Title: Advanced General Relativity - 240403 (afternoon)

Speakers: Eric Poisson

Collection: Advanced General Relativity (PHYS7840)

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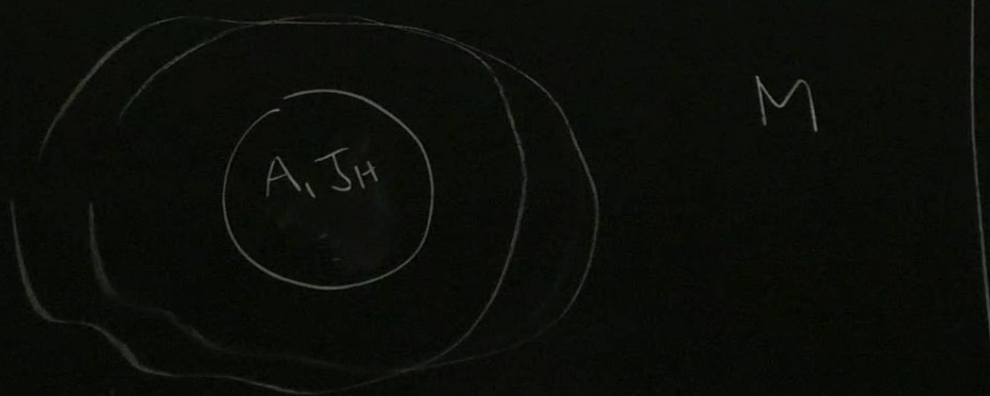
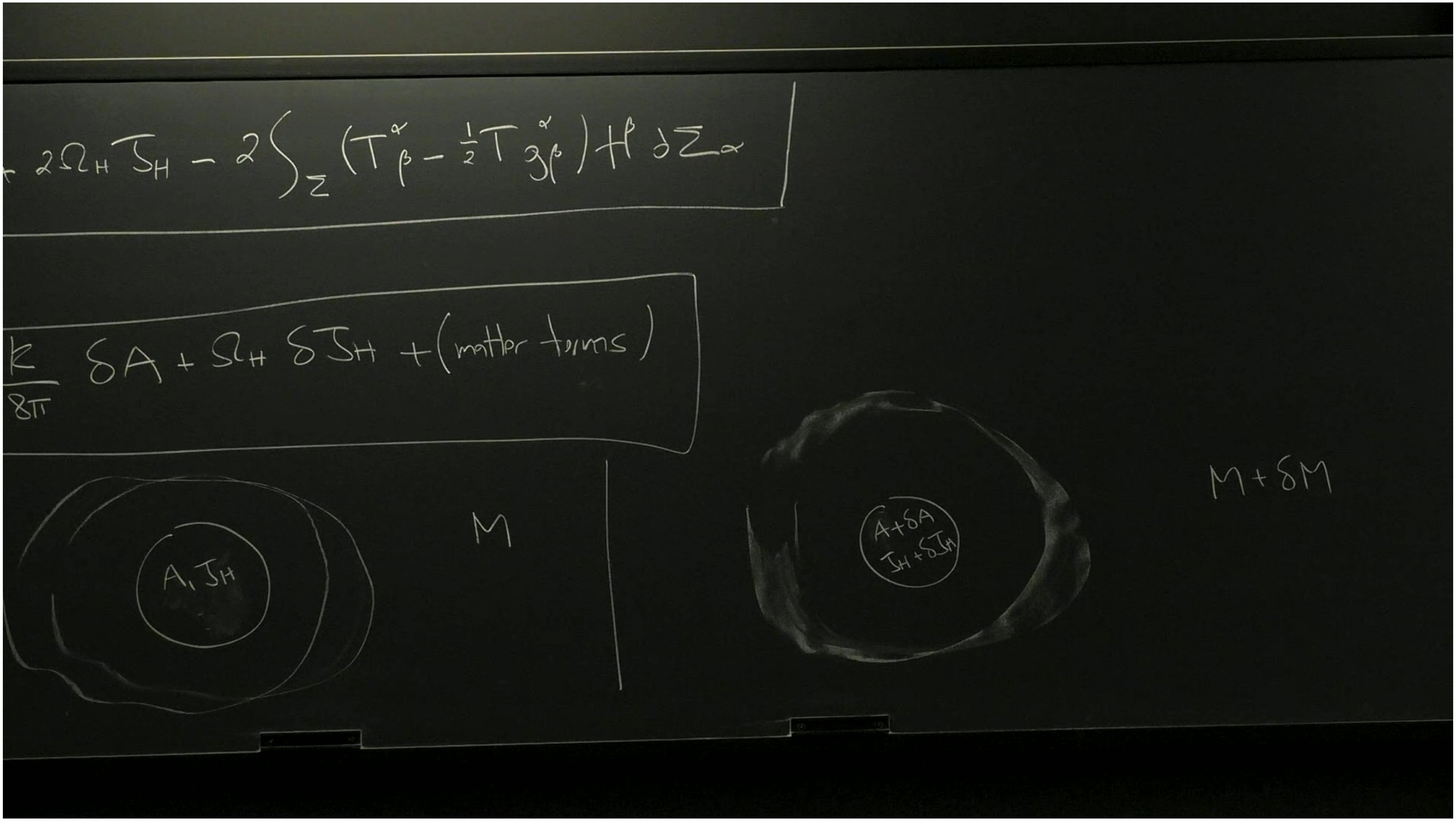
$$\boxed{M = \frac{\kappa}{4\pi} A + 2\Omega_H J_H - 2\int_\Sigma \left(T^\alpha{}_\beta - \tfrac{1}{2} T g^\alpha{}_\beta\right) t^\beta \, d\Sigma_\alpha}$$

