Title: How fast can one route quantum states

Speakers: Chao Yin

Series: Perimeter Institute Quantum Discussions

Date: March 20, 2024 - 11:00 AM

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Abstract: Many quantum platforms naturally host Hamiltonians with power-law or even all-to-all connectivity, which may potentially process quantum information in a way much faster than conventional gate-based models. For such non-geometrically-local Hamiltonians, it is then important to both come up with fast protocols and understand the ultimate limit for realizing various information processing tasks. In this talk, I will first overview this quantum speed limit topic, and then dive into the particular task of quantum routing, i.e. permuting unknown quantum states on the qubits. I aim to show [1] a provably optimal Hamiltonian routing protocol on the star graph that is asymptotically faster than gate-based routing; [2] a lower bound on the time to realize the shift unitary using 1d power-law interactions which, perhaps surprisingly, can be much slower than the time for the conventional Lieb-Robinson light cone to spread across the whole system. The latter result shares interesting connections to the classification of 1d quantum cellular automata and symmetry-protected topological order.

Zoom link

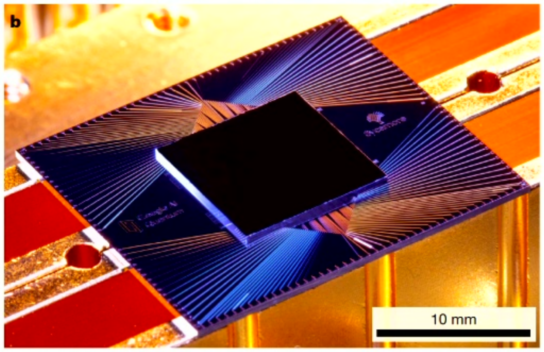
How fast can one route quantum states?

Chao Yin

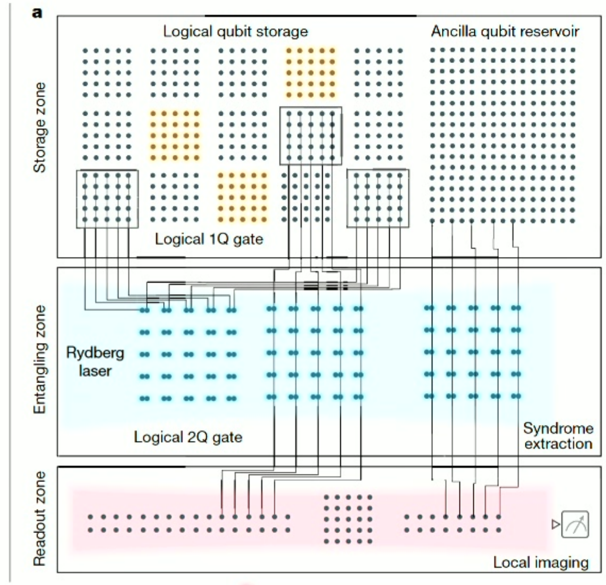
March 20, 2024, at Perimeter Institute

Boulder

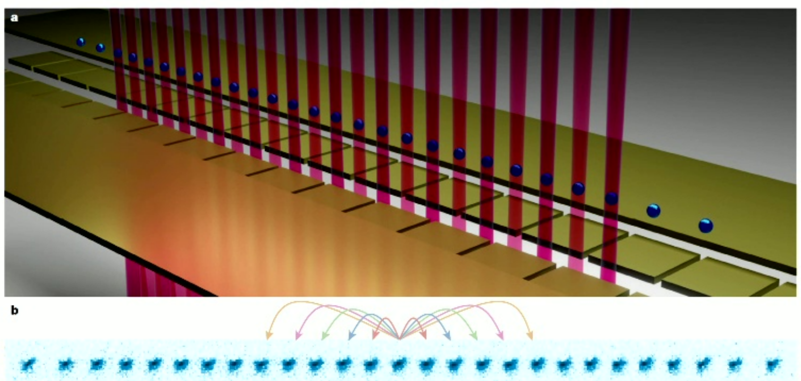
Quantum processors



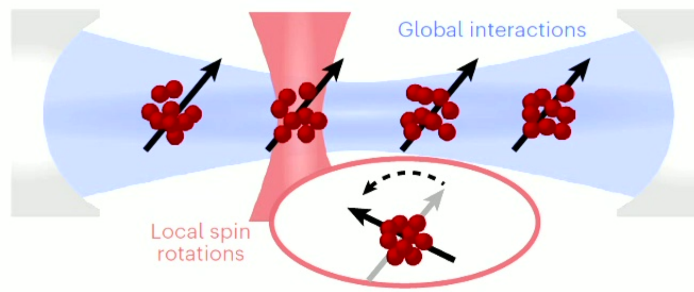
Google's superconducting Sycamore processor



Rydberg atom processor in Lukin group



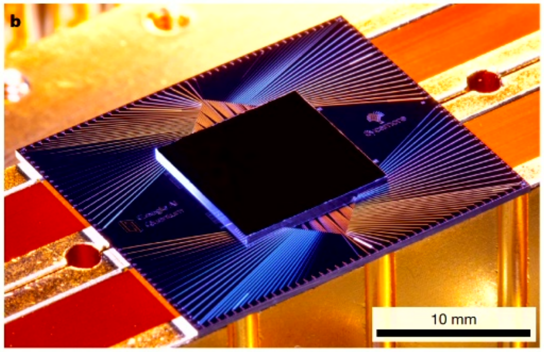
Trapped-ion quantum simulator in Monroe group



Generating entanglement in optical cavity in Schleier-Smith group

1/19

Quantum processors



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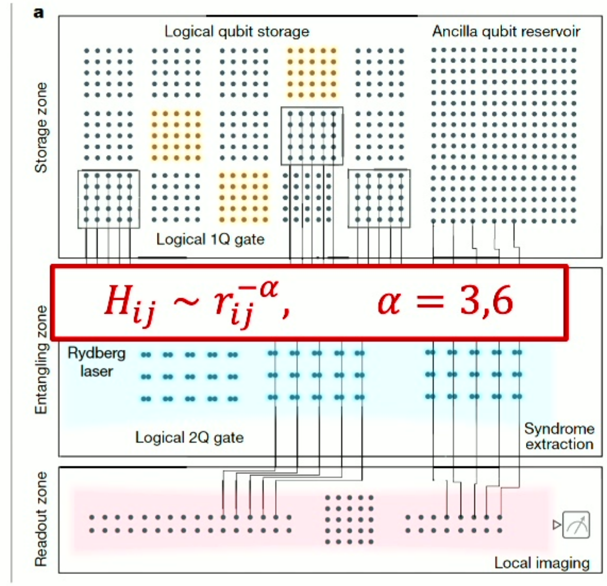

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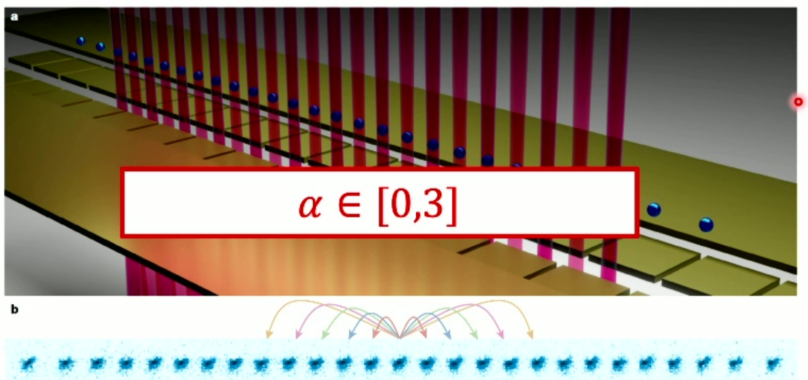

