Title: Strong Gravity Lecture

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Collection: Strong Gravity 2023/24

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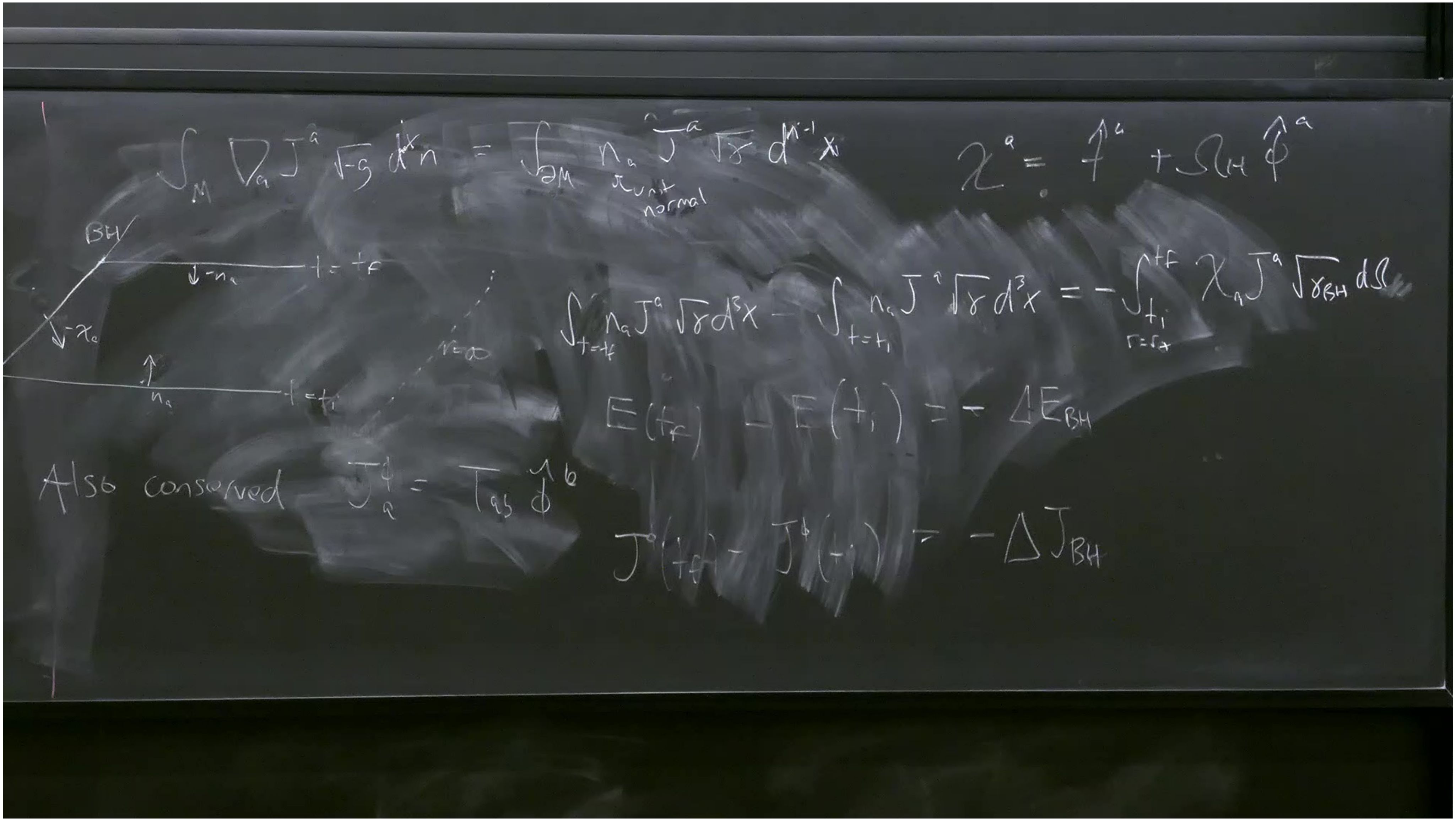
General Penrose Processes