First law

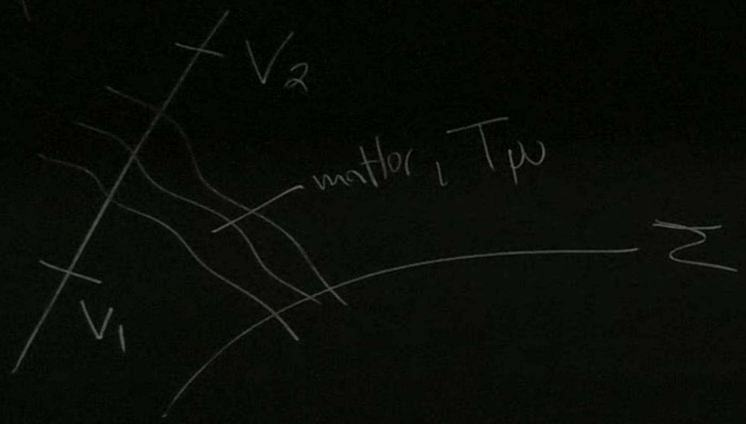
$$\boxed{\delta M = \frac{\kappa}{8\pi} \delta A + \Omega_H \delta J_H + (\text{matter terms})}$$





$+ 2\Omega_H J_H - 2\int_\Sigma \left(T^\alpha{}_\beta - \frac{1}{2} T g^\alpha{}_\beta\right) \xi^\beta d\Sigma_\alpha$
$\frac{\kappa}{8\pi} \delta A + \Omega_H \delta J_H + (\text{matter terms})$
A, J_H
M
$A+\delta A$
$J_H+\delta J_H$
$M+\delta M$

...ical process — BH is stationary before $v = v_1$, after $v = v_2$, changes $v_1 <$



Flux vectors

$$\varepsilon^\alpha = -T^\alpha{}_\beta\, t^\beta$$
$$\ell^\alpha = T^\alpha{}_\beta\, \phi^\beta$$

A, SH

matter, $T_{\mu\nu}$

Σ

$$\mathcal{E}^\alpha = - T^\alpha{}_\beta t^\beta$$

$$l^\alpha = T^\alpha{}_\beta l^\beta$$

$$\nabla_\alpha \mathcal{E}^\alpha = - \nabla_\alpha \left(T^{\alpha\beta} t_\beta \right) = - \underbrace{\left(\nabla_\alpha T^{\alpha\beta}\right)}_{0} t_\beta - \underbrace{T^{\alpha\beta} \nabla_\alpha t_\beta}_{0}$$

$$= 0$$

$$T^{\alpha\beta} = \rho\, u^\alpha u^\beta$$

$$\mathcal{E}^\alpha = \rho\left(-u_\beta t^\beta\right) u^\alpha = \left(\rho \tilde{E}\right) u^\alpha$$

$$\ell^\alpha = \rho\left(u_\beta \phi^\beta\right) u^\alpha = \left(\rho \tilde{L}\right) u^\alpha$$

Mass-energy crossing spacelike hypersurface $\Sigma \equiv \int_\Sigma \mathcal{E}^\alpha \, d\Sigma_\alpha$

$) U^\alpha$

$\Sigma) U^\alpha$

Mass-energy crossing spacelike hypersurface $\Sigma \equiv \int_\Sigma \varepsilon^\alpha \, d\Sigma_\alpha = \delta M$

