

Title: Universal theory of strange metals

Speakers: Subir Sachdev

Series: Quantum Matter

Date: March 14, 2024 - 2:00 PM

URL: <https://pirsa.org/24030117>

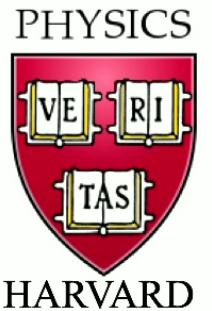
Abstract: Abstract TBA

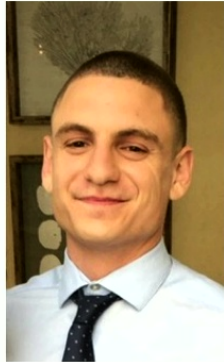
Zoom link

Universal theory of strange metals

Subir Sachdev

Perimeter Institute, Waterloo
March 14, 2024





Ilya Esterlis
Wisconsin



Haoyu Guo
Cornell



Aavishkar Patel
Flatiron



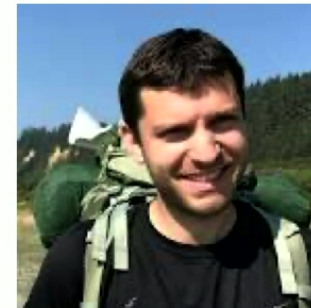
Chenyuan Li
Harvard



Davide Valentini
KIT

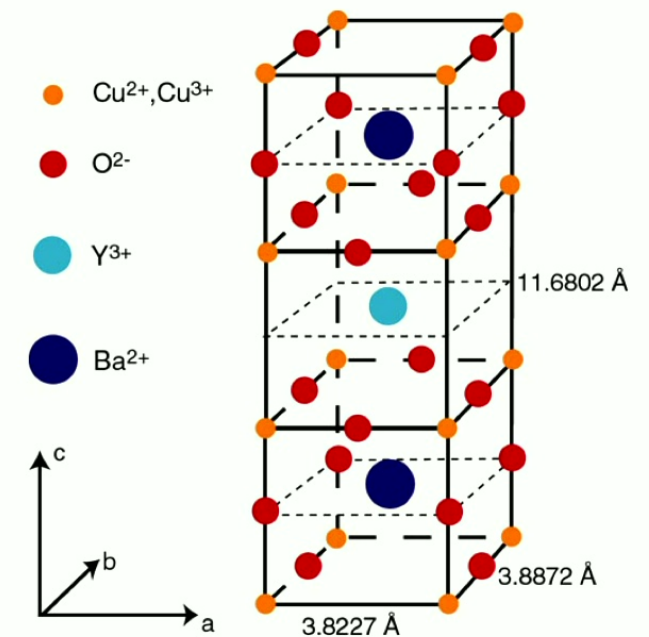
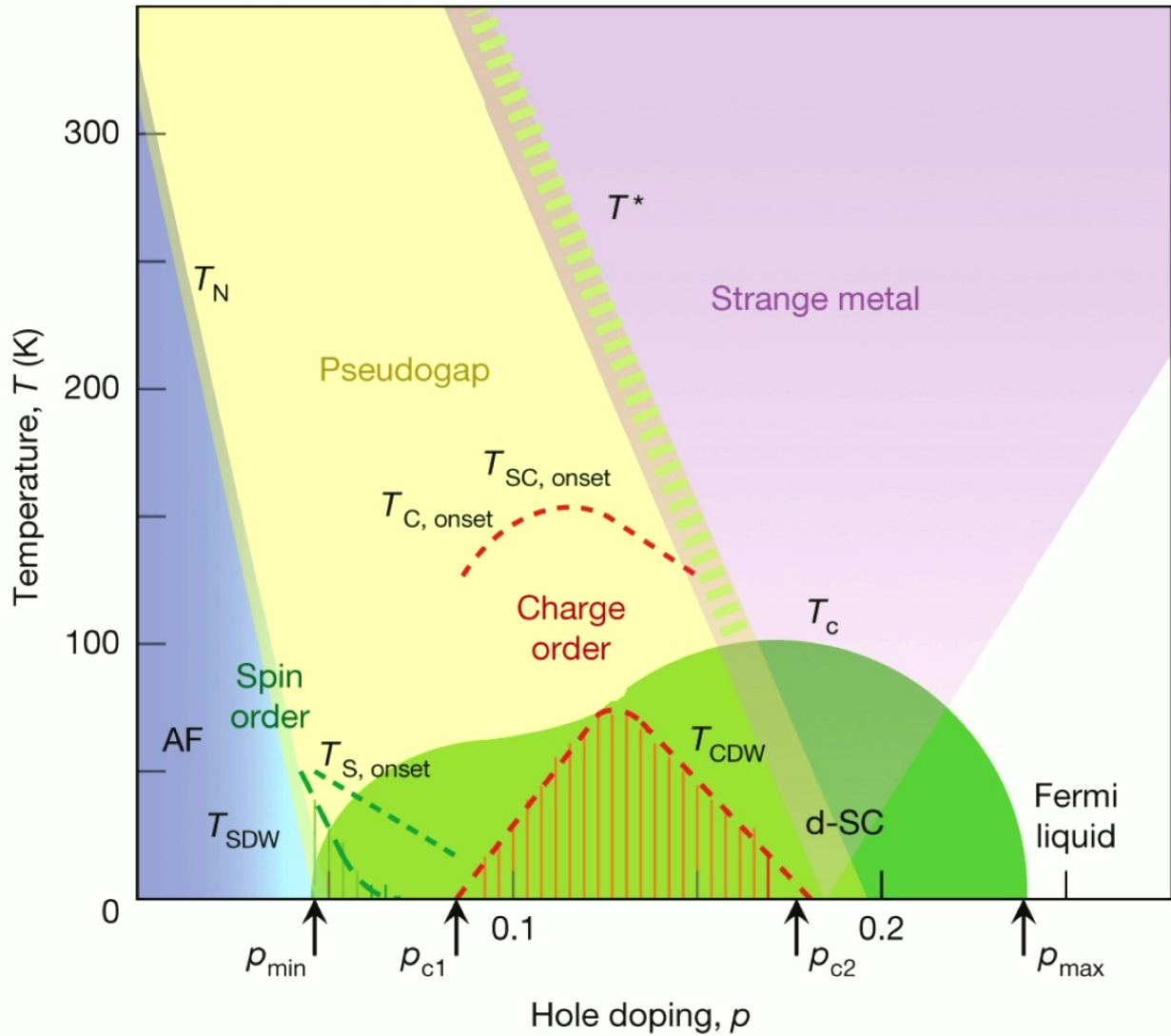


Joerg Schmalian
KIT

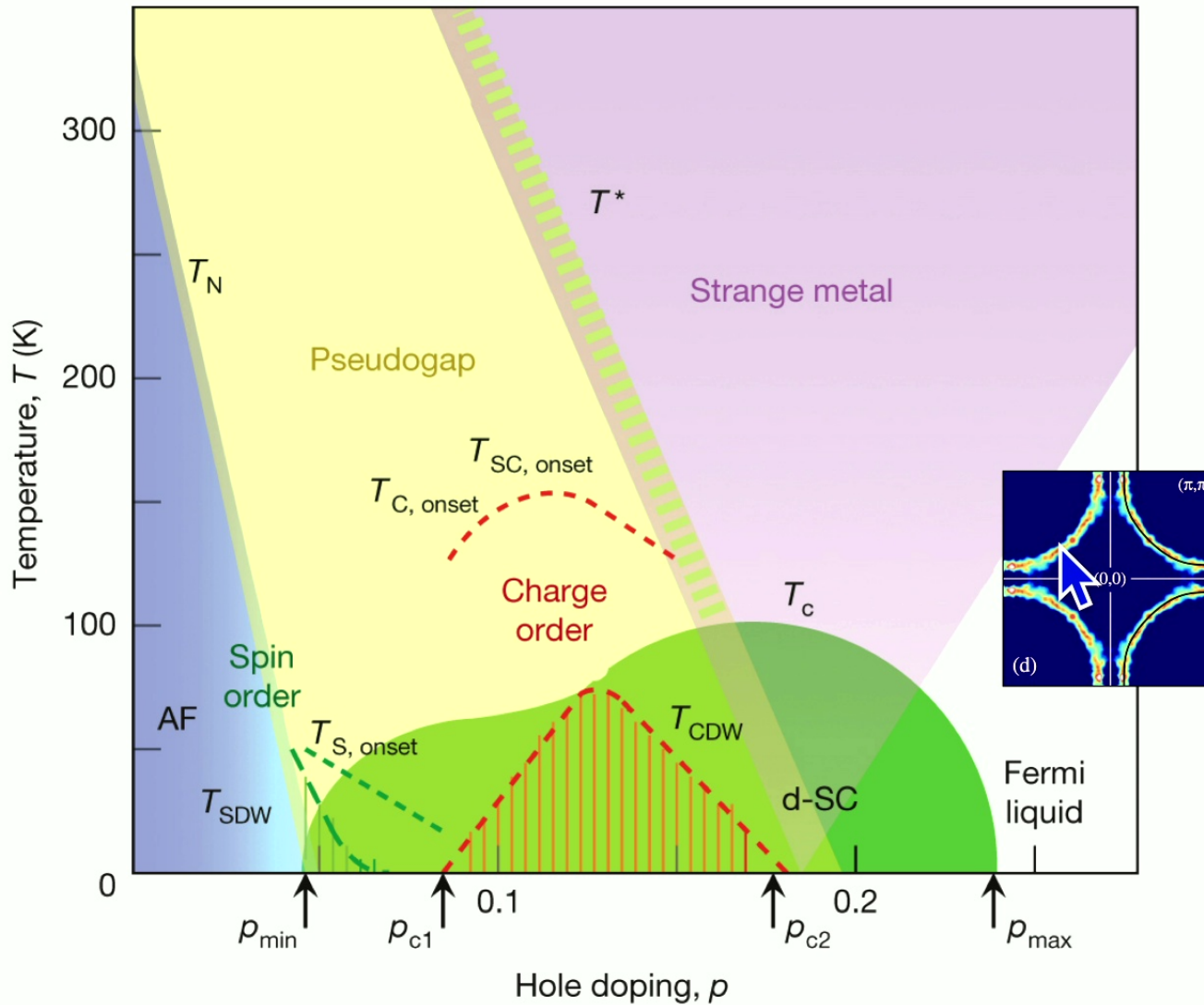


Peter Lunts
Harvard

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

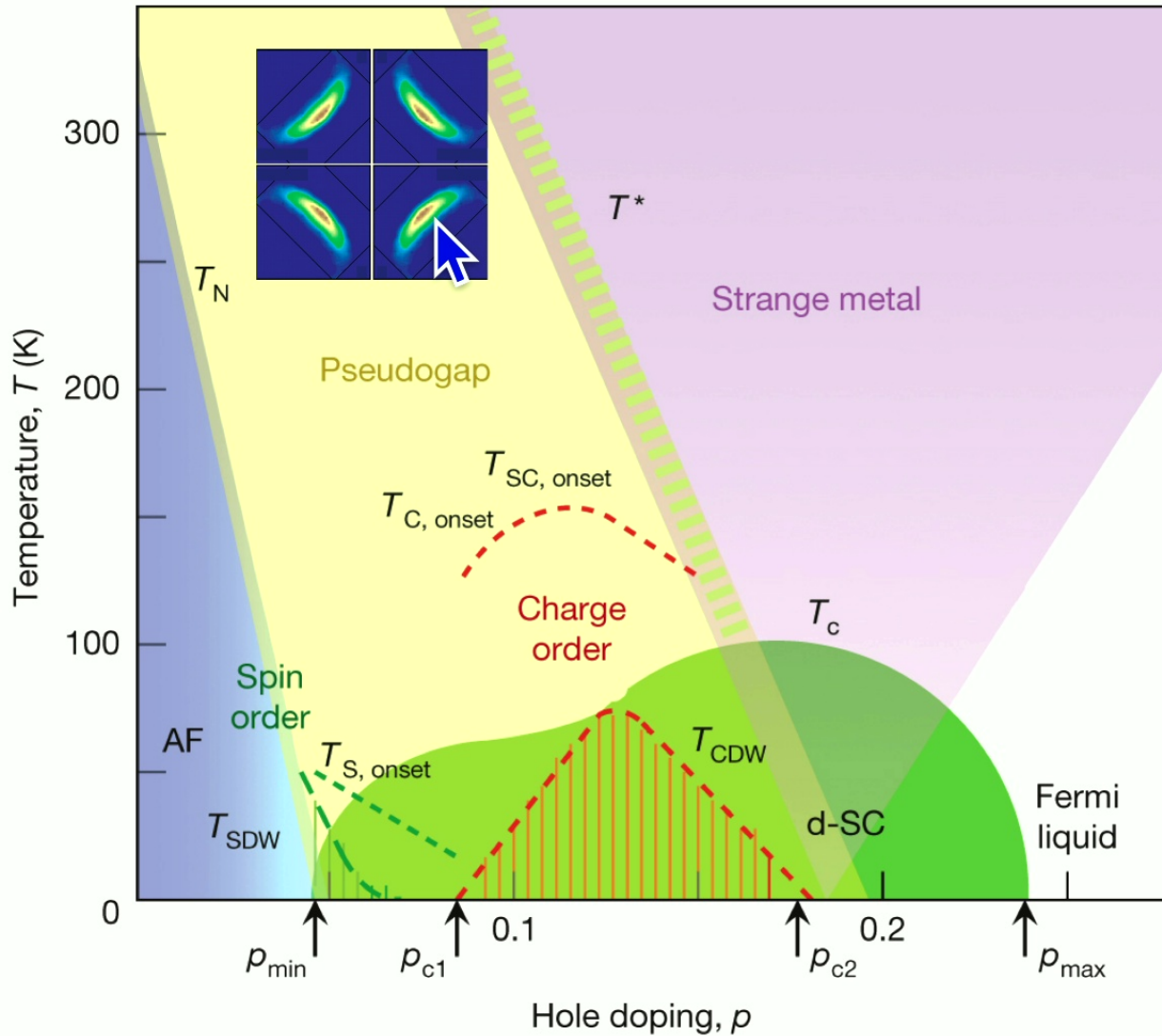


Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



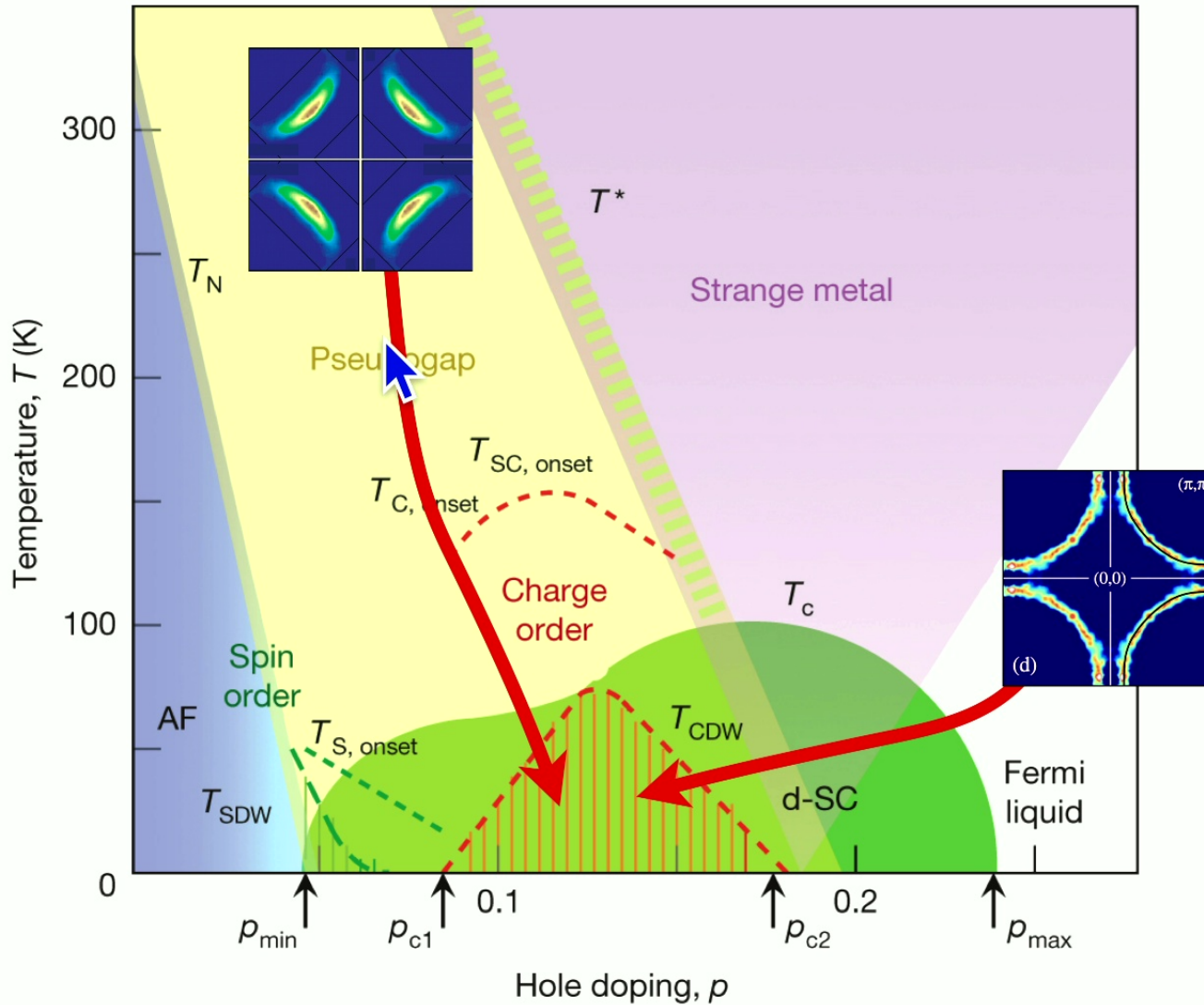
Fermi liquid
in the
overdoped metal

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Pseudogap metal
with “Fermi arcs”

