Title: Classification and characterization of interacting crystalline insulators in (2+1)D

Speakers: Francisco Vladimir Calvera Ciguenas

Series: Quantum Matter

Date: March 19, 2024 - 11:00 AM

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Abstract: In this presentation, I will discuss our latest efforts to classify and characterize insulators in two spatial dimensions in the presence of interactions. I will focus on two examples, both on the square lattice. The first example is of bosonic insulators with symmetry G = p4m x K, where K is an internal symmetry group taken to be U(1), SO(3) or Z_N. The second example is of fermionic insulators with G= p4 x U(1)^f. In both scenarios, we propose a set of invariants that reproduce the classification anticipated through real space constructions and topological field theory (TFT). In doing so, we relate these invariants to coefficients appearing in the response action obtained from TFT. For the fermionic case, we propose a map from the non-interacting classification to the interacting classification.

This talk is based on work done with Naren Manjunath and Maissam Barkeshli.

Zoom link

Classification and Characterization of interacting crystalline insulators

Vladimir Calvera

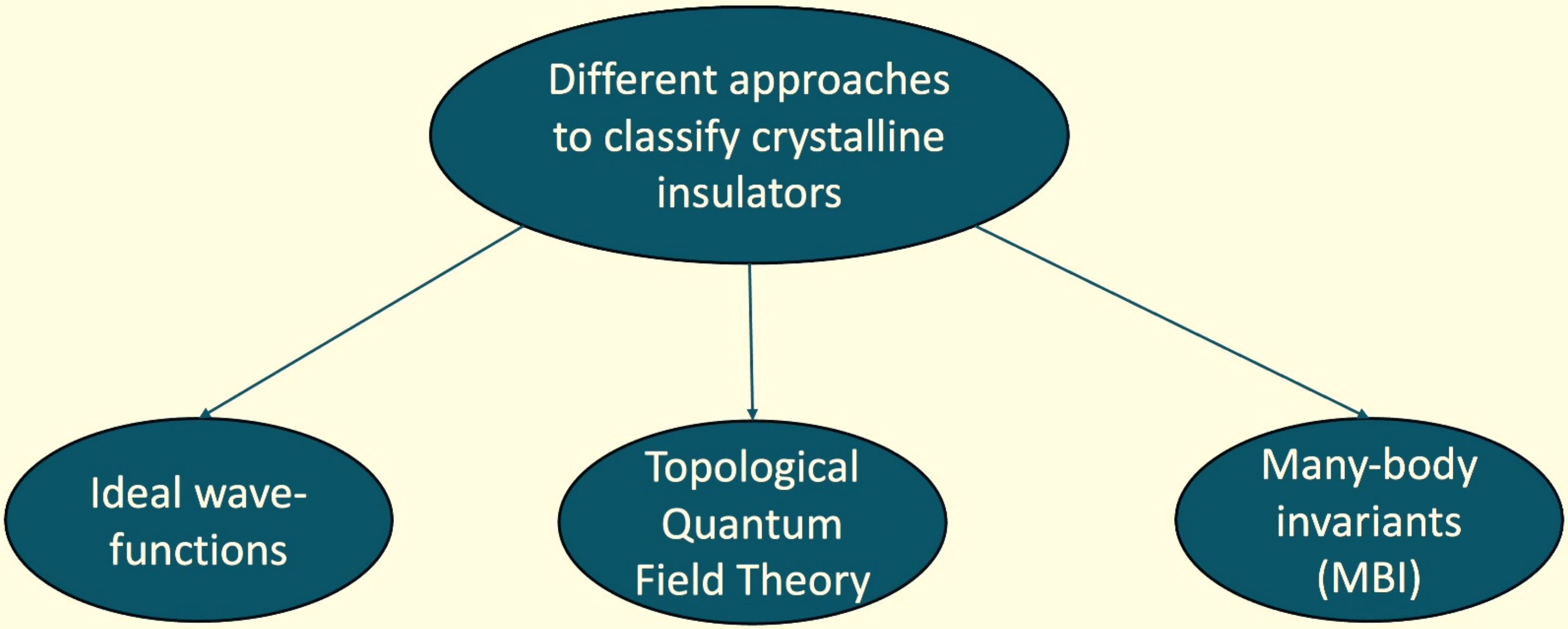
Stanford

Outline

1. Introduction
2. Bosonic example
3. Fermionic example
4. Summary

Introduction

- The classification of phases of matter is enhanced by the presence of symmetries
- For a given symmetry group G:
  - What are the possible phases?
  - What is an appropriate set of labels (invariants)?
  - In practice, how do we compute the labels of a given state?