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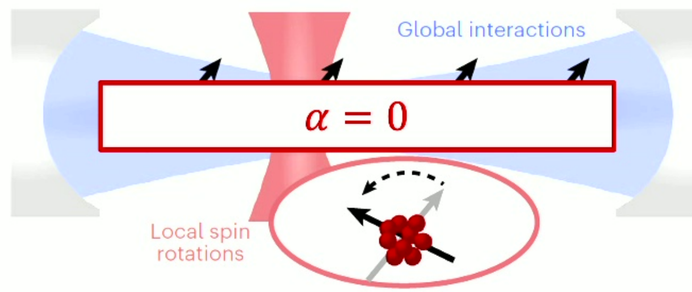

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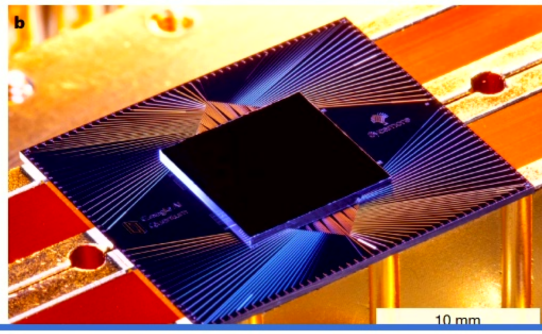
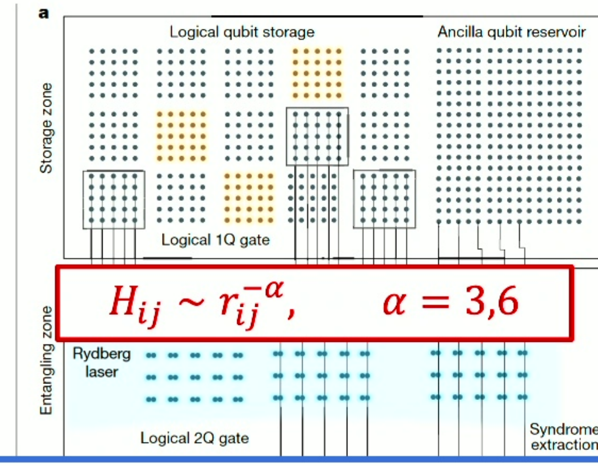
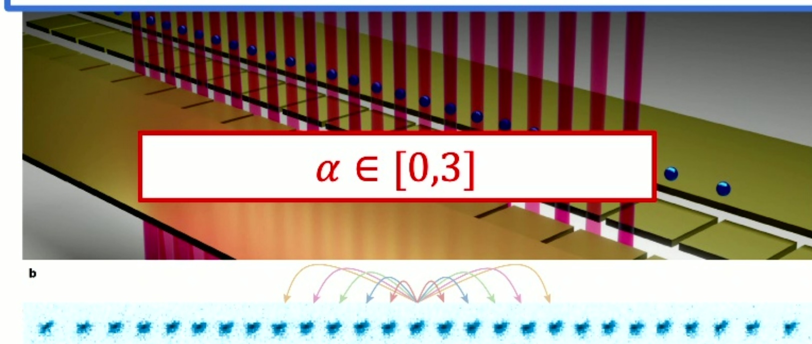




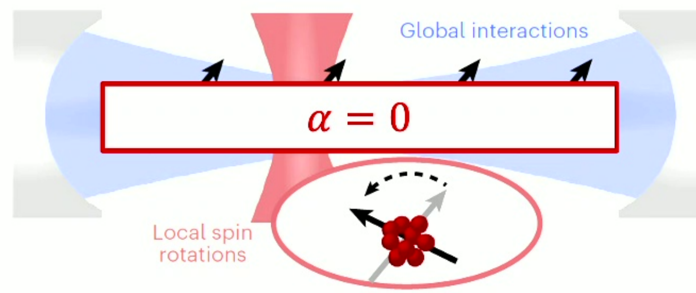
$H_{ij} \sim r_{ij}^{-\alpha}, \qquad \alpha = 3,6$

Question: given Hamiltonian connectivity, how fast can one realize certain quantum information processing task? "quantum speed limit"



$\alpha \in [0,3]$

Trapped-ion quantum simulator in Monroe group



$\alpha = 0$

Generating entanglement in optical cavity in Schleier-Smith group

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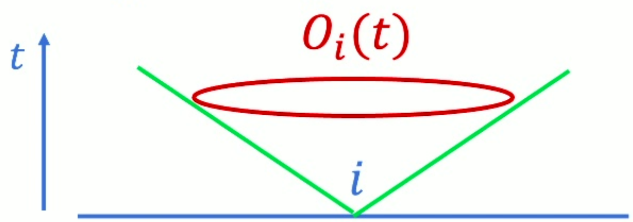
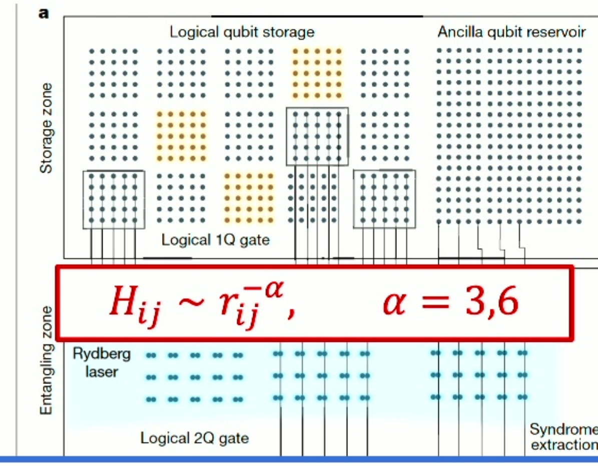
Quantum processors

Geometrically local H: light cone as in circuits





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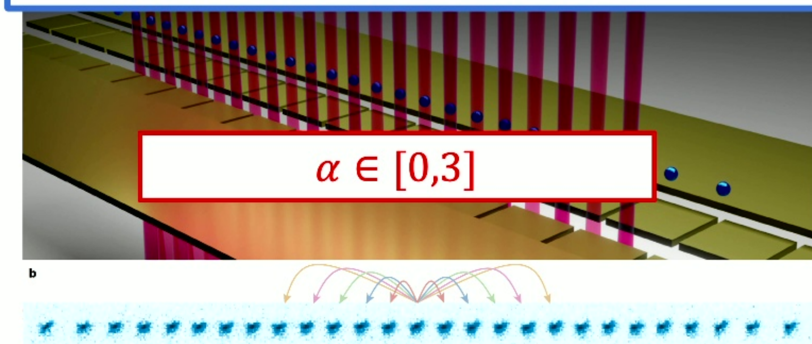