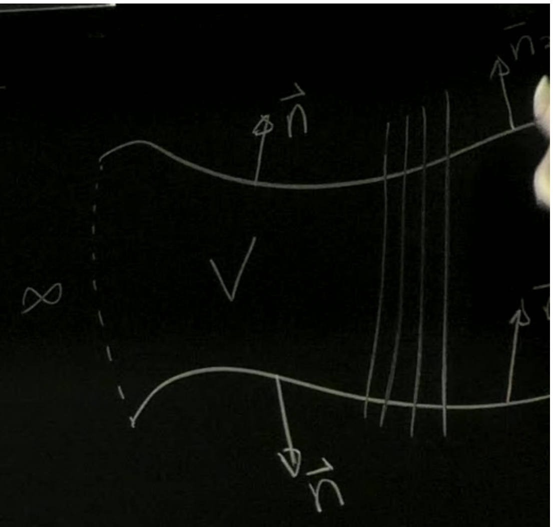
Angular momentum " " " " " " $\equiv \int_\Sigma \ell^\alpha \, d\Sigma_\alpha = \delta J$

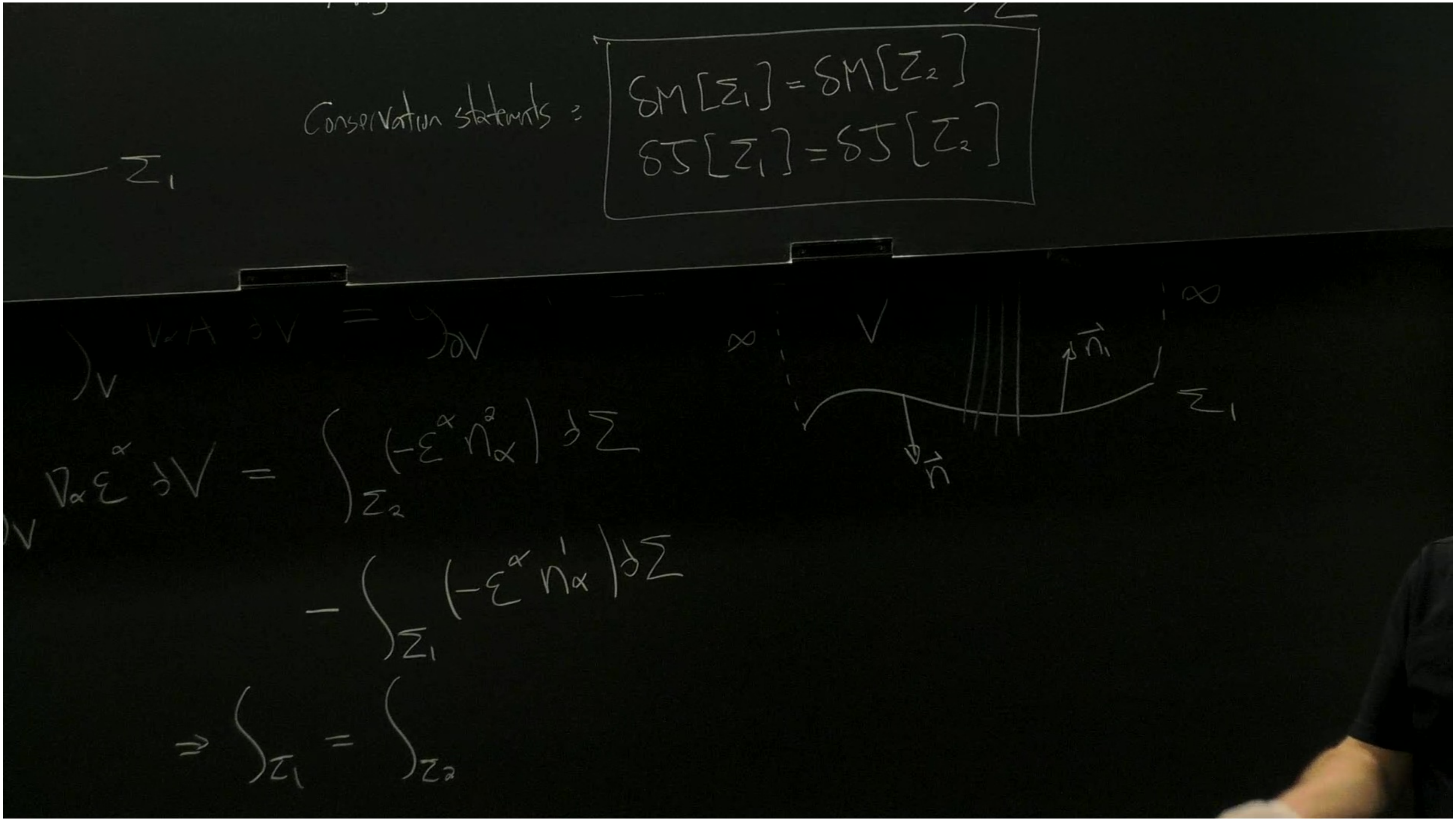
Conservation statements: $\delta M[\Sigma_1] = \delta M[\Sigma_2]$

