Title: Strong Gravity Lecture

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Collection: Strong Gravity 2023/24

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General Penrose Processes

Matter with T_{ab} (ignore backreaction)

$$J_i = -T_{ab}\hat{t}^a\hat{t}^b$$

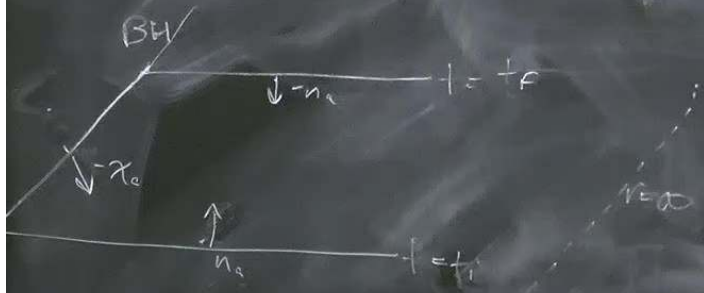
$$\nabla_a J^a = -(\nabla_a T^a_b)\hat{t}^b - T^a_b \nabla_a \hat{t}^b = 0$$

$(\nabla_a T^a_b = 0)$ $\quad$ $(\nabla_a \hat{t}_b = 0)$

$$\int_M \nabla_a J^a \sqrt{-g}\, d^n x = \int_{\partial M} n_a J^a \sqrt{\gamma}\, d^{n-1} x$$