Matter with T_{ab} (ignore backreaction)

$$J_a = -T_{ab}\hat{t}^b$$

$$\nabla_a J^a = -(\nabla_a T^a{}_b)\hat{t}^b - T^a{}_b \nabla_a \hat{t}^b = 0$$

$$\nabla_a T^a{}_b = 0 \qquad (\nabla_{(a}\hat{t}_{b)}) = 0$$

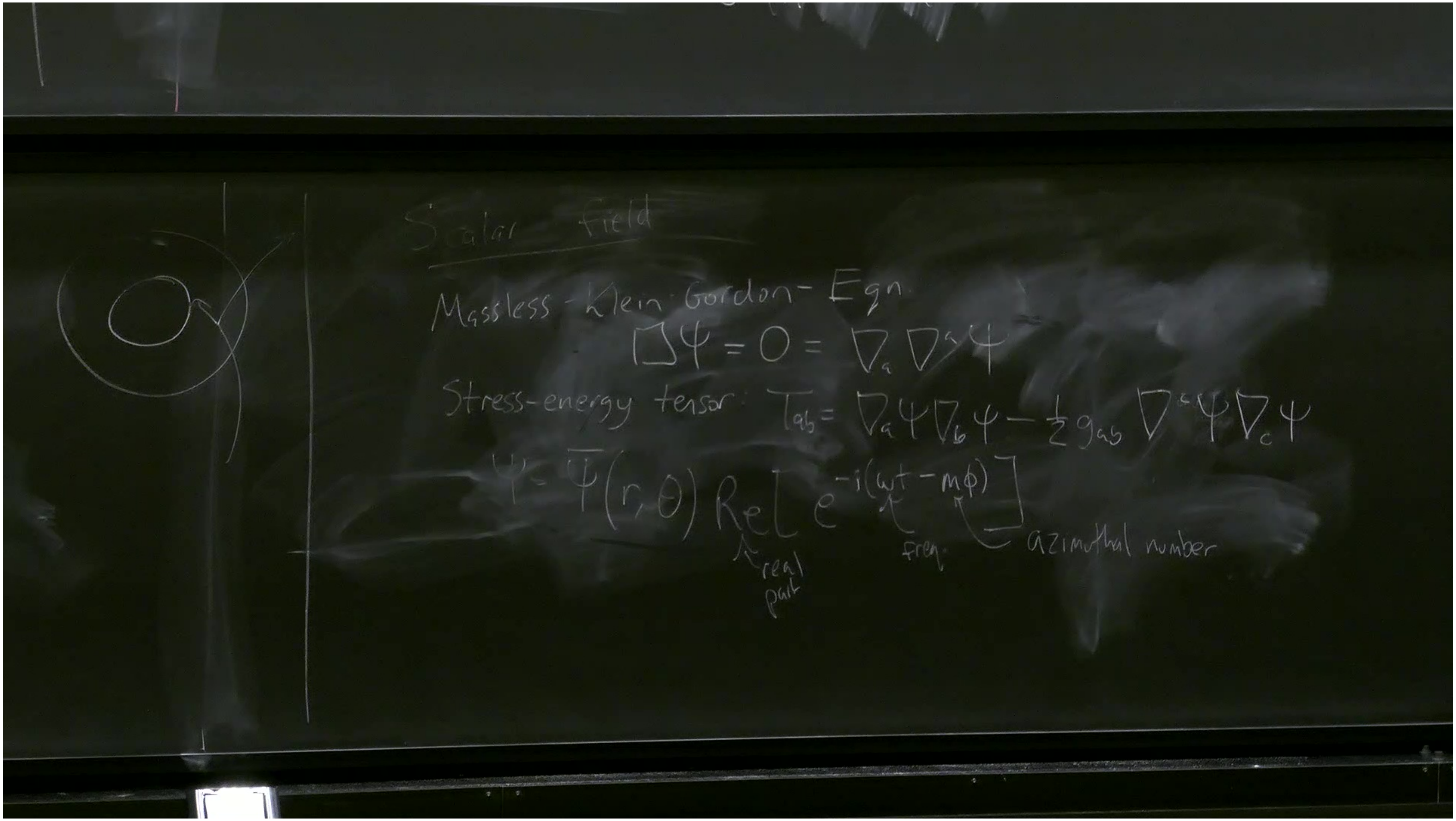

∫_M ∇_a J^a √-g d^n x = ∫_∂M n_a J^a √γ d^(n-1) x
unit normal
χ^a = t^a + Ω_H φ^a
BH
t = t_f
-n_a
-χ_a
r = ∞
n_a
t = t_i
∫_(t=t_f) n_a J^a √γ d^3x − ∫_(t=t_i) n_a J^a √γ d^3x = − ∫_(t_i)^(t_f) χ_a J^a √γ_BH dΩ
r = r_+
E(t_f) − E(t_i) = − ΔE_BH
Also conserved J^φ_a = T_ab φ^b
J^φ(t_f) − J^φ(t_i) = − ΔJ_BH

