Title: Love and Naturalness

Speakers: Mikhail Ivanov

Series: Strong Gravity

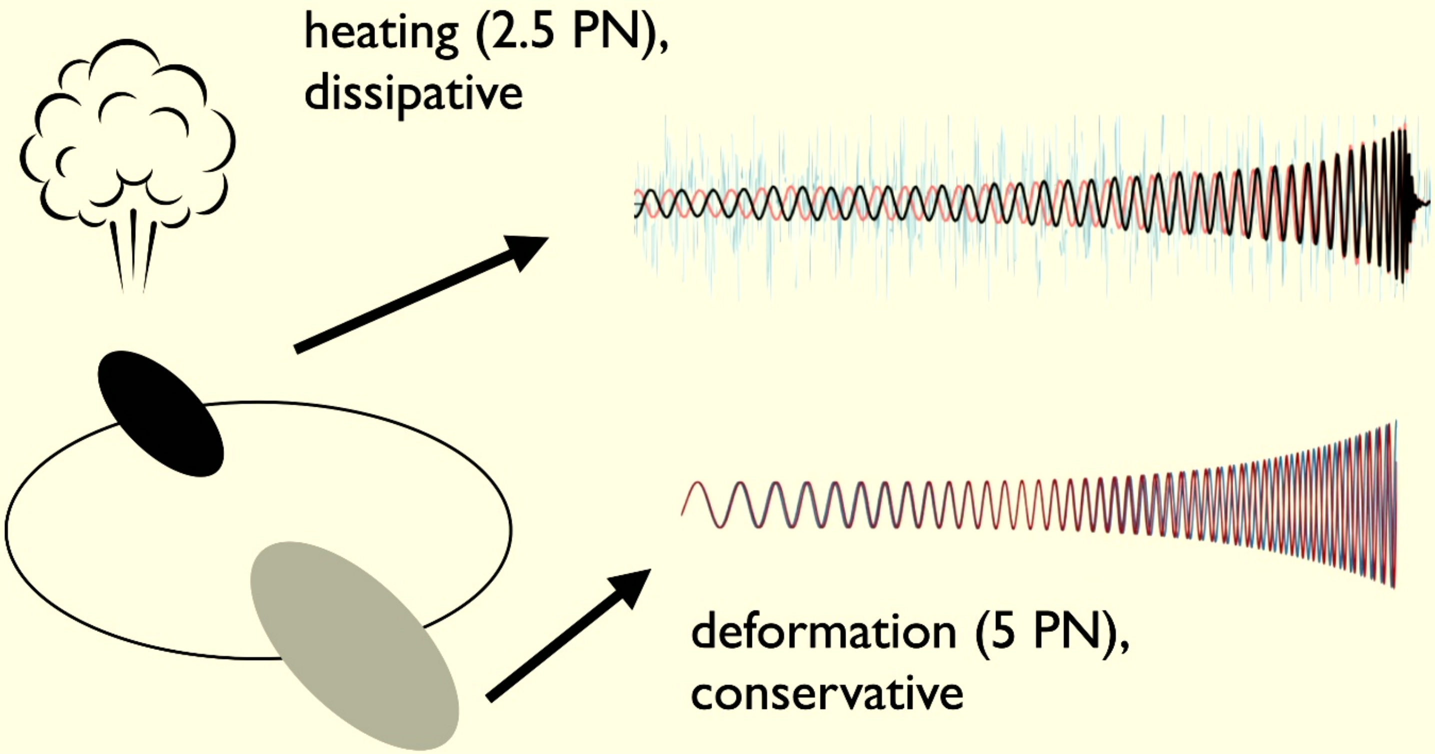
Date: March 28, 2024 - 1:00 PM

URL: https://pirsa.org/24030124

Abstract: Recent progress in gravitational wave astronomy has spurred the development of efficient tools to describe gravitational binary dynamics. One such tool is classical worldline effective field theory (EFT). In the first part of my talk, I will show how to use this EFT for systematic studies of tidal heating and deformations (Love numbers) of compact objects. I will present a gauge-invariant definition of Love numbers and show how to extract them in a coordinate-independent way from scattering amplitudes of the gravitational Raman process. I will show that the worldline EFT exhibits strong fine-tuning when applied to black holes. This gives rise to a naturalness paradox associated with the vanishing of black hole static Love numbers. In the second part of my talk, I will present a new symmetry of black holes (Love symmetry) that elegantly resolves this paradox. The Love symmetry is tightly connected to isometries of extremal black holes that appear in many holographic constructions. It also provides a curious example of IR/UV mixing, which may give insights for other hierarchy problems.

