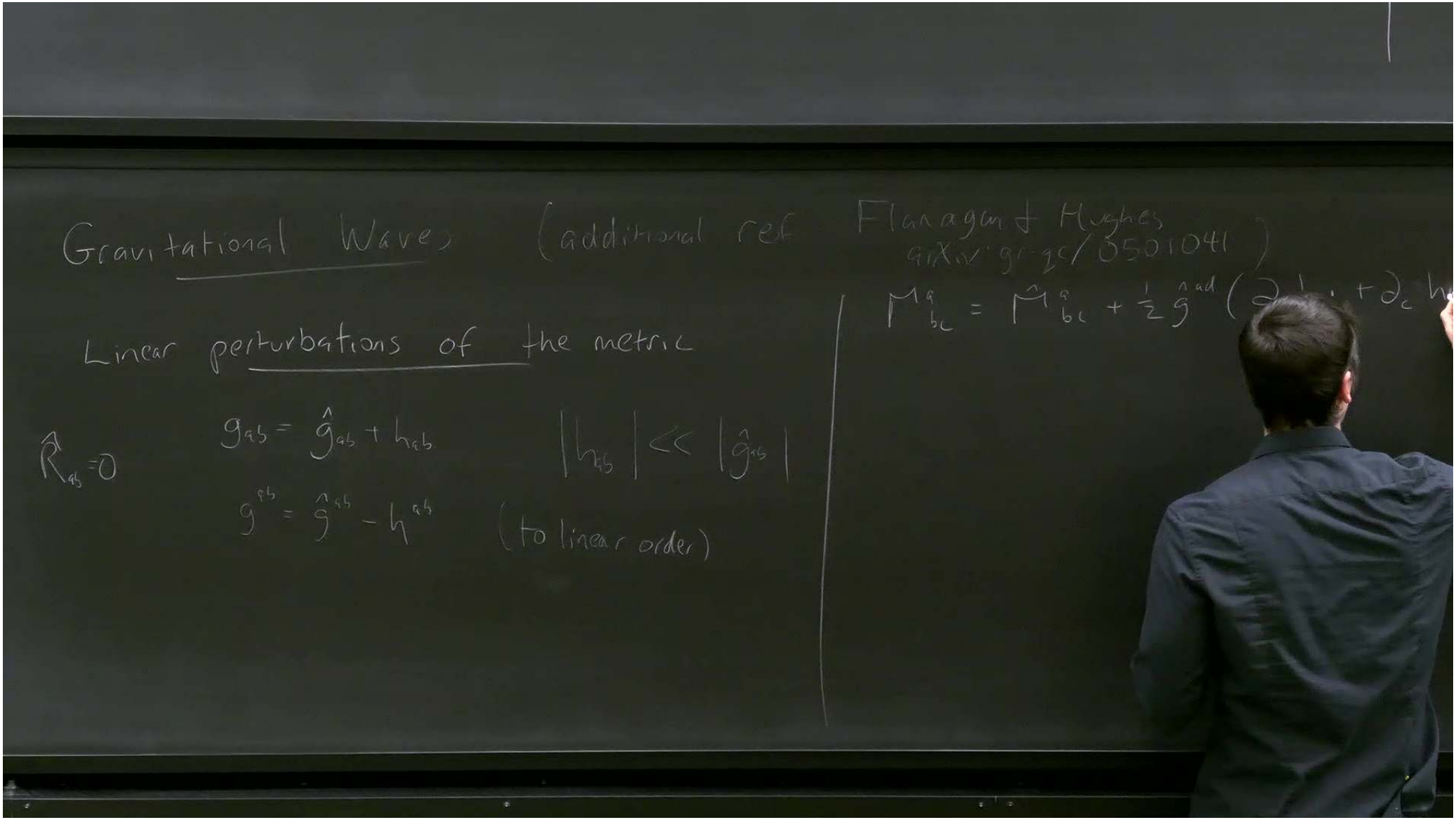


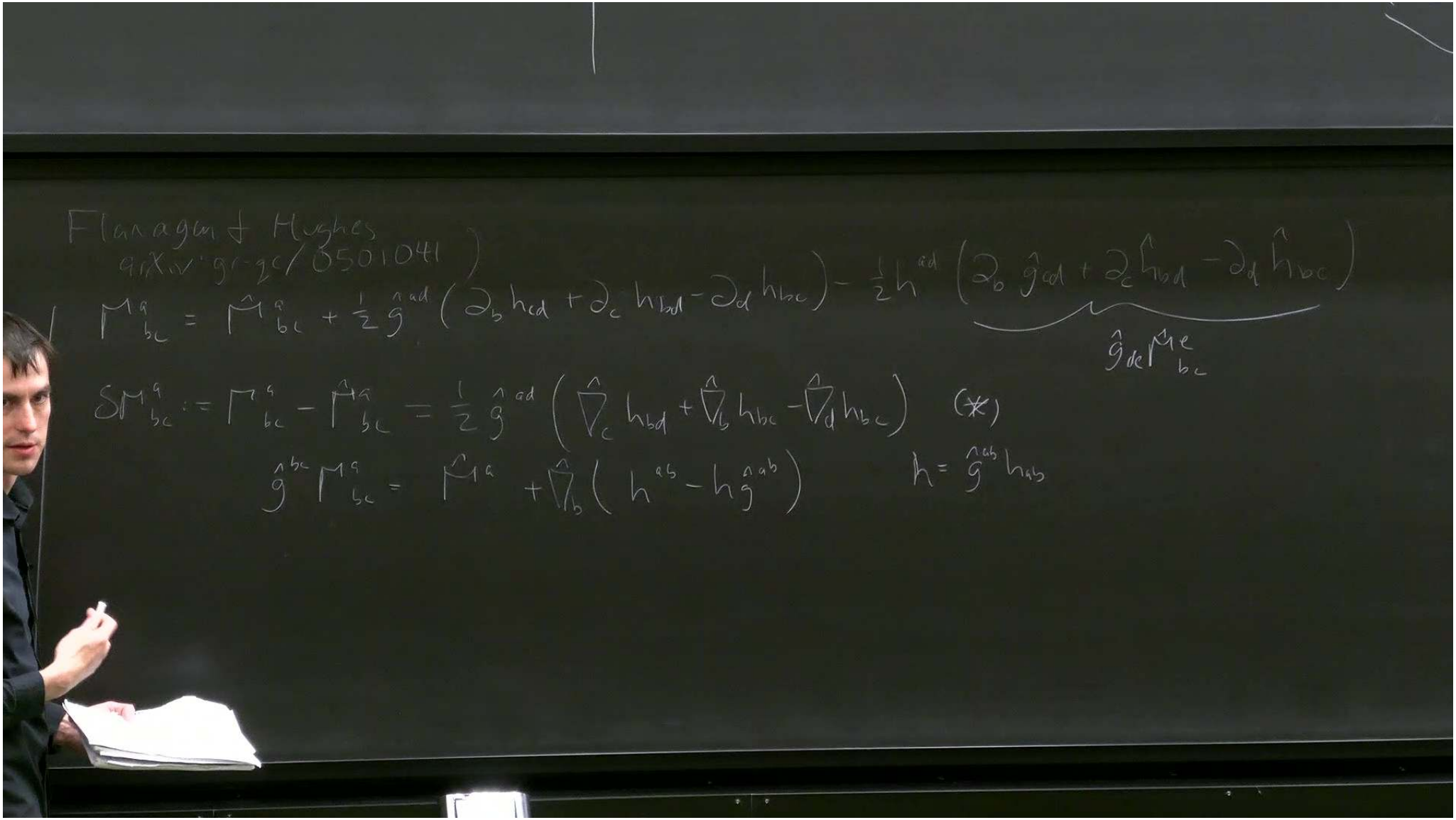
$t < t_{merge}$

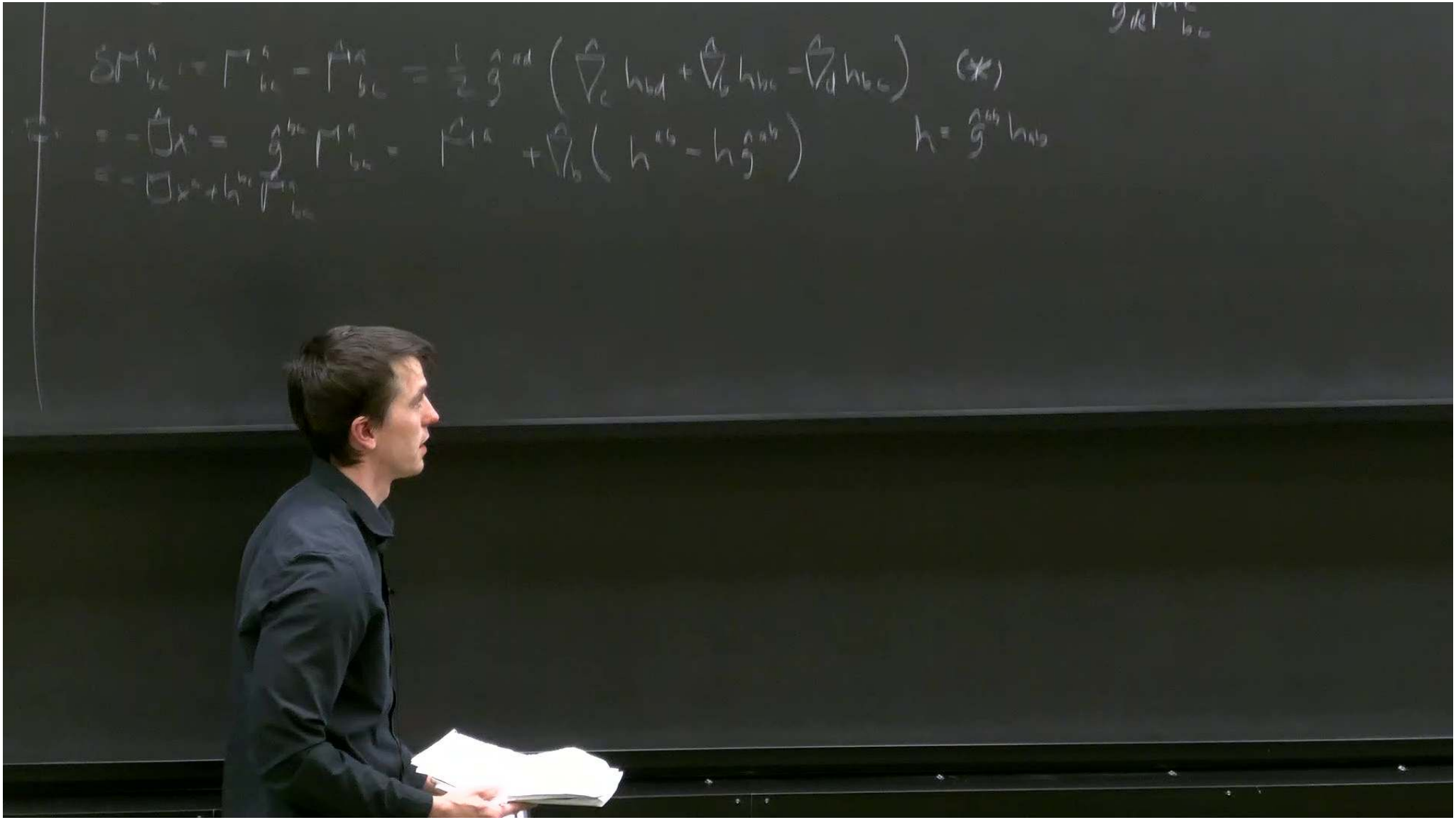
AH_1 AH_2

$t = t_{merge}$ AH

$MOTS_1$ $MOTS_2$


Gravitational Waves (additional ref Flanagan & Hughes gr-qc/0501041)
Linear perturbations of the metric
$\hat{R}_{ab} = 0$
$g_{ab} = \hat{g}_{ab} + h_{ab}$
$|h_{ab}| \ll |\hat{g}_{ab}|$