NEC

$$0 \leq T_{ab} \chi^a \chi^b = \left(-J_b + \Omega_H J^{\phi}_b\right) \chi^b$$

At $r = r_+$

$$0 \leq \Delta E_{B4} - \Omega_H \Delta J_{B4}$$


Scalar field
Massless Klein-Gordon Eqn
$\Box\psi = 0 = \nabla_a \nabla^a \psi$
Stress-energy tensor: $T_{ab} = \nabla_a\psi\nabla_b\psi - \frac{1}{2} g_{ab} \nabla^c\psi\nabla_c\psi$
$\psi = \bar{\Psi}(r,\theta)\, \mathrm{Re}[e^{-i(\omega t - m\phi)}]$
real part
freq.
azimuthal number

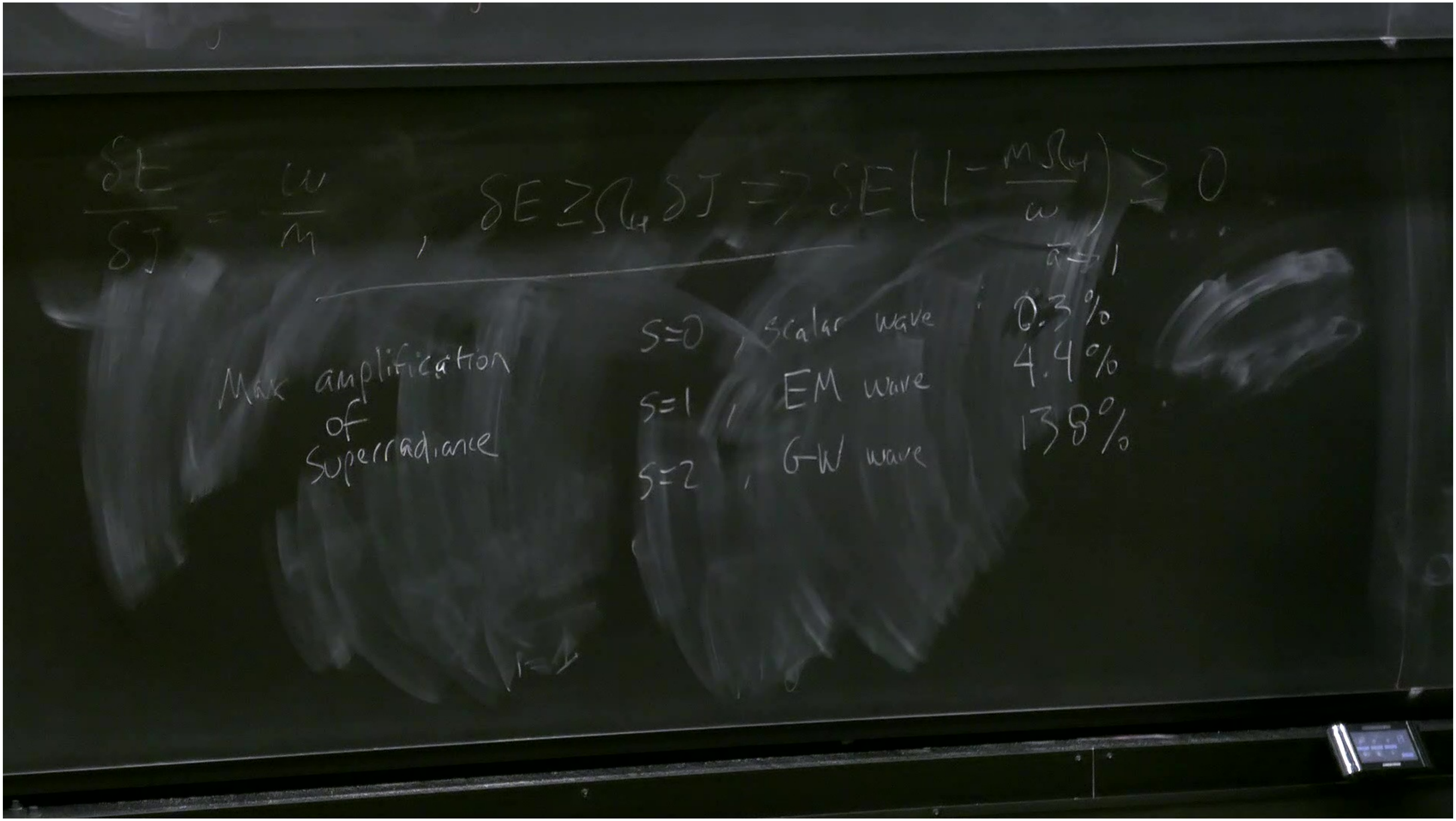
$$\Delta E_{BH} = \int dt \int_{r=r_+} (-\chi^a)\, T_{ab}\, \hat{f}^b \, dS_{BH}$$