Huang, Song, Huang, Hermele, PRB 2017
Song, Fang, Qi, Nature 2020
Zhang, Yang, Qi, Gu, PRR 2022
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Thorngren, Else, PRX 2018
Else, Thorngren, PRB 2019
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Shiozaki, Shapourian, Ryu, PRB (2017)
Zhang, NM, Kobayashi, Barkeshli (2023)
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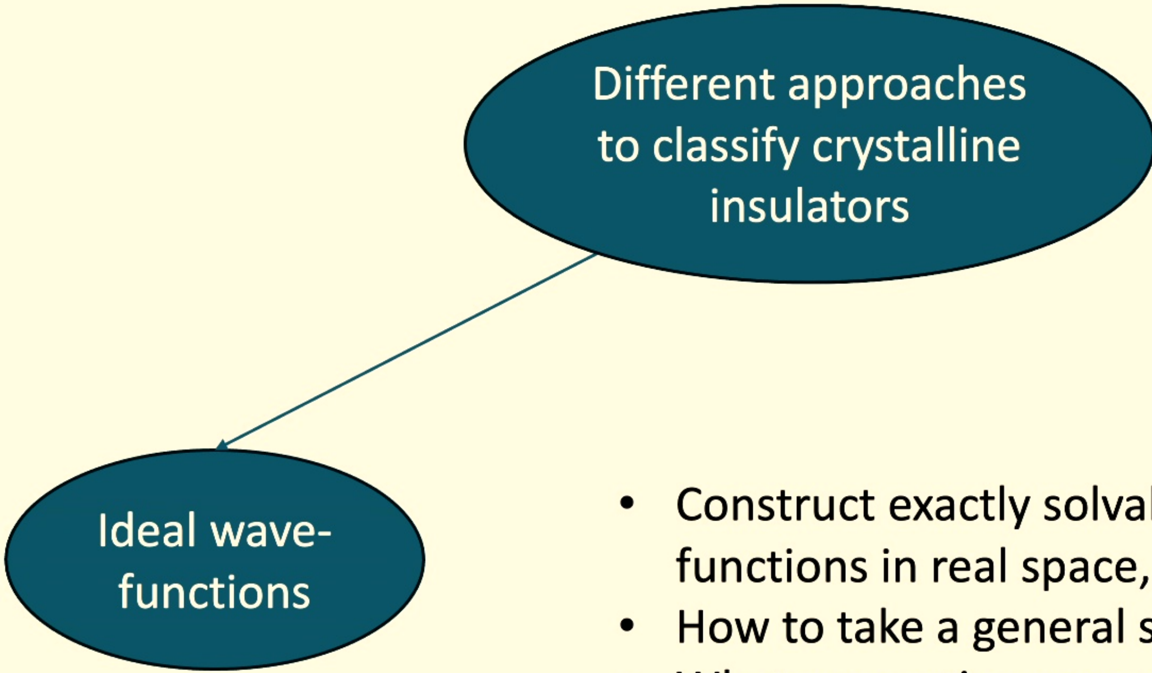


- Construct exactly solvable ground states wave functions in real space, e.g. atomic limits
- How to take a general state into its ideal limit ?
- What properties are expected to remain away from ideal limit?

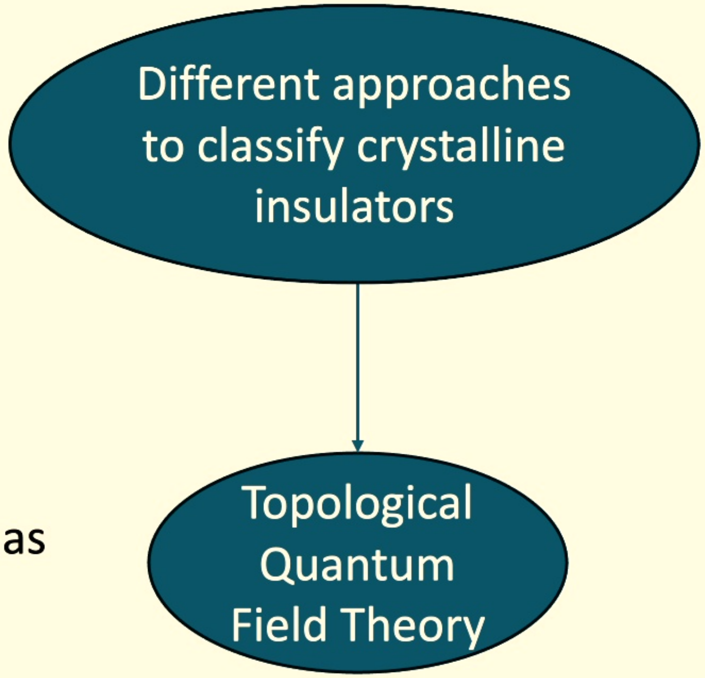
Huang, Song, Huang, Hermele, PRB 2017
Song, Fang, Qi, Nature 2020
Zhang, Yang, Qi, Gu, PRR 2022
...

Different approaches to classify crystalline insulators

Many-body invariants (MBI)

- Define invariants directly from wave-functions or response to symmetry defects
- How many independent invariants?
- What is the relation among invariants?
- Evaluate them on ideal wave-functions!

Shiozaki, Shapourian, Ryu, PRB (2017)
Zhang, NM, Kobayashi, Barkeshli (2023)
...



- Treat lattice symmetries as internal symmetries (Crystalline Equivalence Principle)
- Write down most general action (e.g. group cohomology)

Thorngren, Else, PRX 2018
Else, Thorngren, PRB 2019
...