$g^{ab} = \hat{g}^{ab} - h^{ab}$ (to linear order)
$\Gamma^a_{bc} = \hat{\Gamma}^a_{bc} + \frac{1}{2}\hat{g}^{ad}(\partial_b h_{dc} + \partial_c h$


Flanagan & Hughes
arXiv:gr-qc/0501041
$\Gamma^a_{bc} = \hat{\Gamma}^a_{bc} + \frac{1}{2}\hat{g}^{ad}(\partial_b h_{cd} + \partial_c h_{bd} - \partial_d h_{bc}) - \frac{1}{2}h^{ad}(\partial_b \hat{g}_{cd} + \partial_c \hat{h}_{bd} - \partial_d \hat{h}_{bc})$
$\hat{g}_{de}\hat{\Gamma}^e_{bc}$
$\delta\Gamma^a_{bc} := \Gamma^a_{bc} - \hat{\Gamma}^a_{bc} = \frac{1}{2}\hat{g}^{ad}(\hat{\nabla}_c h_{bd} + \hat{\nabla}_b h_{bc} - \hat{\nabla}_d h_{bc})$ (*)
$g^{bc}\Gamma^a_{bc} = \hat{\Gamma}^a + \hat{\nabla}_b(h^{ab} - h\hat{g}^{ab})$
$h = \hat{g}^{ab}h_{ab}$



$$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^a_{ce}\Gamma^e_{bd} - \Gamma^a_{de}\Gamma^e_{bc}$$

