Title: Towards preparation of scattering wave packets of hadrons on a quantum computer

Speakers: Saurabh Kadam

Series: Perimeter Institute Quantum Discussions

Date: March 13, 2024 - 11:00 AM

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Abstract: Hamiltonian simulation of lattice gauge theories (LGTs) is a non-perturbative method of numerically solving gauge theories that offers novel avenues for studying scattering processes in gauge theories. With the advent of quantum computers, Hamiltonian simulation of LGTs has become a reality. Simulating scattering on quantum computers requires the preparation of initial scattering states in the interacting theory on the quantum hardware. Current state preparation methods involve bridging the scattering states in the free theory to the ones in the interacting theory adiabatically. Such quantum algorithms have limitations when applied to LGTs, and they tend to be computational resource intensive, rendering their implementation a challenge on the noisy intermediate-scale quantum (NISQ) era devices. In this work, we propose a wave packet state preparation algorithm for a 1+1D Z2 LGT coupled to dynamical matter. We show how this algorithm circumvents the adiabatic process by building and implementing the wave packet creation operators directly in the interacting theory using an optimized ansatz consisting of hadronic degrees of freedom in the confined Z2 LGT. Moreover, we numerically confirm the validity of this ansatz for a U(1) LGT in 1+1D. Finally, we demonstrate the viability of our algorithm for NISQ devices by comparing the classical simulation with the results obtained using the Quantinuum H1-1 quantum computer upon a simple symmetry-based noise mitigation technique.

Zoom link

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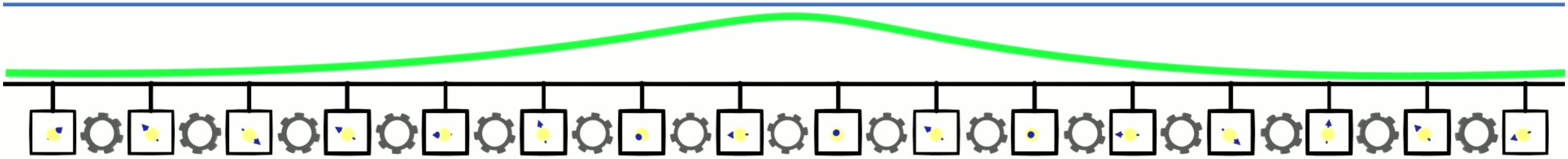
Towards preparation of scattering wave packets of hadrons on a quantum computer

A NISQ Algorithm for Lattice Gauge Theories

arXiv: 2402.00840



Saurabh Kadam[1]

with

Zohreh Davoudi[2] and Chung-Chun Hsieh[2]

Zohreh Davoudi

1. InQubator for Quantum Simulation, University of Washington

2. University of Maryland

Chung-Chun Hsieh

Gauge theories

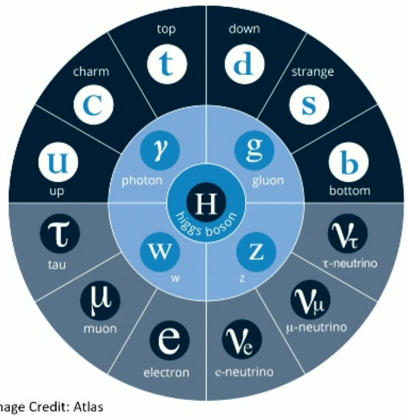
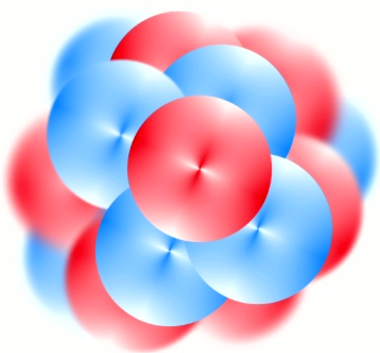


Image Credit: Atlas

Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$

- Forces are described by gauge symmetries
- Weak and strong forces are described by non-Abelian gauge theories

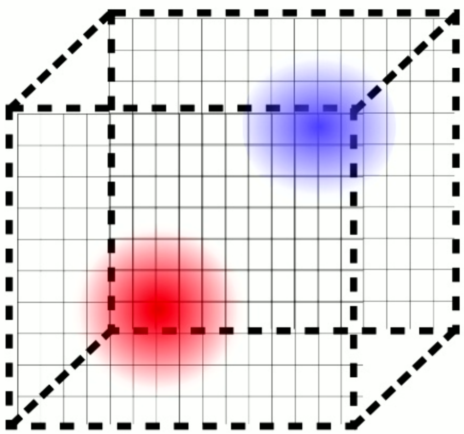


Strong force: Quantum chromodynamics (QCD)

- Nuclear force: Theory of interacting quarks mediated by gluons
- Becomes strongly interacting at low energies
- Requires non-perturbative methods for calculating observables