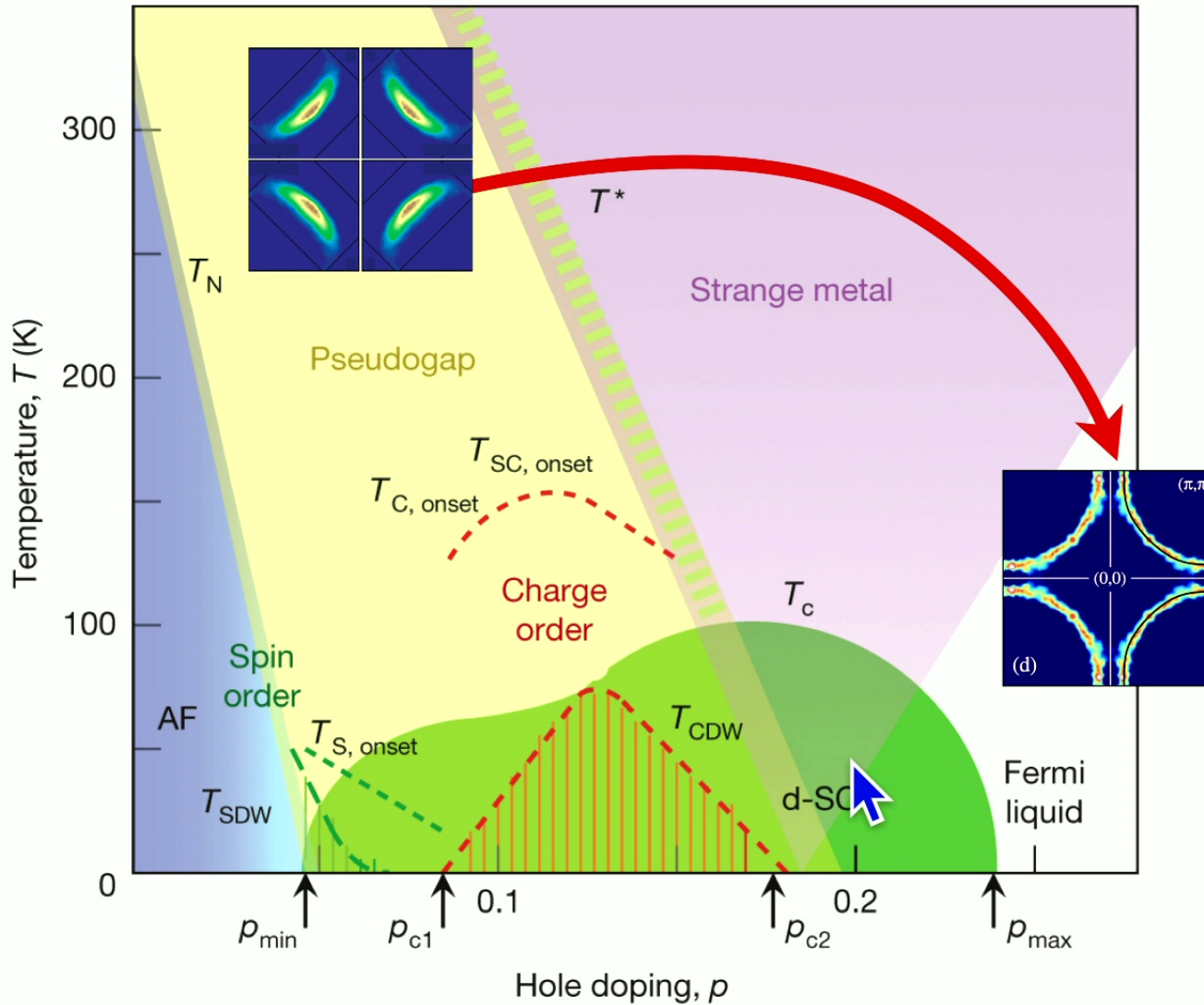
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal as a ‘metastable’ $T = 0$ quantum phase.

Lowest T phases obtained from pseudogap metal should connect smoothly to conventionally order phases obtained from the Fermi liquid.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



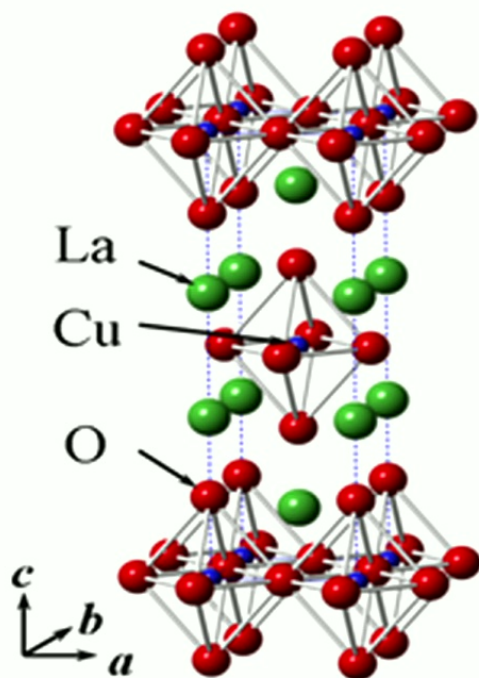
Build a theory for the phase diagram from a theory of the pseudogap metal as a ‘metastable’ $T = 0$ quantum phase.

Strange metal described by quantum critical region of a $T = 0$ quantum transition between the pseudogap metal (FL*) and the Fermi liquid (FL).

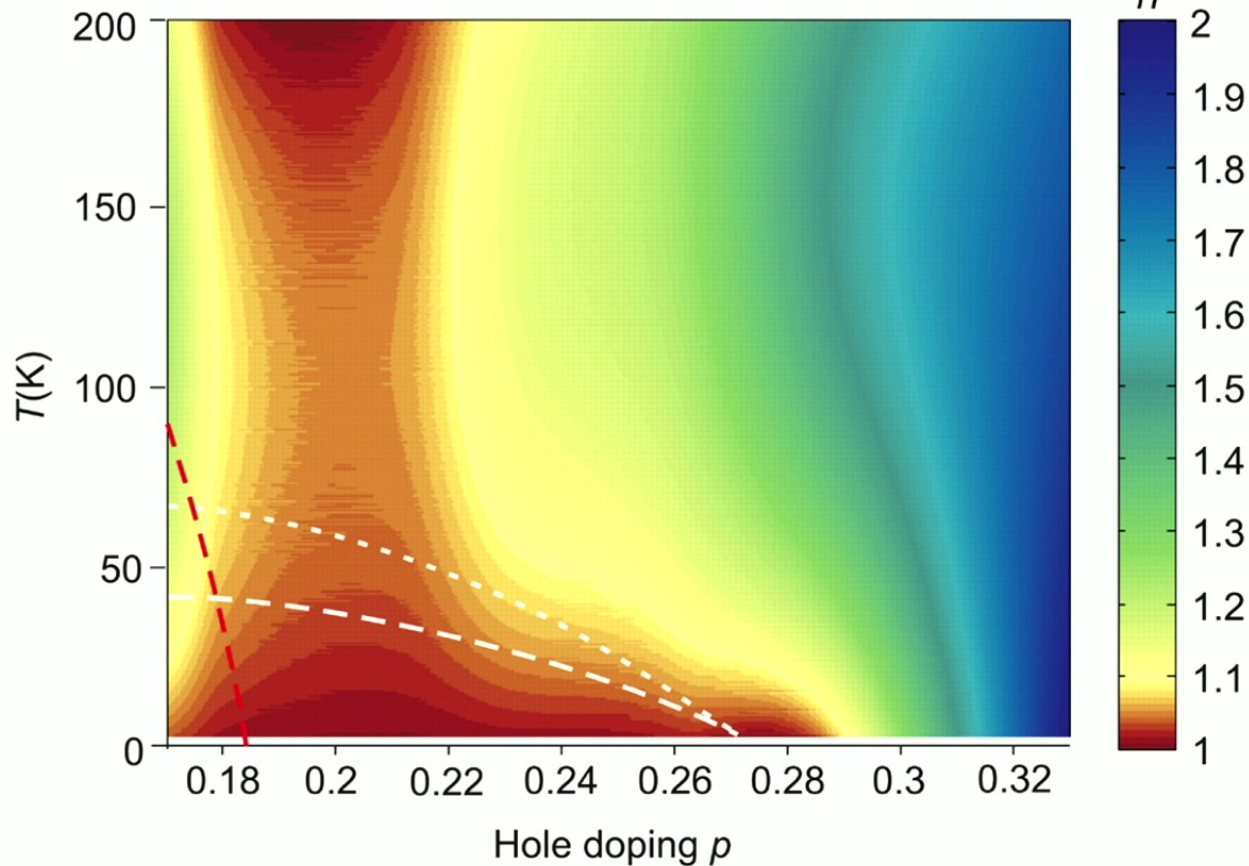
Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

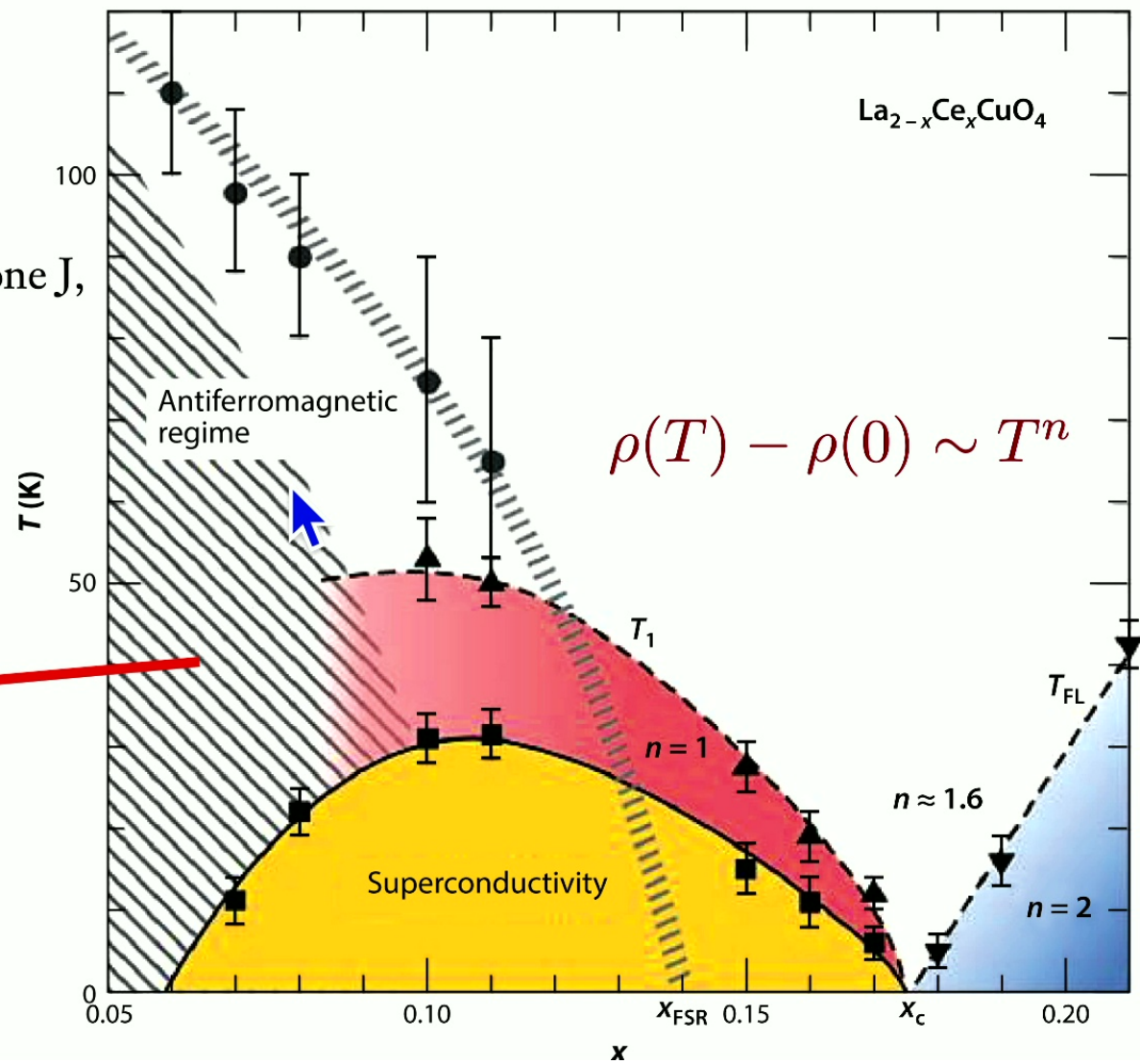
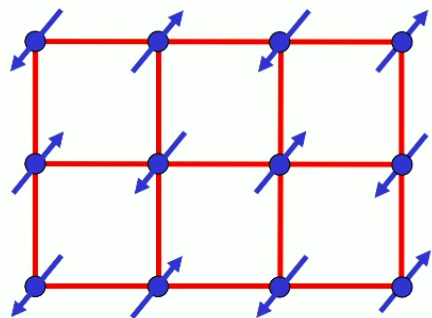
SCIENCE VOL 323 603 2009