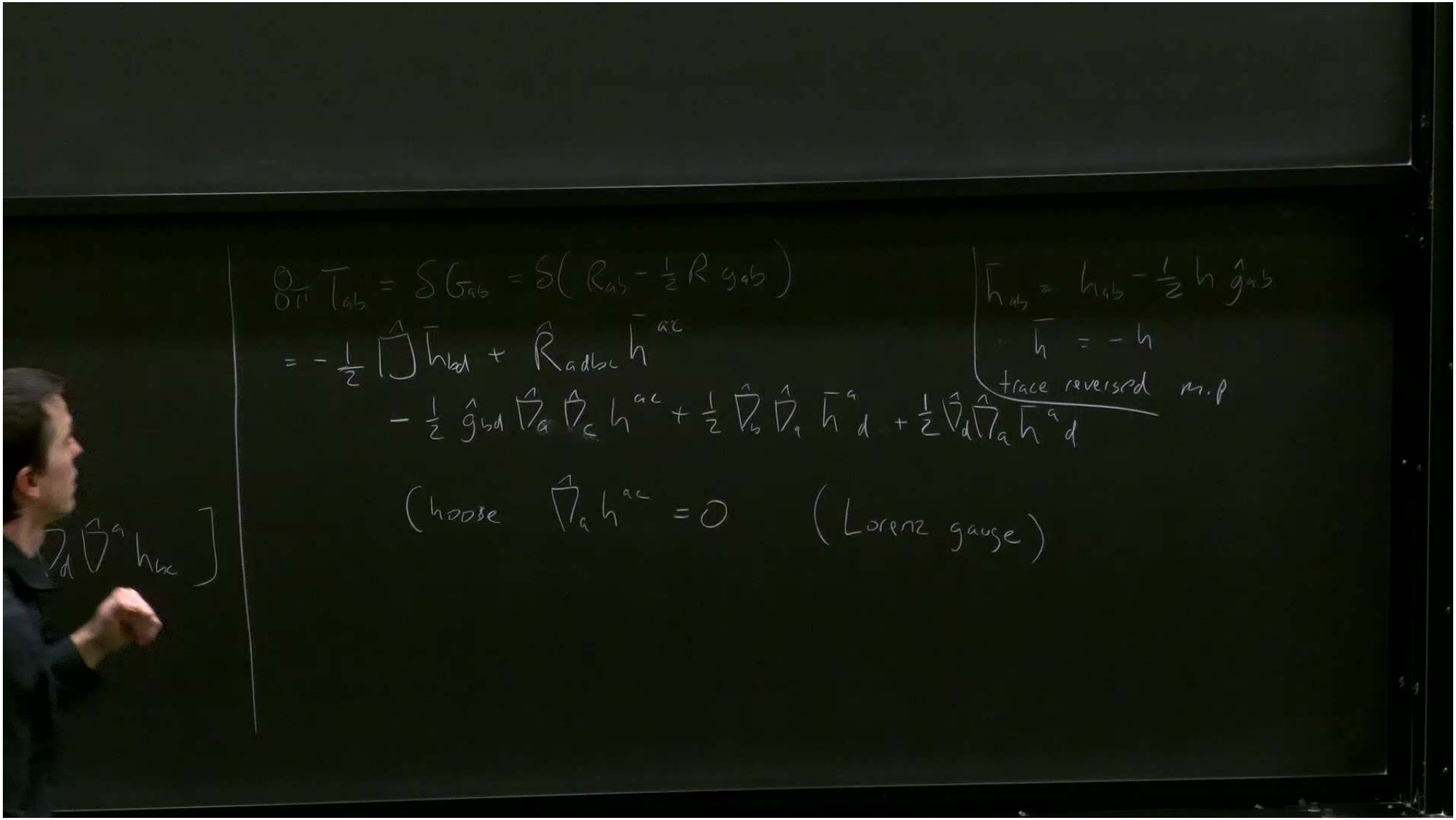
in locally flat coordinates

$$R^a{}_{bcd} = \partial_c \hat{\Gamma}^a_{bd} - \partial_d \hat{\Gamma}^a_{bc} + \partial_c \delta\Gamma^a_{bd} - \partial_d \delta\Gamma^a_{bc}$$

$$= \hat{R}^a{}_{bcd} + \hat{\nabla}_c \delta\Gamma^a_{bd} - \hat{\nabla}_d \delta\Gamma^a_{bc}$$

$$\delta R^a{}_{bcd} = \frac{1}{2}\left[\hat{\nabla}_c\hat{\nabla}_b h^a{}_d + \hat{\nabla}_c\hat{\nabla}_d h^a{}_d - \hat{\nabla}_c\hat{\nabla}^a h_{bd} - \hat{\nabla}_d\hat{\nabla}_b h^a{}_c - \hat{\nabla}_d\hat{\nabla}_c h^a{}_b + \hat{\nabla}_d\hat{\nabla}^a h_{bc}\right]$$

$$\delta R_{bd} = \hat{\nabla}_a\hat{\nabla}_b h^a{}_d) - \frac{1}{2}\hat{\nabla}_a\hat{\nabla}^a h_{bd} - \frac{1}{2}\hat{\nabla}_d\hat{\nabla}_b h$$


$\frac{\delta}{\delta T} T_{ab} = \delta G_{ab} = \delta\left(R_{ab} - \frac{1}{2} R\, g_{ab}\right)$
$= -\frac{1}{2} \hat{\Box} \bar{h}_{bd} + \hat{R}_{adbc} \bar{h}^{ac}$
$- \frac{1}{2} \hat{g}_{bd} \hat{\nabla}_a \hat{\nabla}_c h^{ac} + \frac{1}{2} \hat{\nabla}_b \hat{\nabla}_a \bar{h}^{a}{}_{d} + \frac{1}{2} \hat{\nabla}_d \hat{\nabla}_a \bar{h}^{a}{}_{d}$
$\bar{h}_{ab} = h_{ab} - \frac{1}{2} h\, \hat{g}_{ab}$
$\bar{h} = -h$
trace reversed m.p
(choose $\hat{\nabla}_a h^{ac} = 0$ (Lorenz gauge)
$\hat{\nabla}_d \hat{\nabla}^a h_{bc}$]

$$\frac{1}{2}\hat{g}^{ad}\left(\hat{\nabla}_c h_{bd} + \hat{\nabla}_b h_{bc} - \hat{\nabla}_d h_{bc}\right) \quad (*)$$

$$\hat{\Gamma}^a_{bc} + \hat{\nabla}_b\left(\underbrace{h^{ab} - h\hat{g}^{ab}}_{\bar{h}^{ab}}\right) \qquad h = \hat{g}^{ab} h_{ab}$$

$$+\hat{\nabla}_d \hat{\nabla}^a h_{bc}\Big]$$

$$-\frac{1}{2} g_{bd} \nabla_a \nabla_c h + \frac{1}{2}\nabla_b \nabla_a h_d + \frac{1}{2}\nabla_d \nabla_a h^a{}_d$$