2

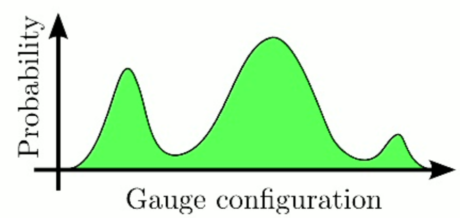
Non-perturbative method of solving QCD



Lattice QCD

- QCD Lagrangian on a discrete spacetime grid and Wick rotate to Euclidean time
- Observables are calculated using the path integral formalism
- Monte Carlo methods for probability distribution of gauge configurations

$$\langle\hat{\mathcal{O}}\rangle = \frac{1}{Z_0}\int\mathcal{D}[\bar{\psi},\psi]\mathcal{D}[A]\; e^{-\int d^4x_E \mathcal{L}_E}\,\mathcal{O}$$



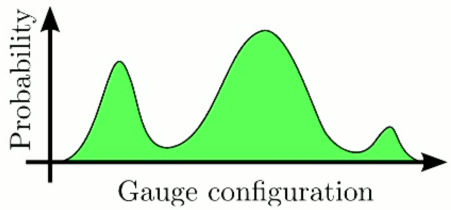
3

Non-perturbative method of solving QCD

FLAG Review 2021

Eur. Phys. J. C 82 (2022) 10, 869

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z_0} \int \mathcal{D}[\bar{\psi}, \psi]\mathcal{D}[A]\; e^{-\int d^4x_E \mathcal{L}_E}\, \mathcal{O}$$



Successes

- ✓ Hadron spectrum and exotic states
- ✓ Hadron form factors
- ✓ Values of quark masses and the strong coupling constant
- ✓ Decay rates and low energy constants
- ✓ Two- and three-body scattering amplitudes

Shortcomings

- ❖ QCD phase diagram:

  Sign problem:

  Loss of probability distribution interpretation
- ❖ Euclidean time:

  Real time evolution of system
- ❖ Many-body processes are harder to obtain

4

Non-perturbative method of solving QCD

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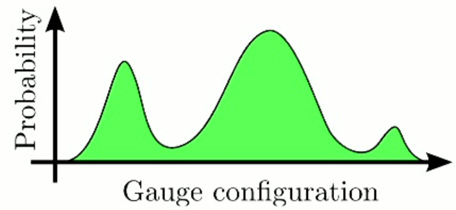


Hamiltonian Formulation

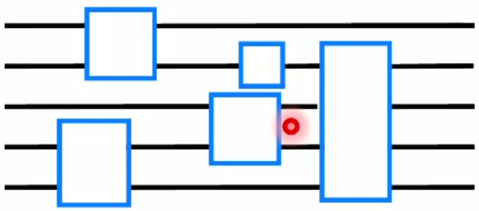
$$\langle\hat{\mathcal{O}(t)}\rangle = \langle 0|e^{iHt}\hat{\mathcal{O}}(0)e^{-iHt}|0\rangle$$

1. No sign problem
2. Both real- and imaginary-time evolution
3. Many-body processes and scattering
4. Hilbert space scales exponentially with the system size

Shortcomings

- QCD phase diagram:
  Sign problem:
  Loss of probability distribution interpretation
- Euclidean time:
  Real time evolution of system
- Many-body processes are harder to obtain

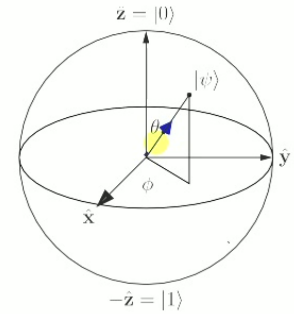
Digital Quantum simulation



❖ Qubits:

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$$

$$|\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \cdots \otimes |\psi\rangle_N \;\sim 2^N$$



Simulating 2D lattice gauge theories on a qudit quantum computer

Michael Meth,[1] Jan F. Haase,[2,3,4] Jinglei Zhang,[2,3] Claire Edmunds,[1] Lukas Postler,[1] Andrew J. Jena,[2,3] Alex Steiner,[1] Luca Dellantonio,[2,3,5] Rainer Blatt,[1,6,7] Peter Zoller,[8,6] Thomas Monz,[1,7] Philipp Schindler,[1] Christine Muschik*,[2,3,9] and Martin Ringbauer*[1]

Simulating one-dimensional quantum chromodynamics on a quantum computer: Real-time evolutions of tetra- and pentaquarks

Yasar Y. Atas*,[1,2,†] Jan F. Haase*,[1,2,3,‡] Jinglei Zhang,[1,2,§] Victor Wei,[1,4] Sieglinde M.-L. Pfaendler,[5] Randy Lewis,[6] and Christine A. Muschik[1,2,7]

Quantum computation of dynamical quantum phase transitions and entanglement tomography in a lattice gauge theory

Niklas Mueller,[1,2,3,*] Joseph A. Carolan,[4] Andrew Connelly,[5] Zohreh Davoudi,[1,6,†] Eugene F. Dumitrescu,[7,‡] and Kübra Yeter-Aydeniz[8]

Scalable Circuits for Preparing Ground States on Digital Quantum Computers: The Schwinger Model Vacuum on 100 Qubits

Roland C. Farrell,[*] Marc Illa,[†] Anthony N. Ciavarella,[‡] and Martin J. Savage[§]

Simulating $\mathbb{Z}_2$ lattice gauge theory on a quantum computer

Clement Charles,[1,2] Erik J. Gustafson,[3,4,5] Elizabeth Hardt,[6,7] Florian Herren,[3] Norman Hogan,[8] Henry Lamm,[3] Sara Starecheski,[9,10] Ruth S. Van de Water,[3] and Michael L. Wagman[3]