$$= -\int dt \int_{r=r_+} \left[\chi^a (\nabla_a \psi)(\nabla_b \psi)\hat{f}^b - \tfrac{1}{2}\chi_a \hat{f}^a \nabla_c \psi \nabla^c \psi\right] dS_{BH}$$

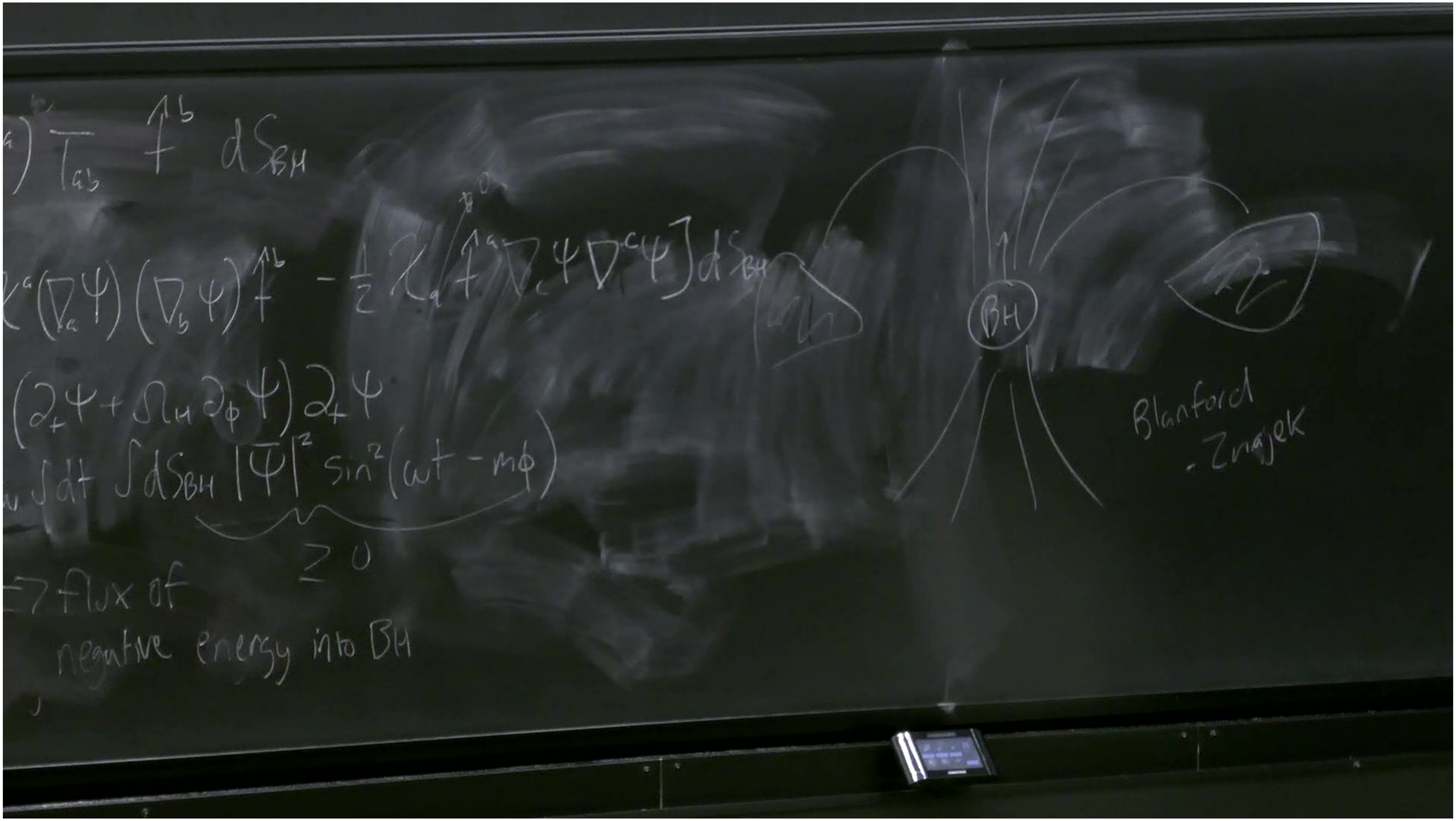
$$= -\int dt \int dS_{BH} \left(\partial_t \psi + \Omega_H \partial_\phi \psi\right)\partial_t \psi$$