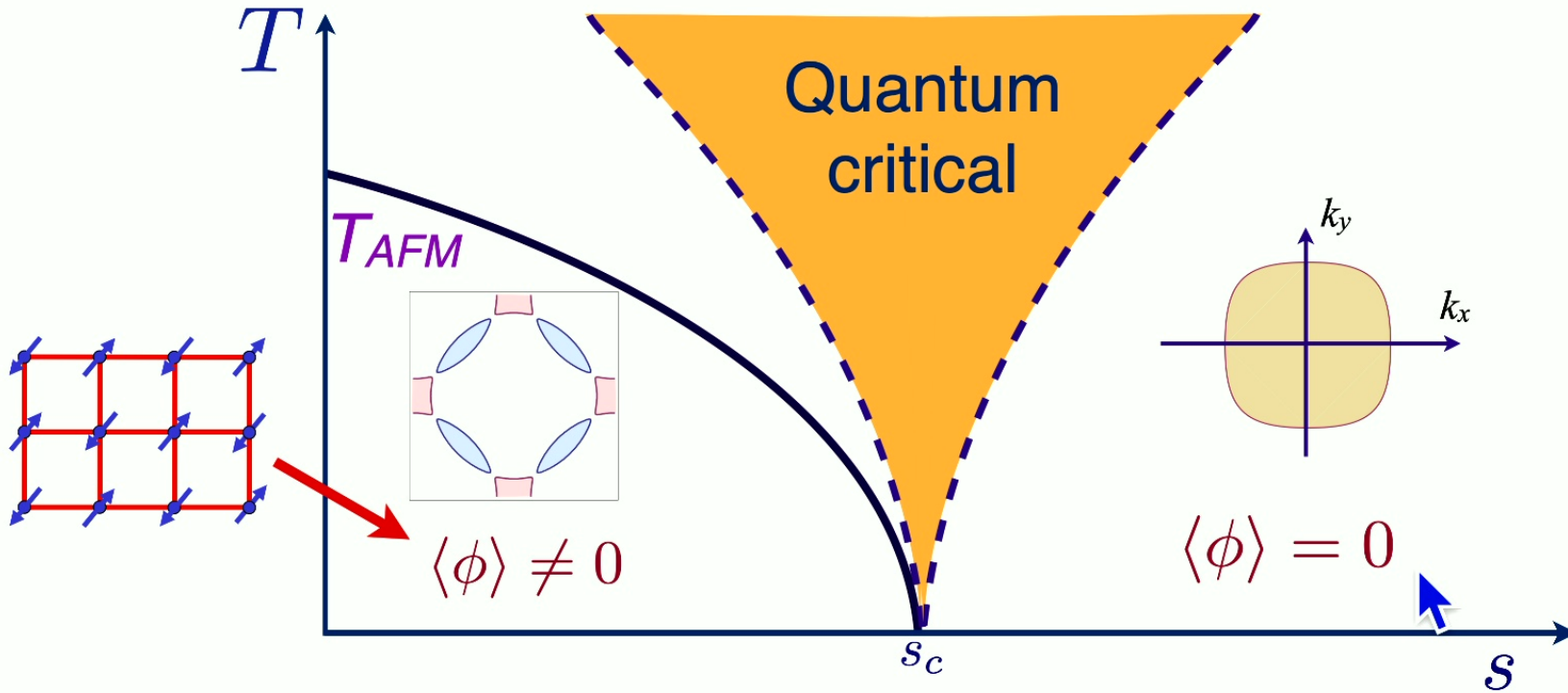
$$\rho(T) - \rho(0) \sim T^n$$



Jin K, Butch NP, Kirshenbaum K, Paglione J,
Greene RL. 2011. *Nature* 476:73–75

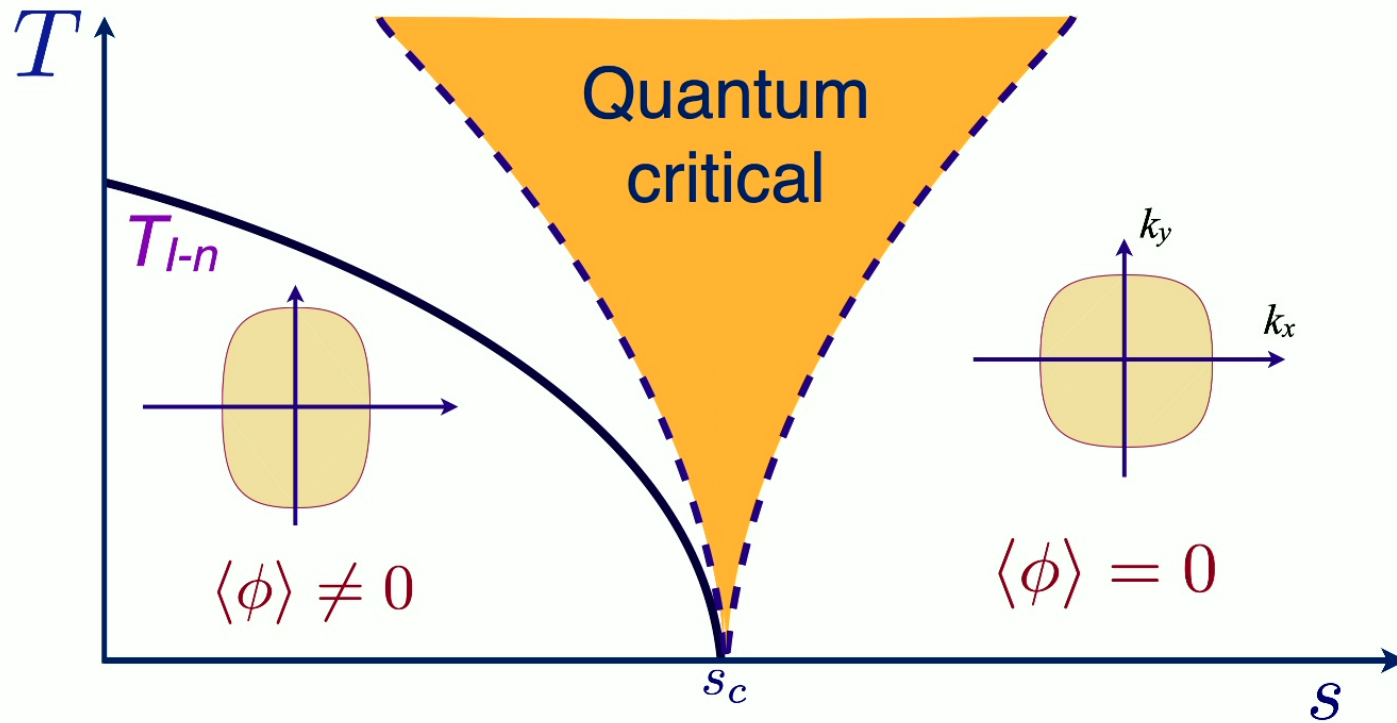


Quantum criticality of AF ordering in a metal



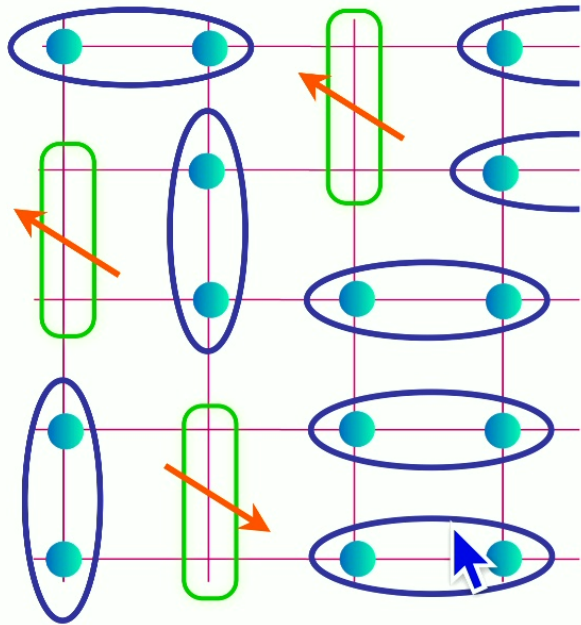
Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlieff, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Quantum criticality of Ising-nematic ordering in a metal



Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlieff, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Pseudogap metal to Fermi liquid in single band model



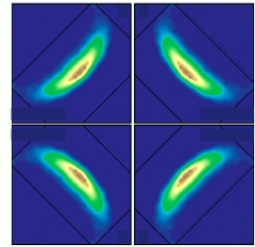
Higgs boson with Φ the fundamental gauge charge of an emergent SU(2) gauge field.

$$\text{blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\text{green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Small Fermi surface of size p + spin liquid.

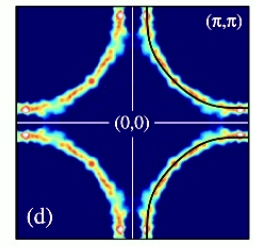
FL*



$$\langle \Phi \rangle \neq 0$$

Large Fermi surface of size $1 + p$

FL

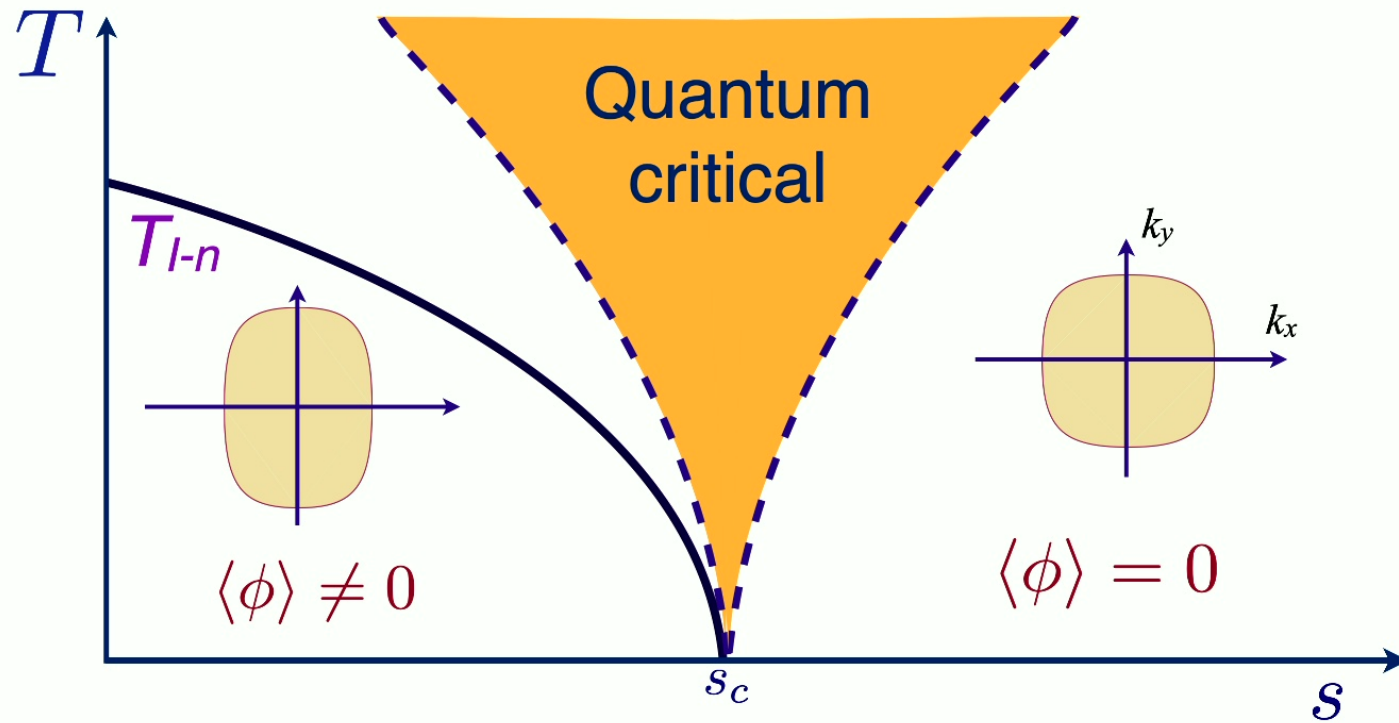


$$\langle \Phi \rangle = 0$$

Ya-Hui Zhang and S.S. *Phys. Rev. Research* **2**, 023172 (2020); *Phys. Rev. B* **102**, 155124 (2020)

doping p →

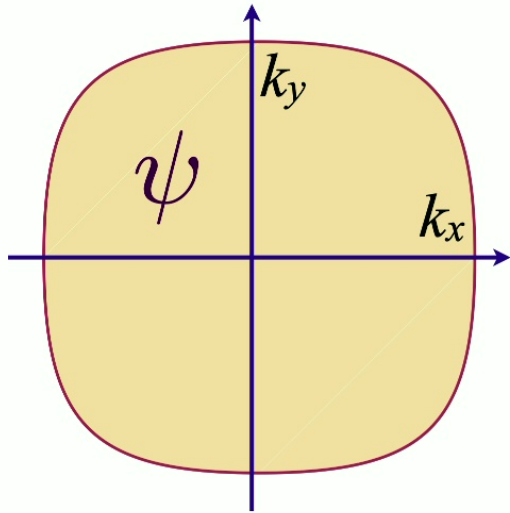
Quantum criticality of Ising-nematic ordering in a metal



Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlieff, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Fermi surface + critical boson with no spatial disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,
 Higgs boson for Fermi-volume changing transition

$$+s [\phi(\mathbf{r})]^2 \quad +g \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

$$+K [\nabla_{\mathbf{r}}\phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4$$

A non-Fermi liquid in the
 electron spectral function
 but a perfect metal in transport!

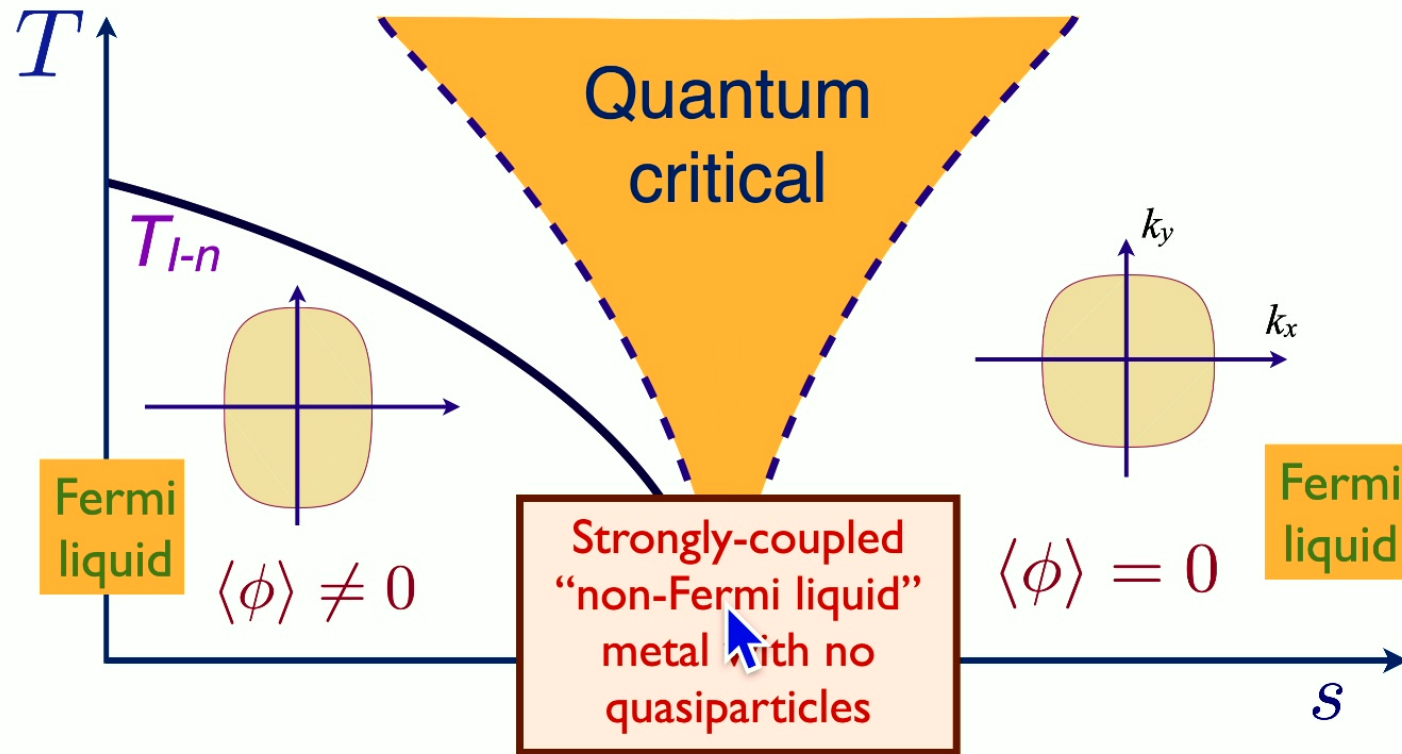


$$\Sigma(\omega) \sim \omega^{2/3}$$

$$\sigma(\omega) = iD/(\omega - \omega_c) + \omega^0 + \dots$$

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S., PRB **106**, 115151 (2022)
 Haoyu Guo, Davide Valentini, J. Schmalian, S.S., Aavishkar Patel, PRB **109**, 075162 (2024)
 D.L. Maslov and A.V. Chubukov, Rep. Prog. Phys. **80**, 026503 (2017)
 Zhengyan Darius Shi, Dominic V. Else, Hart Goldman, T. Senthil, SciPost Phys. **14**, 113 (2023)

Quantum criticality of Ising-nematic ordering in a metal



Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlieff, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

I. Yukawa-SYK model

II. Universal Yukawa-SYK theory in $d=2$ spatial dimensions

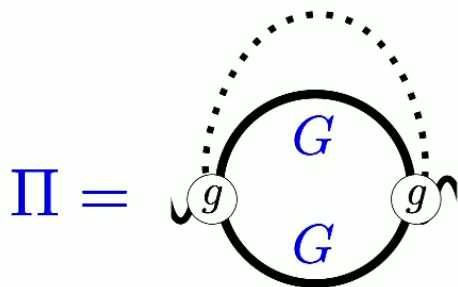
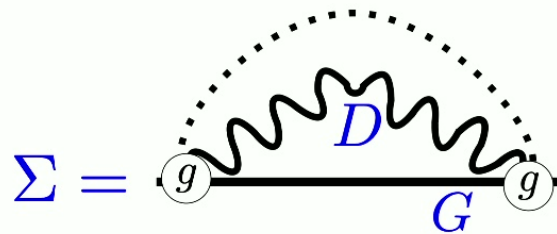
III. Random "mass" Hertz theory at low T

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with $\overline{g_{ij\ell}} = 0$, $\overline{g_{ij\ell}^2} = g^2$.