Quantum Simulations of Hadron Dynamics in the Schwinger Model using 112 Qubits

Roland C. Farrell,[1,*] Marc Illa,[1,†] Anthony N. Ciavarella,[1,2,‡] and Martin J. Savage[1,§]

Nearly-optimal state preparation for quantum simulations of lattice gauge theories

Christopher F. Kane,[1] Niladri Gomes,[2] and Michael Kreshchuk[3]

Fermion-qudit quantum processors for simulating lattice gauge theories with matter

Torsten V. Zache[1,2,3], Daniel González-Cuadra[1,2,3], and Peter Zoller[1,2]

Digital Quantum simulation

- Qubits:

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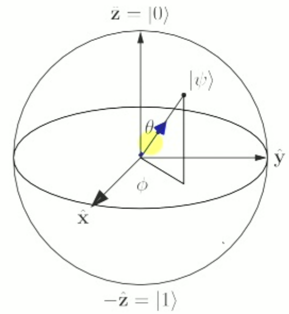
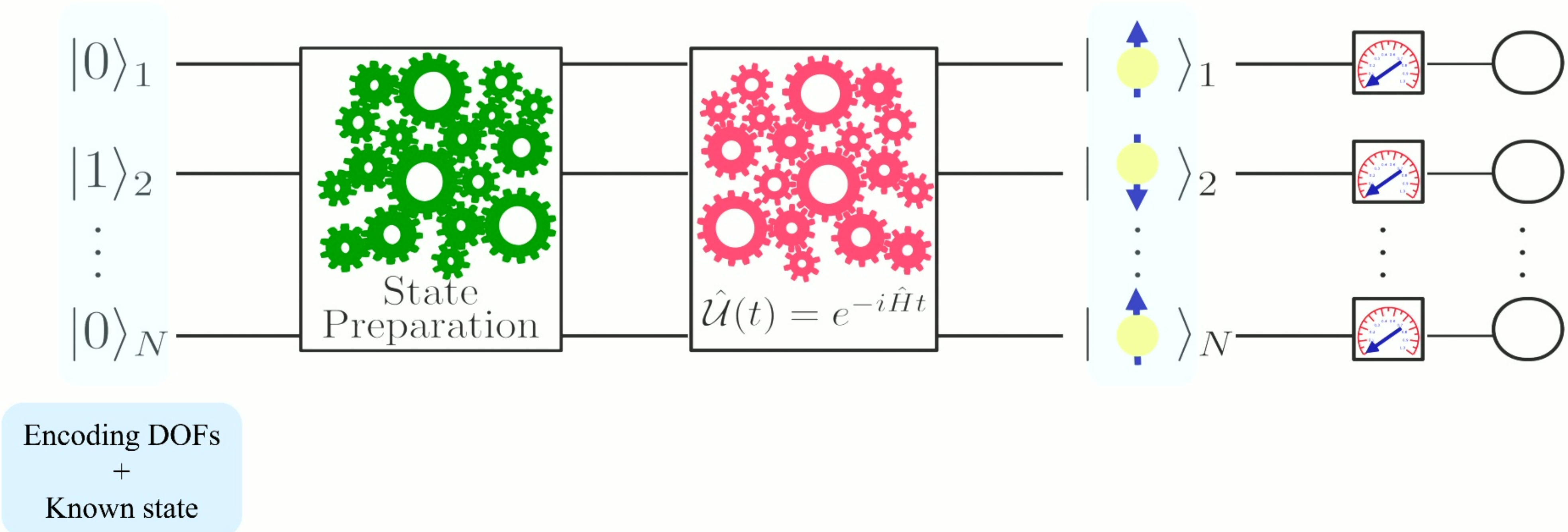


$$|\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \cdots \otimes |\psi\rangle_N \;\sim 2^N$$