$$= (\omega - m\Omega_H)\,\omega \int dt \int \underbrace{dS_{BH} |\Psi|^2 \sin^2(\omega t - m\phi)}_{\geq 0}$$

$0 < \omega < m\Omega_H \Rightarrow$ flux of negative energy into BH


δE/δJ = ω/m , δE ≥ Ω_H δJ ⇒ δE(1 − mΩ_H/ω) ≥ 0
a = 1
Max amplification of Superradiance
s=0 , scalar wave : 0.3%
s=1 , EM wave 4.4%
s=2 , GW wave 138%


Mirror
BH
r=∞


$T_{ab}\ \hat{t}^b\ dS_{BH}$
$\chi^a(\nabla_a\Psi)(\nabla_b\Psi)\hat{t}^b - \frac{1}{2}\chi_a\hat{t}^a\nabla_c\Psi\nabla^c\Psi]dS_{BH}$
$(\partial_t\Psi + \Omega_H\partial_\phi\Psi)\partial_t\Psi$
$\int dt \int dS_{BH} |\Psi|^2 \sin^2(\omega t - m\phi)$
≥ 0
⇒ flux of
negative energy into BH
BH
Blanford
- Znajek

Massive Scalar field

Klein-Gordon Eqn: $\Box\phi = \mu^2 \phi$

$\uparrow$ mass of boson (up to factor of $\hbar$)

Assume, μ^{-1}, $r \gg M_{BH}$ (non-relativistic limit)

$$\Box\phi = \nabla_a\left(g^{ab}\partial_b\phi\right) = \frac{1}{\sqrt{-g}}$$

$$\sqrt{-g} = r^2 \sin\theta$$

$$g^{tt}\partial_t^2 \phi + \partial_i \partial^i \phi - \mu^2 \phi = 0 \quad , \quad g^{tt} \simeq -\left(1+\frac{2M}{r}\right)$$

$$\phi = \frac{1}{\sqrt{2\mu}}\left[\psi(x)\, e^{-i\omega t} + \psi^*(x)\, e^{i\omega t}\right]$$