Leads to fully self-consistent Migdal-Eliashberg equations
 $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

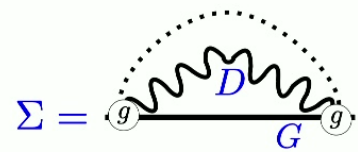


- W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
- J. Murugan, D. Stanford, and E. Witten, JHEP **08**, 146 (2017)
- A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
- E. Marcus and S. Vandoren, JHEP **01**, 166 (2018)
- Yuxuan Wang, PRL **124**, 017002 (2020)
- I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
- Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
- E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
- Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
- W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
- I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

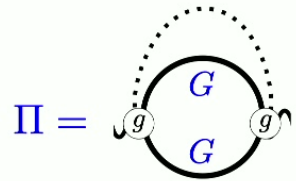
$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with $\overline{g_{ij\ell}} = 0$, $\overline{g_{ij\ell}^2} = g^2$. Large N saddle-point equations:



$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$



Make the low frequency ansatz

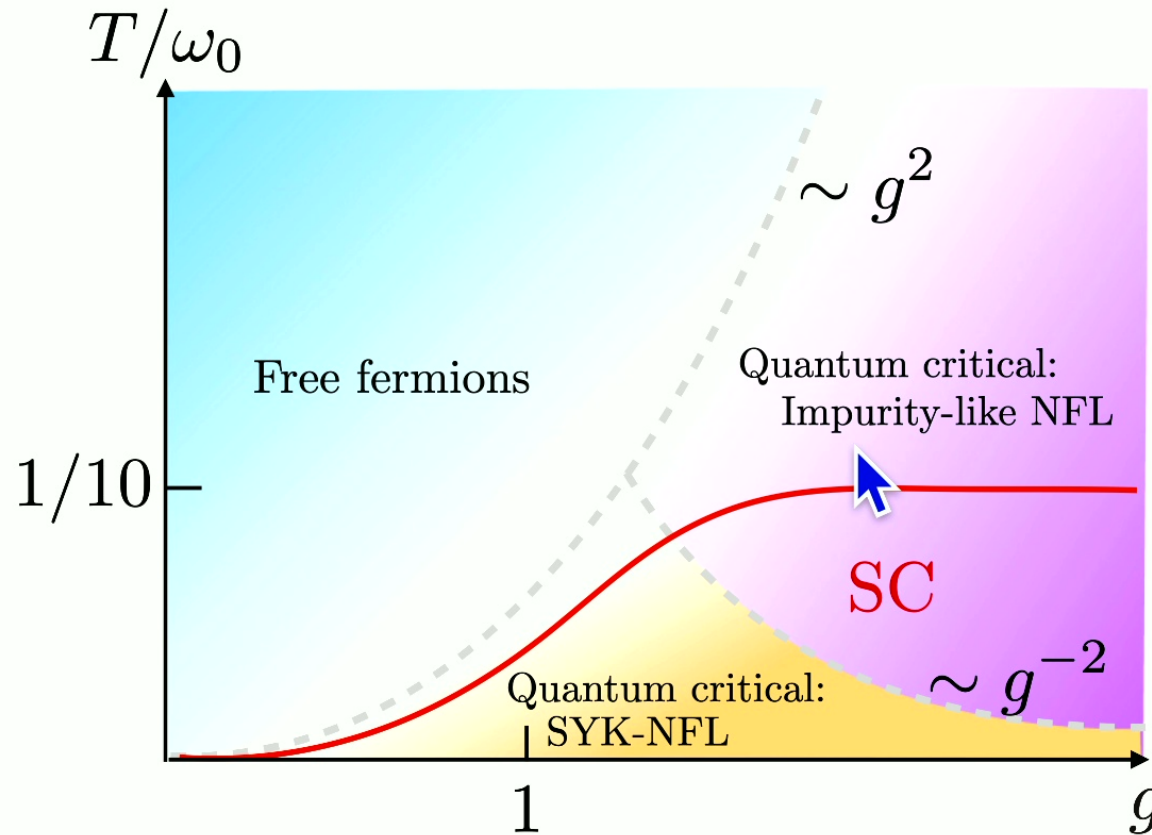
$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037 \dots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK models



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

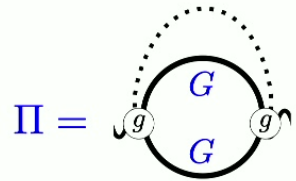
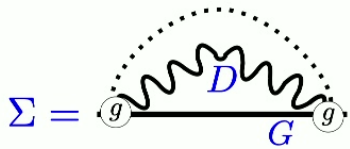
Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with $\overline{g_{ij\ell}} = 0$, $\overline{g_{ij\ell}^2} = g^2$. Large N saddle-point equations:

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$



Make the low frequency ansatz

$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037 \dots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

See also Yuxuan Wang, PRL **124**, 017002 (2020)

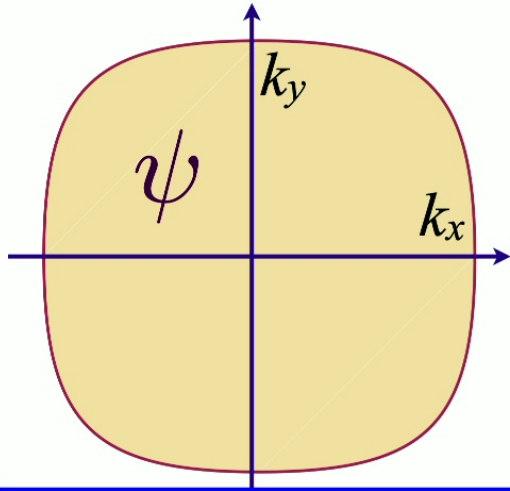
I. Yukawa-SYK model

II. Universal Yukawa-SYK theory in $d=2$ spatial dimensions

III. Random "mass" Hertz theory at low T

Fermi surface + critical boson with potential disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

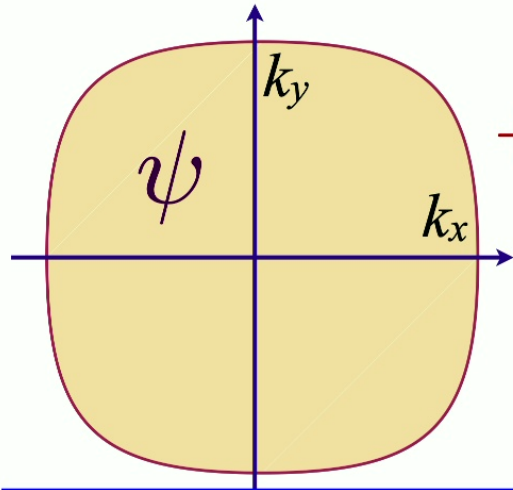
Higgs boson for Fermi-volume changing transition

$$\begin{aligned}
 &+s [\phi(\mathbf{r})]^2 && +g \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) \\
 &+K [\nabla_{\mathbf{r}}\phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 && +v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})
 \end{aligned}$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2\delta(\mathbf{r} - \mathbf{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + +g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

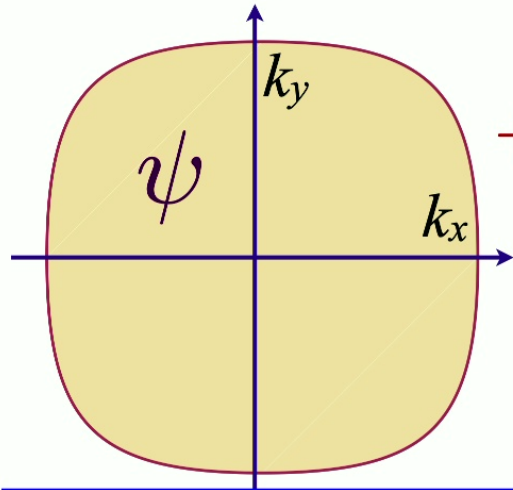
$$+ K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')}$ = $v^2\delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')}$ = $\delta s^2\delta(\mathbf{r} - \mathbf{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + +g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

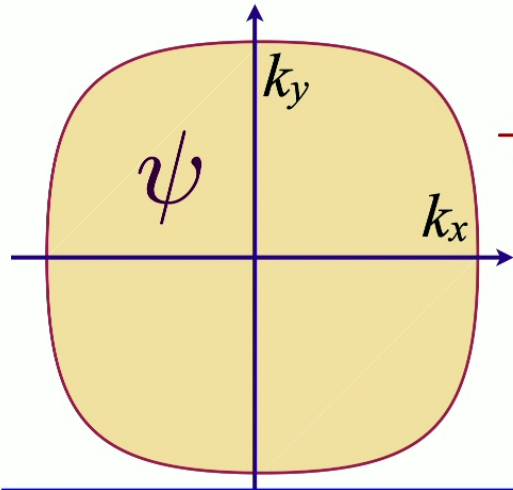
Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')} = \delta s^2 \delta(\mathbf{r} - \mathbf{r}')$

RG analysis (Harris criterion) shows that $\delta s(\mathbf{r})$ is most relevant disorder.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,
 Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + \quad + g \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

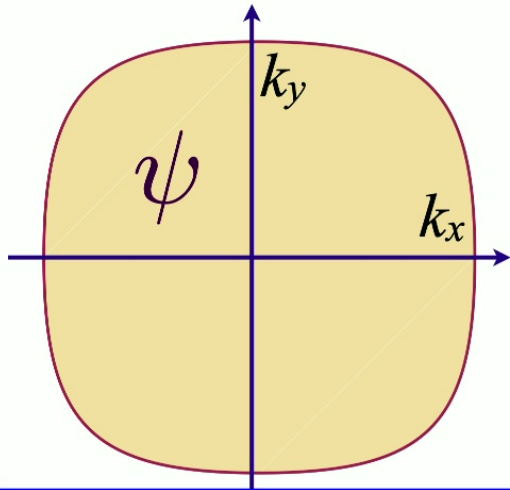
Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')}$ = $v^2\delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')}$ = $\delta s^2\delta(\mathbf{r} - \mathbf{r}')$

RG analysis (Harris criterion) shows that $\delta s(\mathbf{r})$ is most relevant disorder.
 Rescale $\phi(\mathbf{r})$ to obtain a theory with $\delta s(\mathbf{r}) = 0$.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+s [\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

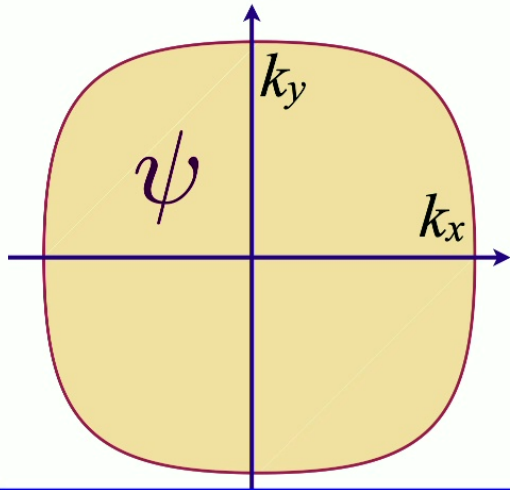
Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

RG analysis (Harris criterion) shows that $\delta s(\mathbf{r})$ is most relevant disorder.
 Rescale $\phi(\mathbf{r})$ to obtain a theory with $\delta s(\mathbf{r}) = 0$.

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+s [\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

Analyze such a theory in a self-averaging manner as in the Yukawa-SYK model.
 Should be applicable as long as eigenmodes of $\phi(\mathbf{r})$ are extended.

Fermi surface + critical boson with potential and interaction disorder

SYK-type self-consistent equations

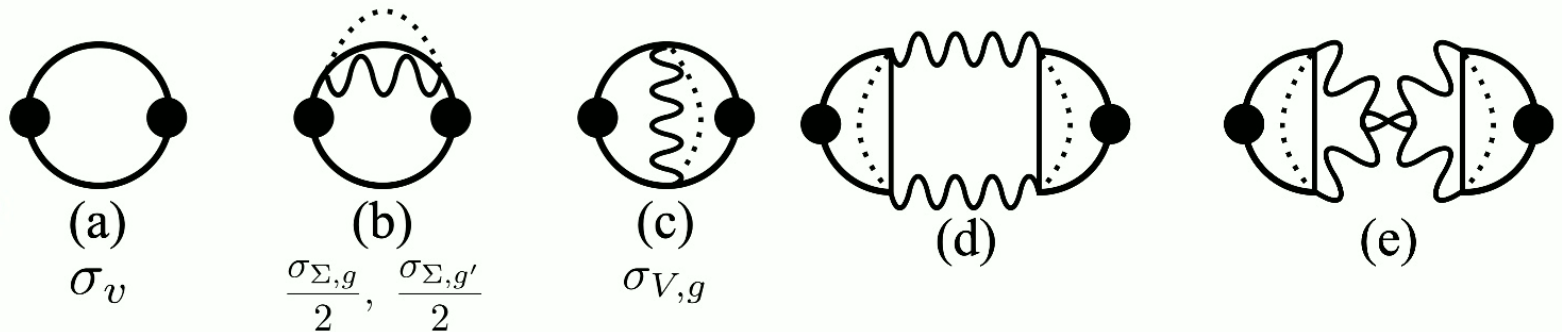
$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Conductivity:



+ all ladders and bubbles.....

Fermi surface + critical boson with potential and interaction disorder

Electron Green's function: $G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

T.J. Reber....D. Dessau, Nature Communications **10**, 5737 (2019)

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S.S., Science **381**, 790 (2023)

Fermi surface + critical boson with potential and interaction disorder

Electron Green's function:
$$G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Conductivity:
$$\sigma(\omega) \sim \frac{1}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

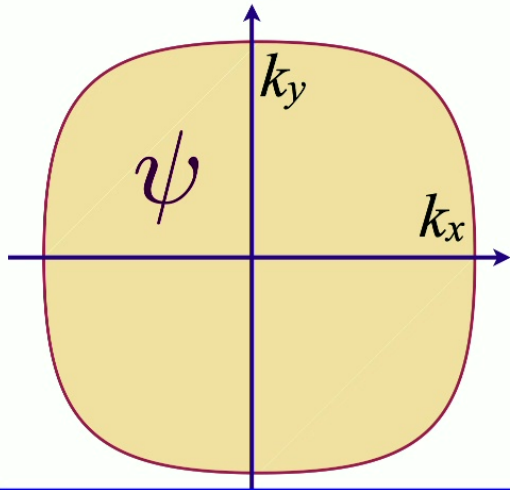
B. Michon.....A. Georges, Nat. Commun. **14**, 3033 (2023)

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.
 Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ;
 Transport insensitive to g ;

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S.S., Science **381**, 790 (2023)

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+s [\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

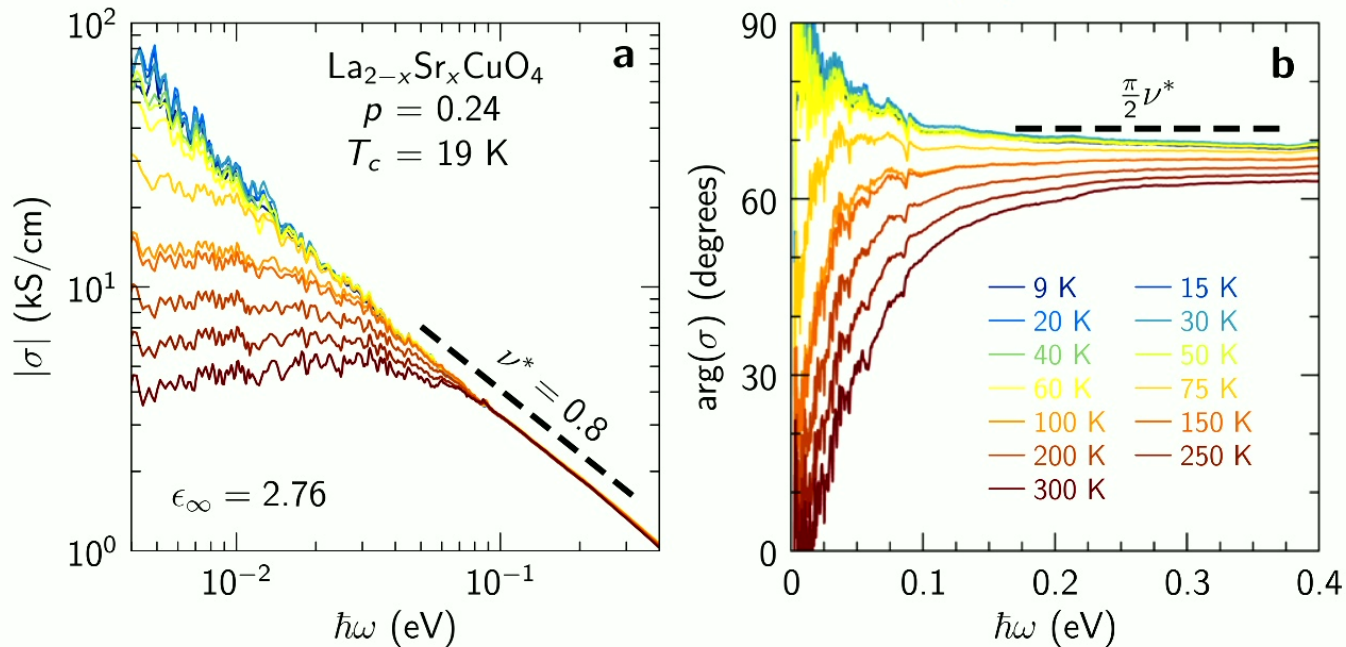
Analyze such a theory in a self-averaging manner as in the Yukawa-SYK model.
 Should be applicable as long as eigenmodes of $\phi(\mathbf{r})$ are extended.