(choose $\hat{\nabla}_a h^{ac} = 0$ (Lorenz gauge)

$$x^a \to x'^a = x^a + \xi^a$$

$$h_{ab} \to h'_{ab} = h_{ab} - 2\hat{\nabla}_{(a}\xi_{b)}$$

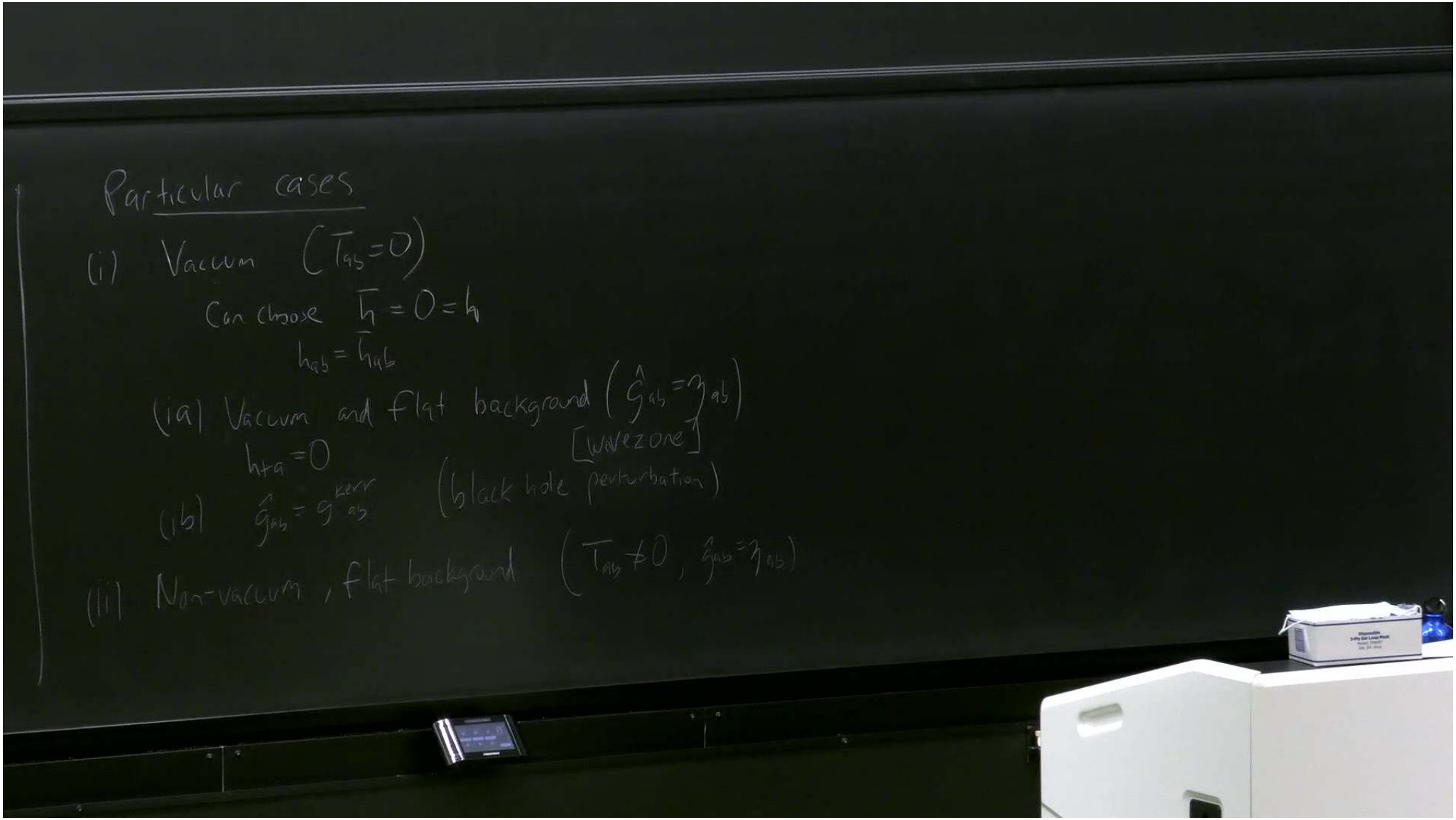
$$\hat{\nabla}_a h^{ab} = (\hat{\nabla}_a h'^{ab} - \hat{\Box}\xi^b$$