n_a: unit normal

$$\chi^a = t^a + \Omega_H \phi^a$$

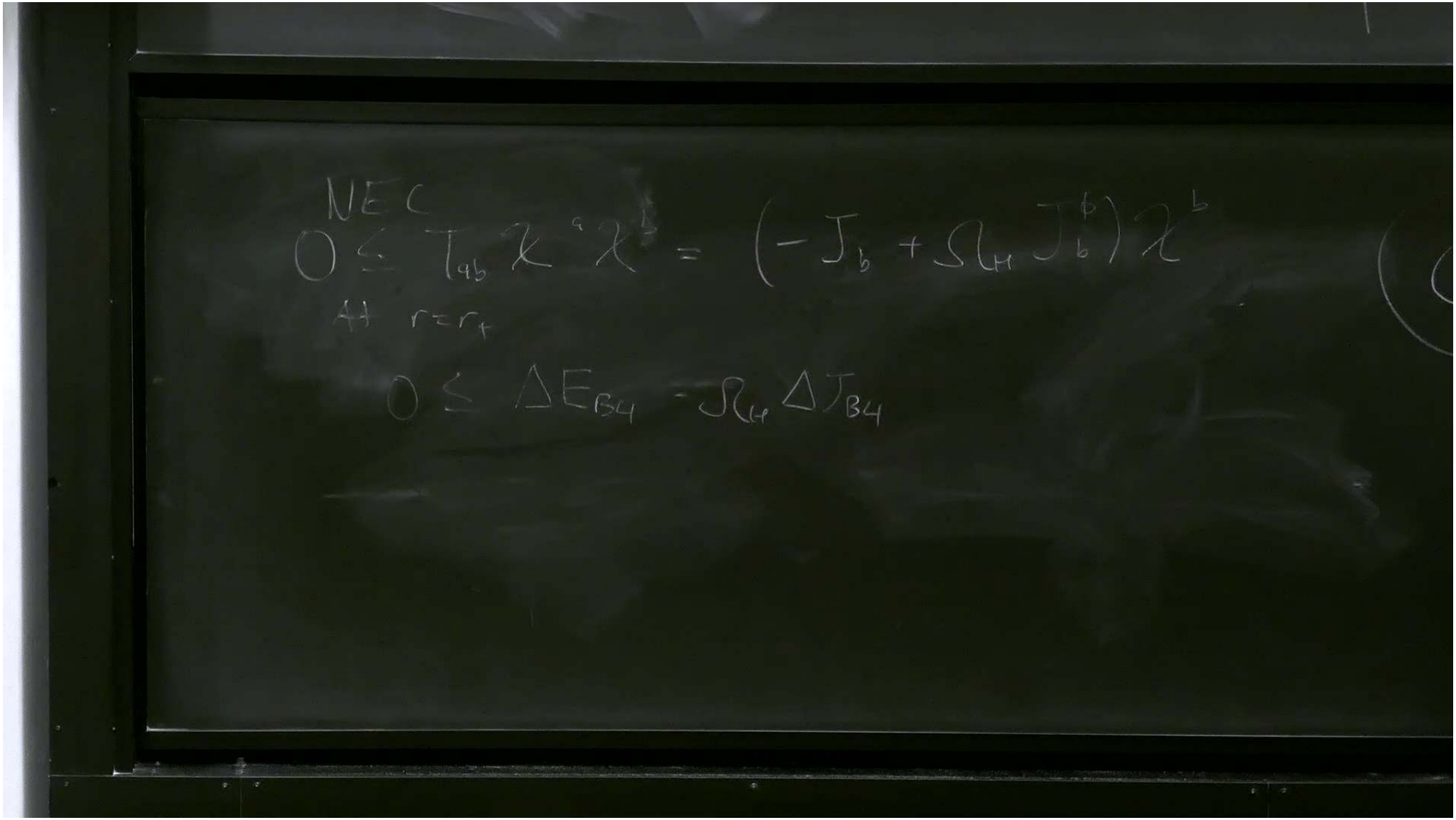


$$\int_{t=t_f} n_a J^a \sqrt{\gamma}\, d^3x - \int_{t=t_i} n_a J^a \sqrt{\gamma}\, d^3x = -\int_{\substack{t_i \\ r=r_+}}^{t_f} \chi_a J^a \sqrt{\gamma_{BH}}\, d\Omega$$