$$\mathcal{L} = \frac{C}{4\pi} A \wedge dA + \ldots$$

Different approaches to classify crystalline insulators

↓

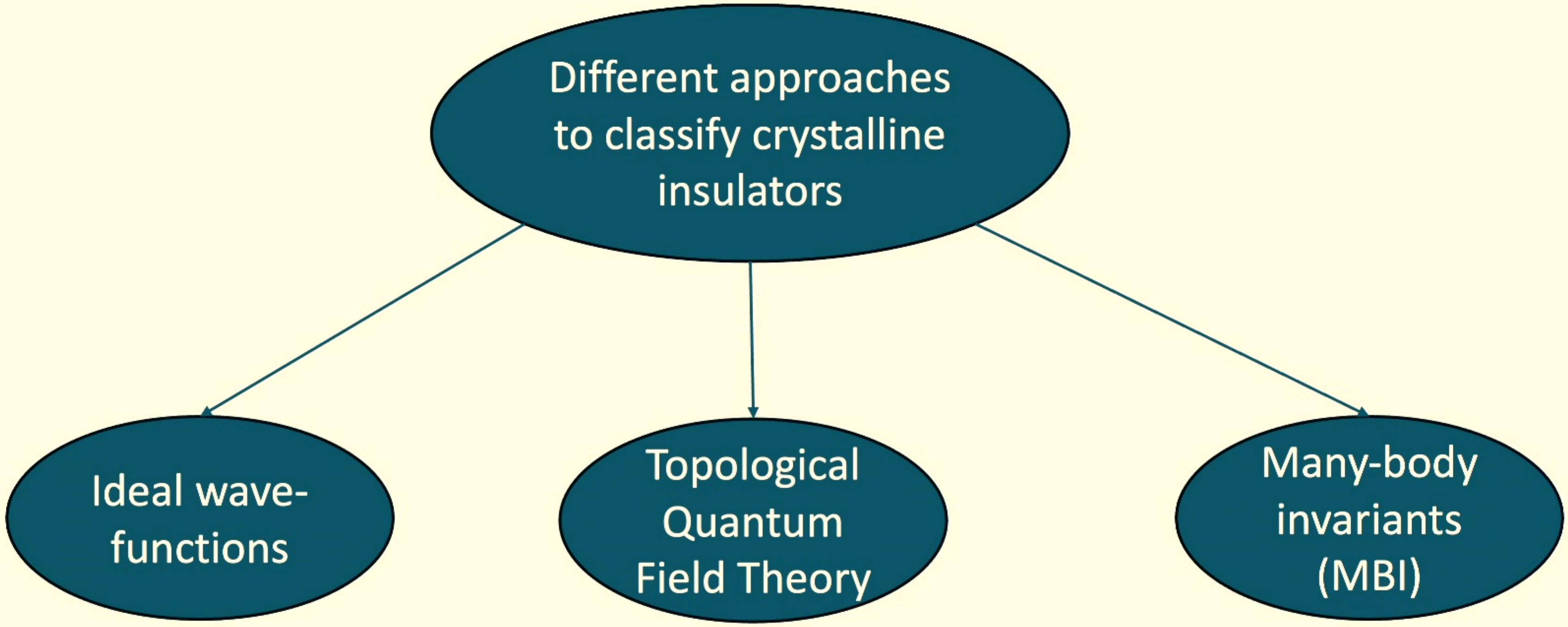
Topological Quantum Field Theory

- Treat lattice symmetries as internal symmetries (Crystalline Equivalence Principle)
- Write down most general action (e.g. group cohomology)

Thorngren, Else, PRX 2018
Else, Thorngren, PRB 2019
...

$$\mathcal{L} = \frac{C}{4\pi} A \wedge dA + \dots$$

- Can relate MBI to coefficients
- Obtain relation MBI!
- Alternative check



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Else, Thorngren, PRB 2019
...

Shiozaki, Shapourian, Ryu, PRB (2017)
Zhang, NM, Kobayashi, Barkeshli (2023)
...

Combine three approaches!

Part 1: crystalline SPTs of bosons/spins

Calvera, Manjunath, Barkeshli : In preparation

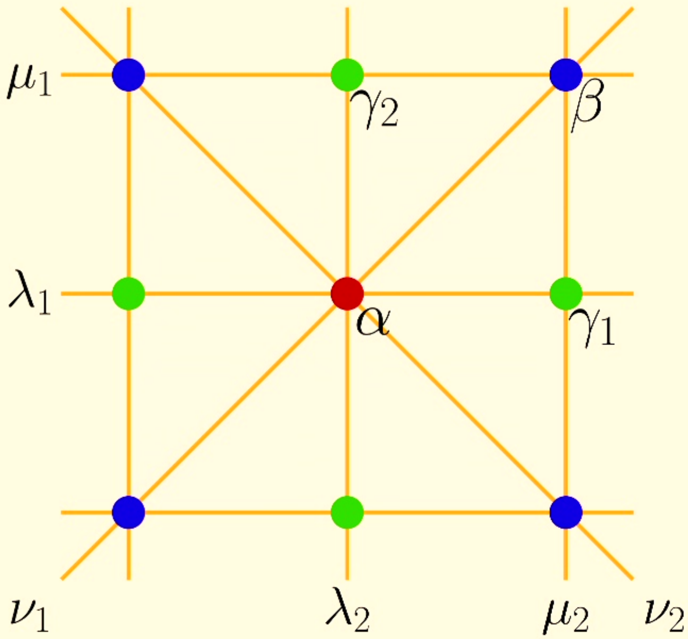
Part 1.1: Pure crystalline invariants

$$G = G_{space} = p4m \cong \mathbb{Z}^2 \rtimes D_4$$

Ideal wave-function

- Characterized by net G_o charge at o, where $o = \alpha, \beta, \gamma_1$

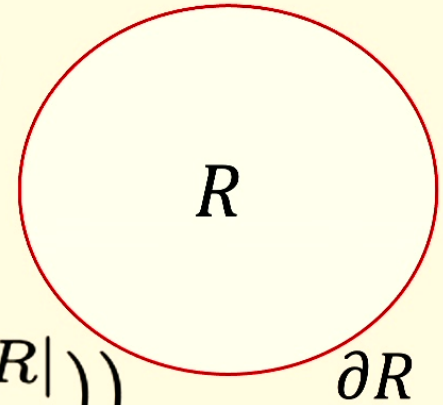
o	G_o	Charge classification
α	D_4	$\mathbb{Z}_2 \times \mathbb{Z}_2$
β	D_4	$\mathbb{Z}_2 \times \mathbb{Z}_2$
γ_1	D_2	$\mathbb{Z}_2 \times \mathbb{Z}_2$