Zoom link

Tides unveil the nature of compact objects

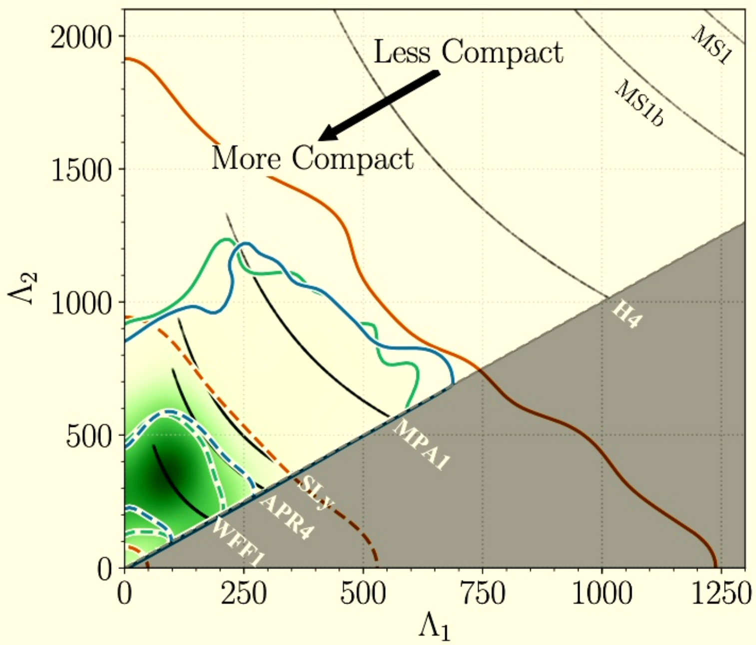


- One can probe EoS for neutron stars

Flanagan, Hinderer (2007) ++

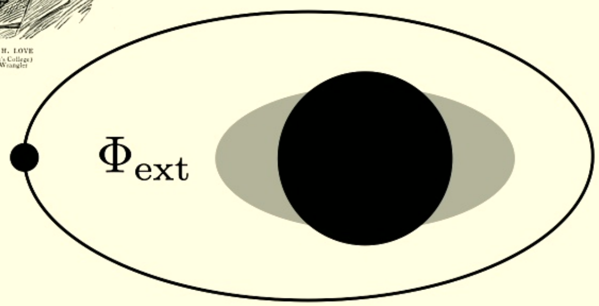
Tidal response of Neutron Stars from GW170817

Ligo/Virgo 1805.11581



Love numbers

Love numbers in Newtonian gravity



$$\Phi = \Phi_0 + \Phi_2 + \Phi_3 + \ldots$$

$$\Phi_0 = -\frac{M}{r}$$

$$\Phi_2 = r^2 E_{ij} n^i n^j + \frac{Q_{ij} n^i n^j}{r^3}$$

tidal source   tidal response

$$\Phi_\ell = r^\ell E_{i_1 \ldots i_\ell} n^{i_1} \ldots n^{i_\ell} + Q_{i_1 \ldots i_\ell} \frac{n^{i_1} \ldots n^{i_\ell}}{r^{\ell+1}}$$

Linear response theory: $Q_\ell = \Lambda_\ell r_s^{2\ell+1} E_\ell$

Love number

$(G = c = \hbar = 1)$

Love numbers in General Relativity

vs.

Fang, Lovelace (2005)
Binnington, Poisson (2009)
Damour, Nagar (2009)

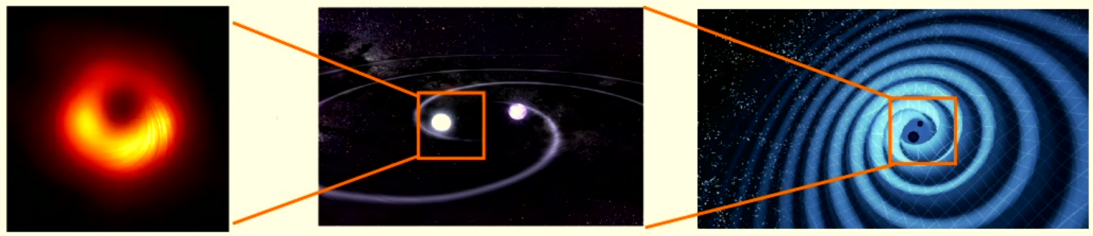
- We want to use a definition that matches the Newtonian one at large distances
- Reparametrization and diff invariant
- Nonlinear GR: is source/response split well defined ?

  spoiler: NO! they mix, producing Log running of LNs

- Worldline effective field theory!

Goldberger, Rothstein (2004,2005)
Kol, Smolkin (2010)

Worldline EFT



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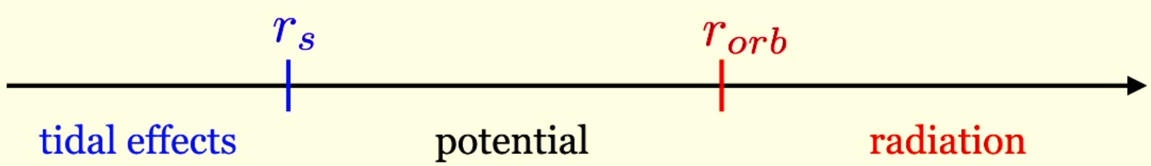
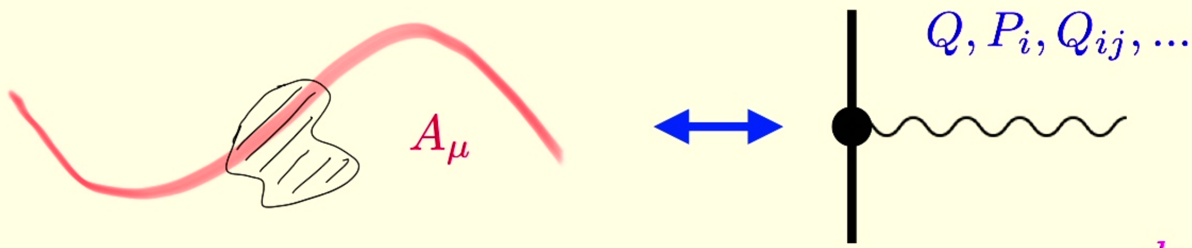
EFT framework: disentangle physics from different scales



at $r \gg r_s$ a black hole is described by the word line effective action

$$S = -m \int ds + \ldots$$

Worldline EFT for E&M



$$u^\mu = \frac{dx^\mu}{d\sigma}$$

$$S_{\text{eff}} = Q \int d\sigma \; u^\mu A_\mu + \chi \int d\sigma \frac{u^\mu u^\lambda F_{\mu\nu} F^\nu_{\ \lambda}}{\sqrt{-u^2}} + \ldots$$