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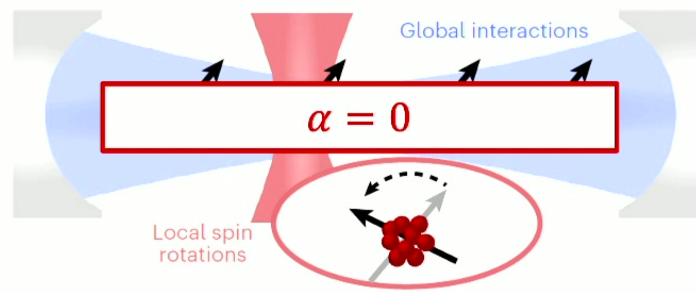

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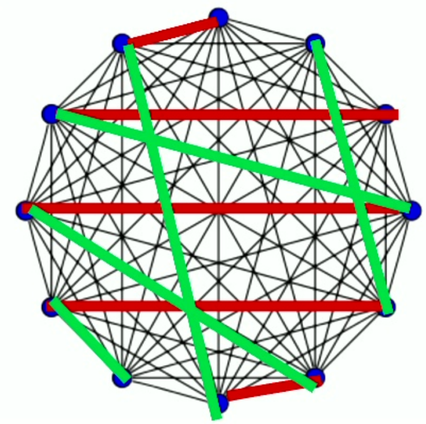
Generating entanglement in optical cavity in Schleier-Smith group

"Hamiltonian advantage"



- All-to-all Hamiltonian: $H(t) = \sum_{ij} J_{ij}^{\alpha\beta}(t) X_i^\alpha X_j^\beta$, $\left\| J_{ij}^{\alpha\beta}(t) \right\| \leq 1$
- Gate-based: e.g. $t = 1, 2, \cdots$
- State preparation task: $U|\text{product}\rangle = |\psi\rangle$, minimal $T = ?$

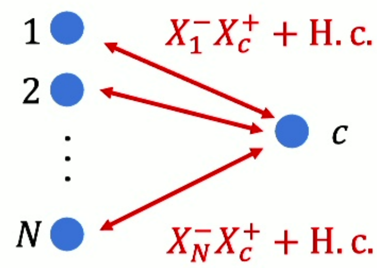
$\lvert\psi\rangle$	Gate $U = U_T \cdots U_1$	Hamiltonian $U = \mathcal{T}\, e^{-i\int_0^T H(t)dt}$
$\lvert GHZ\rangle = \frac{1}{\sqrt{2}}(\lvert 0\cdots 0\rangle + \lvert 1\cdots 1\rangle)$	$\Theta(\log N)$ [1]	$O(1)$ [2]
$\lvert W\rangle = \frac{1}{\sqrt{N}}(\lvert 10\cdots 0\rangle + \lvert 010\cdots 0\rangle + \lvert 00\cdots 1\rangle)$	$\Theta(\log N)$	$O(1/\sqrt{N})$ [3]

[1] Gate lower bound $\Omega(\log N)$: light cone argument
[2] GHZ-like state $T = O(\frac{\log N}{N})$: X. Zhang, Z. Hu and Y.-C. Liu (2023)
[3] A.Y. Guo, ..., A.V. Gorshkov and Z.-X. Gong (2020)

W state $T = O(1/\sqrt{N})$ protocol

A.Y. Guo, …, A.V. Gorshkov and Z.-X. Gong (2020)

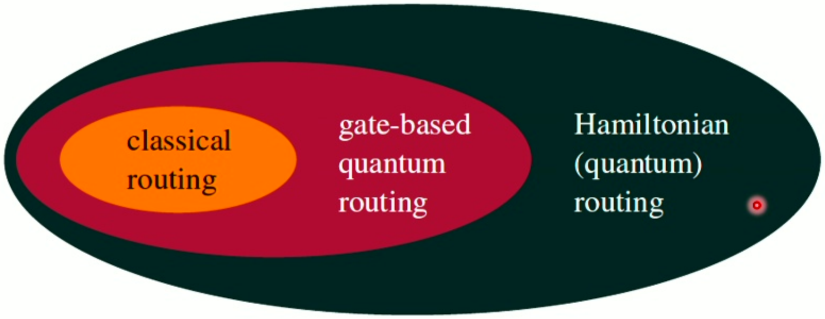
- Start from $|1\rangle_c \otimes |0 \cdots 0\rangle_{1\cdots N}$
- Goal: $|0\rangle_c \otimes |W\rangle_{1\cdots N}$



- $X^+ = |0\rangle\langle 1| = (X^-)^\dagger$
- $H = X_c^+ \sum_i X_i^- + \mathrm{H.c.} = \sqrt{N} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ in the 2d subspace
- $T \sim 1/\sqrt{N}$
- Only use a tiny amount of the couplings
- Open question: Hamiltonian lower bound?

Quantum routing task

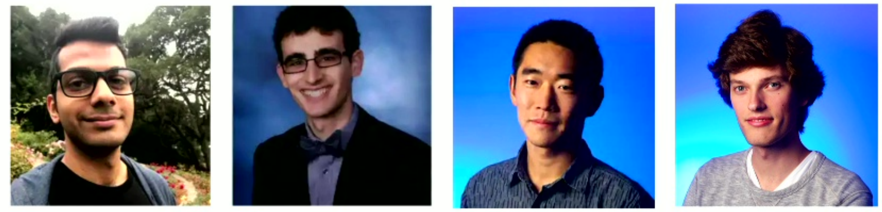
- Realize a target unitary instead of state
- Target unitary U_p: certain permutation of qubits
  - Distributed quantum computation
- Fast Hamiltonian protocol $U_p = \mathcal{T}\, e^{-i\int_0^T H(t)dt}$
  - Separation with gate-based routing?
- Hamiltonian lower bound?
  - Generalize to other tasks?



A. Bapat, A.M. Childs, A.V. Gorshkov and E. Schoute (2023)

Outline/ main results

- Routing on the star graph (ongoing work)
  - Hamiltonian protocol $T = O(\sqrt{N}) \ll T_{gate}$
  - Bound* $T = \Omega(\sqrt{N})$



Dhruv Devulapalli Adam Ehrenberg Andrew Guo Eddie Schoute

Alexey Gorshkov Andrew Childs Andrew Lucas

- The shift unitary in 1d power-law systems (2402.07992)
  - Bound $T \gg$ signaling time across the system
  - Connection to SPT

Andrew Lucas David T. Stephen

- Discussions & outlook

ALFRED P. SLOAN FOUNDATION

5/19

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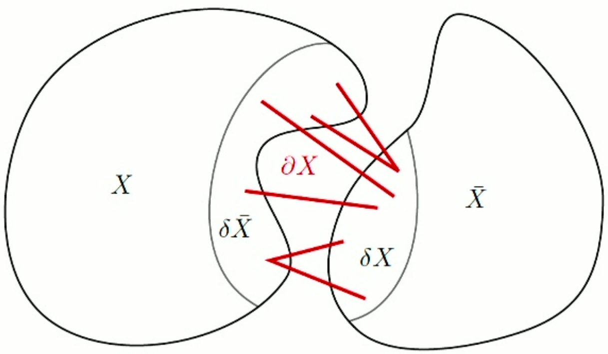
Separation in star graph?