$$E(t_f) - E(t_i) = -\Delta E_{BH}$$

Also conserved $J^a_a = T_{ab}\, \phi^b$

$$J^\phi(t_f) - J^\phi(t_i) = -\Delta J_{BH}$$

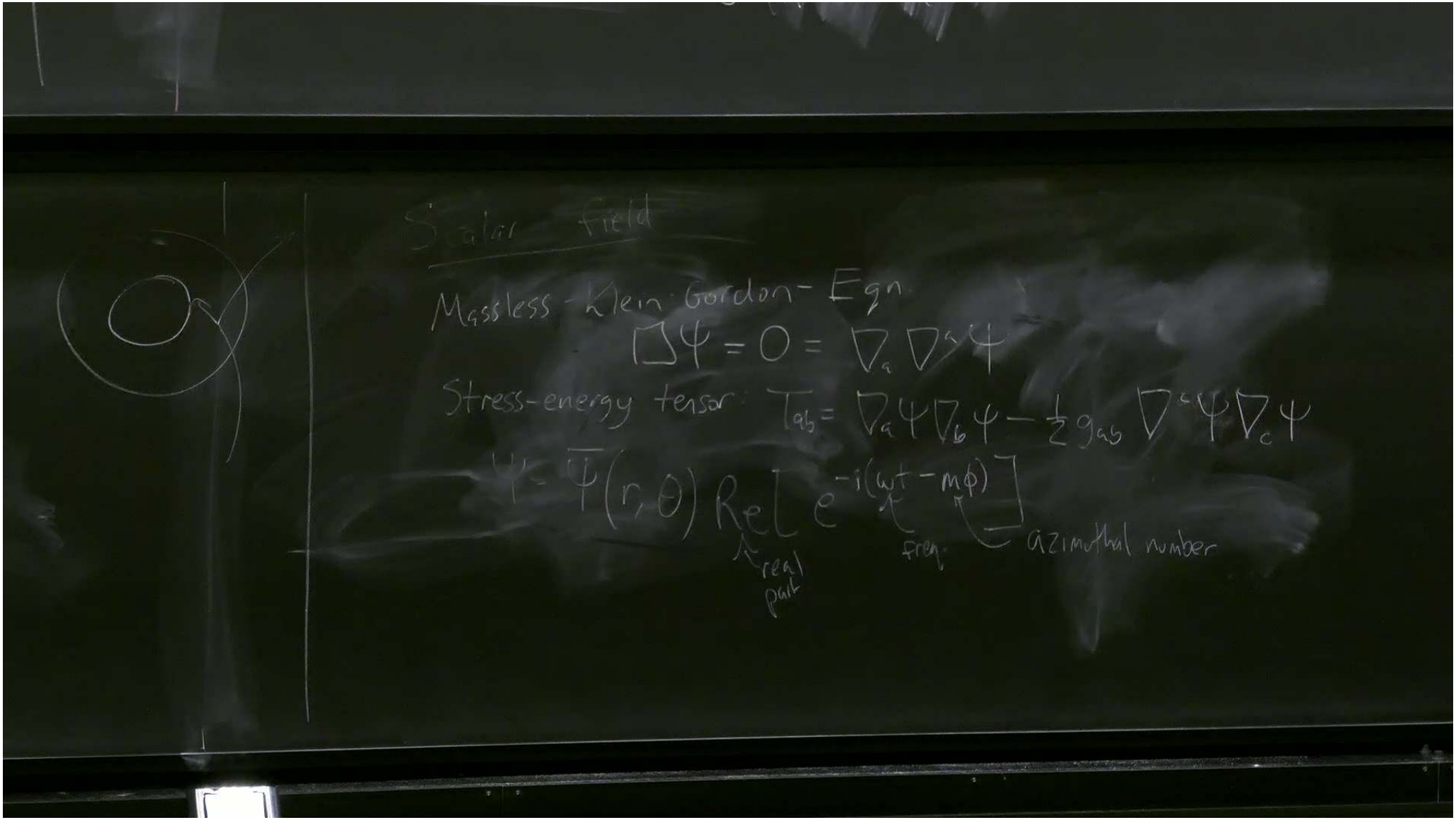


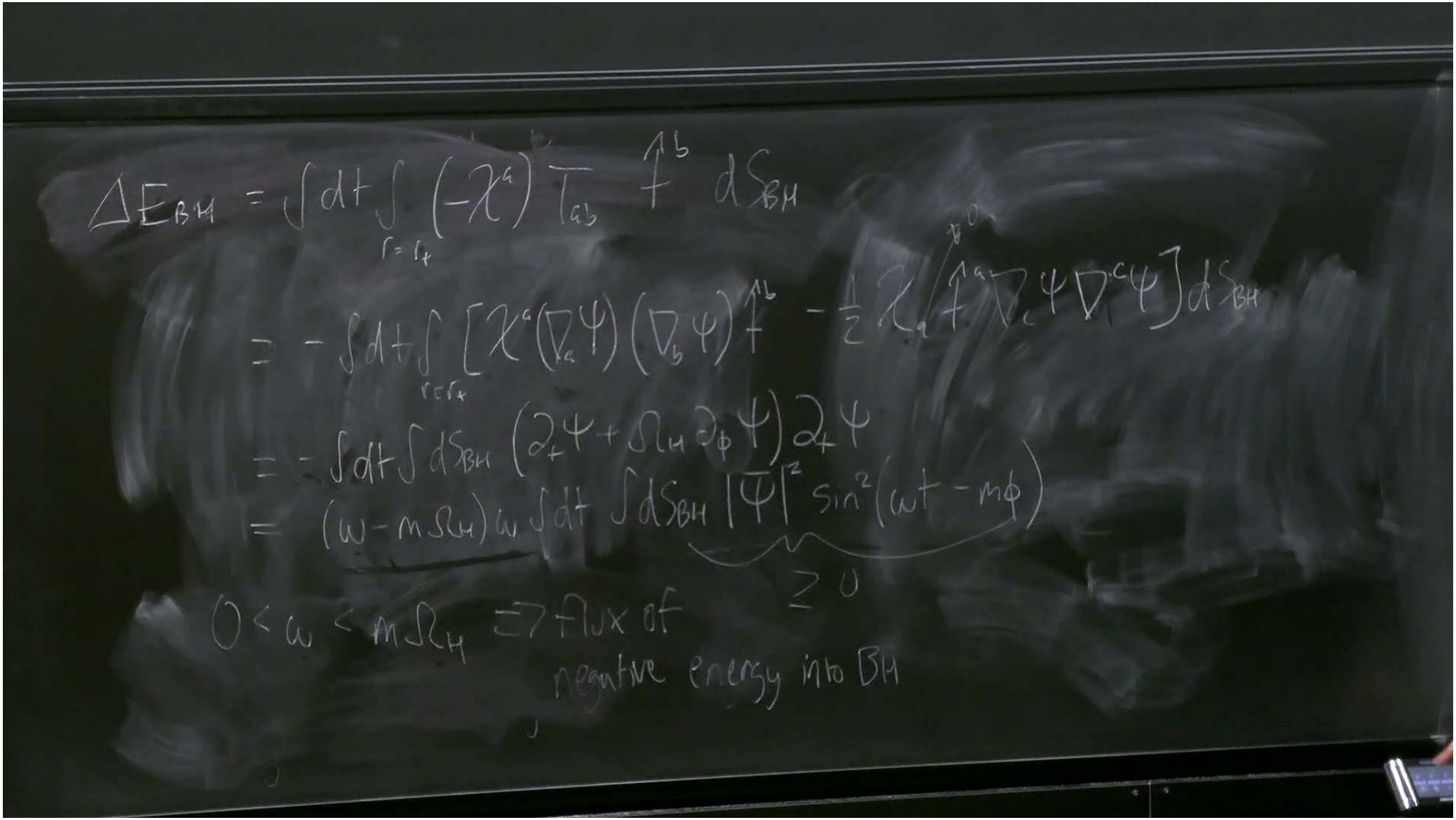
NEC