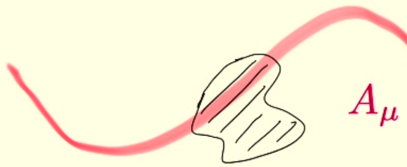
$$S_{\text{eff}} = Q \int ds \; A_0 + \chi \int ds \; E_i E^i + \ldots$$

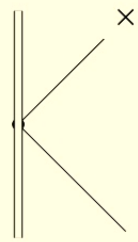
point particle

Finite-size effects

$$\chi \sim [\text{cm}]^3$$

Worldline EFT for E&M





$$E^i_{\rm tot} = \bar{E}^i_{\rm source} + E^i_{\rm response}$$

$$E_r = \underbrace{1}_{\rm source} - \underbrace{\frac{\chi}{r^3}}_{\rm response}$$

$$S_{\rm eff} = \chi \int ds\ E_i E^i - \frac{1}{4}\int d^4x\ F^2_{\mu\nu}$$

$$P^i = \chi E^i$$

Electric field of induced dipole

Polarizability = EFT Wilson coefficient

$$\chi \sim [\mathrm{cm}]^3$$

Including gravity

E&M: $$S_{\text{eff}} = Q \int ds \; A_0 + \chi \int ds \; E_i E^i + \dots$$

Gravity: $$S_{\text{eff}} = -m \int ds \; + \lambda_2 R^5 \int ds \; E_{\mu\nu} E^{\mu\nu}$$

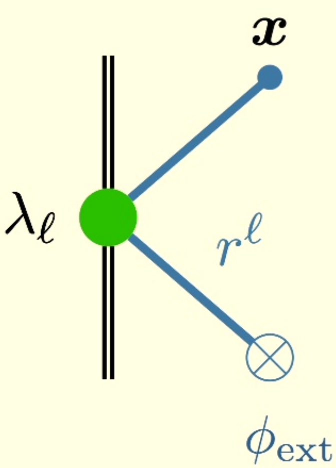
$$E_{ij} = \partial_i \partial_j \Phi \quad \leftarrow \quad u^\mu u^\lambda R_{\mu\nu\lambda\rho}$$

Scalar: $$S_{\text{eff}} = -m \int ds \; + \lambda_2 R^5 \int ds \; (\partial_i \partial_j \Phi)^2$$

In Newtonian gauge:

$$S_{\rm eff} = -m \int ds \;\; + \lambda_2 R^5 \int ds \; (\partial_i \partial_j \Phi)^2$$

Worldline response calculation ⟶ $\lambda_\ell = \Lambda_\ell$



$$\Phi = \Phi_{\rm ext} + \Phi_{\rm resp}$$

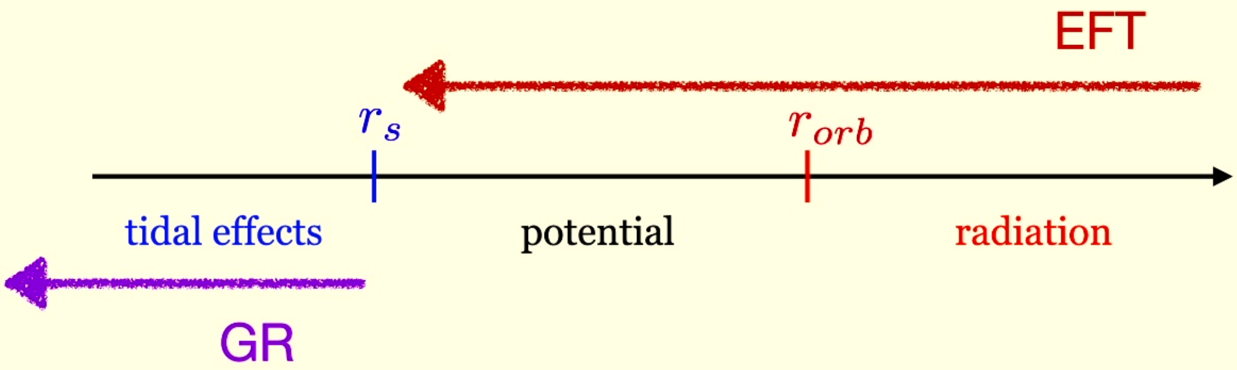
$$\sim r^\ell \times \frac{\lambda_\ell r_s^{2\ell+1}}{r^{2\ell+1}}$$

Love numbers = Wilson coefficients

Unambiguous gauge-invariant definition of Love numbers in GR via matching calculations

Extracting Love

- Match EFT to GR - a UV theory!



Example: (in local asympt. rest frame)

GR:

$$\delta g_{00}\Big|_{\mathrm{GR}}(x) = \frac{BR^2}{r^3}$$

EFT:

$$\delta g_{00}\Big|_{\mathrm{EFT}}(x) = \frac{\lambda_2 R^2}{r^3}$$

- After matching, λ_2 can be used in other calculations, Universality, Predictability!

Doubling down: matching via amplitudes

One can use any observable to match

- "Newtonian potential" = EFT off-shell 1-pf
- Can match in appropriate coordinates/frames

e.g. Poisson (2021a,b)

- Not easy to link it with waveform, e.g. DLN
- Alternative: matching gauge-invariant on-shell scattering amplitudes

Free of gauge, coordinate, and field-redefinition

Universality: once LN are matched, compute GW

Amplitudes: Why is the sky blue ?