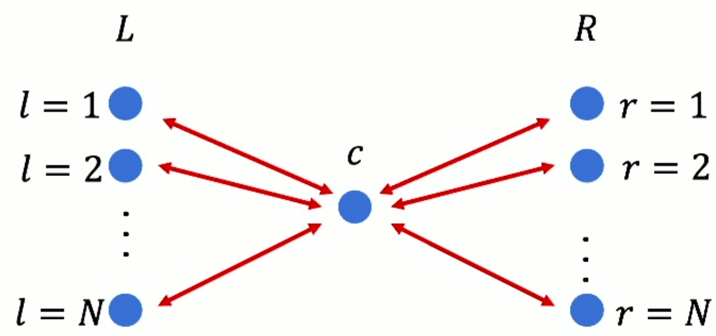
- Theorem: for qubits (fermions) on the vertices Λ of a simple graph

$$T_{gate} \geq \max_{X \subset \Lambda, |X| \leq |\Lambda|/2} \frac{|X|}{2|\delta X|} - 1$$

- Desire bottleneck effect

A. Bapat, A.M. Childs, A.V. Gorshkov and E. Schoute (2023)

- Star graph: e.g. Nitrogen vacancy center + surrounding spins

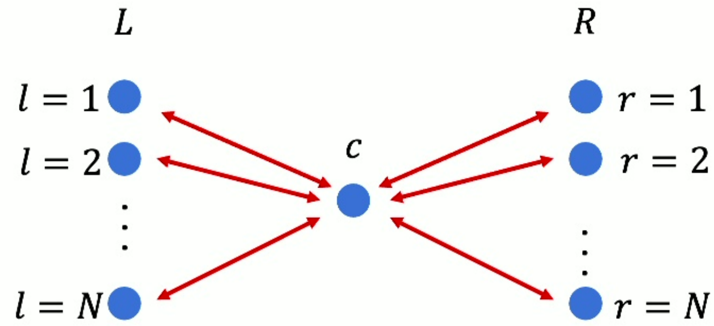


- $T_{gate} = \Omega(N)$
- permuting left N leaves with right N leaves

$$U_p = SWAP_{l=1,r=1} \otimes SWAP_{l=2,r=2} \otimes \cdots$$

Main result 1: $T = O(\sqrt{N})$ Fermionic protocol

- Idea: W-state protocol for fermions
- $H^L_{k=N} = a_c^+ \sum_l a_l + \mathrm{H.c.} = \sqrt{N} a_c^+ b_{k=N} + \mathrm{H.c.}$ Swaps $a_c \leftrightarrow b_k$
- Momentum fermion operators $b_k = \frac{1}{\sqrt{N}} \sum_l e^{\frac{2\pi i}{N} kl} a_l$ $(k = 1, \cdots, N)$



7/19

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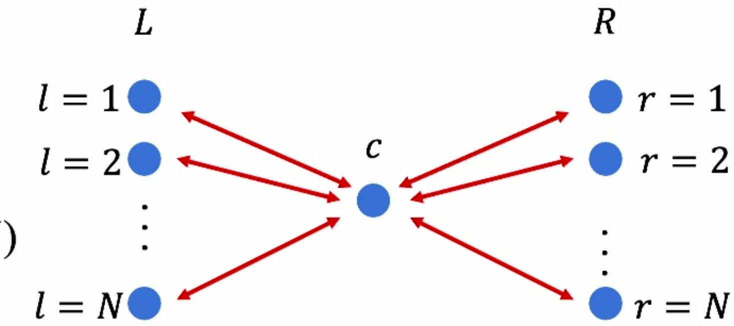
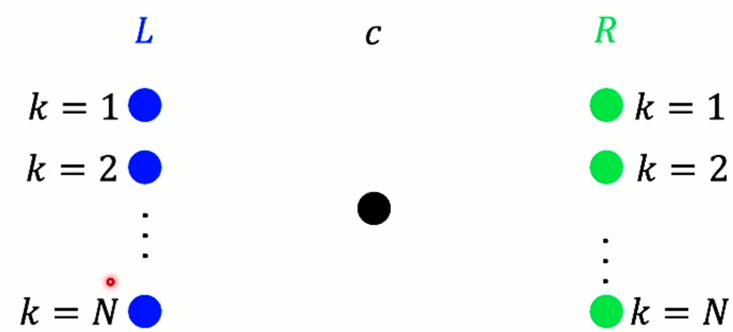


- Permute momentum modes one-by-one: $T = O(\sqrt{N}) \ll \Omega(N)$, separation!



7/19

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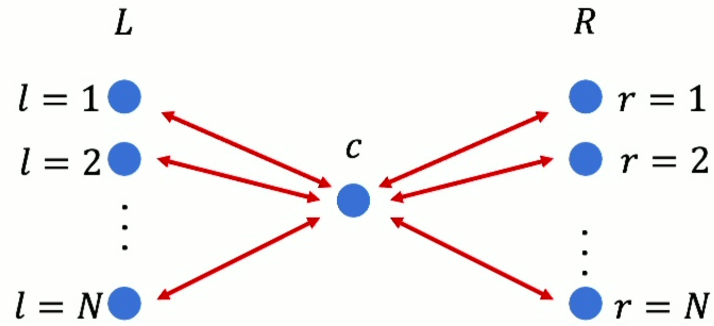
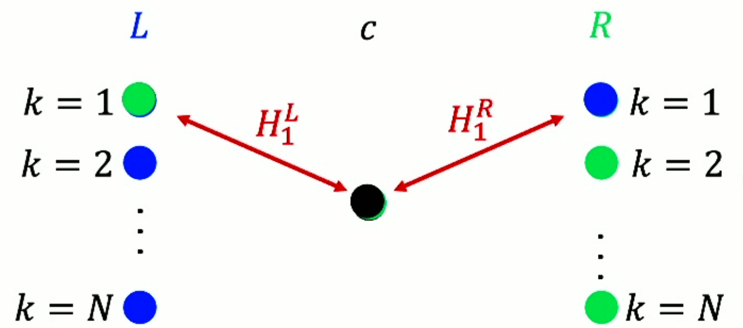


- Permute momentum modes one-by-one: $T = O(\sqrt{N}) \ll \Omega(N)$, separation!
- Also works for one bosonic mode per site
- Open question: qubit protocol?



7/19

Hamiltonian lower bound

- $T = \Omega(1)$ from small incremental entangling theorem *A. Bapat, A.M. Childs, A.V. Gorshkov and E. Schoute (2023)*
  - $T = O(1/N)$ for creating 1 bit of entanglement between L, c
  - $e^{-\frac{i\pi}{2N}(Z_L - N)X_c}|GHZ\rangle_L \otimes |0\rangle_c = |GHZ\rangle_{L,c}$

- Our main result 2:

Theorem: lower bound for star graph routing

For a star graph Hamiltonian $H(t)$ that is piecewise constant:

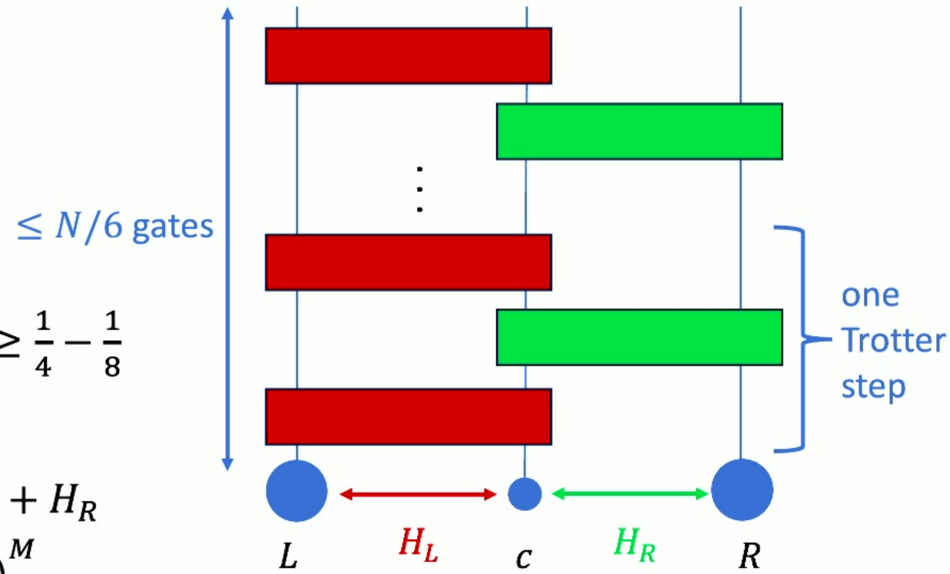
$$H(t) = \begin{cases} H^{(1)}, & t \in [0, t_1) \\ H^{(2)}, & t \in [t_1, t_1 + t_2) \\ \cdots & \end{cases}$$

If the time windows $t_j = \Omega\left(1/\sqrt{N}\right)$, $\forall j$, then routing needs $T = \Omega(N^{\frac{1}{2}-\epsilon})$ where $\epsilon \to 0^+$

- holds for qubits and fermions

Proof sketch: idea

- Approximate $U = e^{-iTH}$ by a "circuit" $\widetilde{U}$, and show 2 things:
  - U is "close" to $\widetilde{U}$, i.e. approximation is good (Hamiltonian simulation)
  - $\widetilde{U}$ is "far" from target U_p
- Frobenius distance $\|A\|_F^2 := 2^{-N}\mathrm{Tr}(A^\dagger A)$
  - If $\|U - \widetilde{U}\|_F \leq \frac{1}{8}$, $\|\widetilde{U} - U_p\|_F \geq \frac{1}{4}$
  - Then $\|U - U_p\|_F = \|(\widetilde{U} - U_p) + (U - \widetilde{U})\|_F \geq \frac{1}{4} - \frac{1}{8}$
- $\widetilde{U}$: k-th-order Trotter-Suzuki expansion for $H = H_L + H_R$
  - E.g. 2nd order $e^{-iTH} \approx \left(e^{-\frac{iT}{2M}H_L} e^{-\frac{iT}{M}H_R} e^{-\frac{iT}{2M}H_L}\right)^M$
  - $M \sim N, k \sim$ large constant

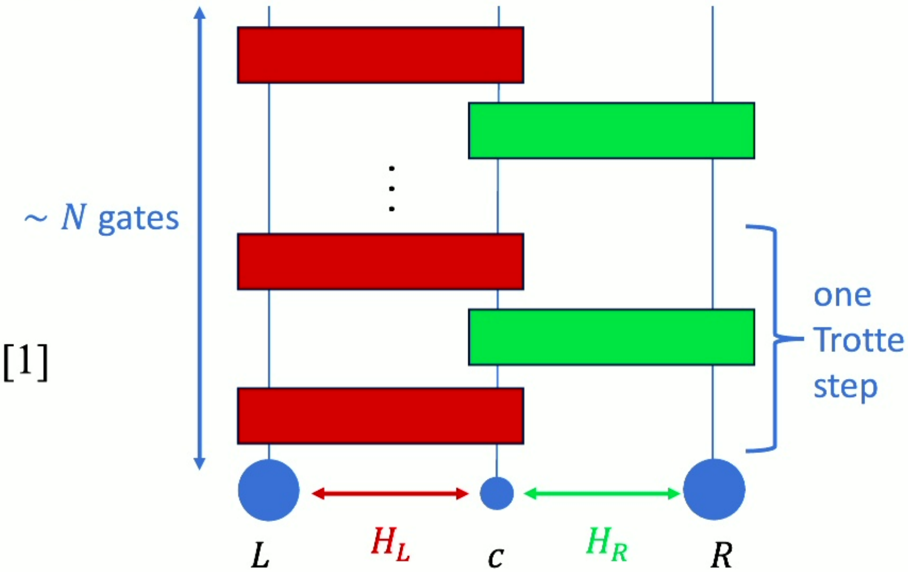


Proof sketch: Frobenius Trotter error

- Goal: if $T \ll \sqrt{N}$, $\|U - \widetilde{U}\|_F \leq 1/8$
- $\|U - \widetilde{U}\|_F \sim N \|e^{-itH} - \widetilde{U}_t\|_F$ where $t \ll 1/\sqrt{N}$
- $\|e^{-itH} - \widetilde{U}_t\|_F \sim t^{k+1} \left\| \left[H_L, \cdots, [H_R, [H_L, H_R]] \right] \right\|_F$ [1]

$$\sim \left(t\sqrt{N}\right)^{k+1}$$

$k+1$ commutators



[1] Q. Zhao, Y. Zhou, A. Shaw, T. Li, A. Childs (2022)

10/19

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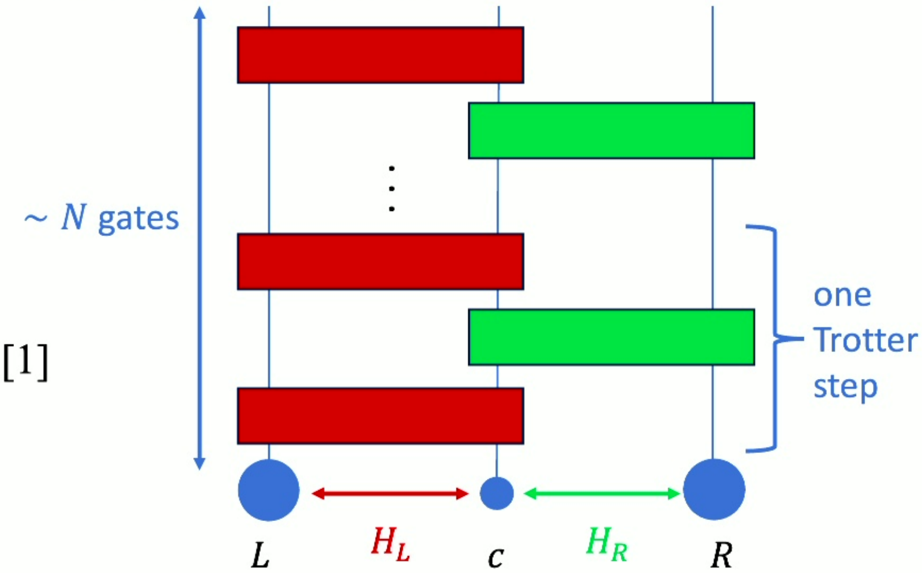
  $\sim (t\sqrt{N})^{k+1}$ — $k+1$ commutators