$\delta J[\Sigma_1] = \delta J[\Sigma_2]$

Gauss theorem : $\int_V \nabla_\alpha A^\alpha \, dV = \oint_{\partial V} A^\alpha \overbrace{d\Sigma_\alpha}^{-n_\alpha d\Sigma}$

$$0 = \int_V \nabla_\alpha \mathcal{E}^\alpha \, dV = \int_{\Sigma_2} \left(-\mathcal{E}^\alpha n^2_\alpha\right)$$




Conservation statements :
$\delta M[\Sigma_1] = \delta M[\Sigma_2]$
$\delta J[\Sigma_1] = \delta J[\Sigma_2]$
Σ_1
∞
V
$\vec{n}_1$
$\vec{n}$
∞
Σ_1
$\int_V \nabla_\alpha \mathcal{E}^\alpha \, dV = \int_{\Sigma_2} (-\mathcal{E}^\alpha n^2_\alpha)\, d\Sigma$
$- \int_{\Sigma_1} (-\mathcal{E}^\alpha n^1_\alpha)\, d\Sigma$
$\Rightarrow \int_{\Sigma_1} = \int_{\Sigma_2}$

$= (\rho \tilde{\Xi}) u^\alpha$

$= (\rho \tilde{\Sigma}) u^\alpha$

Mass-energy crossing spacelike hypersurface $\Sigma \equiv \boxed{\int_\Sigma \varepsilon^\alpha d\Sigma_\alpha = \delta M}$

Angular momentum 〃 〃 〃 〃 $\equiv \boxed{\int_\Sigma \ell^\alpha d\Sigma_\alpha = \delta J}$

Σ_2

Conservation statements: $\boxed{\begin{array}{l} \delta M[\Sigma_1] = \delta M[\Sigma_2] \\ \delta J[\Sigma_1] = \delta J[\Sigma_2] \end{array}}$

Σ_1

$\Rightarrow \int_{\Sigma_1} = \int_{\Sigma_2}$

Mass flowing into horizon $\equiv \delta M_H = \int_H \varepsilon^\alpha \, d\Sigma_\alpha$ $\qquad d\Sigma_\alpha = -K_\alpha \, dv \, dS$

Angular momentum " " " $\equiv \delta J_H = \int_H l^\alpha \, d\Sigma_\alpha$

$$\delta M_H - \Omega_H \delta J_H = \int_H \left(\varepsilon^\alpha - \Omega_H l^\alpha \right) d\Sigma_\alpha$$

$$= - \int_H T^\alpha{}_\beta \Big(\underbrace{t^\beta + \Omega_H l^\beta}_{K^\beta} \Big) \underbrace{d\Sigma_\alpha}_{-K_\alpha \, dv \, dS}$$

$$= \int_H T_{\alpha\beta} K^\alpha K^\beta \, dv \, dS$$

$$= \frac{1}{8\pi} R_{\alpha\beta} k^\alpha k^\beta = \frac{1}{8\pi}\left(-\partial_v \Theta + \kappa\,\Theta \underbrace{-\frac{1}{2}\Theta^2}_{\text{neglect}} - \underbrace{\sigma_{AB}\sigma^{AB}}_{\text{neglect}}\right)$$

$$-\frac{1}{8\pi}\underbrace{\int_H \partial_v\Theta\, dv\, dS}_{\cancel{\int_H \Theta\, dS \Big|_{V_1}^{V_2}}} + \frac{1}{8\pi}\underbrace{\int_H \kappa\,\Theta\, dv\, dS}$$