Scattering of light by neutral atoms (~ nitrogen in the air):

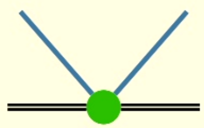
$$S_{\rm eff} = Q \int ds\, A_0 + \chi \int ds\; E_i E^i + \ldots \qquad \chi \sim [\mathrm{cm}]^3$$

$$i\mathcal{A} \sim \chi \omega^2$$

Scattering of GWs by a compact object:

$$S_{\rm eff} = -m \int ds\; + \lambda_2 R^5 \int ds\; E_{\mu\nu} E^{\mu\nu}$$

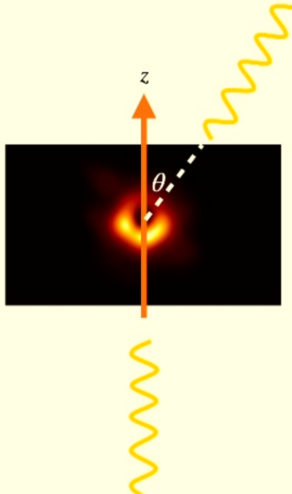
$$i\mathcal{A} =$$

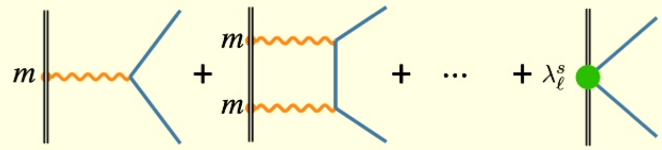


$$\sim \lambda_2 R^5 \omega^4$$

Goldberger, Rothstein (2004,2007)
Zhou, MI (2022)

Matching LN via Raman scattering amplitudes





PM expansion: $G + G^2 + \ldots$

$$\lambda_\ell^s \sim R^{2\ell+1} \propto G^{\frac{2\ell}{D-3}+1}$$

Analytic continuation in angular momentum (Gribov-Froissart) separates LN and PM

$$i\mathcal{A} \sim (Gm) + (Gm)^2\omega + \ldots + (Gm)^n\omega^{n-1} + (Gm)^{2\ell+1}\omega^{2\ell} + \ldots$$

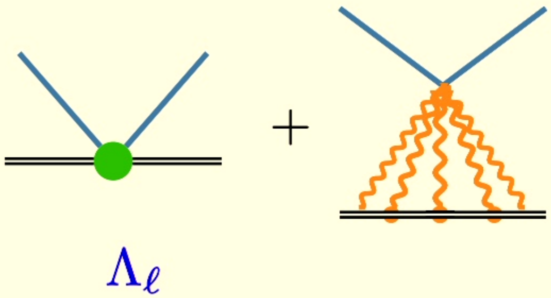
N.B. the limit $\ell \to n \in \mathbb{N}$ leads to singularities if LNs run (there's an actual mixing between LN and PM corrections)

Zhou, MI'22

MI, Li, Parra-Martinez, Zhou'24

Wilsonian naturalness (dimensional analysis):

- Expect $\Lambda_\ell = \mathcal{O}(1)$
- Expect (classical) RG running



- For black holes we can solve the static tidal problem (scattering) in GR exactly

What is Love ? (Schwarzschild)

- D=4: $\Lambda_\ell = 0$ (True for all fields: EM, gravity, scalars)
- Higher dims:

Fang, Lovelace (2005)
Binnington, Poisson (2009)

$$\hat{\ell} = \frac{\ell}{D-3} \qquad \Lambda_\ell = \frac{\hat{\ell}}{\hat{\ell}+1}\frac{\Gamma^2(\hat{\ell})\Gamma^2(\hat{\ell}+2)}{2\pi\,\Gamma(2\hat{\ell}+1)\Gamma(2\hat{\ell}+2)}\tan\pi\hat{\ell}$$

Vanishes for integer $\hat{\ell}$ Runs for half-integer $\hat{\ell}$

Constant otherwise $\hat{\ell}$

Kol, Smolkin (2010) Hui++(2020)
Charalambous (2024)

- This behavior seems very "unnatural" in dimensional analysis

Kerr Love

BH spin: ~~SO~~$(3) \supset SO(2)$

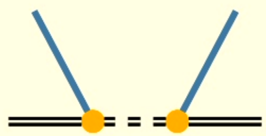
$$S = -m\int ds + r_s^5 \int \lambda^{ijkl}\partial_i\partial_j\Phi\partial_k\partial_l\Phi + \int \mathcal{O}^{ij}\partial_i\partial_j\Phi$$

internal (horizon) DOFs

$$\Phi_{\text{resp}} \sim (\lambda + \langle\mathcal{O}\mathcal{O}\rangle)\Phi_{\text{ext}} \sim \nu(\partial_t + \Omega\partial_\varphi)\Phi_{\text{ext}}$$

Static dissipative response
due to frame dragging !

Ω angular velocity
φ azimutal angl.

$\sigma_{\text{abs}} =$  $\propto (\omega - m\Omega)$ Superradiance

$\text{Im}\langle\mathcal{O}\mathcal{O}\rangle$

can be matched to
inelasticity parameters!

Kerr Love

BH spin: $\cancel{SO}(3) \supset SO(2)$

$$S = -m \int ds + r_s^5 \int \lambda^{ijkl} \partial_i \partial_j \Phi \partial_k \partial_l \Phi + \int \mathcal{O}^{ij} \partial_i \partial_j \Phi$$

internal (horizon) DOFs

$$\Phi_{\text{resp}} \sim (\lambda + \langle \mathcal{O}\mathcal{O} \rangle)\Phi_{\text{ext}} \sim \nu(\partial_t + \Omega \partial_\varphi)\Phi_{\text{ext}}$$

Static dissipative response
due to frame dragging !