- Schematic protocol for scattering



Digital Quantum simulation

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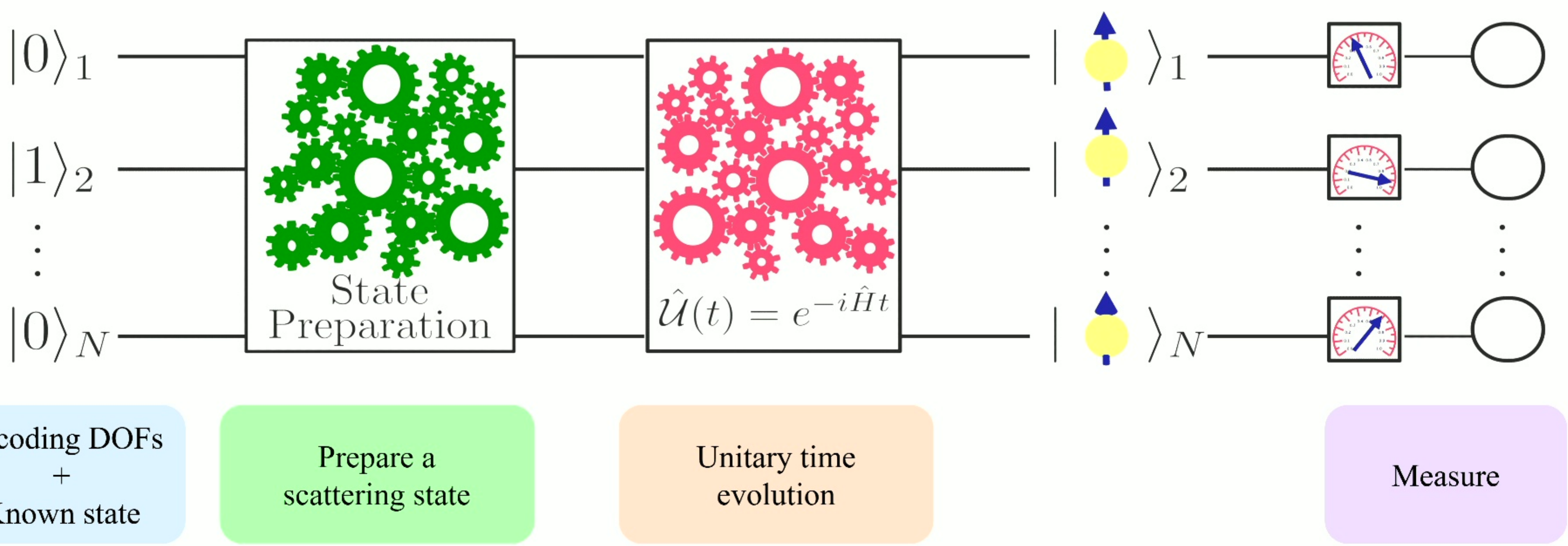
- Schematic protocol for scattering **Example of two shots**



Digital Quantum simulation

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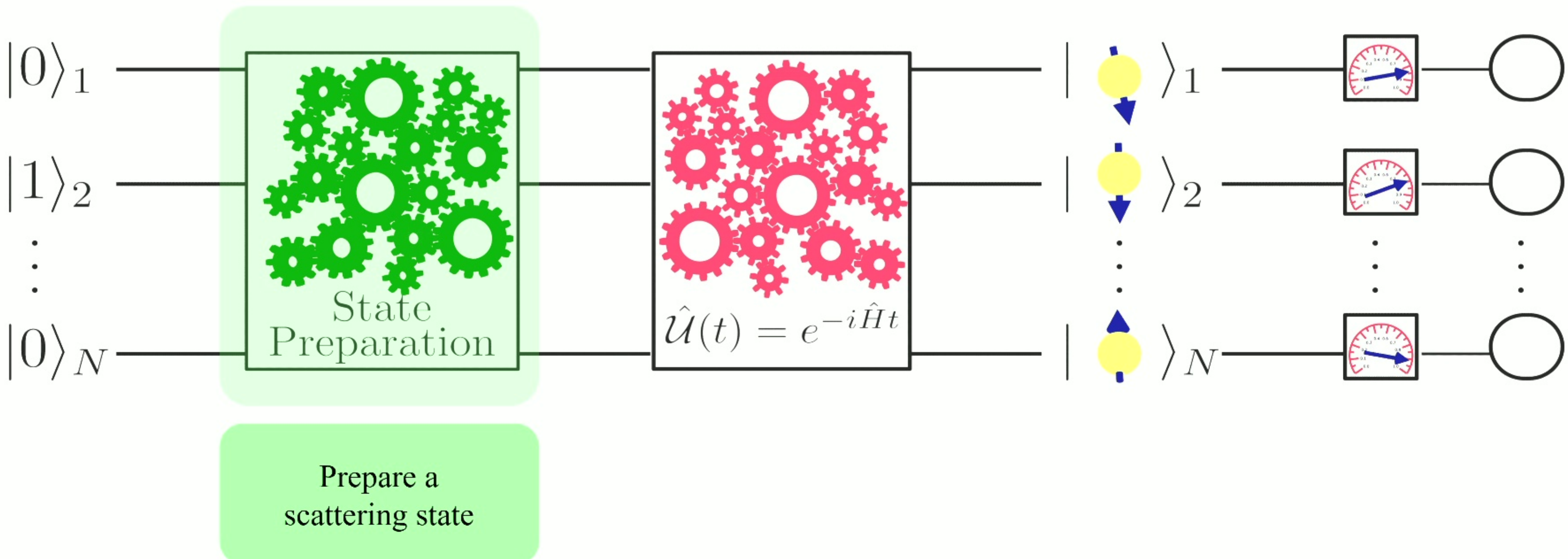
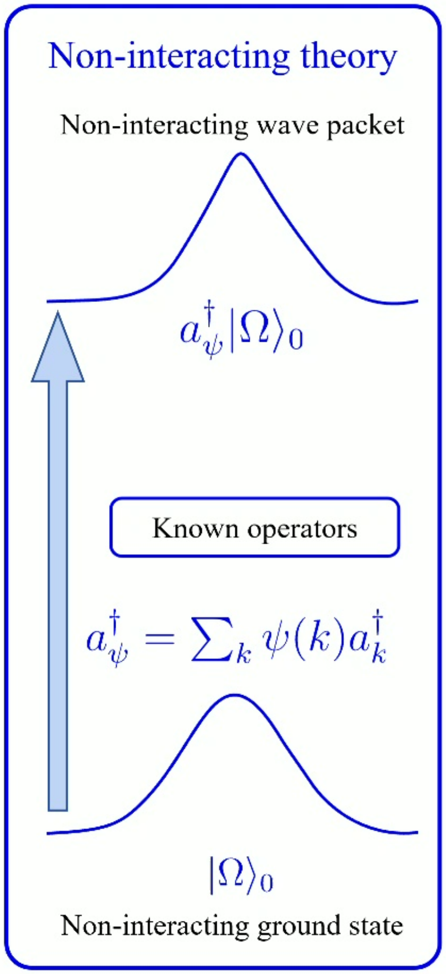
- Schematic protocol for scattering **Example of two shots**