$$dS = \sqrt{\Omega}\, d^2\theta$$

$$\Theta = \frac{1}{\sqrt{\Omega}}\partial_v \sqrt{\Omega}$$

$$\frac{\kappa}{8\pi}\int_H \Theta\, dv\, dS$$

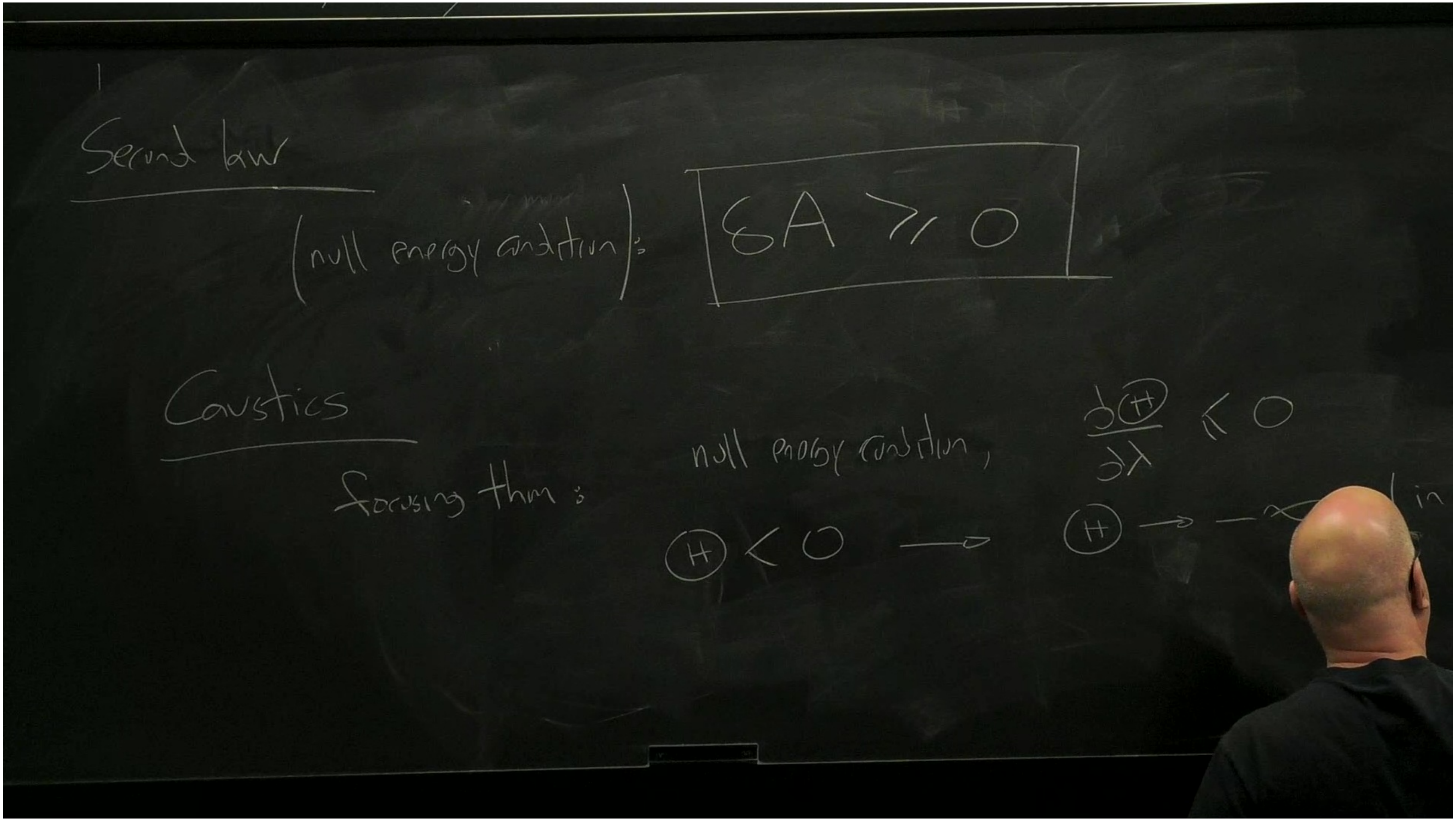
$$= \frac{\kappa}{8\pi}\int_H \partial_v\sqrt{\Omega}\, d^2\theta\, dv$$

$$= \frac{\kappa}{8\pi} \int \sqrt{\Omega}\, d^2\theta \,\Big|_{V_1}^{V_2} = \frac{\kappa}{8\pi}\, \delta A$$

$$\sqrt{\Omega}\, d^2\theta \Big|_{V_1}^{V_2} = \frac{\kappa}{8\pi}\,\delta A$$

$$\boxed{\delta M_H = \Omega_H\, \delta J_H + \frac{\kappa}{8\pi}\,\delta A}$$

$$+\ \Phi_H\, \delta Q$$


Second law
(null energy condition):
δA ≥ 0
Caustics
focusing thm:
null energy condition,
dΘ/dλ ≤ 0
Θ < 0 ⟶
Θ → −∞
(in