Ω angular velocity
φ azimutal angl.

- Kerr Love vanishes
  $\Lambda_\ell = 0$
  Charalambous, Dubovsky, MI (2021)
  see also Chia (2020), Le Tiec++(2020)

- Used in NSBH waveform
  Ligo/Virgo/Kagra 2106.15165

Dynamical Love numbers

- Problem: RG running of LNs (PN loops + local counterterms)
- EFT gives a consistent definition via matching (~coupling running)
- Dynamical LNs of BHs are not zero and run *Chakrabarti ++'13, MI ++'21*

$$C_{\dot{E}^2}\int d\tau\ \dot{E}^{ij}\dot{E}_{ij} \qquad\qquad C_{E\dot{E}}\int d\tau\ \varepsilon^{ijk}\hat{S}_i E_{jl}\dot{E}^l_k$$

- Running can be matched analytically with FG, e.g.

$$\frac{dC_{\dot{E}^2}}{d\log\mu}=\frac{32}{45}m^7G^6 \qquad\qquad \frac{dC_{E\dot{E}}}{d\log\mu}=-\frac{32}{45}\frac{a}{m}m^6G^5$$

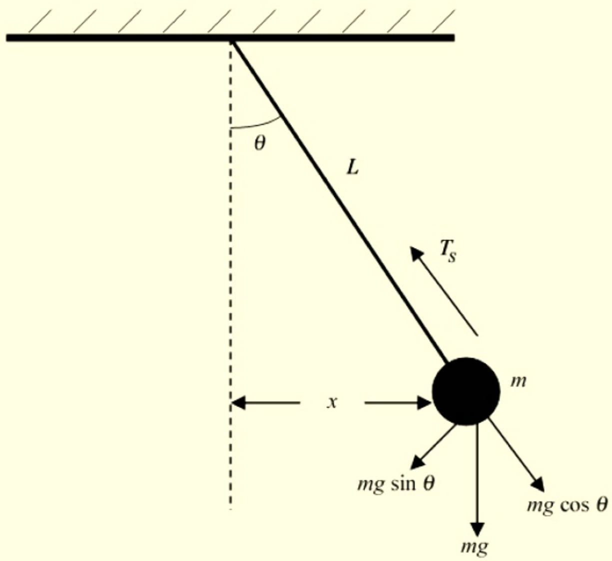
Saketh, Zhou, MI' 23

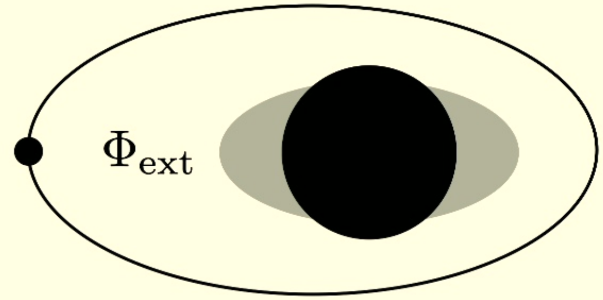
- Or extracted fully from Raman process amplitudes:

$$C_{0,\omega^2}(\bar{\mu})^{\overline{\rm MS}}=-4\pi r_s^3\left[\frac{1}{4\epsilon_{\rm UV}}+\ln(\bar{\mu}r_s)+\frac{35}{24}+\gamma_E\right] \qquad \sim\int\dot{\phi}^2$$

MI, Li, Parra-Martinez, Zhou'24

Naturalness problem





Given: g, L

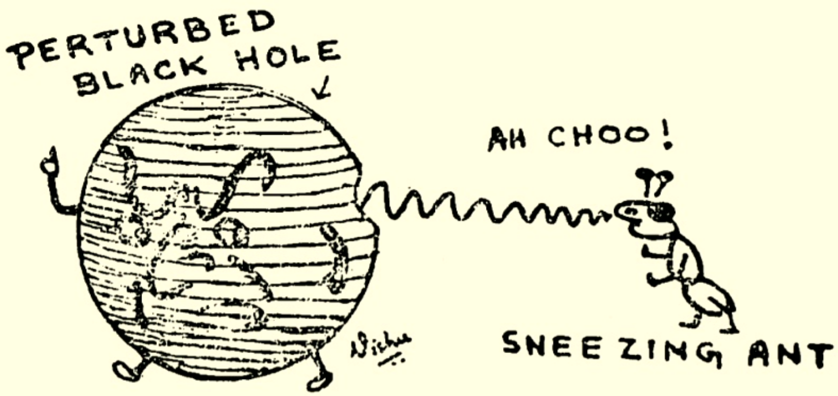
$$T \sim \sqrt{\frac{L}{g}}$$

Given: G_N, R, m

Expect: $\lambda_2 \sim \frac{m}{R} \sim 1$

Find: $\lambda_2 \sim \left(\frac{l_P}{R}\right)^4 \ll 1$

Microscopic calculation: BH perturbations



(c) C. V. Vishveshwara

$\Box\phi = 0$ in Kerr

$$\phi = \Phi(t, r, \varphi) S(\theta) = R(r) S(\theta) e^{-i\omega t + im\varphi}$$