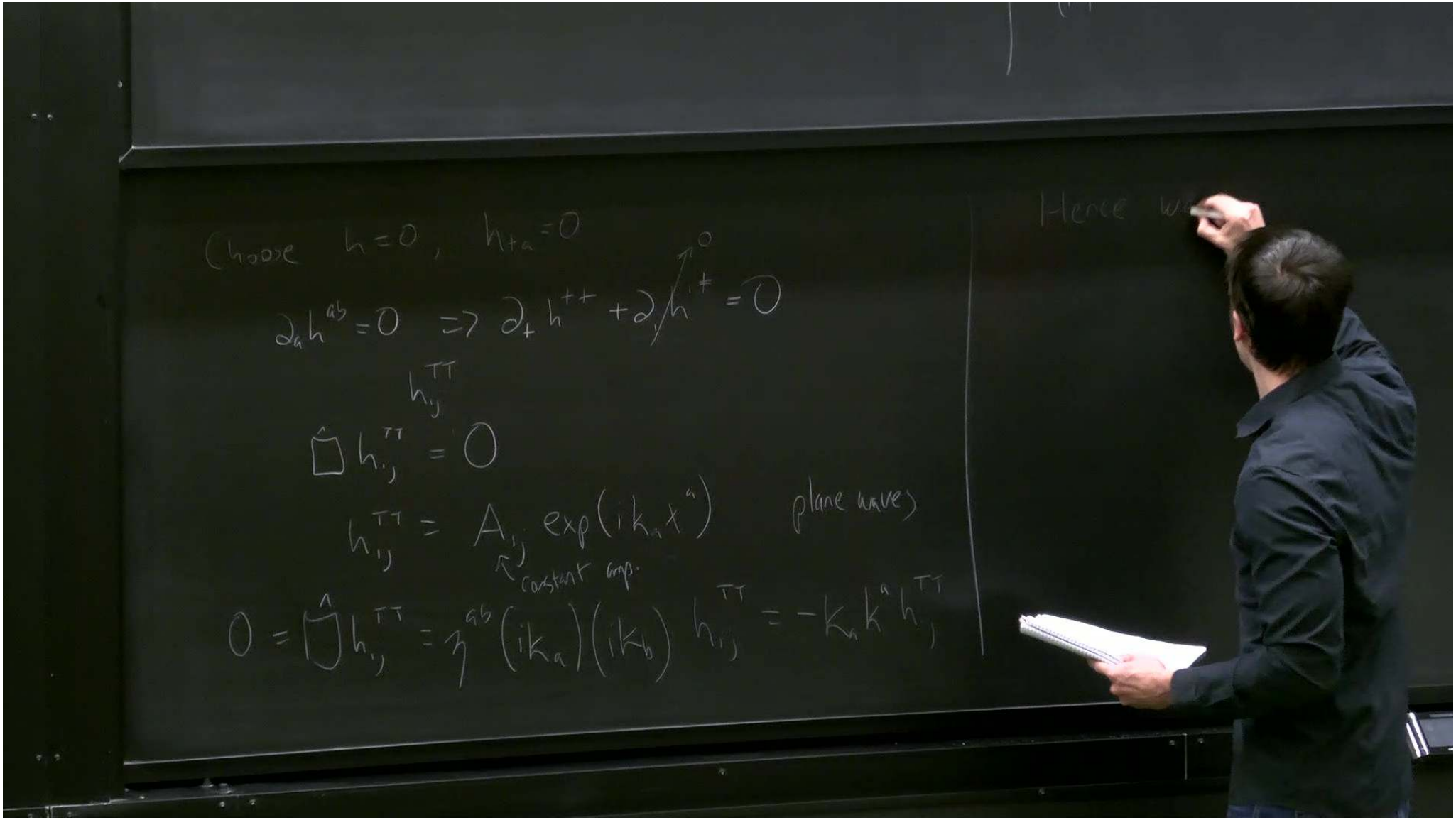
Originally gauge freedom $\xi^a(t,x^i)$