(a) p4m

Huang, Song, Huang, Hermele PRB 2017.

Invariants from partial operations



- Given a state $|\Psi>$, evaluate

$$\langle \Psi | O|_R | \Psi \rangle = \mathrm{e}^{\mathrm{i}\Phi - \gamma|\partial R|} \times (1 + \mathcal{O}(e^{-\epsilon|\partial R|}))$$

where O is a symmetry operator, R is a large region.

Shiozaki, Shapourian, Ryu, PRB (2017)
…

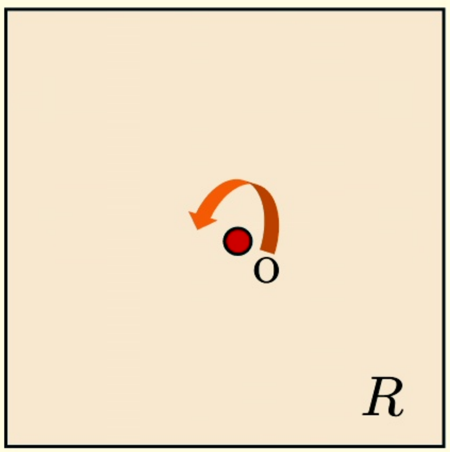
- Invariants are extracted from Φ
- One can argue that Φ should be quantized from the ideal wavefunctions and the entanglement spectrum conjecture
- In practice, we find the quantization to hold in numerical experiments

Li, Haldane, PRL (2008)
Qi, Katsura, Ludwig, PRL (2012)
…

Many body Invariants

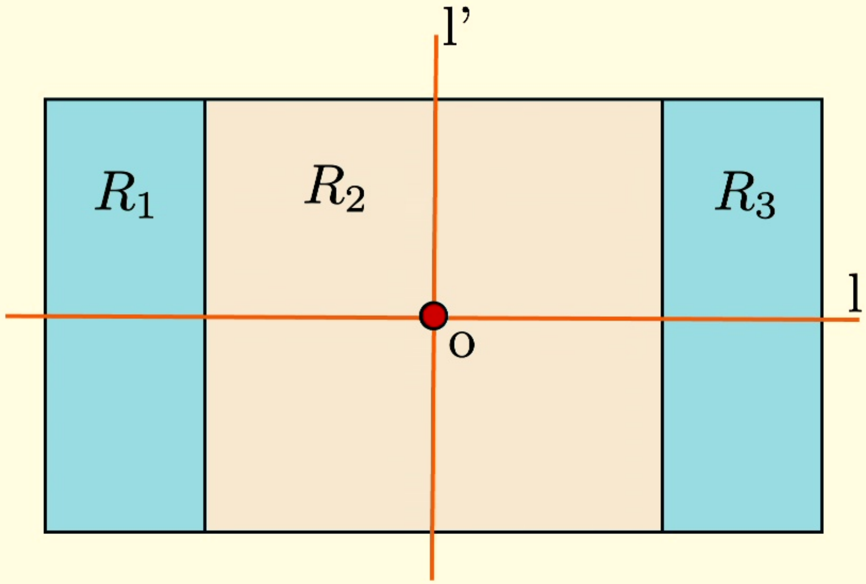
Shiozaki, Shapourian, Ryu, PRB (2017)
VC, Manjunath, Barkeshli: In preparation

Partial rotation (C_M)



$$\langle\Psi|C_M|_R|\Psi\rangle = \mathrm{e}^{\frac{2\pi \mathrm{i}}{M}\Theta_o} \qquad \Theta_o \in \mathbb{Z}_M$$

Partial double reflection (D_2)



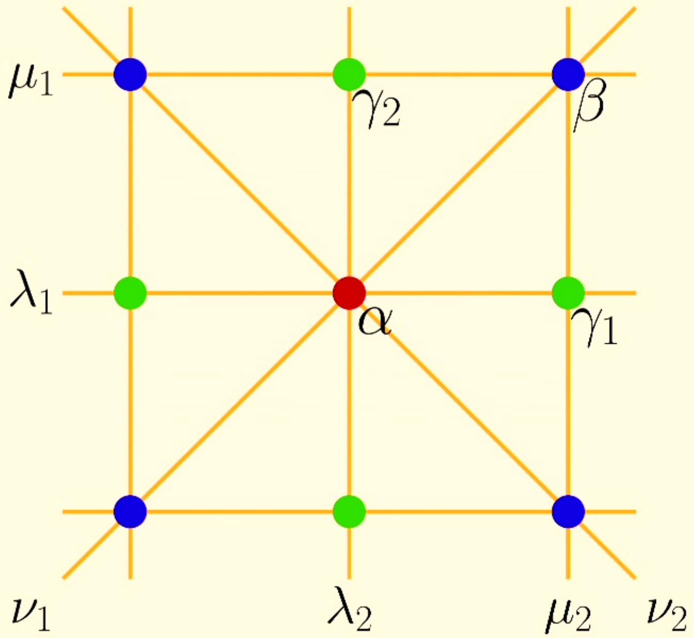
$$\langle\Psi|(r_{\mathrm{l}}|_{R_2})(r_{\mathrm{l}'}|_{R_1\cup R_3})|\Psi\rangle = (-1)^{\Sigma_{o,\mathrm{l}}}$$