Teukolsky (1972)

$$\partial_r(\Delta(r)\partial_r R) + V(\omega, m, r) R = AR$$

$$\Delta = (r - r_+)(r - r_-)$$

r_+, r_- outer (inner) horizons

$$r_\pm = M \pm \sqrt{M^2 - a^2}$$

$$A = \ell(\ell + 1) + \mathcal{O}(\omega\Omega)$$

angular eigenvalues

Static Solution

$$R = \left(\frac{r - r_+}{r - r_-}\right)^{iZ} {}_2F_1\left(-\ell, \ell + 1; 1 + 2iZ; \frac{r_+ - r}{r_+ - r_-}\right)$$

$$Z = \frac{r_+^2 + a^2}{r_+ - r_-} m\Omega$$

- Vanishing of Love numbers follows from quasi - polynomial form of R:

$$R\Big|_{r\to\infty} \propto \underbrace{r^\ell + r^{\ell-1} + ... + 1}_{\text{growing mode}} + \underbrace{0}_{\text{decaying tail}}$$

- The same form of the solution holds for near zone (~ low frequency)

Scattering: near zone expansion

Starobinsky (1965), Maldacena, Strominger (1997), Castro, Maloney, Strominger (2010)

near zone

$\omega r \ll 1$

r

near horizon region

$r \sim r_+$

far zone

$r \gg r_+$

At low frequencies, near zone covers horizon and overlaps with the asymptotically far region

$$V = V_0 + \epsilon V_1$$

- Formal parameter $\epsilon = 1$ In what follows $\epsilon = 0$
- $V_0 \gg V_1$ in the near zone ~ multipole expansion
- $V_1 = 0$ exact at zero frequency

Let's choose the split as follows

$$V_0 = \frac{(2Mr_+)^2}{\Delta}\left((\omega - \Omega m)^2 - 4\omega\Omega m \frac{r - r_+}{r_+ - r_-}\right)$$

$$\beta = \frac{4Mr_+}{r_+ - r_-}$$

$$\beta = (2\pi T_H)^{-1}$$

$$V_1 = \frac{2M(\omega a m \beta + 4M^2\omega^2 r_+)}{r_+(r - r_-)} + \omega^2(r^2 + 2Mr + 4M^2)$$

$$\phi \propto e^{-i\omega t + i m\varphi}$$

… and a symmetry appears

$$L_0 = -\beta\partial_t\,,$$

$$L_{\pm 1} = e^{\pm\beta^{-1}t}\left(\mp\Delta^{1/2}\partial_r + \beta\partial_r(\Delta^{1/2})\partial_t + \frac{a}{\Delta^{1/2}}\partial_\phi\right)$$

Love symmetry

$$L_0 = -\beta\partial_t\,,$$

$$L_{\pm 1} = e^{\pm\beta^{-1}t}\left(\mp\Delta^{1/2}\partial_r + \beta\partial_r(\Delta^{1/2})\partial_t + \frac{a}{\Delta^{1/2}}\partial_\phi\right)$$

$a = J/M$
$\beta = (2\pi T_H)^{-1}$

- Regular at the horizon
- Satisfy SL(2,R) algebra

$$[L_n, L_m] = (n-m)L_{n+m}\,, \quad n, m = -1, 0, 1\,.$$

- Near zone Teukolsky can be rewritten

$$\mathcal{C}_2\Phi = \ell(\ell+1)\Phi$$

Casimir $\mathcal{C}_2 \equiv L_0^2 - \frac{1}{2}(L_{-1}L_1 + L_1L_{-1})$

- All properties of Love numbers follow from SL(2,R) representation theory

The Black Hole Is the Atom of the 21st Century

- R. Dijkgraaf (and others)

https://www.ias.edu/ideas/dijkgraaf-EHT-black-hole

Solving black hole perturbations
just like the Hydrogen atom!

SO(3)xSO(3) (Runge-Lenz)

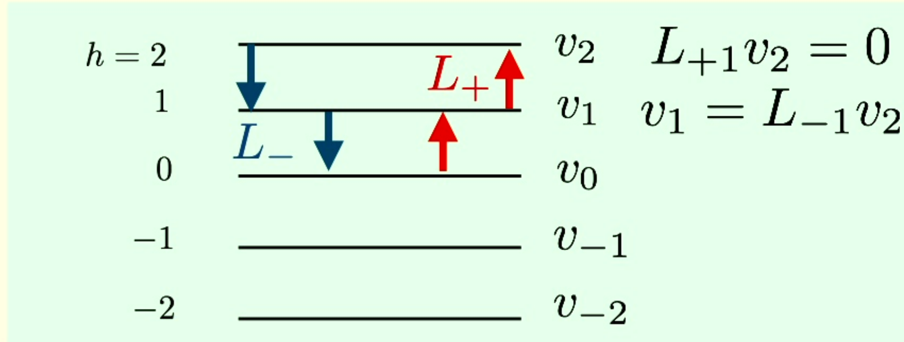
Highest weight banishes Love

each vector in SL(2,R)
has a weight h

$$L_0 v_h = -h v_h$$

$$L_0 = -\beta \partial_t$$

Static $h = 0$



Integer ℓ → Static solution belongs to a highest weight rep

As such, $v_0 = L_{-1}^{\ell} v_\ell$ → $L_{+1}^{\ell+1} v_0 = L_{+1} v_\ell = 0$