- Need large k, although get intuition from $k = 1$
  - $e^{-itH} - e^{-itH_L} e^{-itH_R} = -\frac{1}{2} t^2 [H_L, H_R] + O(t^3)$
  - $\|[H_L, H_R]\|_F \sim \|\sum_{lr} X_l X_r X_c\|_F \sim N$

    Orthogonal w.r.t. operator inner product $2^{-N}\mathrm{Tr}(A^\dagger B)$

[1] Q. Zhao, Y. Zhou, A. Shaw, T. Li, A. Childs (2022)

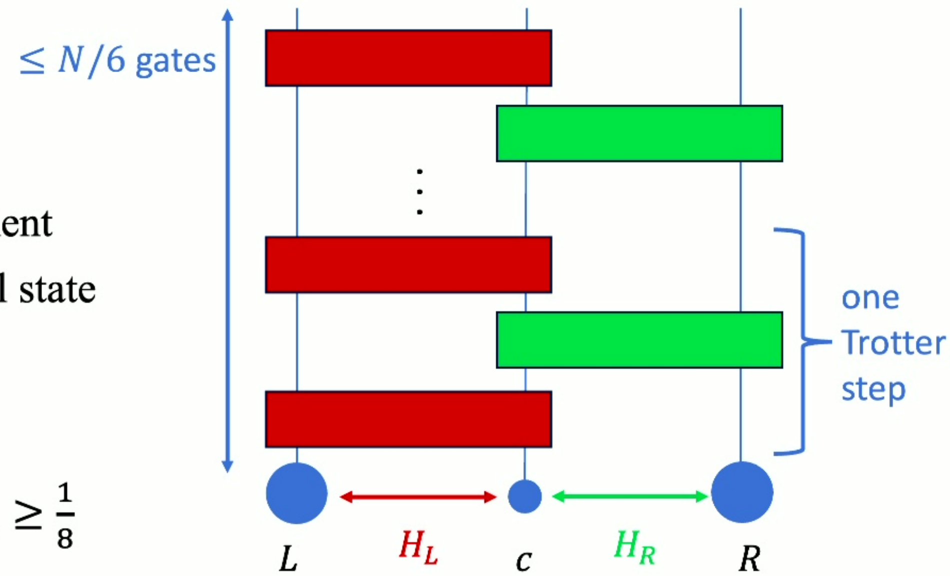
Proof sketch: circuit does not realize U_p

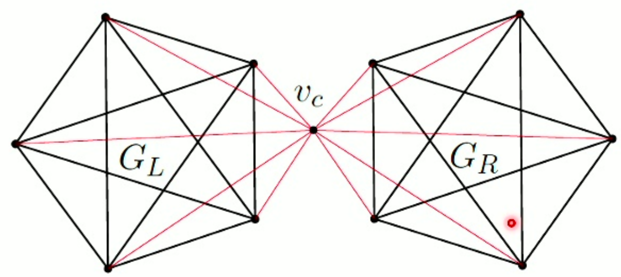
- $\|\widetilde{U} - U_p\|_F \geq 1/4$
  - For any initial state $\rho_z \coloneqq |z\rangle_{L,c} \otimes I_R$
  - $\widetilde{U}$ and U_p create different amount of entanglement
  - Varying z: $\widetilde{U}$ and U_p act differently on a typical state

- Put together: if $T \ll \sqrt{N}$,

$$\|U - \widetilde{U}\|_F \leq \frac{1}{8}, \|\widetilde{U} - U_p\|_F \geq \frac{1}{4} \;\Rightarrow\; \|U - U_p\|_F \geq \frac{1}{8}$$

- Generalizations
  - Piecewise constant $H(t)$
  - General time-dependent case?
  - Vertex barbell graph

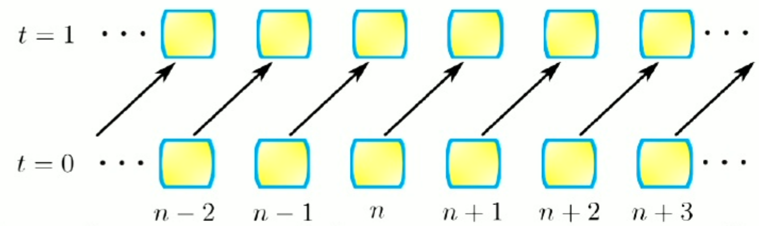




A. Bapat, A.M. Childs, A.V. Gorshkov and E. Schoute (2023)

11/19

The shift unitary



- A specific permutation U_{sh}

A review of Quantum Cellular Automata, Terry Farrelly (2020)

- Index theory of quantum cellular automata (QCA): $T_{gate} = \Theta(L)$
  - $U_{sh} = SWAP_{1,2}\, SWAP_{2,3} \cdots SWAP_{L-1,L}$
  - Applications: classifying phases of Floquet systems, disentangler for topological order …

- $H(t) = \sum_{ij} J_{ij}^{\alpha\beta}(t) X_i^{\alpha} X_j^{\beta}$, $\left\| J_{ij}^{\alpha\beta}(t) \right\| \leq |i-j|^{-\alpha}$
- Index theory for approximate locality-preserving unitaries

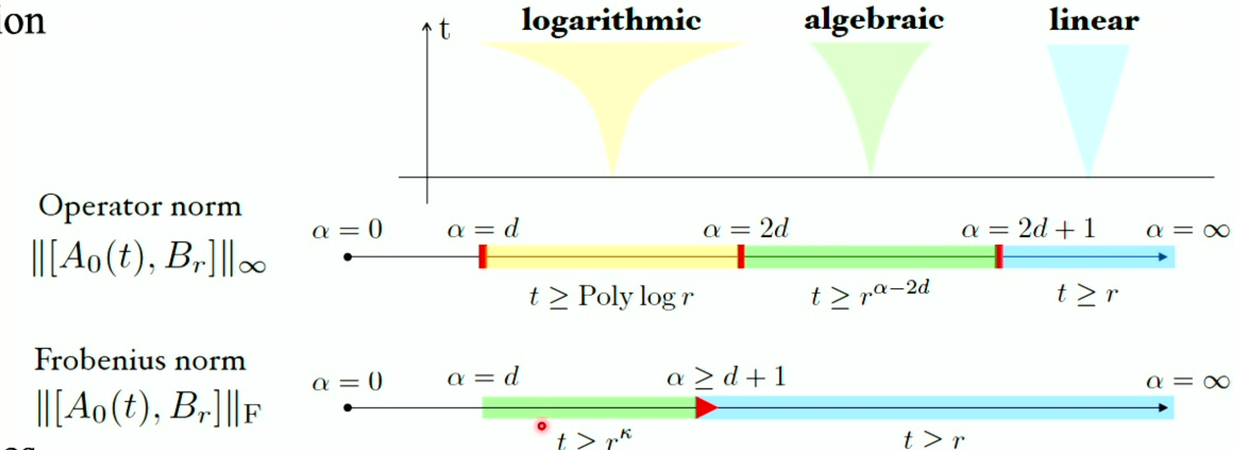
Daniel Ranard, Michael Walter and Freek Witteveen (2022)

- Caveat
  - Cannot treat periodic boundary condition
  - L scaling of T unclear
  - Rely on Lieb-Robinson light cone

13/19

Operator growth in power-law interactions

- Lieb-Robinson light cone $\|[A_0(t), B_r]\| \ll 1,\ t \leq t(r)$
  - Matching protocols
  - Signaling/ generating correlation