$$(\Sigma_{o,\mathrm{l}}, \Sigma_{o,\mathrm{l}'}) \in \mathbb{Z}_2 \times \mathbb{Z}_2$$

Many body Invariants

- A complete set of invariants is

$$\left(\frac{\Theta_\alpha}{2}, \frac{\Theta_\beta}{2}, \Theta_{\gamma_1}, \Sigma_{\alpha,\lambda_1}, \Sigma_{\gamma_1,\lambda_1}, \Sigma_{\beta,\mu_1}\right) \in \mathbb{Z}_2^6$$



(a) p4m

VC, Manjunath, Barkeshli: In preparation

TQFT approach

- Introduce the following origin dependent gauge fields ($\mathrm{o} = \alpha$ or β):

$$\text{Translations:} \quad \frac{1}{2\pi}\vec{R}_\mathrm{o} \in \mathbb{Z}^2$$

$$\text{Rotations:} \quad \omega_\mathrm{o} \in \frac{2\pi}{M_\mathrm{o}}\mathbb{Z}; \qquad \omega \sim \omega + 2\pi$$

$$\text{Reflections:} \quad \sigma_\mathrm{o} \in \mathbb{Z}; \qquad \sigma \sim \sigma + 2$$

- The general action we find is

$$\mathcal{L} = k_{1\mathrm{o}} 2\omega_\mathrm{o} \frac{\bar{\mathrm{d}}\omega_\mathrm{o}}{2\pi} + k_{2\mathrm{o}}\pi\sigma_\mathrm{o}\frac{\bar{\mathrm{d}}\omega_\mathrm{o}}{2\pi} + k_{3\mathrm{o}}(\vec{R}_\mathrm{o}\cdot \boldsymbol{M})\frac{\bar{\mathrm{d}}\omega_\mathrm{o}}{2\pi} + k_{4\mathrm{o}}\pi\sigma_\mathrm{o}\frac{\bar{\mathrm{d}}\vec{R}_\mathrm{o}\cdot\boldsymbol{M}}{2\pi} + k_{5\mathrm{o}}2\omega_\mathrm{o}\mathrm{A}_{\mathrm{XY}} + k_{6\mathrm{o}}\pi\sigma_\mathrm{o}\mathrm{A}_{\mathrm{XY}}$$

($k_{jo} \epsilon \mathbb{Z}_2$, $\mathrm{M} = \left(\frac{1}{2}, \frac{1}{2}\right)$, A_{XY}:area form, $\bar{d}$: twisted differential)

VC, Manjunath, Barkeshli: In preparation

TQFT approach

- Introduce the following origin dependent gauge fields (o $= \alpha$ or β):

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($k_{jo} \epsilon \mathbb{Z}_2$, $\mathrm{M} = \left(\frac{1}{2}, \frac{1}{2}\right)$, A_{XY}:area form, $\bar{d}$: twisted differential)

VC, Manjunath, Barkeshli: In preparation

Part 1.2: Mixed crystalline invariants

$$\mathrm{G} = p4m \times K$$

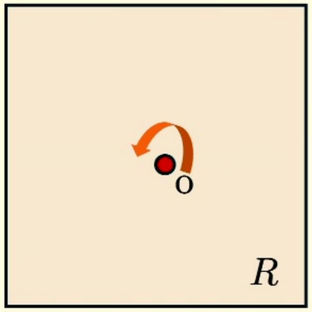
$$K = U(1), SO(3), \mathbb{Z}_2$$

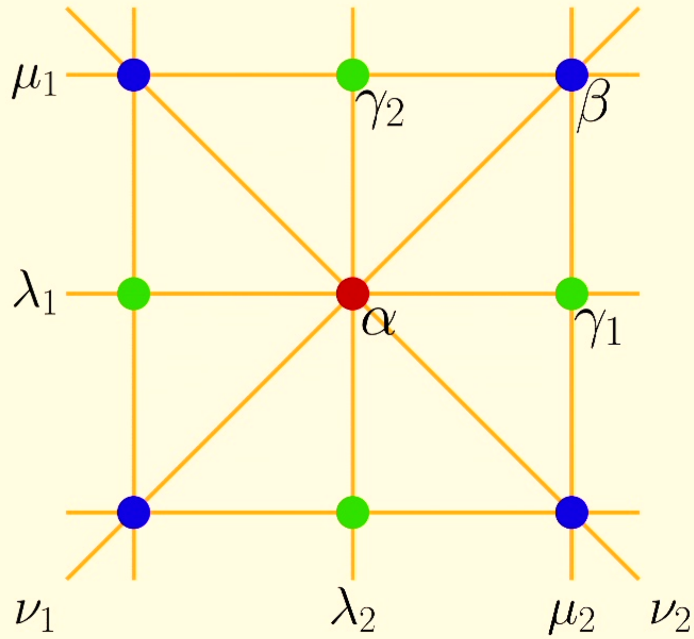
$K = U(1)$

- Ideal wave-functions are atomic limits characterized by
  - Net charge at o modulo order of rotation: Q_o
  - Filling per unit cell ν