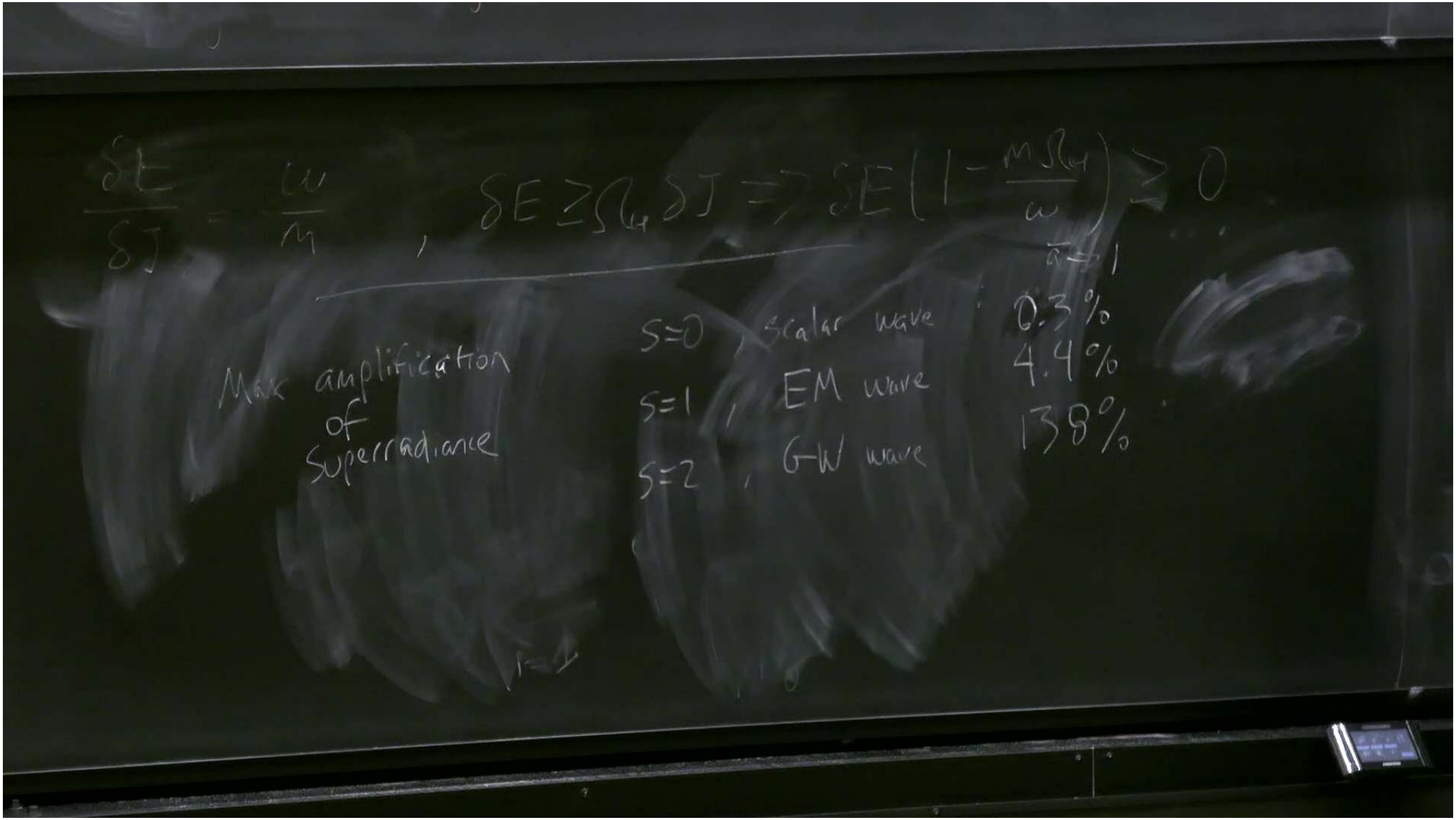
$$0 \leq T_{ab} \chi^a \chi^b = \left(-J_b + \Omega_H J_b^\phi\right) \chi^b$$