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

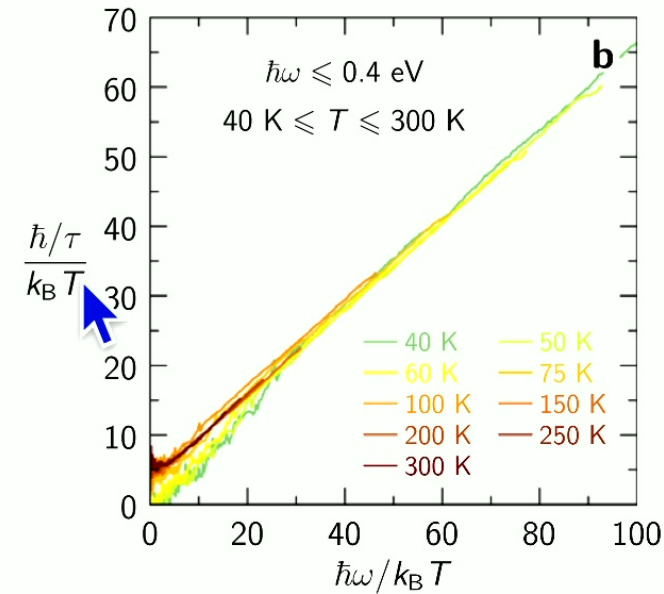
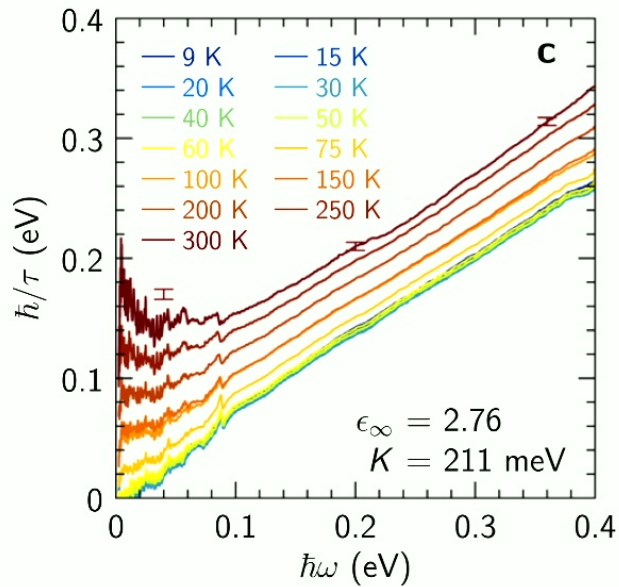


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$



Fermi surface + critical boson with potential and interaction disorder

Electron Green's function:
$$G(\omega) \sim \frac{1}{\omega \frac{m^*(\omega)}{m} - \varepsilon(\mathbf{k}) + i \left(\frac{1}{\tau_e} + \frac{1}{\tau_{\text{in}}(\omega)} \right) \text{sgn}(\omega)}$$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

Conductivity:
$$\sigma(\omega) \sim \frac{1}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

B. Michon.....A. Georges, Nat. Commun. **14**, 3033 (2023)

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.
 Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ;
 Transport insensitive to g ;

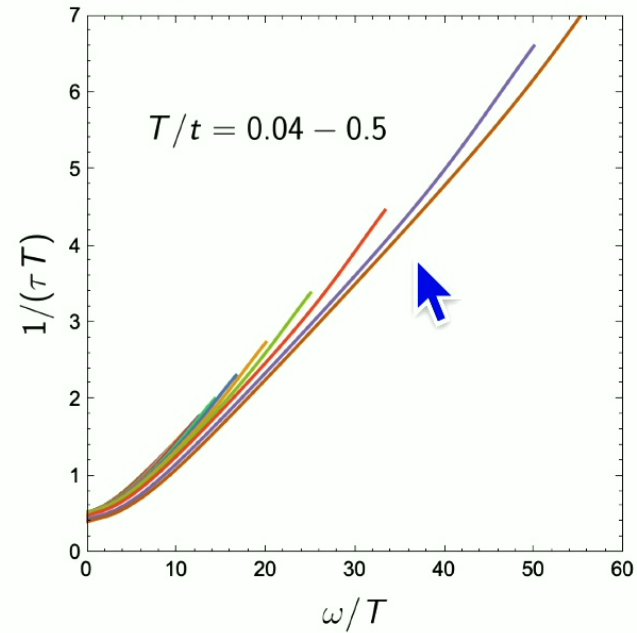
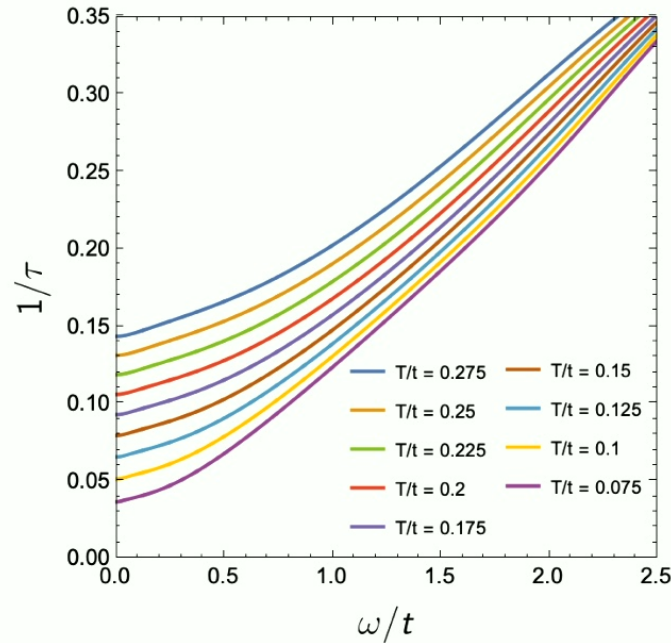
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S.S., Science **381**, 790 (2023)

Strange metal and superconductor in the two-dimensional Yukawa-SYK model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentini, Jorg Schmalian, S.S., Ilya Esters, to appear

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$



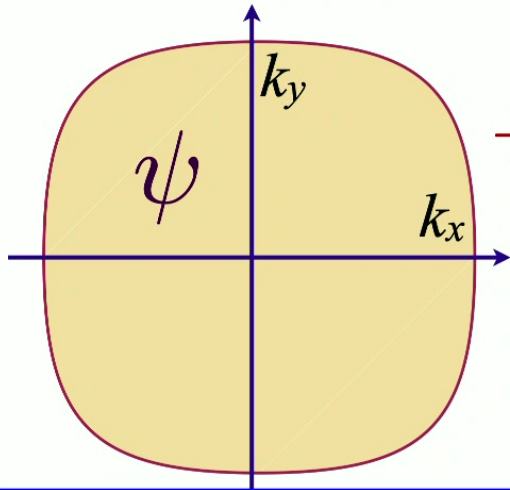
I. Yukawa-SYK model

**II. Universal Yukawa-SYK theory
in $d=2$ spatial dimensions**

III. Random "mass" Hertz theory at low T

Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



A critical boson ϕ
e.g. Ising-nematic order,
 spin-density wave order,

Higgs boson for Fermi-volume changing transition

$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r})$$

$$+ K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')} = \delta s^2 \delta(\mathbf{r} - \mathbf{r}')$

RG analysis (Harris criterion) shows that $\delta s(\mathbf{r})$ is most relevant disorder.
 Mapping of $\delta s(\mathbf{r})$ to $g'(\mathbf{r})$ only works if eigenmodes of $\phi(\mathbf{r})$ are extended.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \left(\frac{s + s'_j}{2} \phi_{ja}^2 + \frac{u}{4M} (\phi_{ja}^2)^2 \right) \right]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_\Omega \sum_j (\gamma|\Omega| + \Omega^2/c^2) |\phi_{ja}(i\Omega)|^2,$$

where $a = 1 \dots M$ is a flavor index for an order parameter with $O(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\bar{\mathcal{S}}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\bar{s}'_j}{2} \phi_{ja}^2 \right]$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev,
Phys. Rev. Lett. **101**, 035701 (2008).

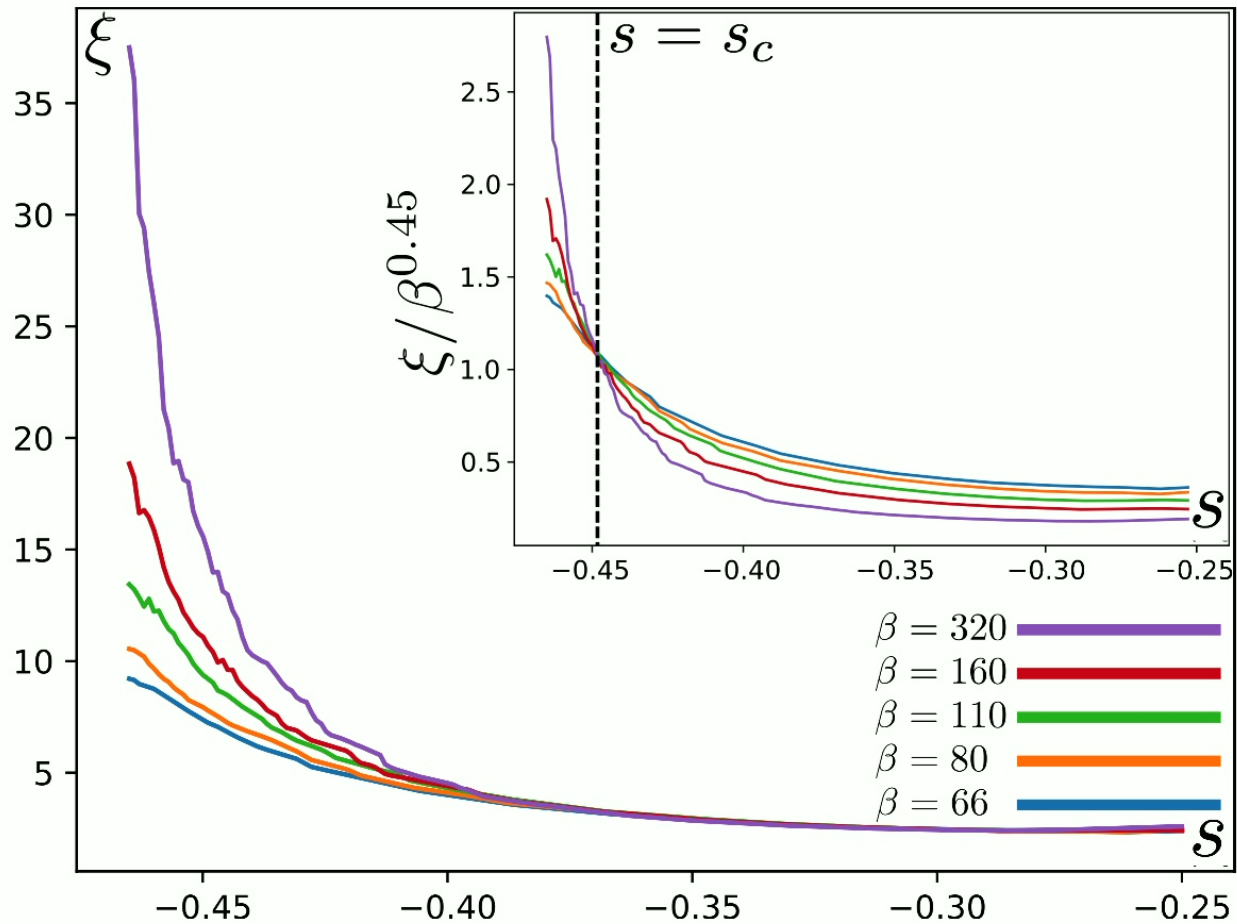
$$\bar{s}'_j = s + s'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\bar{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + s'_j + uT \sum_\Omega \sum_\alpha \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}$$

where e_α and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\bar{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \dots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS to appear, arXiv:2312.06751

Bosonic eigenmodes in random mass Hertz theory

ϕ correlation length ξ



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \left(\frac{s + s'_j}{2} \phi_{ja}^2 + \frac{u}{4M} (\phi_{ja}^2)^2 \right) \right]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j (\gamma|\Omega| + \Omega^2/c^2) |\phi_{ja}(i\Omega)|^2,$$

where $a = 1 \dots M$ is a flavor index for an order parameter with $O(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\bar{\mathcal{S}}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\bar{s}'_j}{2} \phi_{ja}^2 \right]$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev,
Phys. Rev. Lett. **101**, 035701 (2008).

$$\bar{s}'_j = s + s'_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\bar{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + s'_j + uT \sum_{\Omega} \sum_{\alpha} \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma|\Omega| + \Omega^2/c^2 + e_{\alpha}}$$

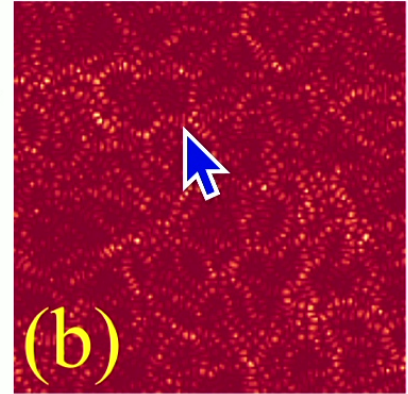
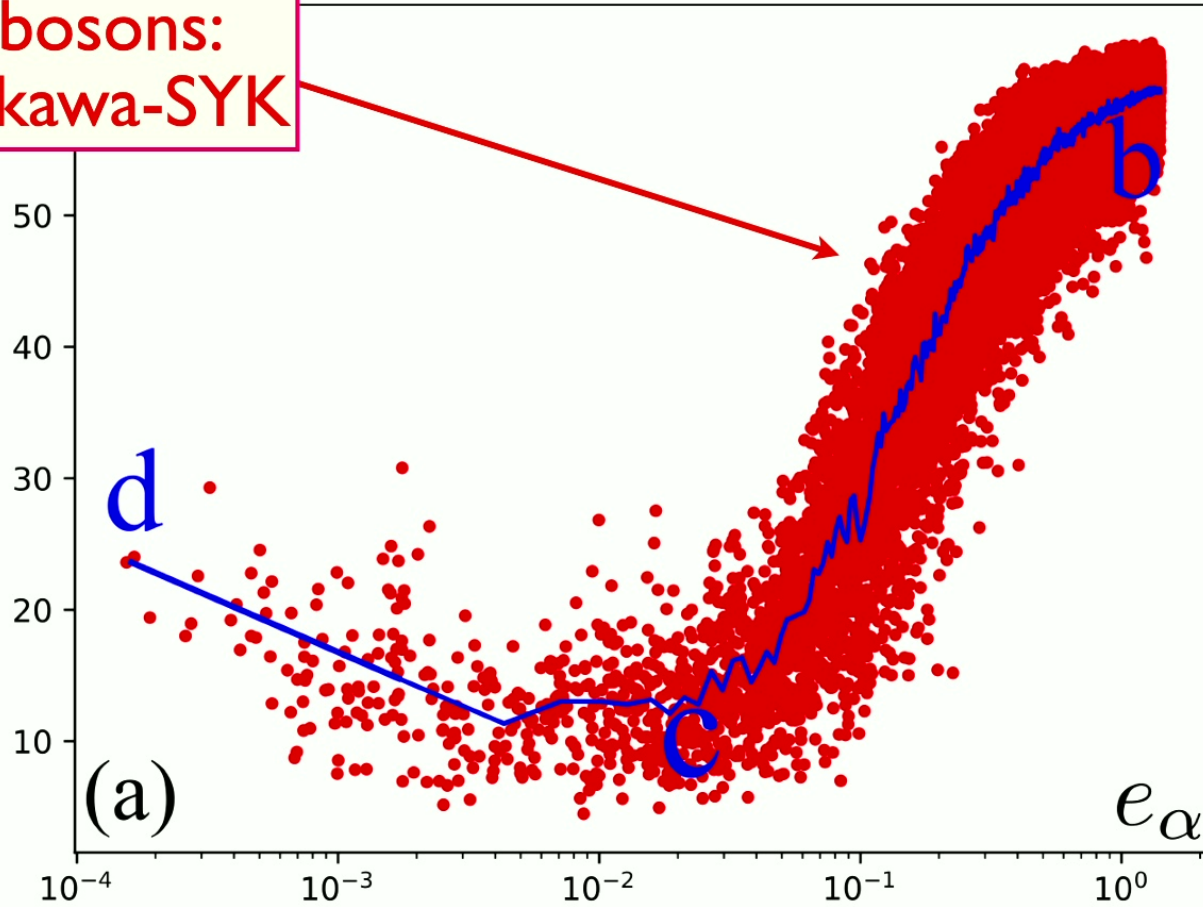
where e_{α} and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\bar{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \dots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS to appear, arXiv:2312.06751