Proven in $d = 1$ and conjectured for higher d

- Other tasks: Hierarchy of light cones
  - Frobenius light cone: single-qubit routing $X_0(t) = X_r$

Speed limits and locality in many-body quantum dynamics,
Chi-Fang Chen, Andrew Lucas, ***Chao Yin*** *(2023)*

14/19

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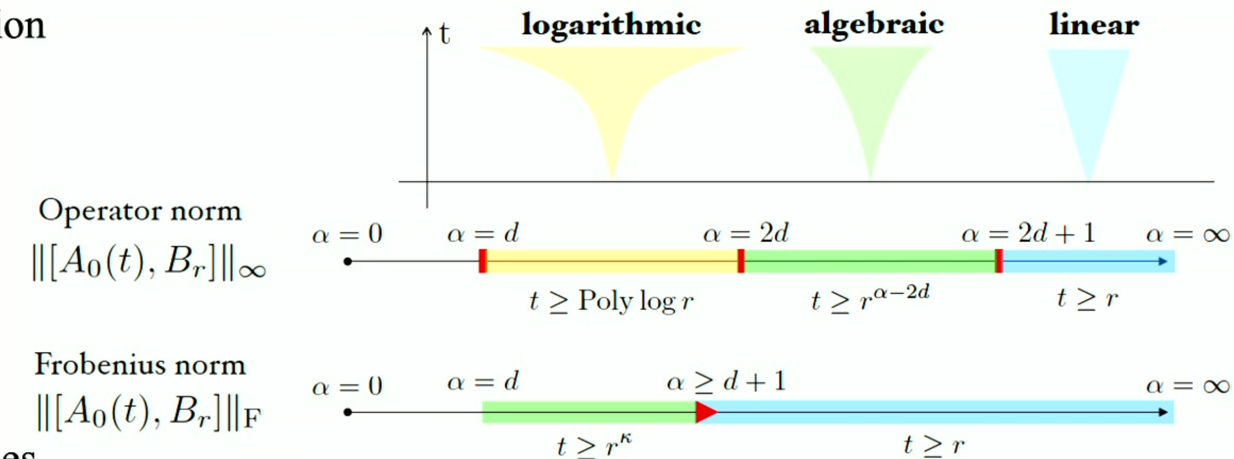


Proven in $d = 1$ and conjectured for higher d

- Other tasks: Hierarchy of light cones
  - Frobenius light cone: single-qubit routing $X_0(t) = X_r$
  - shift: moving N qubits one site ≈ moving 1 qubit N sites?

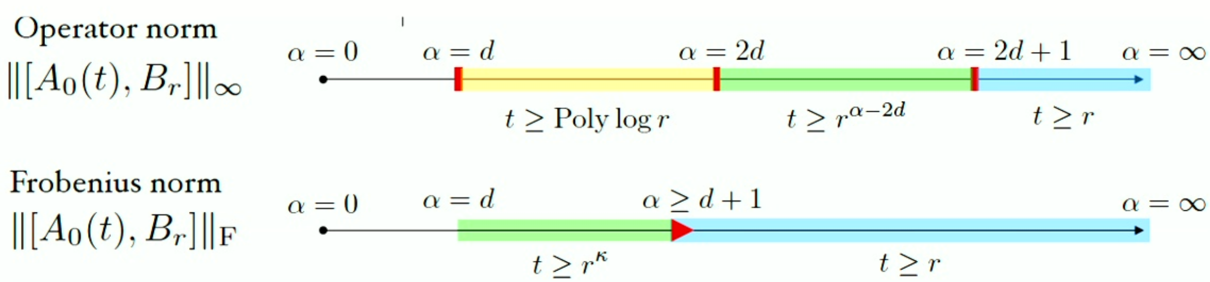
Speed limits and locality in many-body quantum dynamics,
Chi-Fang Chen, Andrew Lucas, ***Chao Yin*** *(2023)*

Our main result

Theorem: lower bound for realizing shift using power-law interaction

A α-power-law Hamiltonian $H(t)$ on the ring with length L cannot generate U_{sh} for time

$$T \leq \begin{cases} C'L - C & \alpha \geq 4 \\ C'L^{(\alpha-1)/3} - C & 3 < \alpha < 4 \\ C'L^{(\alpha-2)(\alpha-1)/(2\alpha-3)-\epsilon} & 2+1/\sqrt{2} < \alpha \leq 3 \\ C'L^{1/2-\epsilon} - C & 2 \leq \alpha < 2+1/\sqrt{2} \\ C'L^{(\alpha-1)/2} - C & 1 < \alpha < 2 \end{cases}.$$



15/19

Our main result

Theorem: lower bound for realizing shift using power-law interaction

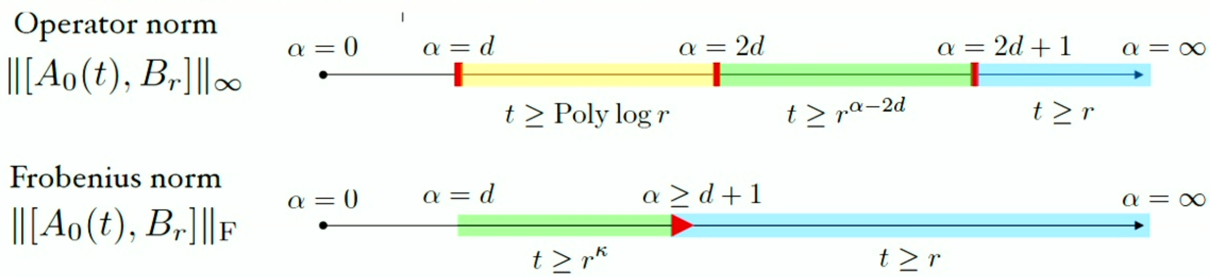
A α-power-law Hamiltonian $H(t)$ on the ring with length L cannot generate U_{sh} for time

$$T \leq \begin{cases} C'L - C & \alpha \geq 4 \\ C'L^{(\alpha-1)/3} - C & 3 < \alpha < 4 \\ C'L^{(\alpha-2)(\alpha-1)/(2\alpha-3)-\epsilon} & 2 + 1/\sqrt{2} < \alpha \leq 3 \\ C'L^{1/2-\epsilon} - C & 2 \leq \alpha < 2 + 1/\sqrt{2} \\ C'L^{(\alpha-1)/2} - C & 1 < \alpha < 2 \end{cases}.$$

Based on Lieb-Robinson light cone (first three cases)

Based on Frobenius light cone (last two cases)