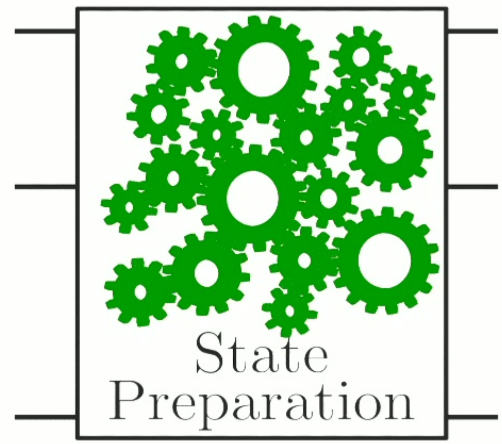


The Pioneering work

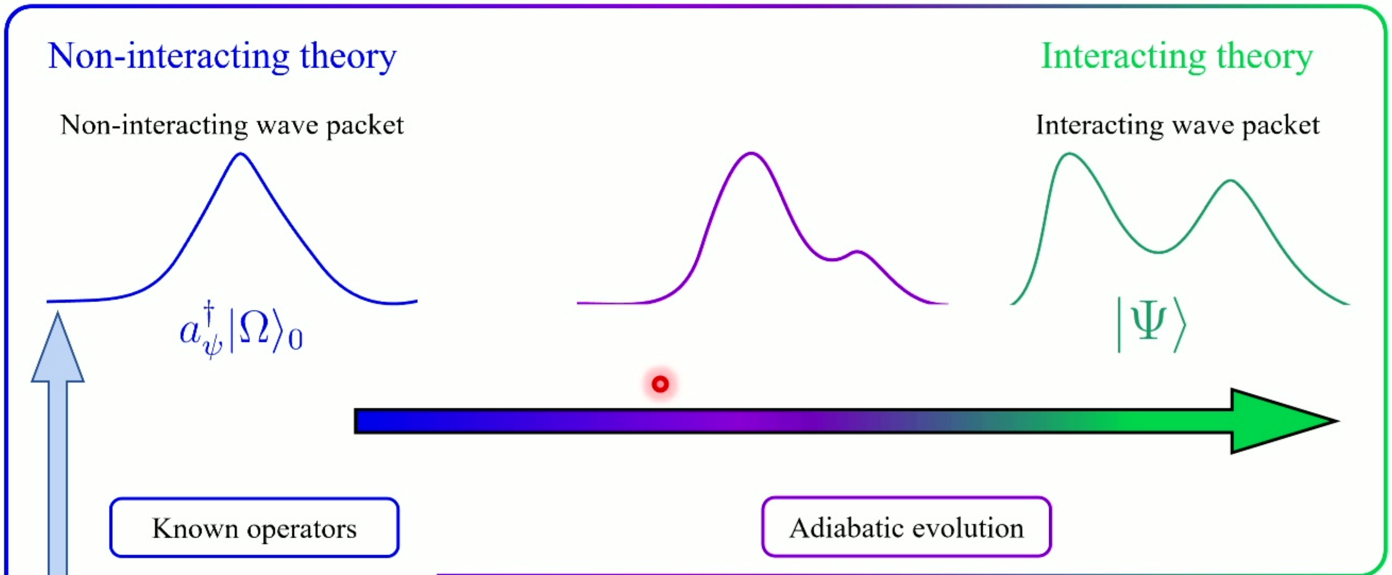
Jordan, Lee, and Preskill (JLP)

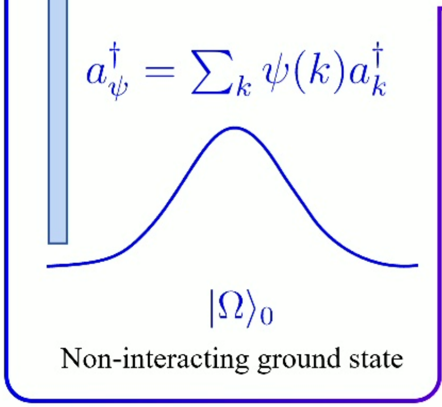
Science 336, 1130-1133 (2012)

Quantum Info. and Comp. 14, 1014-80 (2014)



The Pioneering work





Shortcomings

- Resource cost: expensive for the NISQ era devices
- Phase transition: long-range modes
- Confinement: interacting dofs are different