  Classification is $\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}$
- Q_o can be detected using a dressed partial rotation

$$\langle \Psi | C_M |_R | \Psi \rangle = \mathrm{e}^{\frac{2\pi \mathrm{i}}{M} \Theta_o}$$





(a) p4m

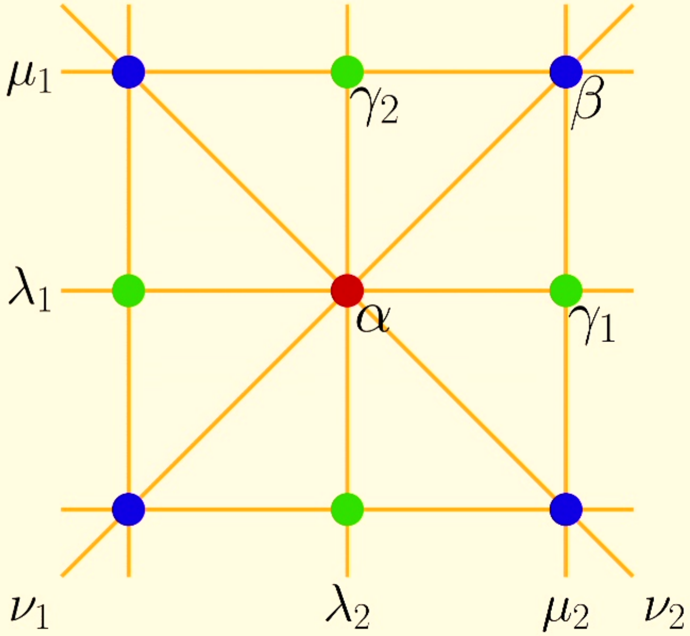
Song, Fang, Qi: Nat Com 2020
VC, Manjunath, Barkeshli: In preparation

$K = U(1)$

- The respective action is

$$\mathcal{L} = m_{1o} A \frac{\bar{\mathrm{d}}\omega_o}{2\pi} + m_{2o} A \frac{\bar{\mathrm{d}}\vec{R}\cdot \boldsymbol{M}}{2\pi} + m_{3o} A\, \mathrm{A}_{\mathrm{XY}}$$

where $(m_{1o}, m_{2o}, m_{3o}) \epsilon\, \mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}$



(a) p4m

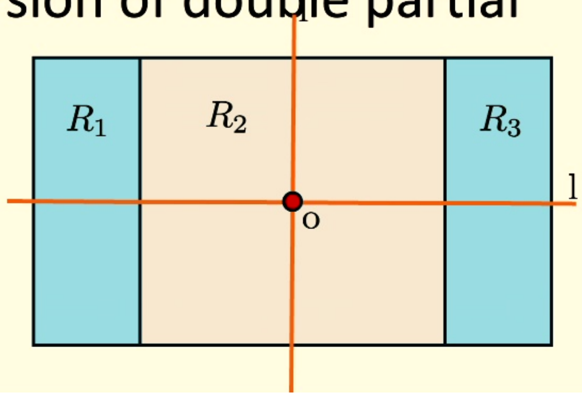
Song, Fang, Qi: Nat Com 2020

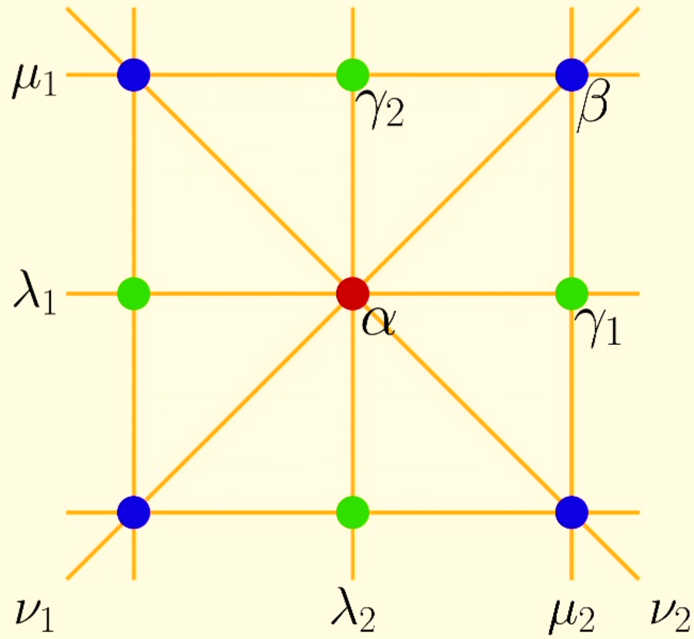
VC, Manjunath, Barkeshli: In preparation

$K = SO(3)$

- Ideal wave-functions are AKLT chains on high symmetry lines (λ, μ, ν)

  Classification is $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
- Can be detected by
  - Partial reflection with twisted boundary conditions; <u>or</u>
  - Dressed version of double partial reflection