At $r = r_+$

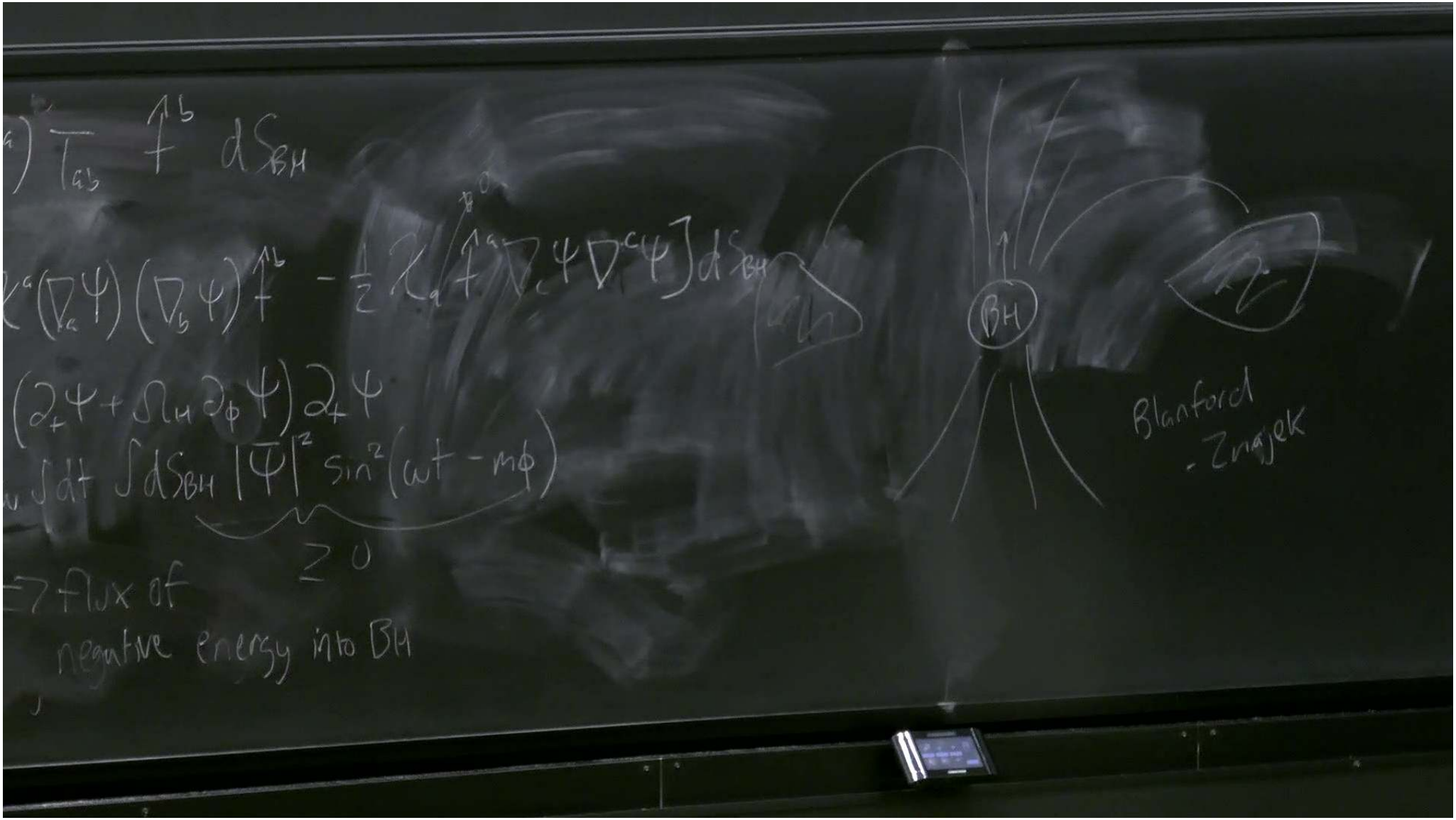
$$0 \leq \Delta E_{BH} - \Omega_H \Delta J_{BH}$$


Scalar field
Massless - Klein Gordon - Eqn
$\Box\psi = 0 = \nabla_a \nabla^a \psi$
Stress-energy tensor $T_{ab} = \nabla_a\psi\nabla_b\psi - \frac{1}{2} g_{ab} \nabla^c\psi\nabla_c\psi$
$\psi = \Psi(r,\theta)\, \mathrm{Re}\left[e^{-i(\omega t - m\phi)} \right]$
real part
freq.
azimuthal number


$\Delta E_{BH} = \int dt \int_{r=r_+} (-\chi^a) T_{ab} \hat{t}^b \, dS_{BH}$
$= -\int dt \int_{r=r_+} [\chi^a (\nabla_a \psi)(\nabla_b \psi) \hat{t}^b - \frac{1}{2} \chi_a \hat{t}^a \nabla_c \psi \nabla^c \psi] dS_{BH}$
$= -\int dt \int dS_{BH} (\partial_t \psi + \Omega_H \partial_\phi \psi) \partial_t \psi$
$= (\omega - m\Omega_H) \omega \int dt \int dS_{BH} |\Psi|^2 \sin^2(\omega t - m\phi)$
≥ 0
$0 < \omega < m\Omega_H \Rightarrow$ flux of
negative energy into BH