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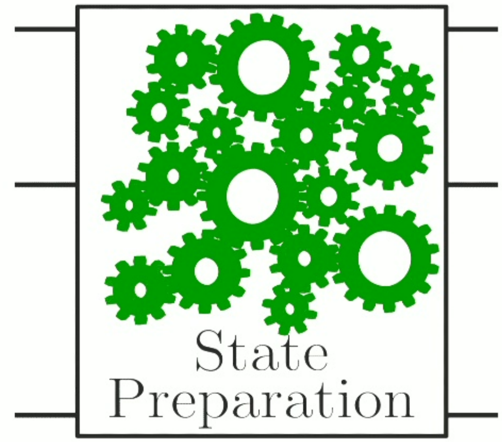

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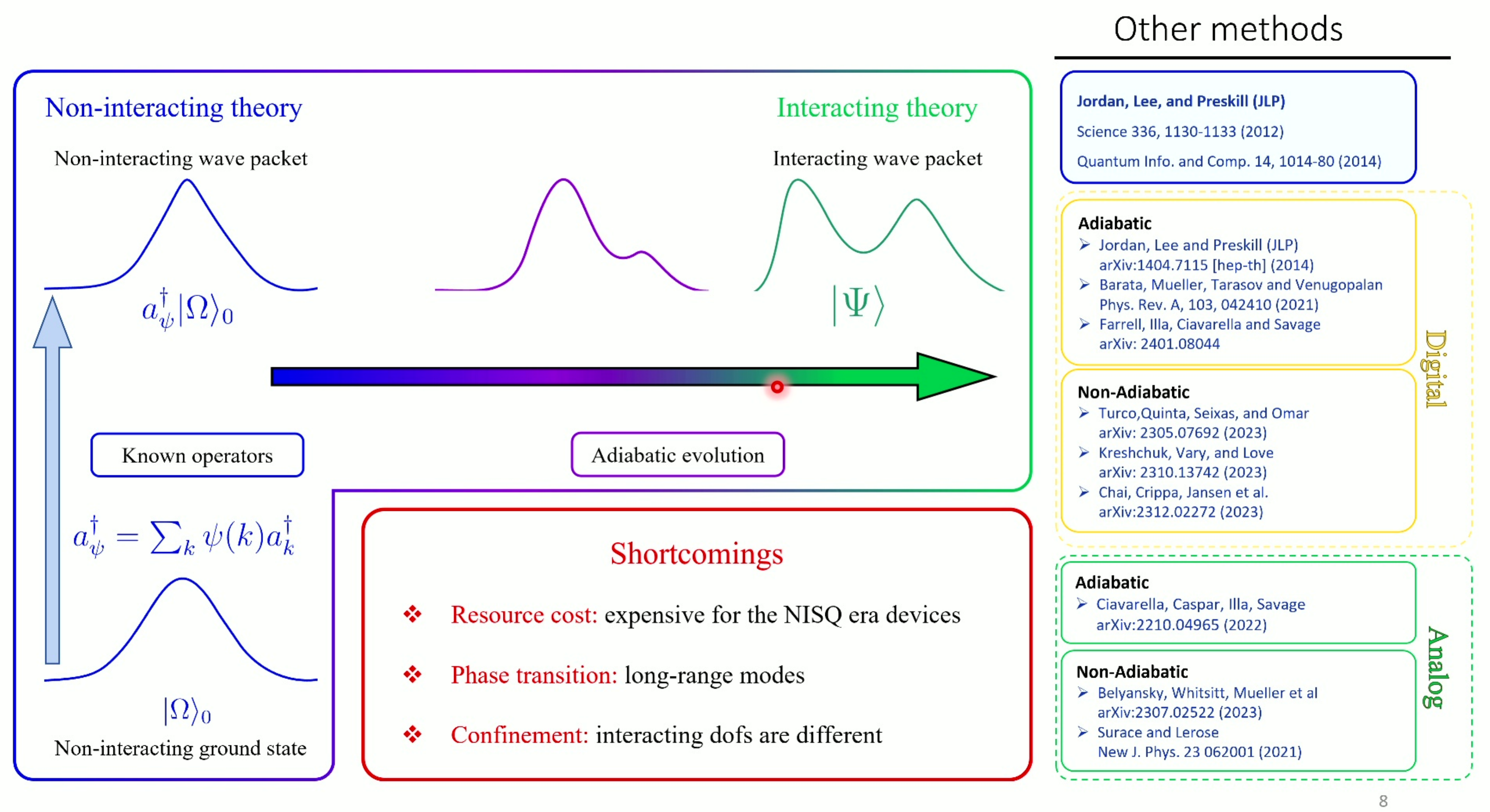
Other methods
Non-interacting theory
Interacting theory
Non-interacting wave packet
Interacting wave packet
$a_\psi^\dagger|\Omega\rangle_0$
$|\Psi\rangle$
Known operators
Adiabatic evolution
$a_\psi^\dagger = \sum_k \psi(k) a_k^\dagger$
$|\Omega\rangle_0$
Non-interacting ground state
Shortcomings
❖ Resource cost: expensive for the NISQ era devices
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Digital
Adiabatic
➢ Jordan, Lee and Preskill (JLP) arXiv:1404.7115 [hep-th] (2014)
➢ Barata, Mueller, Tarasov and Venugopalan Phys. Rev. A, 103, 042410 (2021)
➢ Farrell, Illa, Ciavarella and Savage arXiv: 2401.08044
Non-Adiabatic
➢ Turco,Quinta, Seixas, and Omar arXiv: 2305.07692 (2023)
➢ Kreshchuk, Vary, and Love arXiv: 2310.13742 (2023)
➢ Chai, Crippa, Jansen et al. arXiv:2312.02272 (2023)
Analog
Adiabatic
➢ Ciavarella, Caspar, Illa, Savage arXiv:2210.04965 (2022)
Non-Adiabatic
➢ Belyansky, Whitsitt, Mueller et al arXiv:2307.02522 (2023)
➢ Surace and Lerose New J. Phys. 23 062001 (2021)
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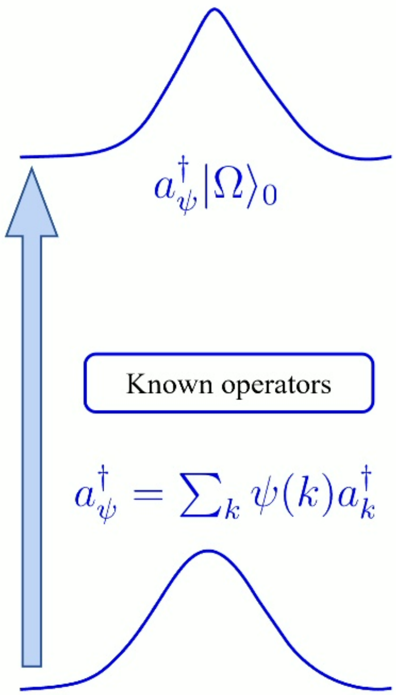
Our approach