(a) p4m

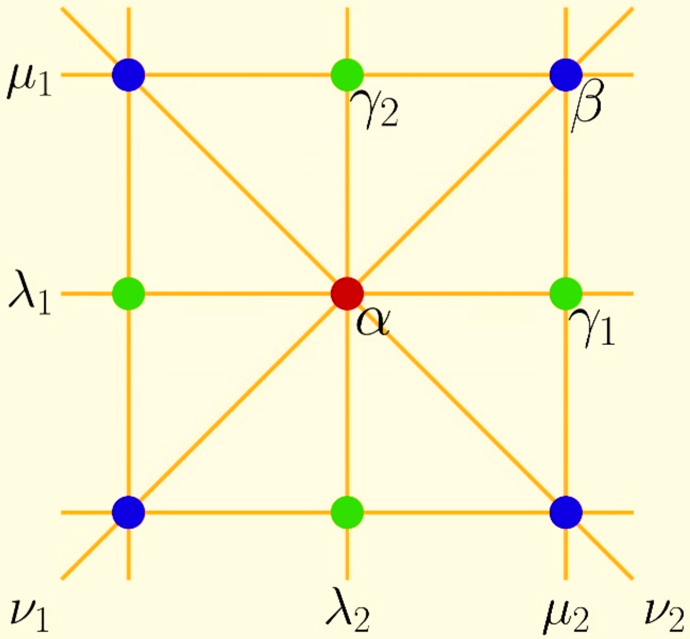
Song, Fang, Qi: Nat Com 2020

<u>VC</u>, Manjunath, Barkeshli: In preparation

$K = \mathbb{Z}_2$

- Ideal wave-functions are a combination of:
  - atomic insulators ($\mathbb{Z}_2^3$)
  - AKLT chains ($\mathbb{Z}_2^3$)

  Net classification is $\mathbb{Z}_2^6$
- Can be detected by partial symmetry operations



(a) p4m

Song, Fang, Qi: Nat Com 2020

VC, Manjunath, Barkeshli: In preparation

Part 1.3: General wallpaper group

- The story is similar for wallpaper groups with enough rotations:
  - Ideal wavefunctions are build from atomic insulators and AKLT chains
  - States can be detected using appropriate partial symmetry operations.

VC, Manjunath, Barkeshli: In preparation

Part 2: Crystalline insulators with orientation-preserving wallpaper groups

Manjunath, Calvera, Barkeshli: PRB (2024)

Part 2: Crystalline insulators with orientation-preserving wallpaper groups

Manjunath, Calvera, Barkeshli: PRB (2024)

This talk: $G = p4 \times U(1)^f$

Interacting classification

- Classification is given by

$$\mathbb{Z}^3 \times \mathbb{Z}_8 \times \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$$

- The action is

$$\mathcal{L} = \frac{C}{4\pi} A \wedge dA + \frac{\mathscr{S}_o}{2\pi} A \wedge d\omega + \frac{\ell_o - c_-/12}{4\pi} \omega \wedge d\omega$$
$$+ \frac{\vec{\mathscr{P}}_o}{2\pi} A \wedge \vec{T} + \frac{\kappa}{2\pi} A \wedge A_{XY} + \frac{\vec{\mathscr{P}}_{s,o}}{2\pi} \omega \wedge \vec{T} + \frac{\kappa_s}{2\pi} \omega \wedge A_{XY} + \ldots$$

- Ideal wave-functions: atomic insulators + Chern insulators

Manjunath, VC, Barkeshli, PRB (2024)
Zhang, et al., PRR (2022)
Zhang, et al., PRL (2023)

Many-body invariants

- The integer valued invariants are

$$\left(C, \frac{C - c_-}{8}, \kappa\right) \in \mathbb{Z}^3$$

  - c_-: chiral central charge.
  - The other two appear in the Streda's formula for the filling per unit cell

$$\nu = C\frac{\phi}{2\pi} + \kappa.$$

Many-body invariants

- The other invariants

$$\mathbb{Z}_8 \times \mathbb{Z}_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$$

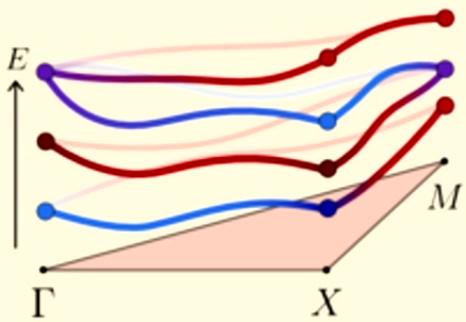
can be detected using a fermionic version of partial rotations ($\Theta_o^{+/-}$)

- $\Theta_o^{+/-}$ can be related to the response coefficients.
- Numerical check for the discrete shift prediction.

Manjunath, VC, Barkeshli, PRB (2024)
Zhang, et al., PRL (2022)
Zhang, et al., PRX (2023)

Recap of non-interacting fermions

- Non-interacting crystalline fermions understood to large extend using the single particle wave-functions in momentum space



Kruthoff et al, Band structure combinatorics, 2017
Watanabe, Po, Vishwanath, Symmetry indicators, 2017
Bradlyn, Cano, Bernevig et al, Topological quantum chemistry, 2020
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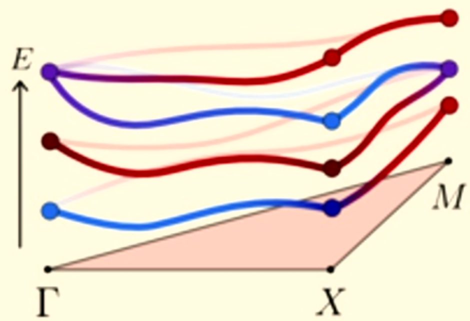


Kruthoff et al, Band structure combinatorics, 2017
Watanabe, Po, Vishwanath, Symmetry indicators, 2017
Bradlyn, Cano, Bernevig et al, Topological quantum chemistry, 2020
...