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

Extended bosons:
physics of Yukawa-SYK

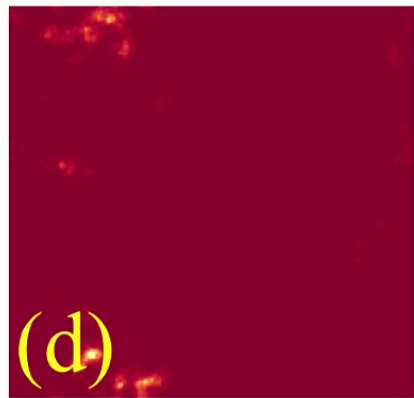


Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751

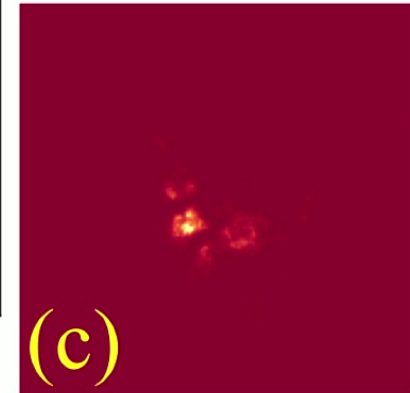
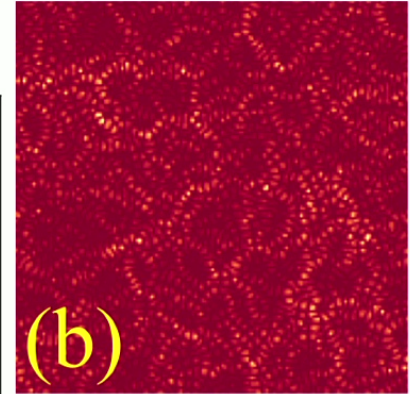
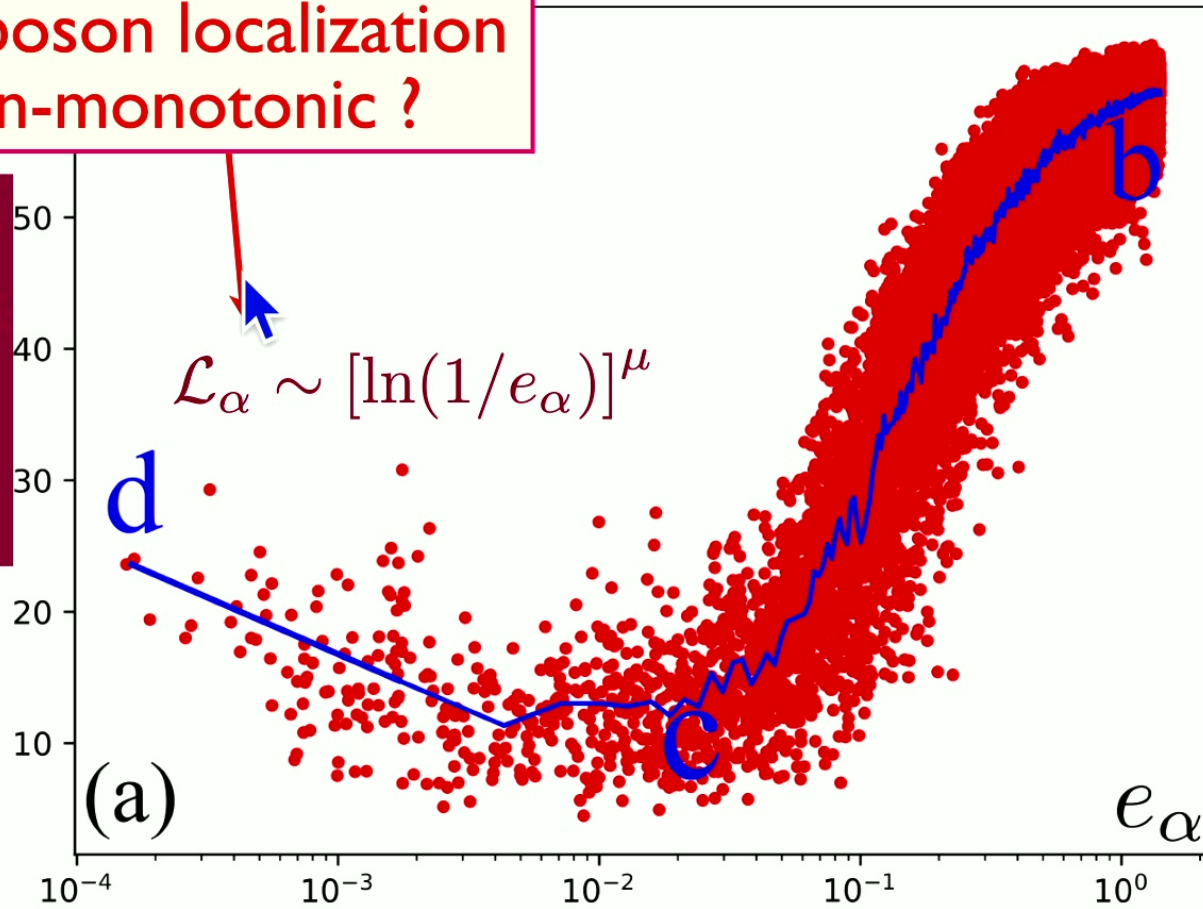
Landau-damped bosonic eigenmodes with random mass

ϕ eigenmodes localization length \mathcal{L}_α

Why is the boson localization length non-monotonic ?



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751



Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

$$\mathcal{S}_b = \int d\tau \left(- \sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[\frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4} (\phi_{ja}^2)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

Strong disorder RG identical to that for the RTFIM

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\tilde{s}_2 = 2 \frac{s_2 s_3}{J_{23}}$$

$$H_{\text{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

Numerically studied in $d=2$ by
O. Motrunich, S.-C. Mau, D.A. Huse and D.S. Fisher,
PRB **61** (2000) 1160

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

Strong disorder RG identical to that for the RTFIM

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\tilde{s}_2 = 2\frac{s_2s_3}{J_{23}}$$

$$H_{\text{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

Numerically studied in $d=2$ by
O. Motrunich, S.-C. Mau, D.A. Huse and D.S. Fisher,
PRB **61** (2000) 1160

Key fact: Similarity of classical $O(N \geq 2)$ chain with $1/r^2$ interactions,
and classical Ising chain with short-range interactions.

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

$$\mathcal{S}_b = \int d\tau \left(- \sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[\frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4} (\phi_{ja}^2)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

Strong disorder RG identical to that for the RTFIM

$$\tilde{J}_{ij} = J_{ij} + \frac{J_{i2}J_{2j}}{s_2}$$

$$\tilde{s}_2 = 2\frac{s_2s_3}{J_{23}}$$

$$H_{\text{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

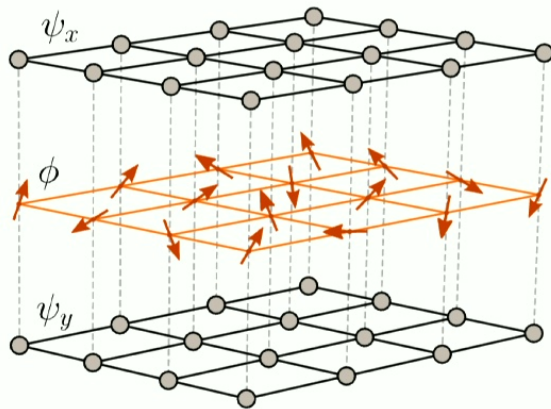
Numerically studied in $d=2$ by
O. Motrunich, S.-C. Mau, D.A. Huse and D.S. Fisher,
PRB **61** (2000) 1160

Key fact: Similarity of classical $O(N \geq 2)$ chain with $1/r^2$ interactions,
and classical Ising chain with short-range interactions.

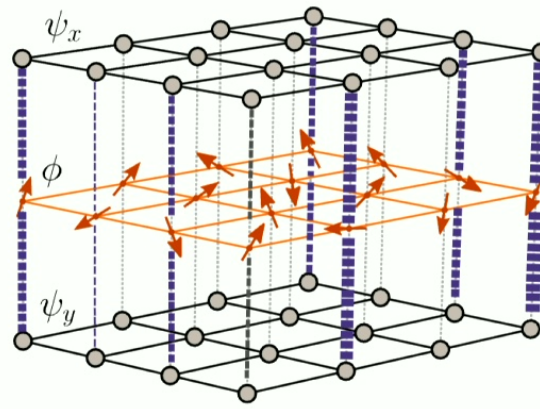
Model for sign-free QMC

$$\mathcal{S} = \int d\tau \sum_{\sigma=\uparrow,\downarrow} \sum_{\alpha=0,1} \sum_{j=1}^2 \sum_{\vec{r},\vec{r}'} \psi_{\alpha,\sigma,j,\vec{r}}^\dagger [\partial_\tau - (-1)^\alpha \mu - \delta_{\vec{r},\vec{r}'} - t_{\alpha,\vec{r},\vec{r}'}] \psi_{\alpha,\sigma,j,\vec{r}'}$$

$$+ \int d\tau \sum_{\vec{r}} \left[\frac{1}{c^2} (\partial_\tau \vec{\phi}_{\vec{r}})^2 + \frac{1}{2} (\nabla \vec{\phi}_{\vec{r}})^2 + \frac{r}{2} (\vec{\phi}_{\vec{r}})^2 + \frac{u}{4} (\vec{\phi}_{\vec{r}})^4 \right] + \sum_{\sigma,\sigma'=\uparrow,\downarrow} \sum_{j=1}^2 \int d\tau \sum_{\vec{r}} \boxed{g'(\vec{r}) e^{i\vec{Q}_{\text{AF}} \cdot \vec{r}}} \vec{\phi}_{\vec{r}} \cdot [\psi_{0,\sigma,\vec{r}}^\dagger \vec{\tau}_{\sigma,\sigma'} \psi_{1,\sigma',\vec{r}} + \text{H.c.}]$$



coupling $g = \text{const}$



coupling $g = \text{spatially random}$

$g = 0$

Two-band structure: Berg, Metlitsl Sachdev, Science 338 1606-1609 (2012).

Aavishkar A. Patel, Peter Lunts, M. Albergo **(to appear)**



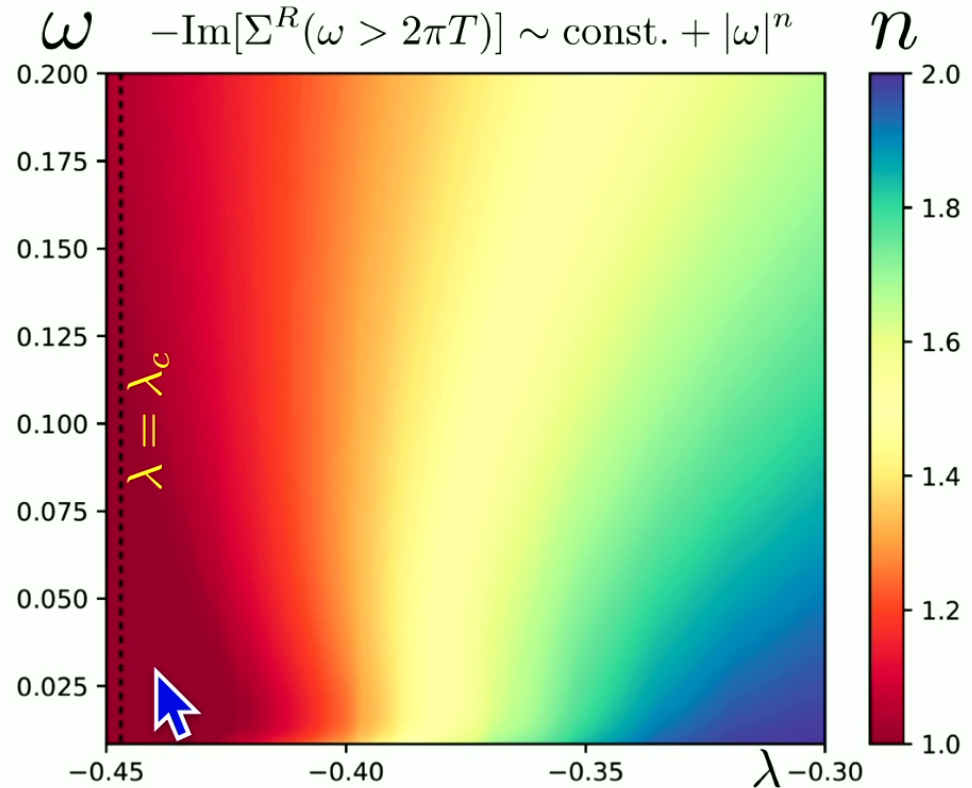
Bosonic eigenmodes in random mass Hertz theory

Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha, \Omega} \frac{\text{sgn}(\omega + \Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}.$$



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751



$L = 160, \beta = 800, 10$ disorder samples

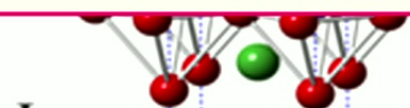
Extended region in λ with $n \approx 1$ - a strange metal *phase*

Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

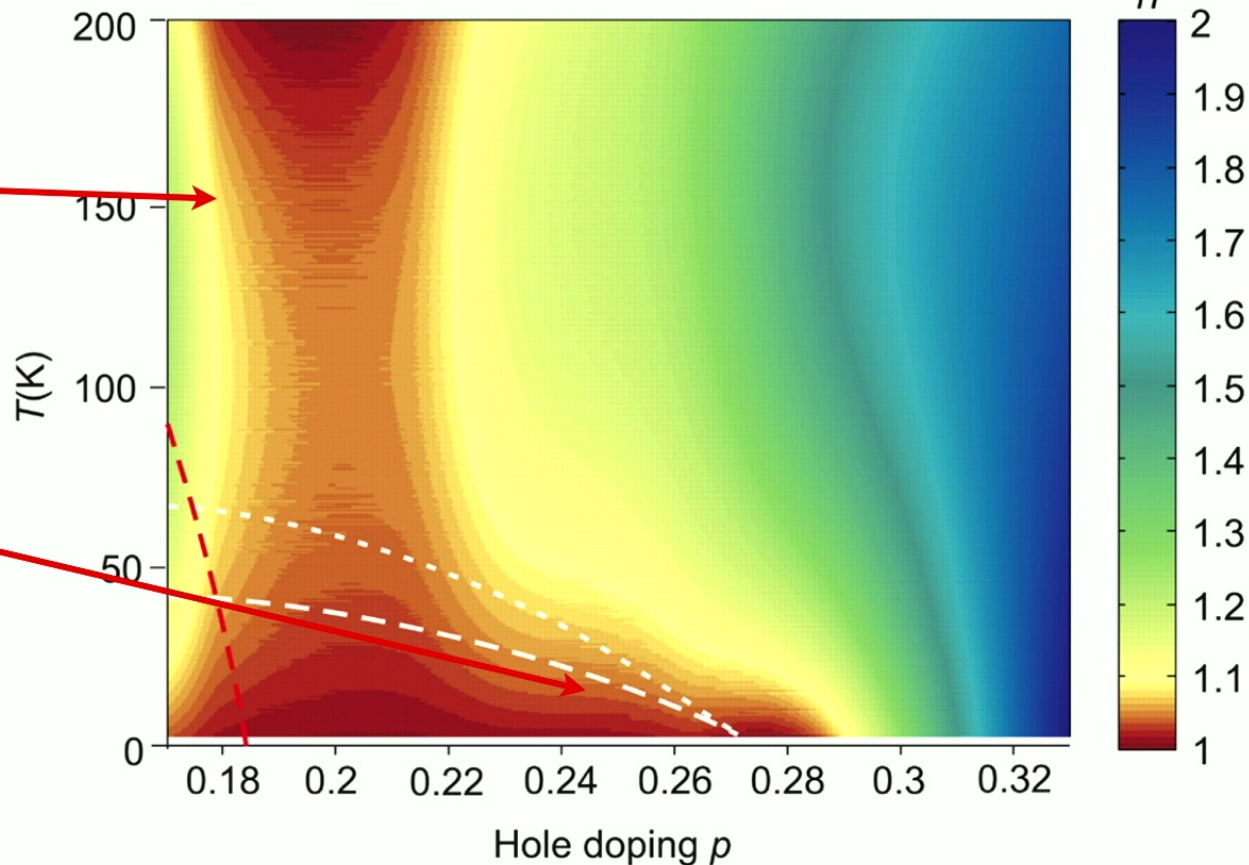
R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

Extended bosons:
physics of Yukawa-SYK



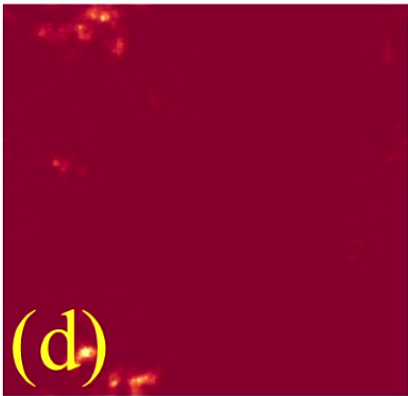
Localized overdamped bosons, but extended fermions