Non-interacting theory

Non-interacting wave packet

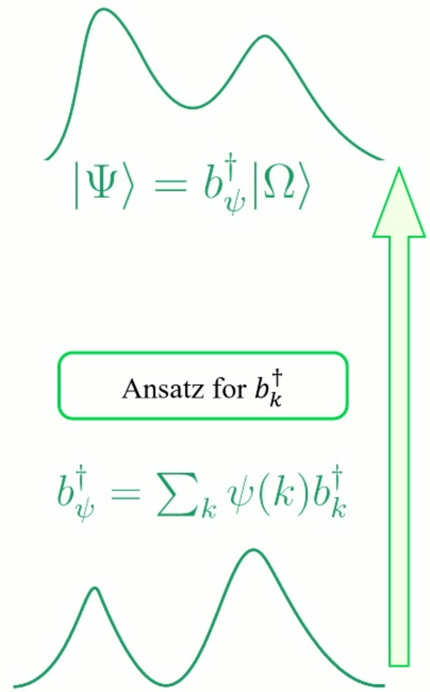


$|\Omega\rangle_0$

Non-interacting ground state

Interacting theory

Interacting wave packet



$|\Omega\rangle$

Interacting ground state

Rigobello, Notarnicola, Magnifico, and Montangero
Phys. Rev. D 104, 114501 (2021)

- Start with the interacting ground state
- Use an ansatz to build the interacting creation operators
- Act the wave packet creation operator on the interacting ground state

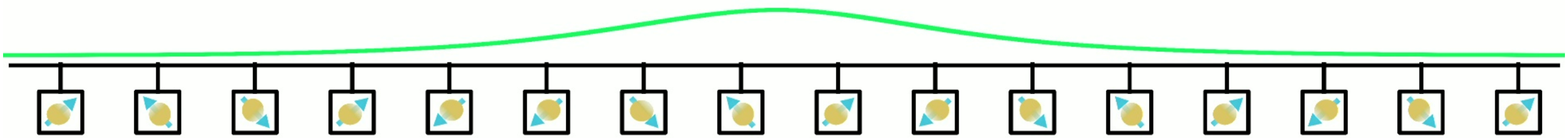
Outline

Preparation of scattering wave packets

- **Model:** *Z_2 (and U(1)) lattice gauge theory in 1+1D with matter*
- **Method:** *Construction of creation operators*
- **Mapping :** *Quantum algorithm and circuit*
- **Measurements:** *Hardware results from Quantinuum H1-1*
- **Moreover:** *Conclusions and outlook*

Model

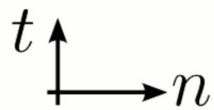
Z_2 Lattice Gauge Theory (LGT) in 1+1D with dynamical matter



Z_2 LGT in 1+1D

Motivation

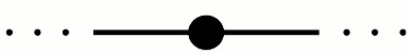
- Confined Theory
- $Z_N \xrightarrow{N\to\infty} U(1)$