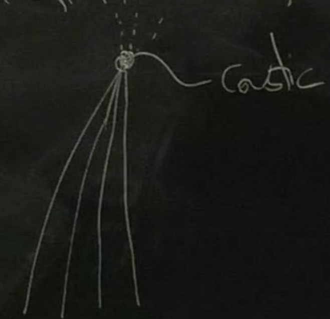
ll energy condition): $\boxed{\delta A \geq 0}$

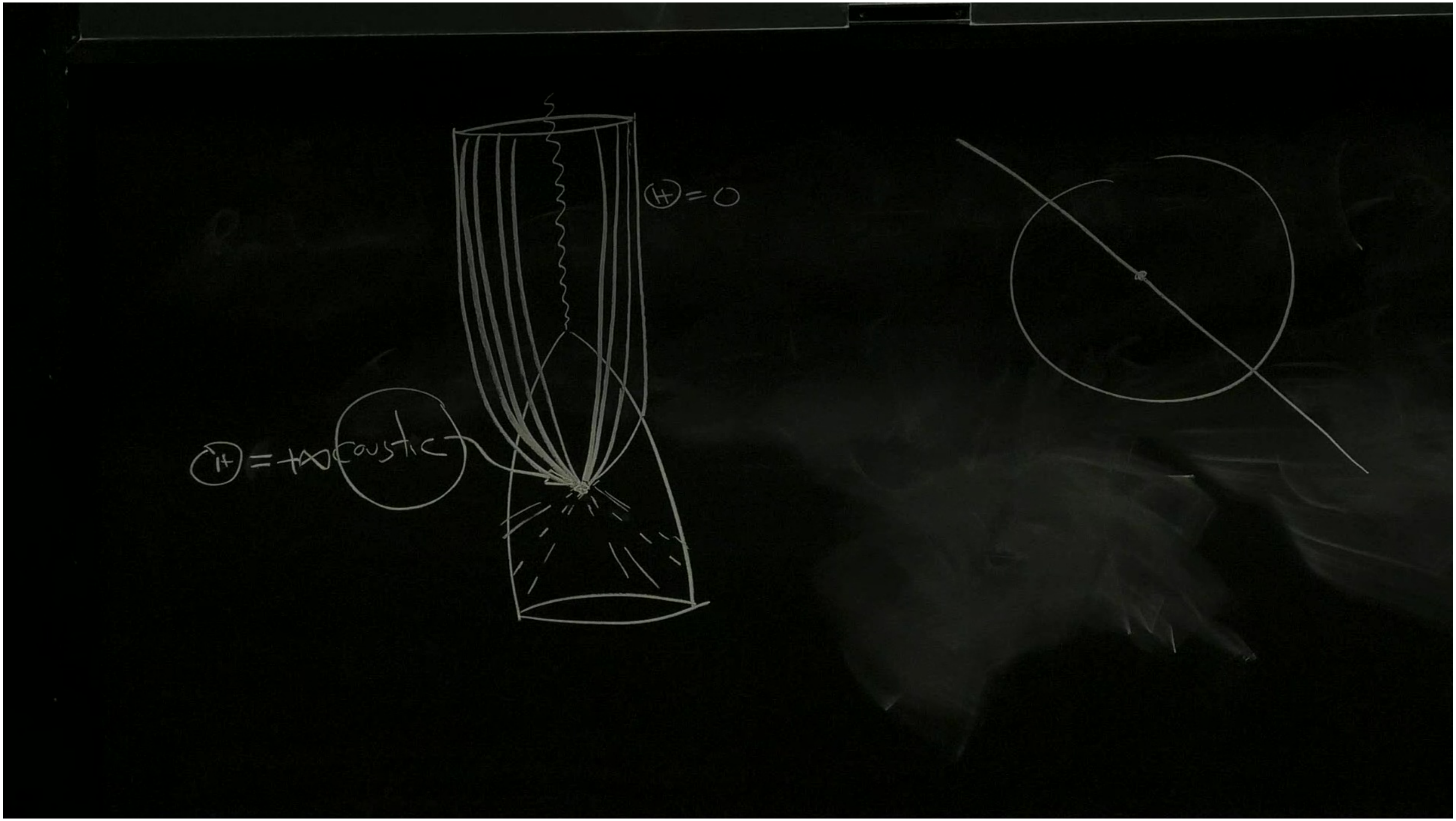
focusing thm:

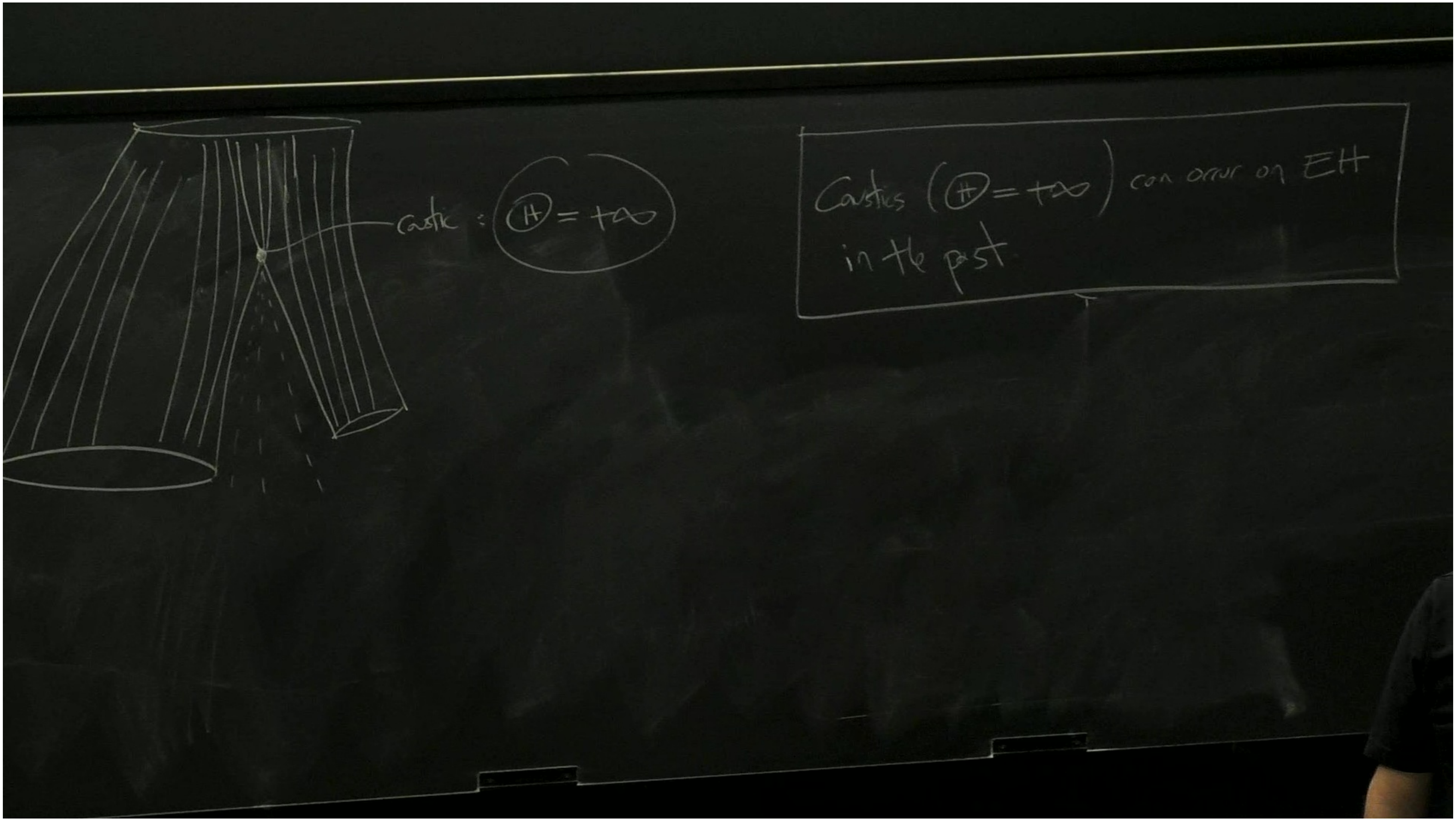
null energy condition, $\frac{d\Theta}{d\lambda} \leq 0$