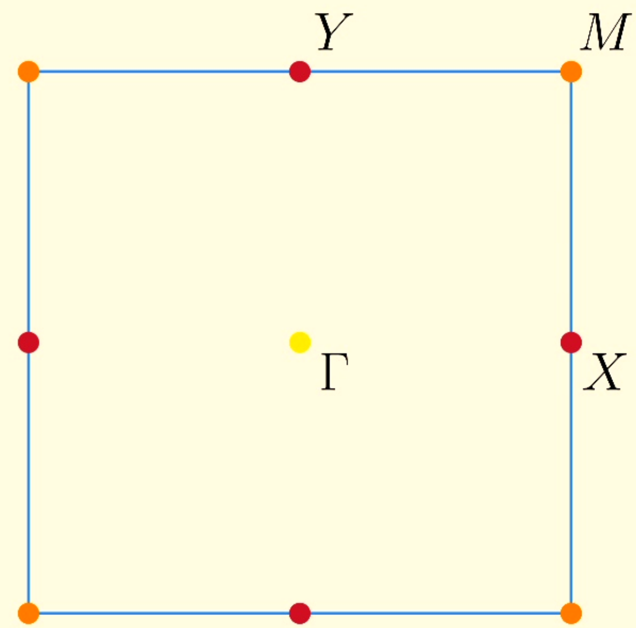
- Needs a K-theory to check that the above approach is `complete’

Recap of non-interacting fermions

- For $G = U(1)^f \times p4$, invariants are

$$C, \#\Gamma_1, \#\Gamma_2, \#\Gamma_3, \#\Gamma_4$$
$$\#M_2, \#M_3, \#M_4, \#X_2$$

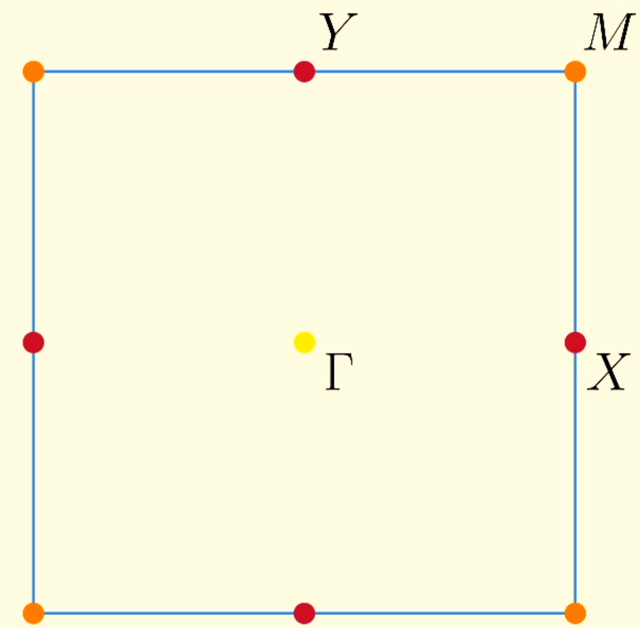
C: Chern number, $\#K_n$: degeneracy of n-th eigenvalue at K.



Manjunath, VC, Barkeshli, PRB (2024)

Non-interacting to Interacting phases

$$\begin{pmatrix} \kappa \\ \Theta^+_\alpha \\ \Theta^+_\beta \\ \Theta^-_\alpha \\ \Theta^-_\beta \\ \Theta^-_\gamma \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & \frac{1}{2} & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & \frac{1}{2} & 0 \\ -1 & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & \frac{7}{2} & \frac{13}{4} & 1 & \frac{1}{4} & \frac{5}{2} \\ -1 & 0 & 0 & 0 & 0 & \frac{13}{4} & 3 & \frac{9}{4} & \frac{3}{2} \\ -1 & 0 & 0 & 0 & 0 & \frac{5}{4} & 0 & \frac{5}{4} & 0 \end{pmatrix} \begin{pmatrix} C \\ \#\mathbf{\Gamma}^{(4)}_{1,\alpha} \\ \#\mathbf{\Gamma}^{(4)}_{2,\alpha} \\ \#\mathbf{\Gamma}^{(4)}_{3,\alpha} \\ \#\mathbf{\Gamma}^{(4)}_{4,\alpha} \\ [\boldsymbol{M}^{(4)}_{2,\alpha}] \\ [\boldsymbol{M}^{(4)}_{3,\alpha}] \\ [\boldsymbol{M}^{(4)}_{4,\alpha}] \\ [\boldsymbol{X}^{(2)}_{2,\alpha}] \end{pmatrix}$$



Manjunath, VC, Barkeshli, PRB (2024)

Summary/Outlook

- We have studied invertible states in the presence of wallpaper symmetries and proposed many body invariants to detect the respective phases.
- We obtained an explicit map from non-interacting to interacting fermionic phases for insulators with orientiation preserving wallpaper groups.
- It would be interesting to generalize the above map in the presence of reflections and time-reversal as new non-interacting phases arise.