→ $L_{+}^{\ell+1} v_0 \propto \partial_r^{\ell+1} v_0 = 0$

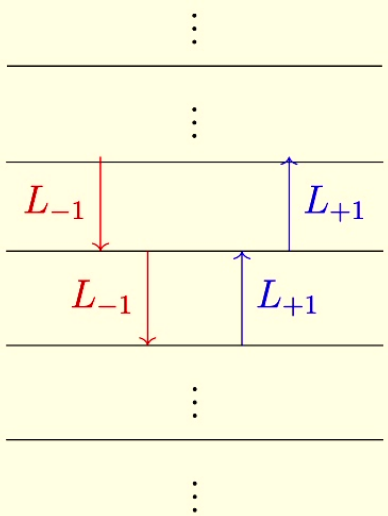
v_0 Polynomial in r

$v_{0,\,\text{Guess}} \propto r^{\ell} + \ldots + \frac{\text{Love}}{r^{\ell+1}}$ but we have $v_0 \propto r^{\ell} + \ldots + r^0$

→ Love is zero

- Higher dimensions: generic

$$\hat{\ell} = \frac{\ell}{D-3}$$



Infinite reps, no running because of the EFT power counting

Charalambous (2024)

- $\hat{\ell}$ half-integer

Infinite reps, EFT power counting allows running!

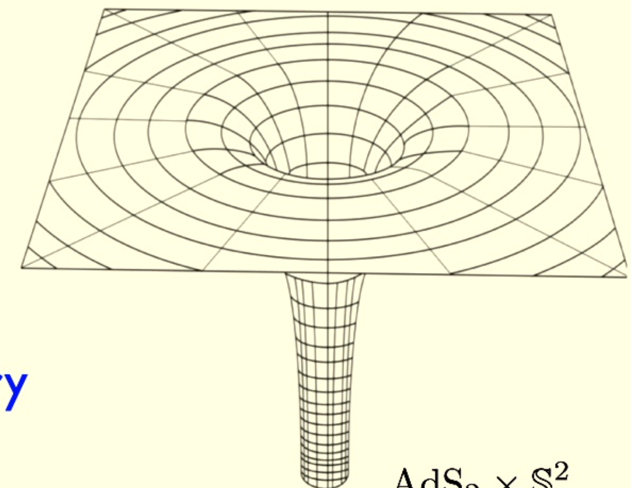
cf. $\hat{\ell}$ integer, Love symmetry forbids running, but EFT OK

Charalambous, Dubovsky, MI (2022)

Love and Near horizon isometries (NHE)

- The story is identical for charged BH. In the extremal limit $Q = M \; (T_H = 0)$ Love symmetry reduces to the near-horizon isometry

$$\lim_{Q \to M} SL(2,\mathbb{R})_{\text{Love}} = SL(2,\mathbb{R})_{\text{NH}}$$

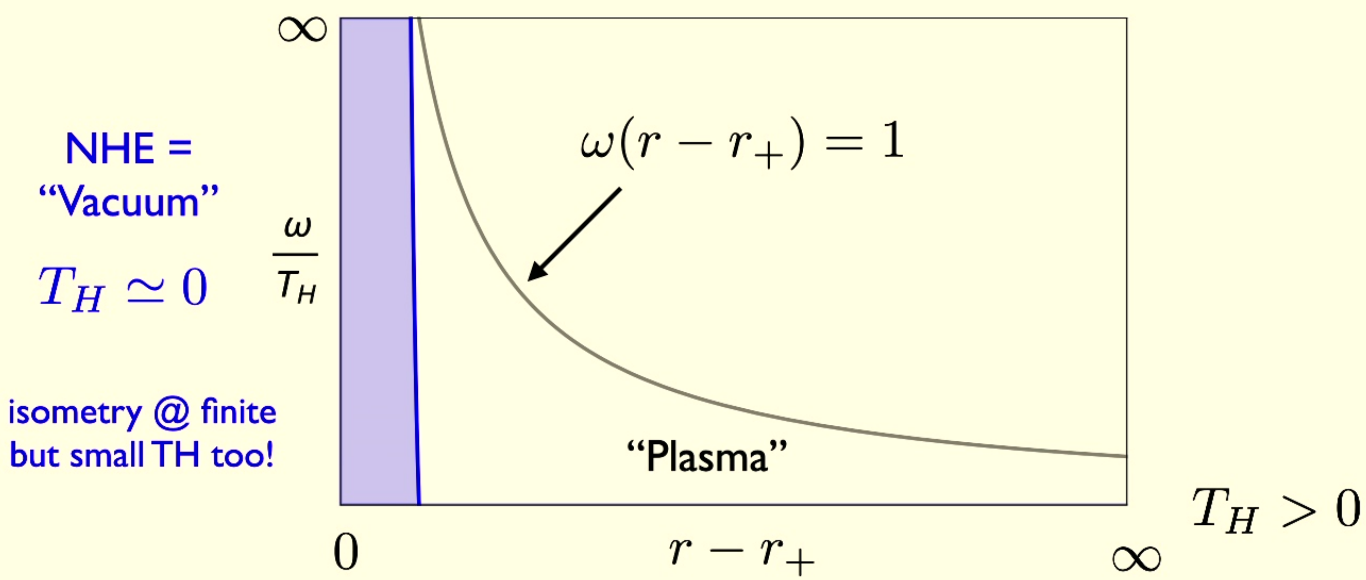


$\mathrm{AdS}_2 \times \mathbb{S}^2$

- Love symmetry = broken near-horizon isometry (holography!)