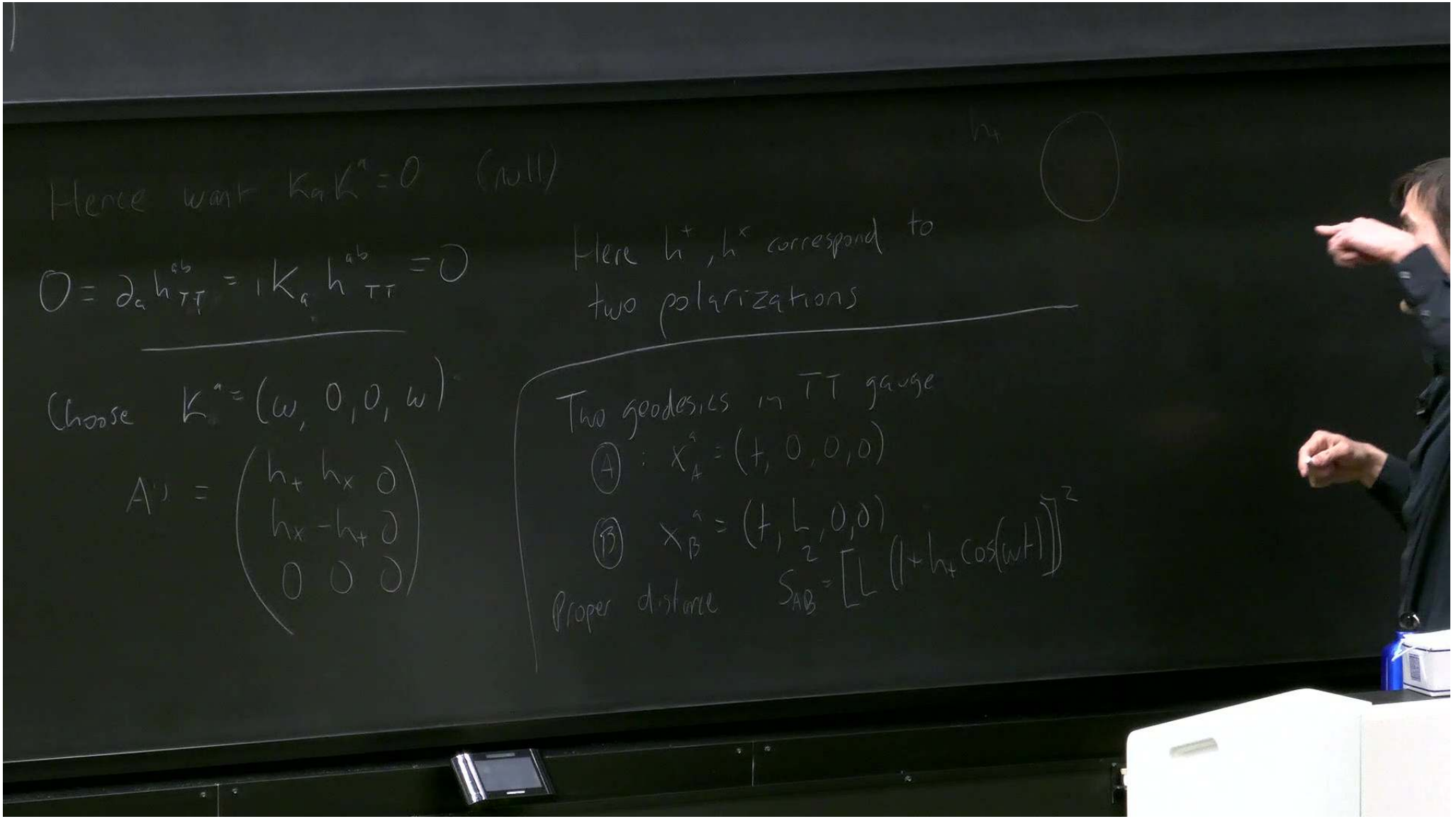
With Lorenz gauge $\xi^a(t=0,x^i)$, $\partial_t \xi^a(t=0,x)$