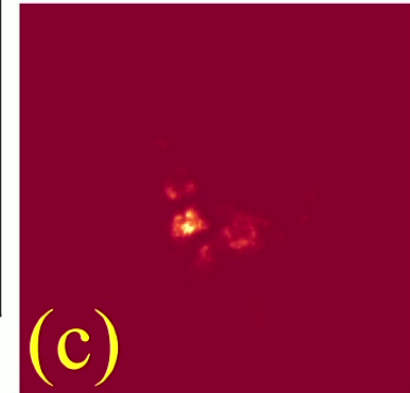
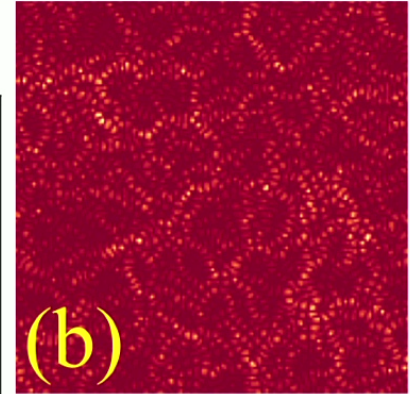
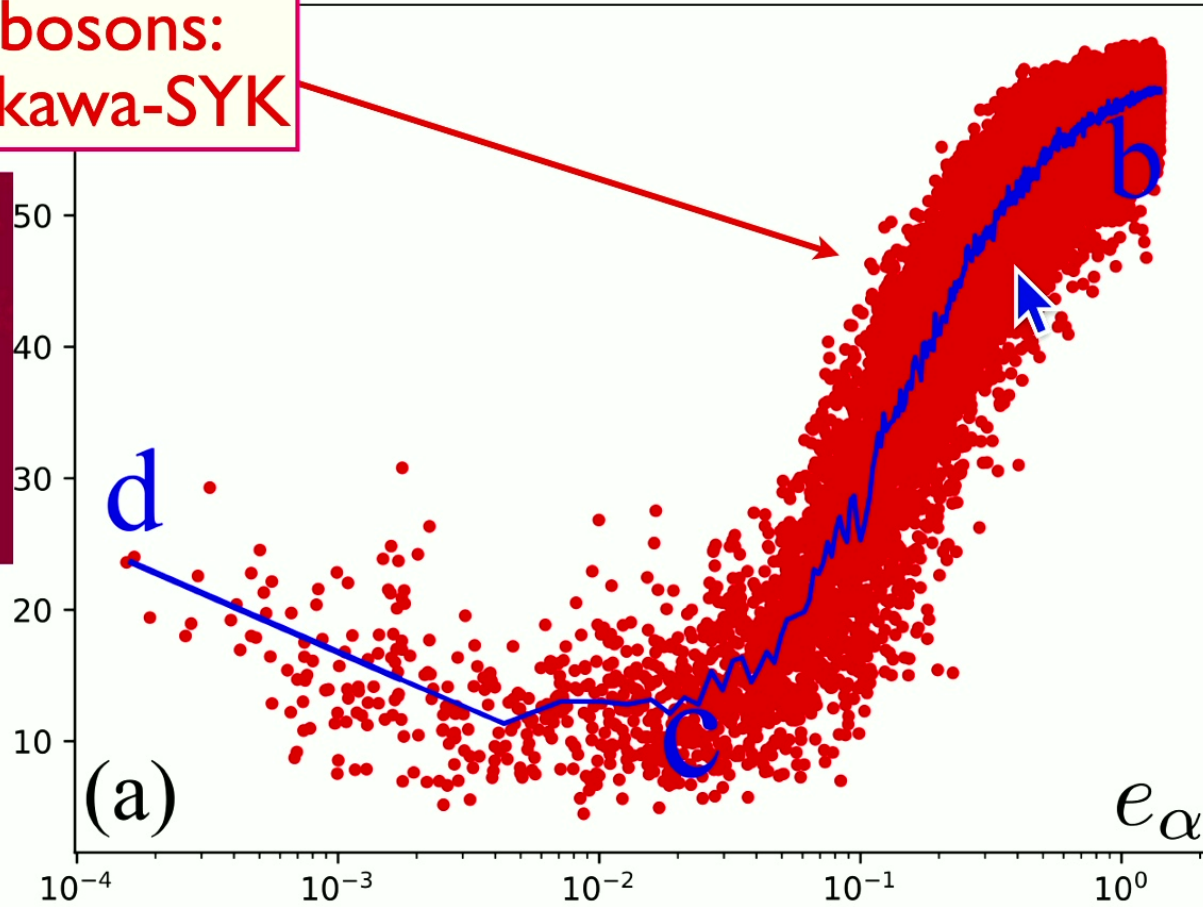
Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

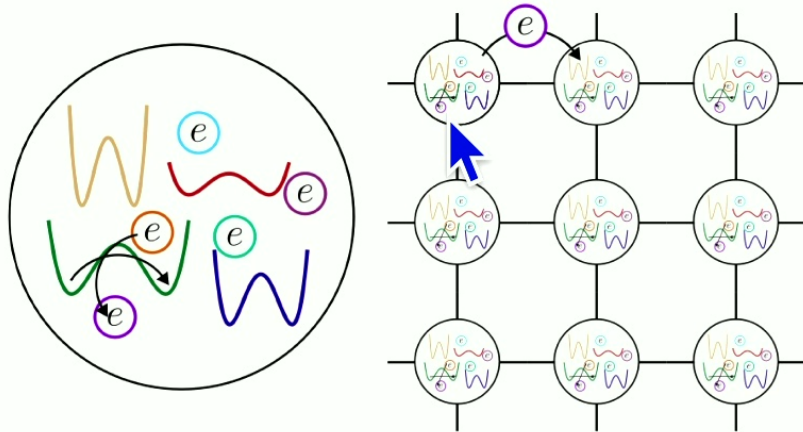
Extended bosons:
physics of Yukawa-SYK



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751



Tuneable non-Fermi liquid phase in electronic glasses

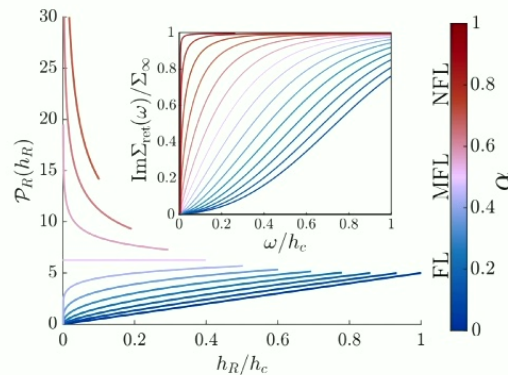


generalization of the Sachdev-Ye-Kitaev approach to two-level systems in glasses



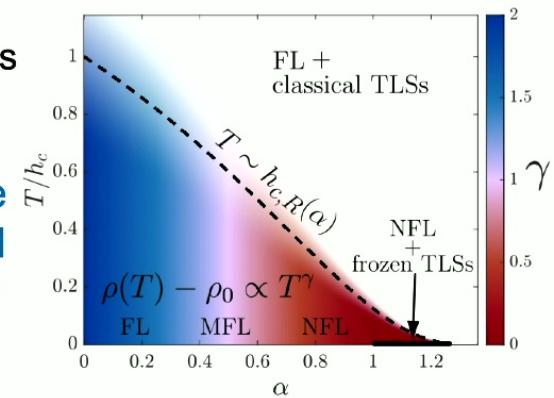
controlled approach to obtain a tunable Non-Fermi liquid state

strong renormalizations of the TLS distribution function



non-Fermi liquid physics in electronic glasses

Is this relevant to the physics of correlated quantum materials?



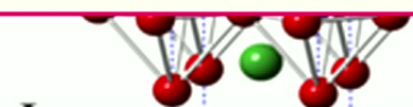
Noga Bashan, Evyatar Tulipman, Jörg Schmalian, Erez Berg, arXiv:2310.07768

Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

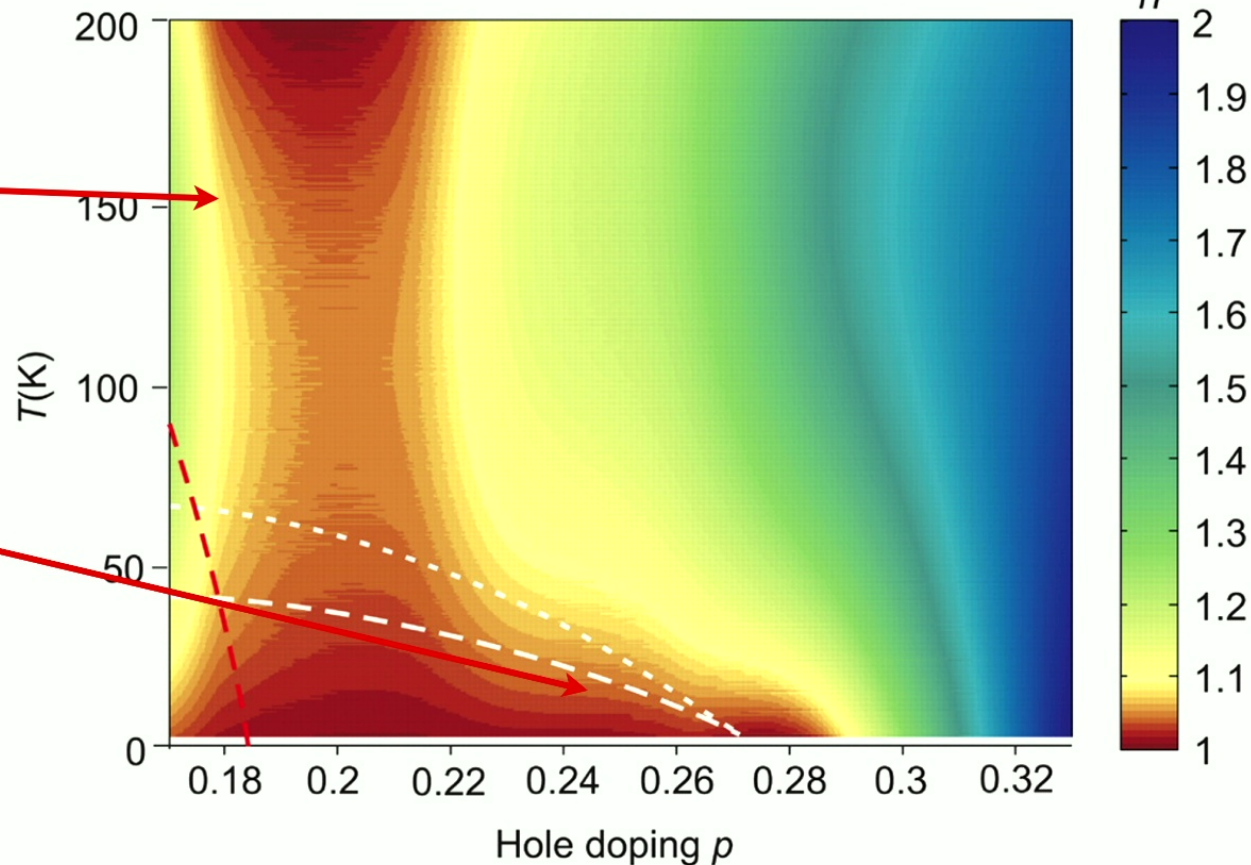
R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

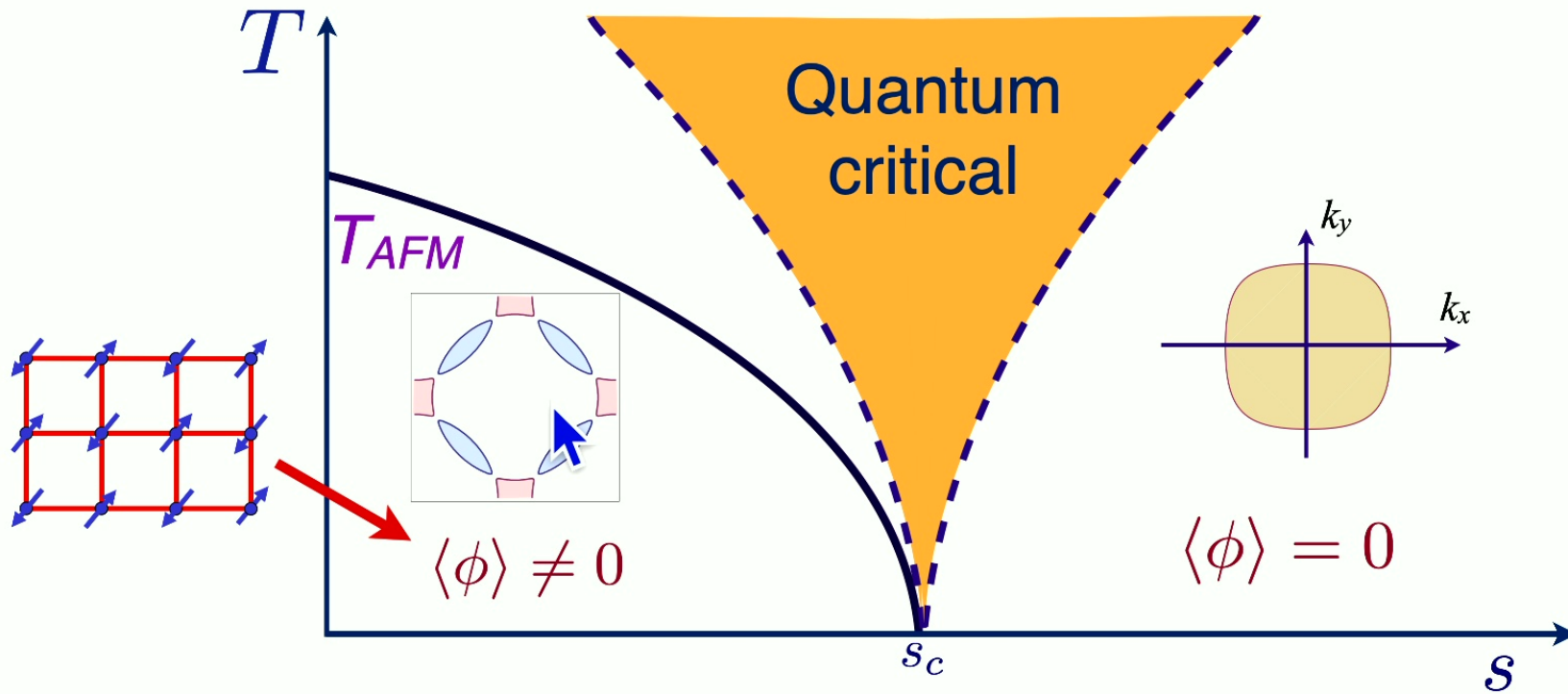
Extended bosons:
physics of Yukawa-SYK



Localized overdamped bosons, but extended fermions

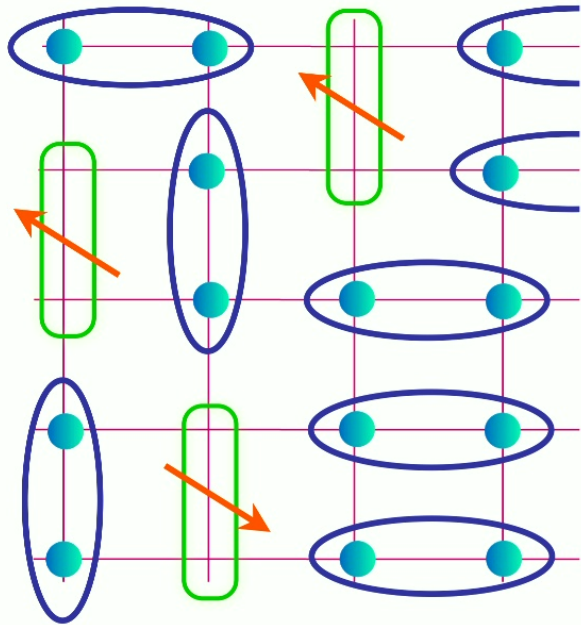


Quantum criticality of AF ordering in a metal



Francisco Borges, Anton Borissov, Ashutosh Singh, Andres Schlieff, Sung-Sik Lee
Annals of Physics 450, 169221 (2023)

Pseudogap metal to Fermi liquid in single band model



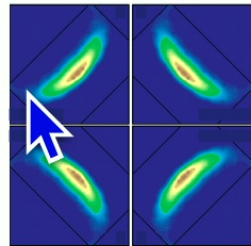
Higgs boson with Φ the fundamental gauge charge of an emergent SU(2) gauge field.

$$\text{blue oval} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

$$\text{green oval} = (|\uparrow\circ\rangle + |\circ\uparrow\rangle) / \sqrt{2}$$

Small Fermi surface of size p + spin liquid.