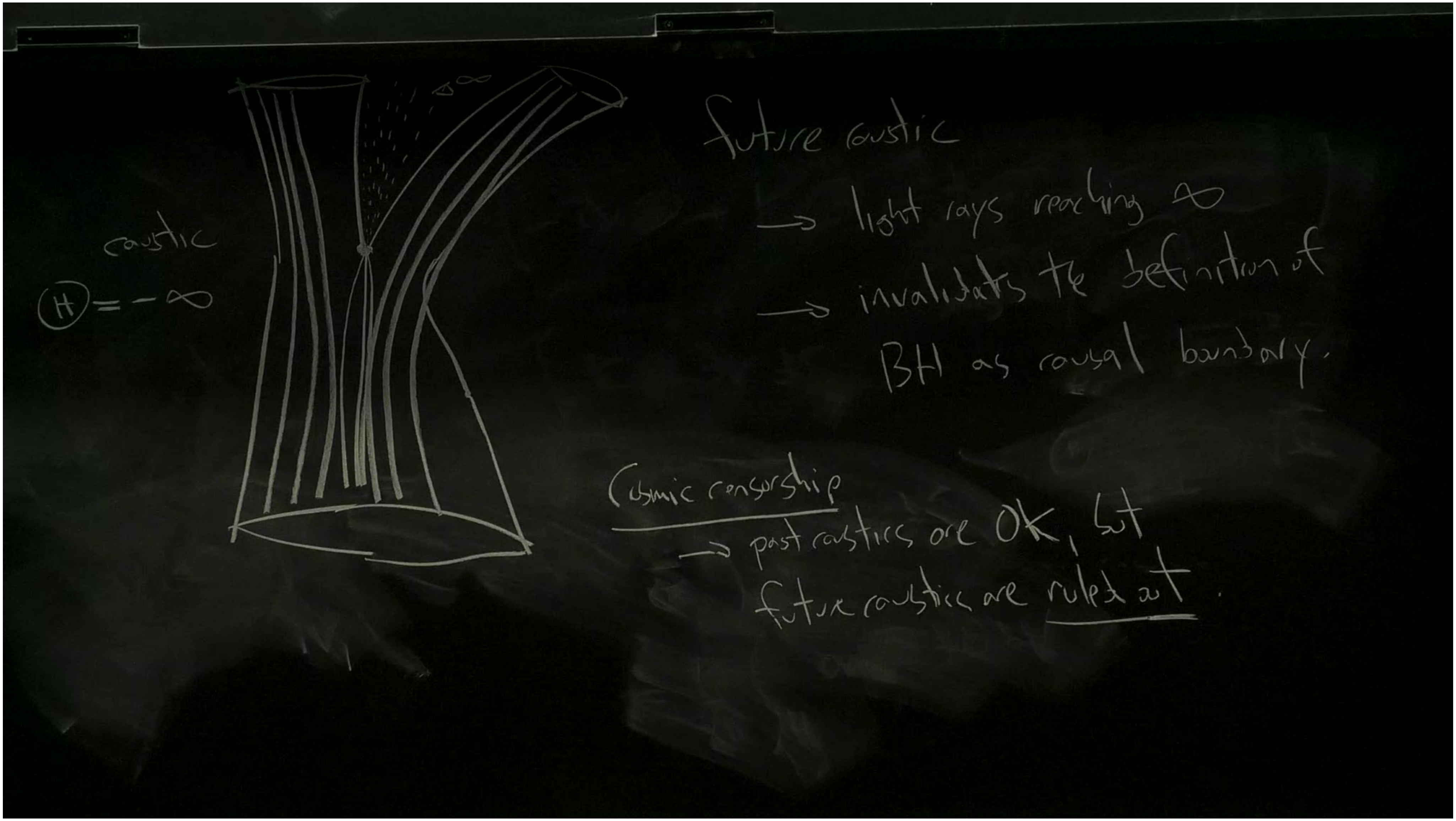
$\Theta < 0 \longrightarrow \Theta \to -\infty$ (in finite λ)

$\longrightarrow$ caustic




caustic


caustic : $\Theta = +\infty$
Caustics ($\Theta = +\infty$) can occur on EH in the past.


∞
caustic
Θ = −∞
future caustic
→ light rays reaching ∞
→ invalidates the definition of BH as causal boundary.
Cosmic censorship
→ past caustics are OK, but future caustics are ruled out.

two statements : 1- Θ cannot go to $-\infty$ on EH (cosmic censorship)

2- $\Theta \to -\infty$ when at some initial time, $\Theta < 0$

$$\frac{\partial A}{\partial V} = \int_H \Theta \, \partial S$$

two statements :

1- H cannot go to $-\infty$ on EH (cosmic censorship)

2- $H \to -\infty$ when at some initial time, $H < 0$

$$\Bigg\} \quad H \geq 0$$

$$\frac{\partial A}{\partial V} = \int_H \underbrace{H}_{\geq 0} \, \delta S \geq 0 \quad \Rightarrow \quad \boxed{\delta A \geq 0}$$