$\frac{\delta E}{\delta J} = \frac{\omega}{m}$, $\delta E \geq \zeta_4 \delta J \Rightarrow \delta E \left(1 - \frac{m \zeta_4}{\omega}\right) \geq 0$
$a = 1$
Max amplification of Superradiance
$s=0$, Scalar wave 0.3%
$s=1$, EM wave 4.4%
$s=2$, G-W wave 138%

Mirror
BH
r=∞


BH
Blanford
- Znajek

Massive Scalar field

Klein-Gordon Eqn: $\Box\phi = \mu^2\phi$

μ: mass of boson (up to factor of $\hbar$)

Assume, μ^{-1}, $r >> M_{BH}$ (non-relativistic limit)

$$\Box\phi = \nabla_a(g^{ab}\partial_b\phi) = \frac{1}{\sqrt{-g}}$$

$\sqrt{-g} = r^2 \sin\theta$

$g^{tt}\partial_t^2\phi + \partial_i\partial^i\phi - \mu^2\phi = 0$, $g^{tt} \simeq -\left(1+\frac{2M}{r}\right)$

$\phi = \frac{1}{\sqrt{2\mu}}\left[\psi(x)e^{-i\omega t} + \psi^*(x)e^{i\omega t}\right]$

$\omega \approx \mu$

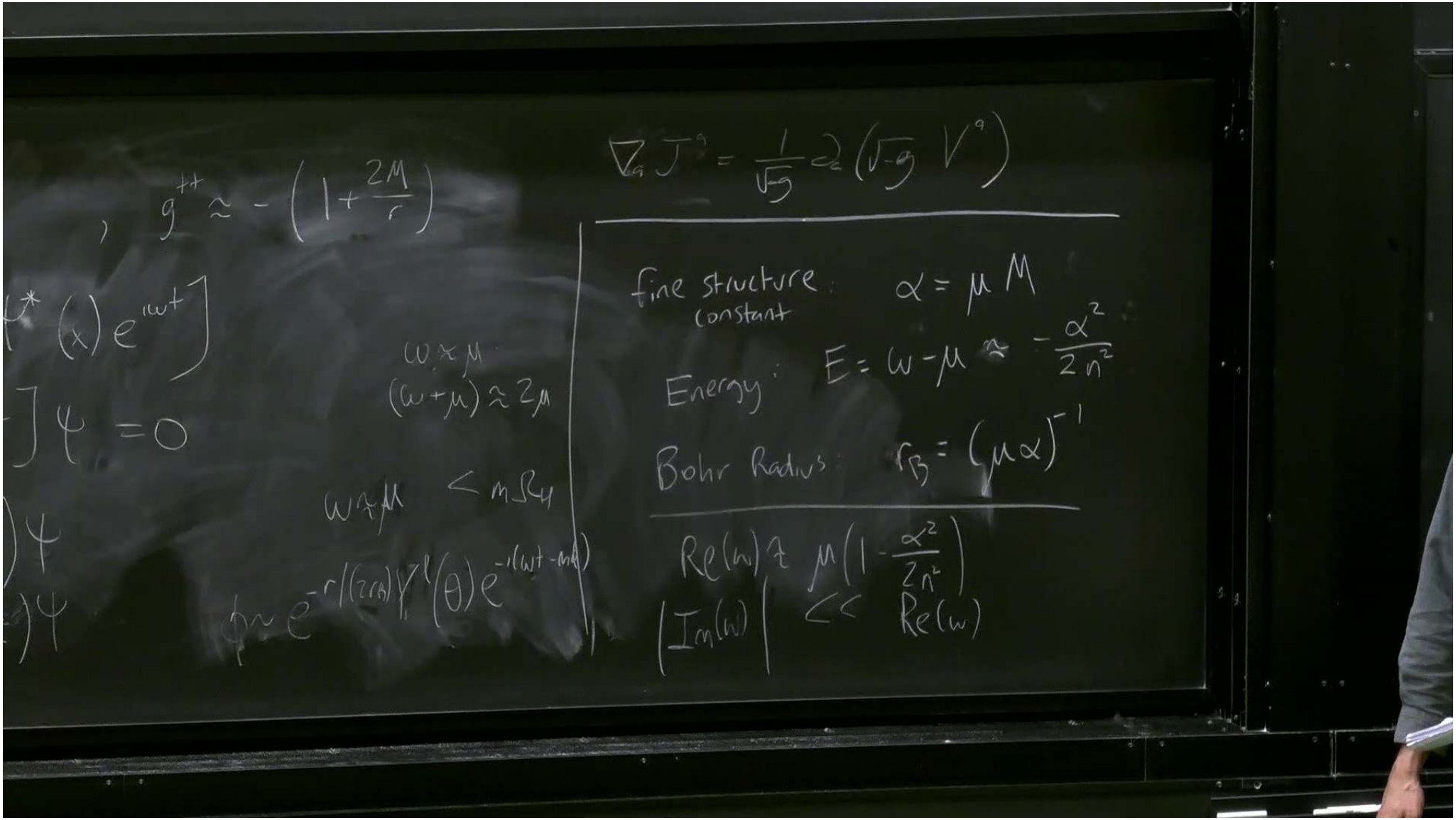
$(\omega+\mu) \approx 2\mu$

$\left[-\left(1+\frac{2M}{r}\right)(-i\omega)^2 + \partial_i\partial^i - \mu^2\right]\psi = 0$

(limit)

$(\omega^2-\mu^2)\psi = \left(-\partial_i\partial^i - \frac{2M\omega^2}{r}\right)\psi$

$(\omega-\mu)\psi = \left(-\frac{1}{2\mu}\partial_i\partial^i - \frac{\mu M}{r}\right)\psi$


$g^{tt} \approx -\left(1+\frac{2M}{r}\right)$