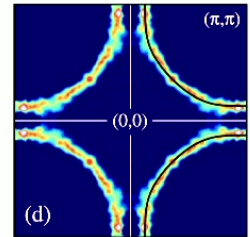
FL*



$$\langle \Phi \rangle \neq 0$$

Large Fermi surface of size $1 + p$

FL



$$\langle \Phi \rangle = 0$$

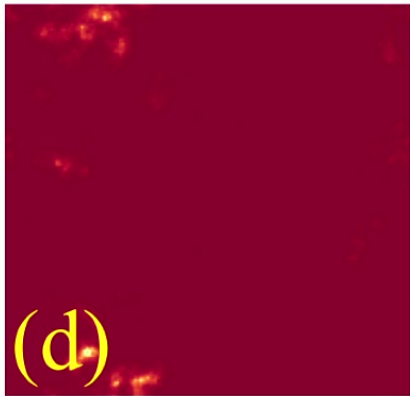
Ya-Hui Zhang and S.S. *Phys. Rev. Research* **2**, 023172 (2020); *Phys. Rev. B* **102**, 155124 (2020)

doping p →

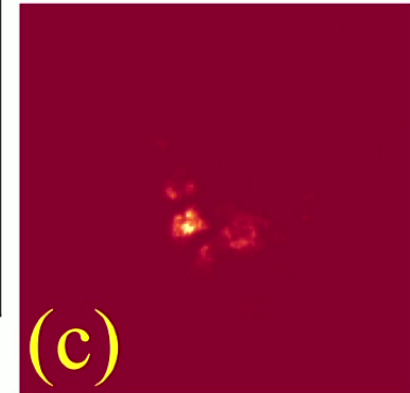
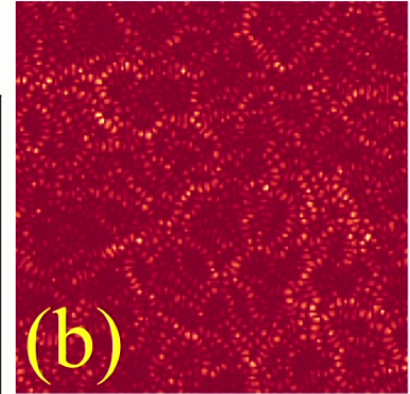
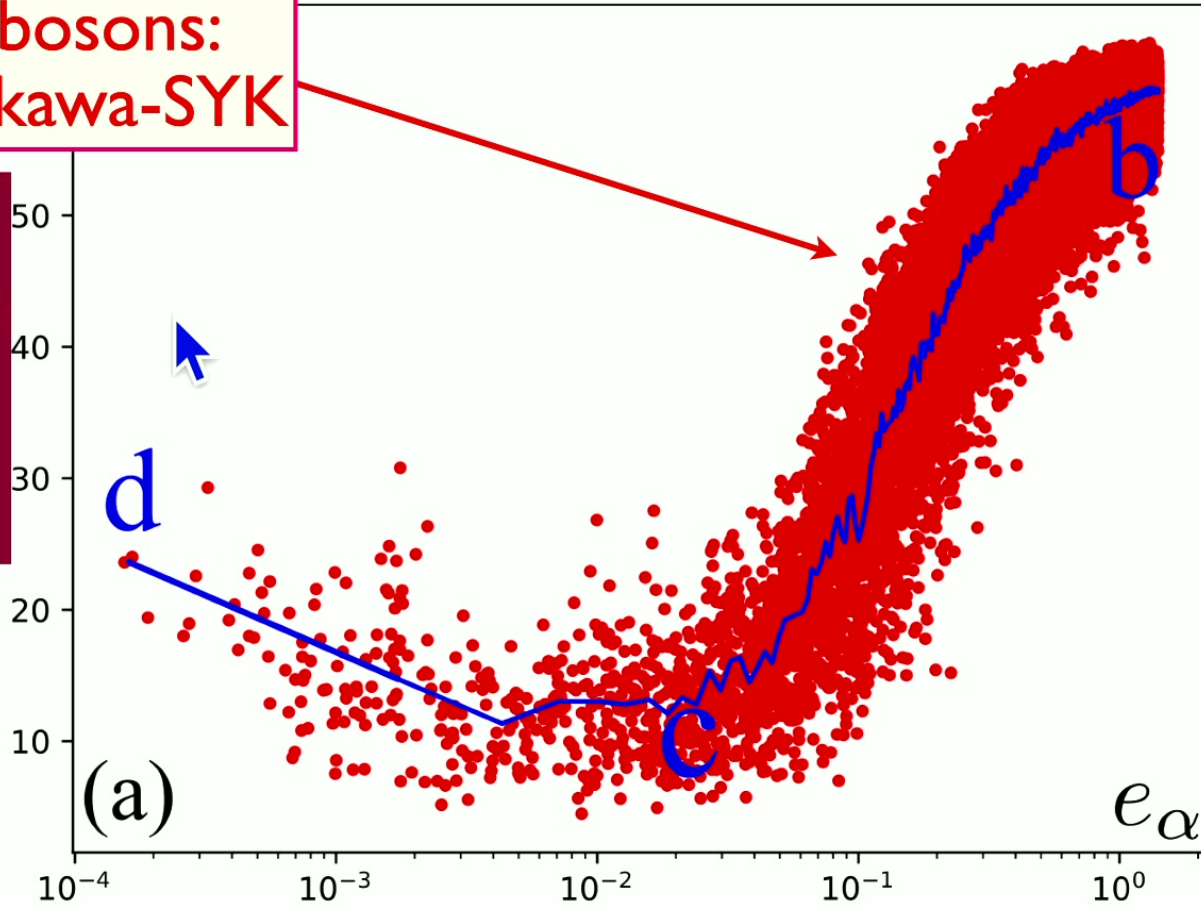
Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

Extended bosons:
physics of Yukawa-SYK



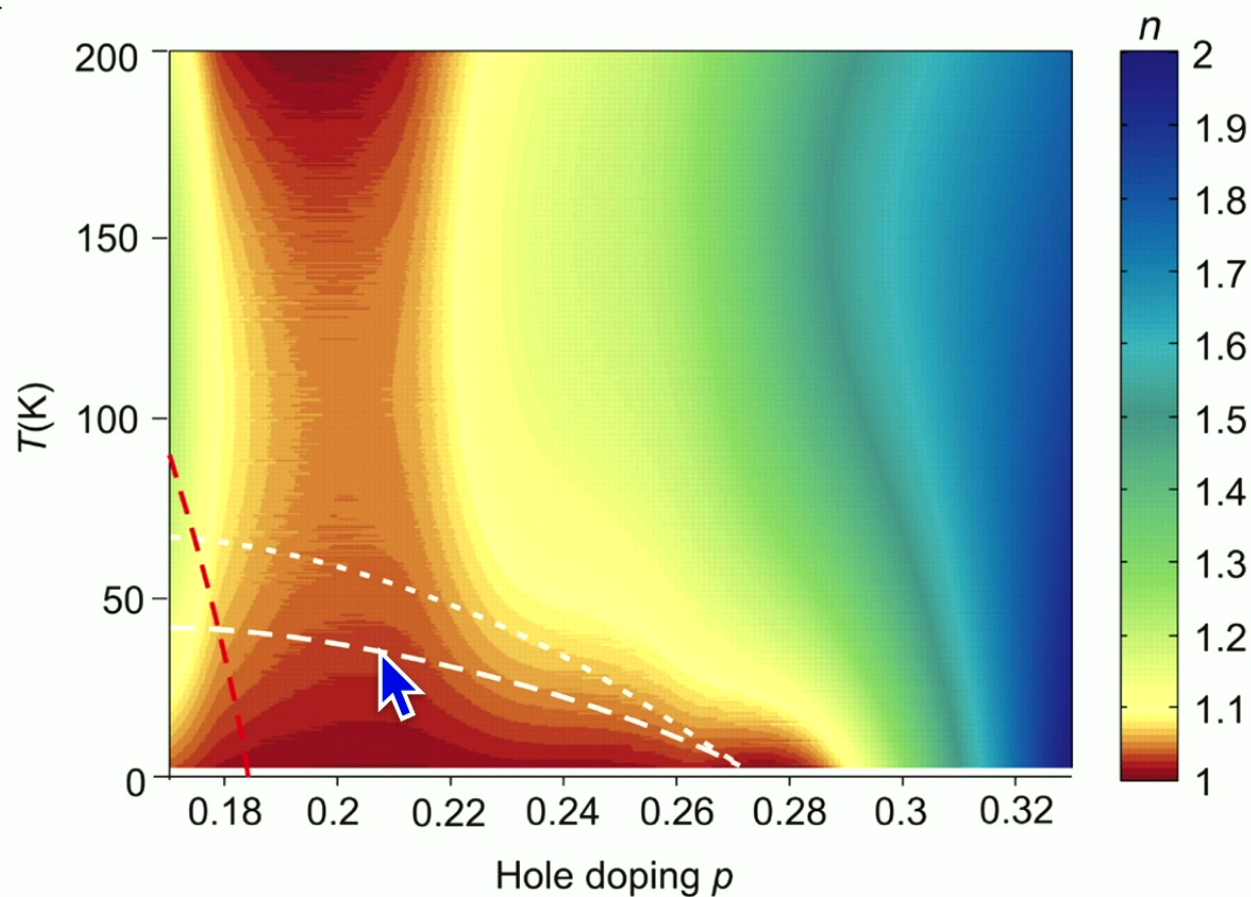
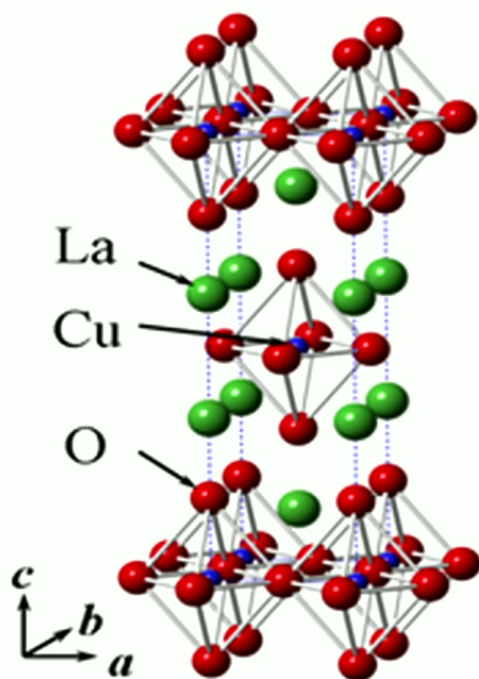
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS to appear,
arXiv:2312.06751



Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

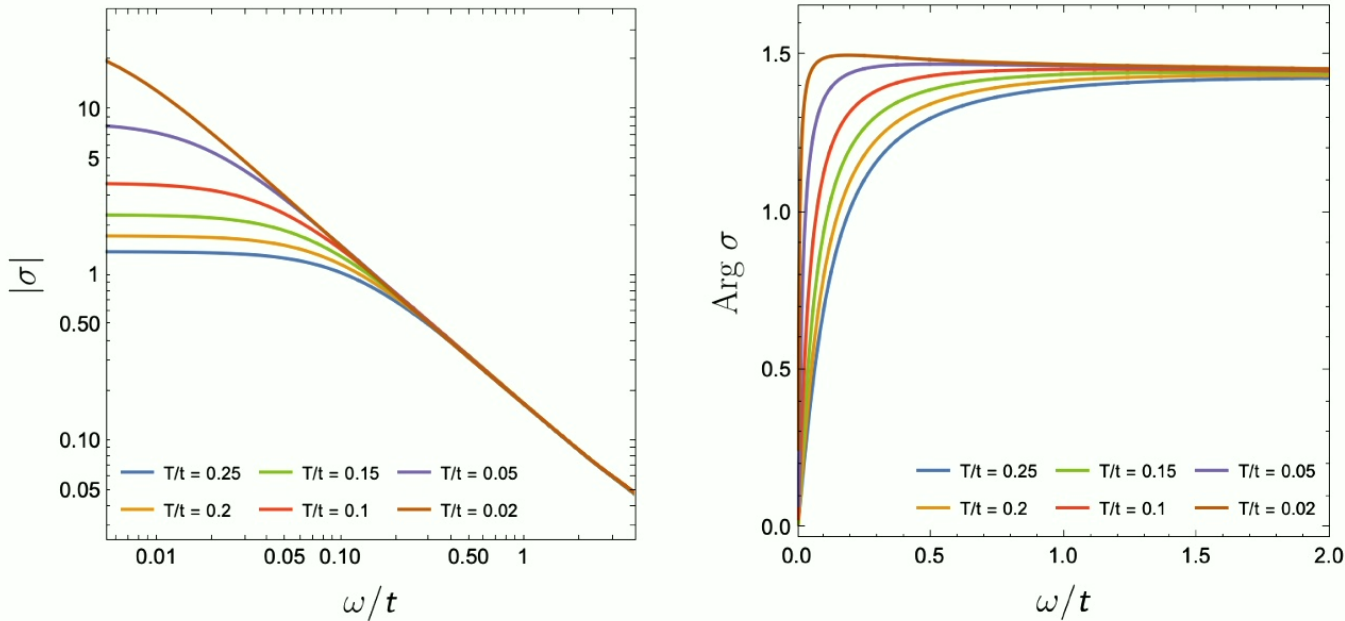


Strange metal and superconductor in the two-dimensional Yukawa-SYK model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentini, Jorg Schmalian, S.S., Ilya Esters, to appear

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$



Fermi surface + critical boson with potential and interaction disorder

SYK-type self-consistent equations

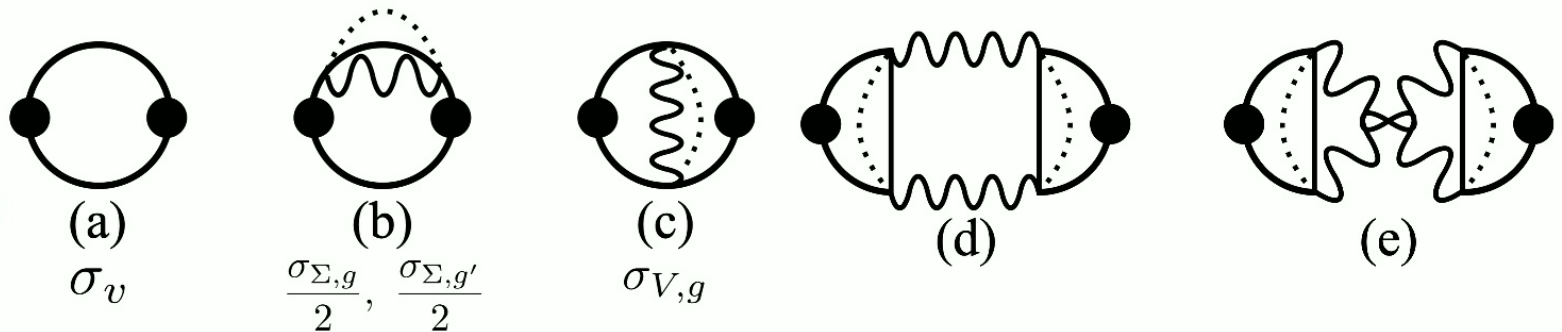
$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Conductivity:



+ all ladders and bubbles.....

Fermi surface + critical boson with potential and interaction disorder

SYK-type self-consistent equations

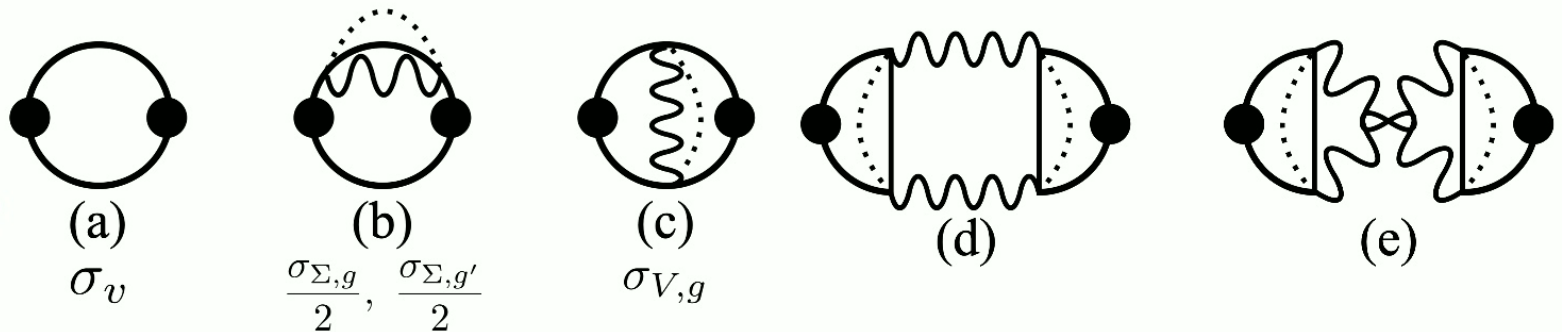
$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

Conductivity:



+ all ladders and bubbles.....