Regime of validity ~ holography

Params: $T_H, \omega, (r - r_+)$



- Love symmetry exists as a consequence of NHE for 2d BH solutions
- Love symm = broken NHE

The world of Love

- Kerr black holes have more symmetries, e.g. Starobinsky

$$L_0^{\mathrm{Star}} = -\beta\left(\partial_t + \Omega\partial_\phi\right) ,$$

Charalambous, Dubovsky, MI (2021)

$$L_{\pm1}^{\mathrm{Star}} = e^{\pm t/\beta}\left[\mp\sqrt{\Delta}\partial_r + \partial_r\left(\sqrt{\Delta}\right)\beta\left(\partial_t + \Omega\partial_\phi\right) \mp s\frac{r - r_\mp}{\sqrt{\Delta}}\right]$$

- Infinte Love: $\mathrm{SL}\left(2,\mathbb{R}\right) \ltimes \hat{U}\left(1\right)_{\mathcal{V}}$

$$L_a \to L_a + \alpha v_a \partial_\varphi$$

$$v_a \in SL(2,\mathbb{R})$$

- Love symmetries in higher dimensions

Charalambous (2024)

Charalambous, MI (2023)

- Love symmetries of 5D Myers-Perry black holes

$$\mathrm{SL}(2,\mathbb{R}) \ltimes \hat{U}(1)^2$$

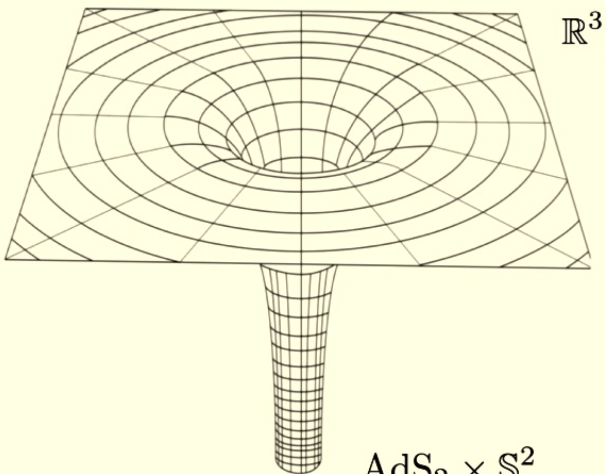
LNs are not zero, unless BHs are equirotating or perturbations have equal magnetic numbers

- Love symmetries of p-branes: NE isometries

Charalambous, MI (to appear)

e.g. p=1 $\quad \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R}) \simeq \mathrm{AdS}_3$

Kerr isometries



$$\mathrm{AdS}_2 = SL(2,\mathbb{R})$$

Bardeen, Horowitz (1998)

$$Q = M$$

RN: $\lim_{Q\to M} SL(2,\mathbb{R})_{\text{Love}} = SL(2,\mathbb{R})_{\text{NH}}$

$$J = M^2$$

Kerr: $\lim_{a\to M} SL(2,\mathbb{R})_{\text{Love}} \ltimes \hat{U}(1) \supset SL(2,\mathbb{R})_{\text{NH}}$

$$L_a \to L_a + \alpha v_a \partial_\varphi \qquad v_a \in SL(2,\mathbb{R})$$

"infinite Love": Love, Starobinsky, and NH algebras

- Could be a larger symmetry structure for Schwarzschild ?

Hui, Joyce, Penco, Santoni, Solomon (2021)

Kol, Guevarra (2023)

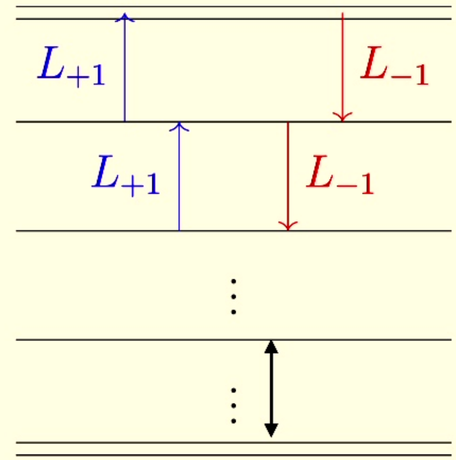
Triumph of Naturalness?

- UV miracle!

- Symmetry between massive (UV) states ("QNMs") and static solutions (IR)

$$L_0 = -\beta\partial_t\,,$$
$$L_{\pm 1} = e^{\pm\beta^{-1}t}\left(\mp\Delta^{1/2}\partial_r + \beta\partial_r(\Delta^{1/2})\partial_t + \frac{a}{\Delta^{1/2}}\partial_\phi\right)$$

Love mixes IR and UV



$$\Delta E = \beta^{-1} \sim M$$

- Massive states are integrated out in pp EFT

- IR/UV mixing as a solution to CC problem - ?

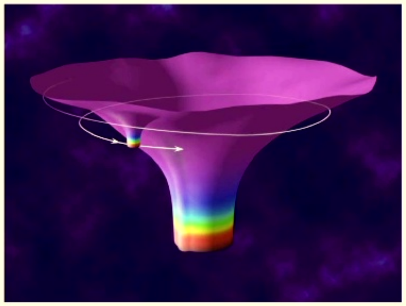
Practical applications

- Love symmetry is approximate ~ Chiral symmetry

$$V = V_0 + \epsilon V_1$$

~ Love spurion

- Analogs of Gell-Mann-Okubo relations for time-dependent tidal responses and QNMs!
- High order solutions of the Teukolsky equation $\mathcal{O}((\omega M)^n)$
- Extreme mass-ratio inspirals with LISA
- Precision waveform calculations!



Credit: http://bhptoolkit.org

Summary

- ✓ Love numbers - worldline EFT Wilson coefficients
- ✓ Scattering amplitudes help extract them
- ✓ Love Symmetry explains vanishing of Love numbers
- ✓ Holographic interpretation via NH isometries
- ✓ A new tool for precision GW science!

Thank you!