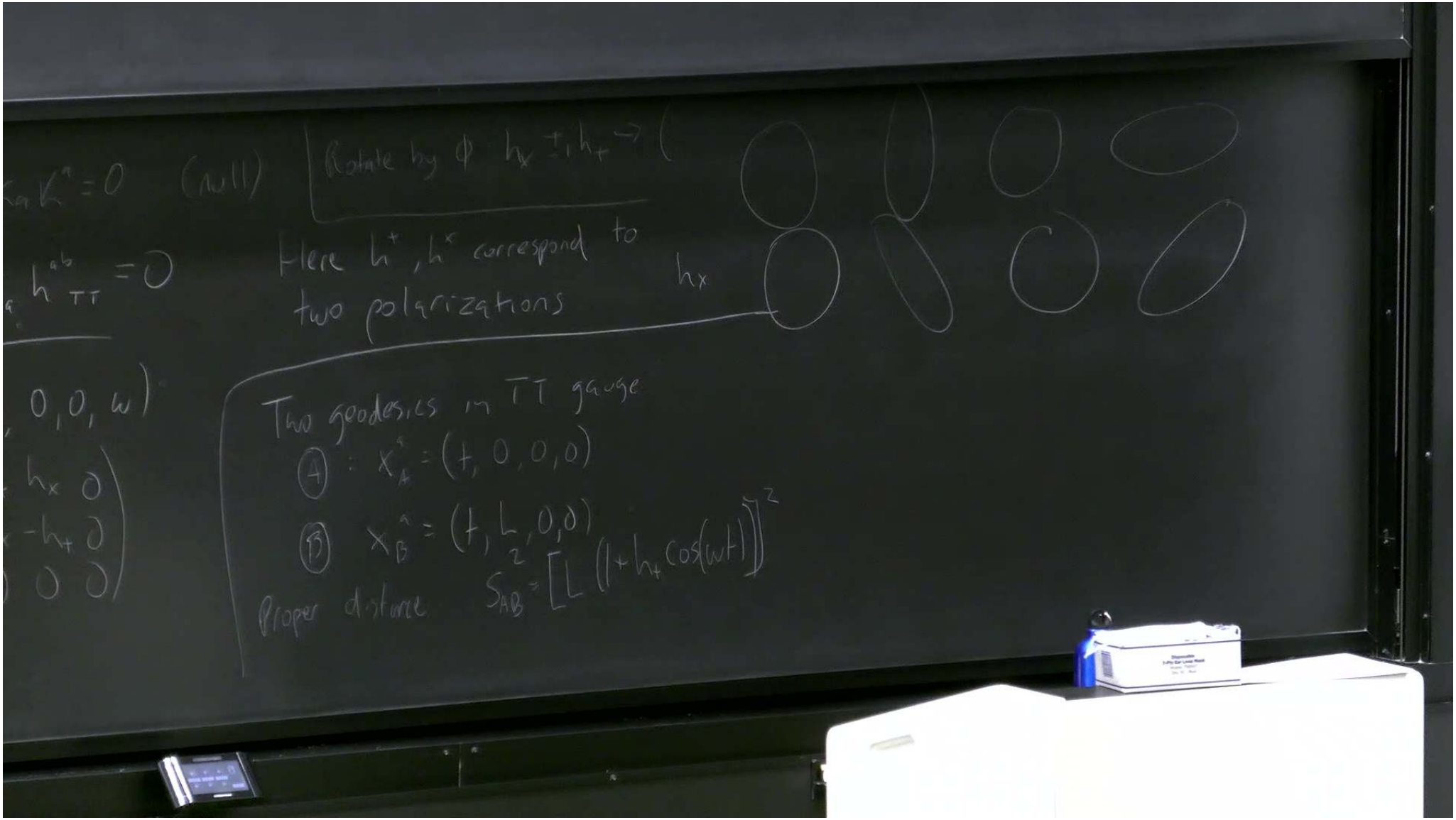
Particular cases

(i) Vacuum ($T_{ab}=0$)


Particular cases
(i) Vacuum ($T_{ab}=0$)
Can choose $\bar{h}=0=h$
$h_{ab}=\bar{h}_{ab}$
(ia) Vacuum and flat background ($\hat{g}_{ab}=\eta_{ab}$)
$h_{ta}=0$
[wavezone]
(ib) $\hat{g}_{ab}=g^{Kerr}_{ab}$ (black hole perturbation)
(ii) Non-vacuum, flat background ($T_{ab}\neq 0$, $\hat{g}_{ab}=\eta_{ab}$)


Choose $h=0$, $h_{ta}=0$
$\partial_a h^{ab}=0 \Rightarrow \partial_t h^{tt} + \partial_i h^{it} = 0$
h_{ij}^{TT}
$\Box h_{ij}^{TT} = 0$
$h_{ij}^{TT} = A_{ij} \exp(i k_a x^a)$ plane waves
constant amp.
$0 = \Box h_{ij}^{TT} = \eta^{ab}(ik_a)(ik_b) h_{ij}^{TT} = -k_a k^a h_{ij}^{TT}$
Hence


Hence want $k_a k^a = 0$ (null)
$0 = \partial_a h^{ab}_{TT} = i k_a h^{ab}_{TT} = 0$
Here h^+, $h^\times$ correspond to two polarizations
Choose $k^a = (\omega, 0, 0, \omega)$
$A^{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$
Two geodesics in TT gauge
(A) : $x^a_A = (t, 0, 0, 0)$
(B) $x^a_B = (t, \frac{L}{2}, 0, 0)$
Proper distance $S_{AB} = \left[L\left(1 + h_+ \cos(\omega t)\right)\right]^2$


$K^a = 0$ (null)
$h^{ab}_{TT} = 0$
Rotate by ϕ: $h_x \pm i h_+ \rightarrow$
Here h^+, $h^\times$ correspond to two polarizations
h_x
Two geodesics in TT gauge
A: $x^a_A = (t, 0, 0, 0)$
B: $x^a_B = (t, L, 0, 0)$
Proper distance $S_{AB} = \left[L\left(1 + h_+ \cos(\omega t)\right)\right]^2$

$$\frac{8}{8\pi} \bar{T}_{ab} = \delta G_{ab} = \delta\left(R_{ab} - \frac{1}{2} R\, g_{ab}\right)$$

$$= -\frac{1}{2} \hat{\Box} \bar{h}_{bd} + \hat{R}_{adbc}\, \bar{h}^{ac}$$

$$- \frac{1}{2} \hat{g}_{bd} \hat{\nabla}_a \hat{\nabla}_c h^{ac} + \frac{1}{2} \hat{\nabla}_b \hat{\nabla}_a$$

$$+ \hat{\nabla}_d \hat{\nabla}^a h_{bc} \Big]$$

(choose $\hat{\nabla}_a h^{ac} = 0$

$$\hat{\Box} \bar{h}_{bd} - 2 \hat{R}_{adbc}\, \bar{h}^{ac} = -16\pi$$

$$\hat{\Box} \bar{h} = -16\pi T$$

$$h_x \pm i h_+ \to \left(h_x \pm i h_+\right) e^{\mp 2i\phi}$$