- Much slower than signaling across the whole system

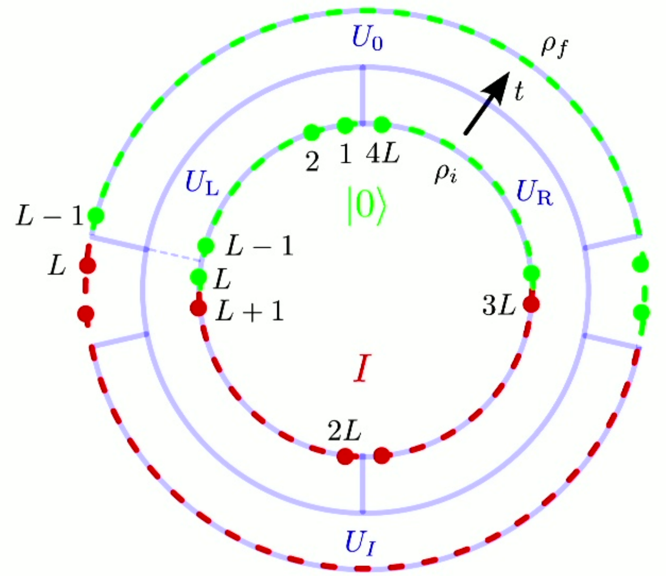


- Conjecture:

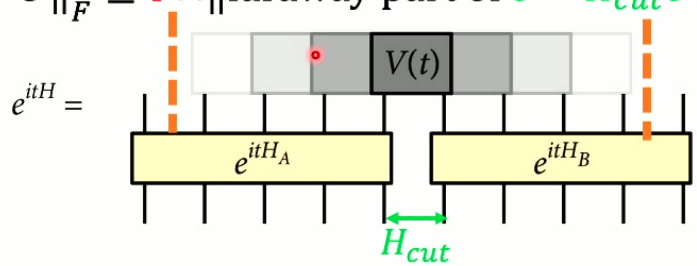
$$T \leq \begin{cases} C'L - C & \alpha \geq 2 \\ C'L^{\alpha-1} - C & 1 < \alpha < 2 \end{cases}.$$

Proof sketch

- Similar idea as before
  - $U = e^{-iTH} \approx \widetilde{U} := (U_I \otimes U_0) \cdot (U_L \otimes U_R)$
  - $\|\widetilde{U} - U_{sh}\|_F \geq 1/4$



- "light-cone improved Trotter expansion"
  - $\|U - \widetilde{U}\|_F \leq T \times \|\text{faraway part of } e^{itH} H_{cut} e^{-itH}\|_F$ + (throw away faraway couplings)



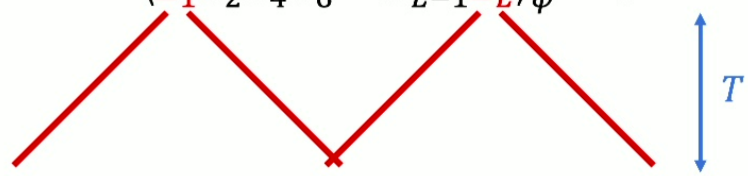
Tobias J. Osborne (2006)
J. Haah, M.B. Hastings, R. Kothari, G.H. Low (2020)
M. C. Tran, …, A.M. Childs, A.V. Gorshkov (2019)

16/19

An alternative perspective connected to SPT

Yichen Huang, Xie Chen (2015)

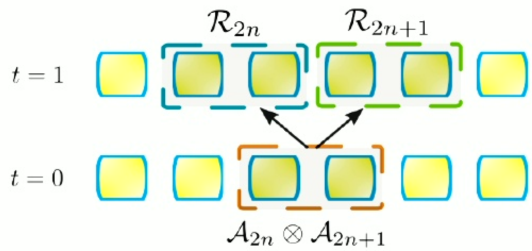
- Symmetry-protected topological order (SPT) requires linear-depth symmetric circuit
  - 1d cluster state $|\psi\rangle$ stabilized by $Z_{i-1}X_iZ_{i+1}$, $\mathbb{Z}_2\times\mathbb{Z}_2$ symmetry $\prod_{i\ \mathrm{even/odd}} X_i$
  - String order parameter $\langle Z_1 X_2 X_4 X_6 \cdots X_{L-1} Z_L \rangle_\psi = 1$
  - If $|\psi\rangle = U_{symmetric}|+\cdots+\rangle$
  - only evolve end points $\Rightarrow T = \Omega(L)$



- Operator Hilbert space spanned by $|I), |X), |Y), |Z)$
- Heisenberg evolution $\Rightarrow$ super-unitary $\mathcal{U}$
- super-density-matrix $\mathcal{J} = |I)(I| + |X)(X| + |Y)(Y| + |Z)(Z|$
- String super-density-matrix $\mathcal{J}_1 \otimes \mathcal{J}_2 \otimes \cdots \otimes \mathcal{J}_L \otimes |I)_{L+1}(I| \otimes |I)_{L+2}(I| \otimes \cdots$
- Built-in "symmetry" $\mathcal{U}|I \cdots I) = 0 \ \Rightarrow$ only evolve end points

Discussion: New strategy for lower bound?

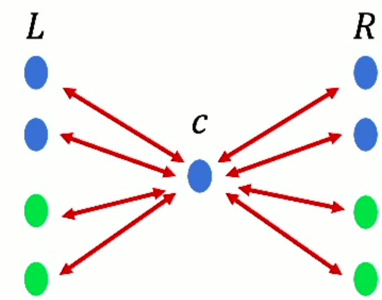
- Routing bounded by Frobenius light cone without the extra factor T?
    - "support of algebras" in index theory of QCA and algebraic approach to quantum scrambling



Terry Farrelly (2020)
D. Gross, V. Nesme, H. Vogts, R.F. Werner (2012)
Paolo Zanardi (2022)

- Even better bound from worst-case state instead of an average state?

- Ancilla-assisted routing $U|0\cdots0\rangle_a \otimes |\psi\rangle = |\phi\rangle_a \otimes U_p|\psi\rangle$: Frobenius strategy will fail
    - Shift becomes trivial: even (odd) sites to left (right)
    - Star graph: swapping $N/2$ left qubits with right can be much faster ??



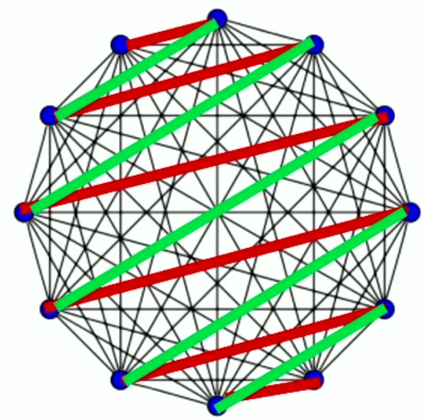
18/19

Conclusion

- Quantum speed limit for signaling, generating entanglement, quantum routing …
- Star graph: gate-based routing $T_{gate} = \Omega(N)$ while Hamiltonian routing $T = \Theta(\sqrt{N})$*
- 1d power-law interaction: the shift unitary needs $T = \Omega(L^{\kappa})$ if $\alpha > 1$

Outlook

- lower bounds: new strategy? other tasks?
- Trotter-Suzuki expansion for time-dependent Hamiltonian
- 1d shift for $\alpha \leq 1$? All-to-all gate protocol $T = O(1)$



Conclusion

- Quantum speed limit for signaling, generating entanglement, quantum routing …
- Star graph: gate-based routing $T_{gate} = \Omega(N)$ while Hamiltonian routing $T = \Theta(\sqrt{N})$*
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Thank you!

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- lower bounds: new strategy? other tasks?
- Trotter-Suzuki expansion for time-dependent Hamiltonian
- 1d shift for $\alpha \leq 1$? All-to-all gate protocol $T = O(1)$
- Robustness of protocols
- Measurement + adaptive feedback