$\omega \approx \mu$

$(\omega+\mu) \simeq 2\mu$

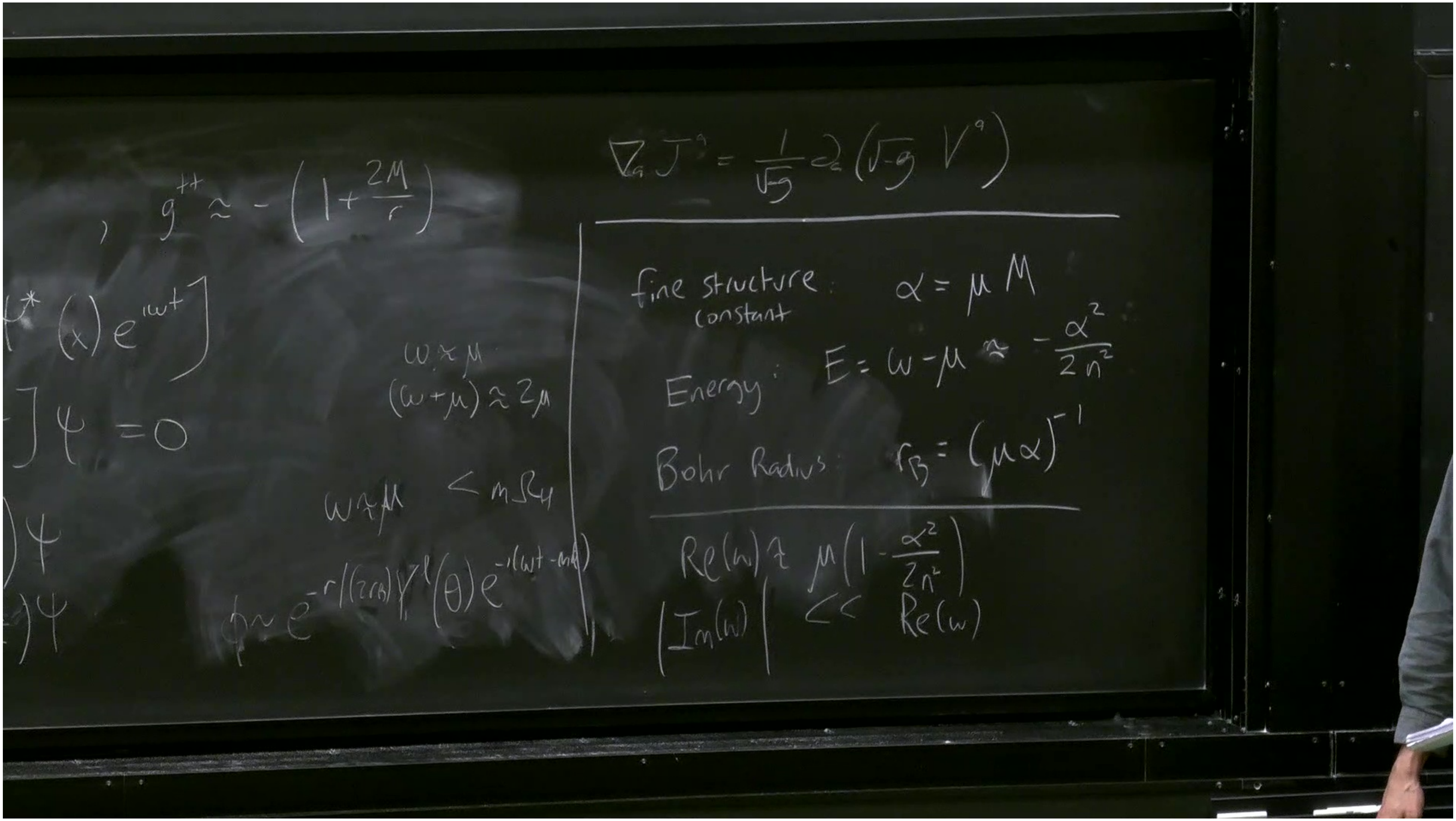
h)