$\nabla_a J^a = \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g}\, J^a)$
$\psi^*(x) e^{i\omega t}$
$J\psi = 0$
$\omega \approx \mu$
$(\omega+\mu) \approx 2\mu$
$\omega \neq \mu < m_e$
$\psi \sim e^{-r/(2r_B)} Y_l^l(\theta) e^{-i(\omega t - m\phi)}$
fine structure constant: $\alpha = \mu M$
Energy: $E = \omega - \mu \approx -\frac{\alpha^2}{2n^2}$
Bohr Radius: $r_B = (\mu\alpha)^{-1}$
$\mathrm{Re}(\omega) \approx \mu\left(1-\frac{\alpha^2}{2n^2}\right)$
$|\mathrm{Im}(\omega)| \ll \mathrm{Re}(\omega)$

$$\alpha = \mu M = 0.1 \left(\frac{\mu}{10^{-12}\,\mathrm{eV}}\right)\left(\frac{M_{BH}}{10\,M_\odot}\right)$$

Gordon Eqn $\Box\psi = \mu\,\phi$

mass of boson (up to factor of $\hbar$)

$$\phi = \frac{1}{\sqrt{2\mu}}\left[\psi(x)\,e^{i\omega t} + \psi^*(x)\,e^{i\omega t}\right]$$

$$\left[-\left(1+\frac{2M}{r}\right)(-i\omega)^2 + \partial_i\partial^i - \mu^2\right]\psi = 0$$

μ^{-1}, $r \gg M_{BH}$ (non-relativistic limit)

$$\nabla_a\left(g^{ab}\partial_b\phi\right) = \frac{1}{\sqrt{-g}}\,\partial_a\left(\sqrt{-g}\,g^{ab}\partial_b\phi\right)$$

$$(\omega^2-\mu^2)\psi = \left(-\partial_i\partial^i - \frac{2M\omega^2}{r}\right)\psi$$

$$(\omega-\mu)\psi = \left(-\frac{1}{2\mu}\partial_i\partial^i - \frac{\mu M}{r}\right)\psi$$