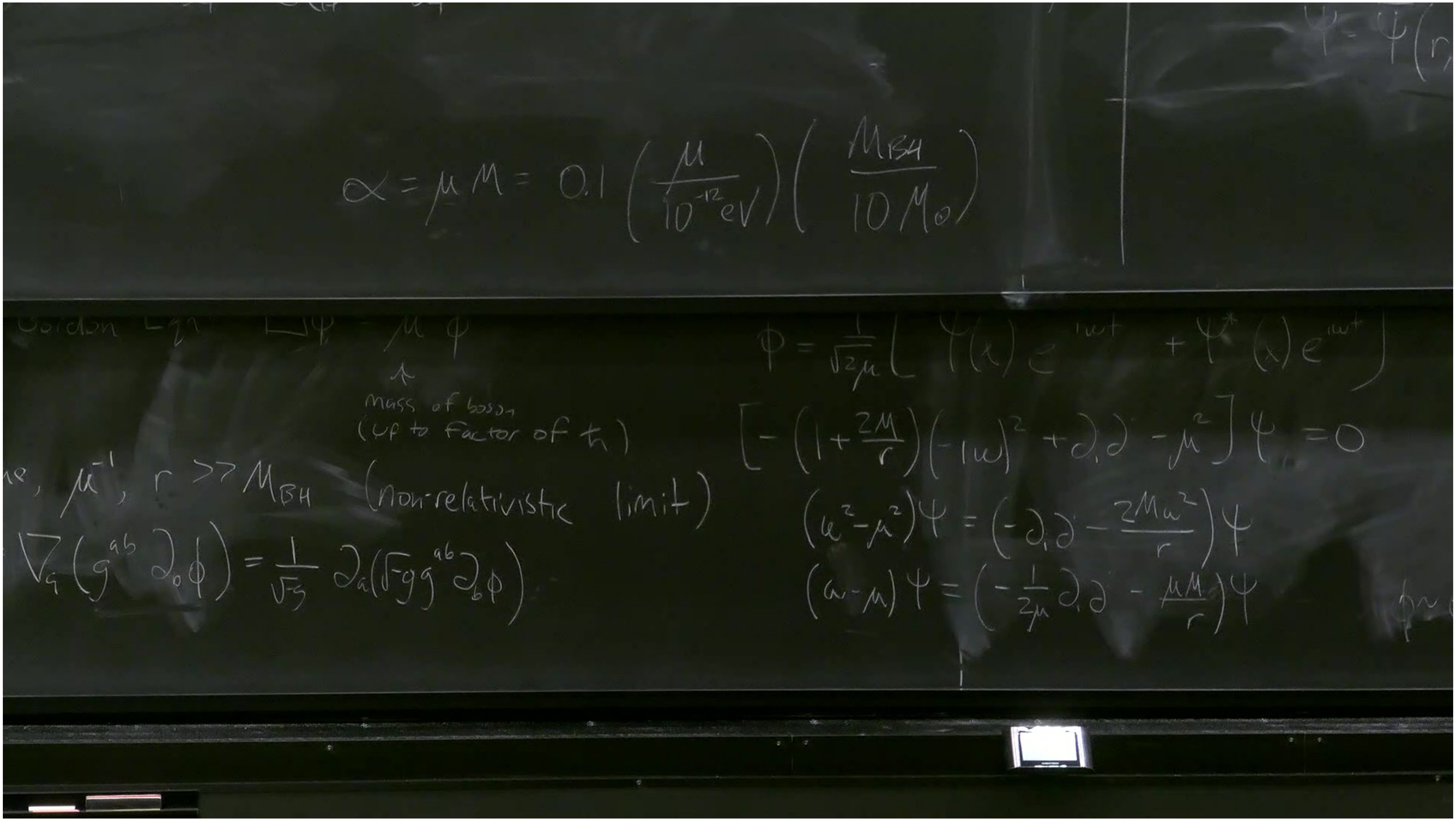
$$\left[-\left(1+\frac{2M}{r}\right)(-i\omega)^2 + \partial_i\partial^i - \mu^2\right]\psi = 0$$

limit)

$$(\omega^2-\mu^2)\psi = \left(-\partial_i\partial^i - \frac{2M\omega^2}{r}\right)\psi$$

$$(\omega-\mu)\psi = \left(-\frac{1}{2\mu}\partial_i\partial^i - \frac{\mu M}{r}\right)\psi$$


fine structure constant: α = μ M
Energy: E = ω - μ ≈ - α²/2n²
Bohr Radius: r_B = (μα)⁻¹
Re(ω) ≈ μ(1 - α²/2n²)
|Im(ω)| << Re(ω)
ω ≈ μ
(ω + μ) ≈ 2μ



$\alpha = \mu M = 0.1 \left(\frac{\mu}{10^{-12} eV}\right)\left(\frac{M_{BH}}{10 M_\odot}\right)$
mass of boson
(up to factor of ħ)
μ^{-1}, $r >> M_{BH}$ (non-relativistic limit)
$\nabla_a (g^{ab} \partial_b \phi) = \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} g^{ab} \partial_b \phi)$
$\phi = \frac{1}{\sqrt{2\mu}} [\psi(x) e^{-i\omega t} + \psi^*(x) e^{i\omega t}]$
$[-(1 + \frac{2M}{r})(-i\omega)^2 + \partial_i \partial^i - \mu^2] \psi = 0$
$(\omega^2 - \mu^2)\psi = (-\partial_i \partial^i - \frac{2M\omega^2}{r})\psi$
$(\omega - \mu)\psi = (-\frac{1}{2\mu}\partial_i \partial^i - \frac{\mu M}{r})\psi$