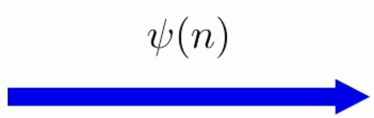


Continuous time

Discretized space

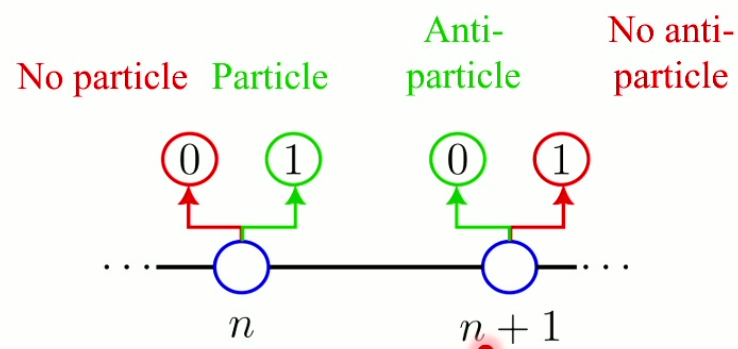
Fermionic DOFs





Staggered Fermion

Dirac sea interpretation



Matter → Even sites   Anti-matter → Odd sites

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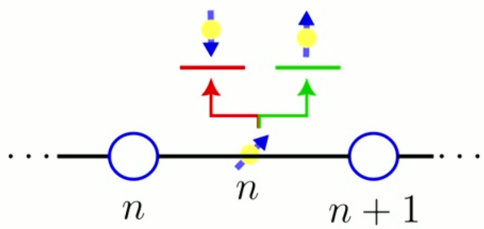
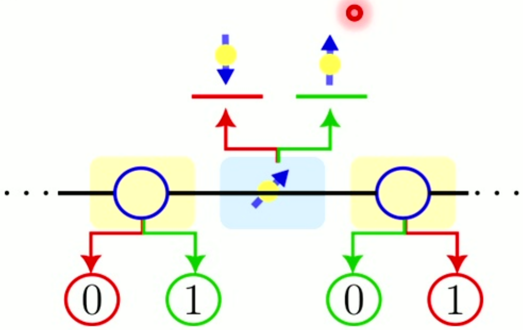
Z_2 LGT in 1+1D

Bosonic DOFs



$$H_E = \epsilon \sum_n \tilde{\sigma}_n^Z$$

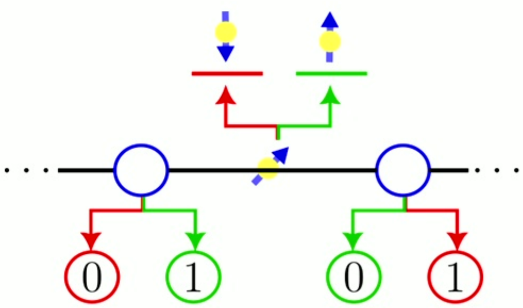
Fermionic + bosonic DOFs



$$H_I = \sum_n \psi_n^\dagger \tilde{\sigma}_n^X \psi_{n+1} + \text{H.c.}$$

Z_2 LGT in 1+1D

Fermionic + bosonic DOFs



Are all 2^{2N} states physical states?

No!!

Z_2 LGT Hamiltonian

$$H_E = \epsilon \sum_n \tilde{\sigma}_n^Z$$

$$H_I = \sum_n \psi_n^\dagger \tilde{\sigma}_n^X \psi_{n+1} + \text{H.c.}$$

$$H_M = m_f \sum_n (-1)^n \psi_n^\dagger \psi_n$$

$$H_{Z_2} = H_E + H_I + H_M$$

Physical Hilbert space

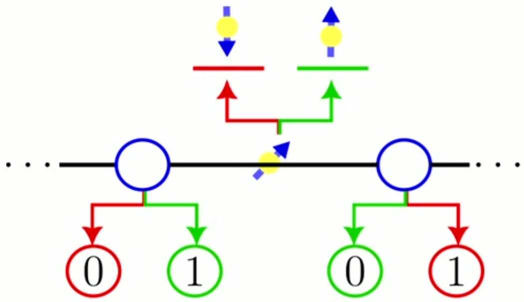
Gauge invariance $\rightarrow$ Gauss's law

$$G_n |\psi\rangle_{\text{Phys}} = |\psi\rangle_{\text{Phys}} \quad \forall n$$

$$G_n = \tilde{\sigma}_{n-1}^Z \tilde{\sigma}_n^Z e^{i\pi\left(\psi_n^\dagger \psi_n - \frac{1-(-1)^n}{2}\right)}$$

Z_2 LGT in 1+1D

Fermionic + bosonic DOFs



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Z_2 LGT Hamiltonian

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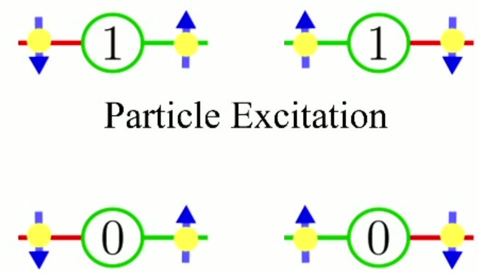


Particle Excitation

Anti-particle Excitation

Z_2 LGT in 1+1D

Periodic boundary condition with charge 0 sector

Z_2 LGT Hamiltonian

$$H_E = \epsilon \sum_n \tilde{\sigma}_n^Z$$

$$H_I = \sum_n \psi_n^\dagger \tilde{\sigma}_n^X \psi_{n+1} + \text{H.c.}$$

$$H_M = m_f \sum_n (-1)^n \psi_n^\dagger \psi_n$$

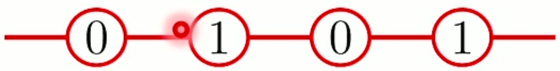
$$H_{Z_2} = H_E + H_I + H_M$$

Consequences

- Physical Hilbert space is a tiny fraction of the full Hilbert space

	Hilbert Space	
# Sites	Possible	Physical
4	256	12
6	4096	40

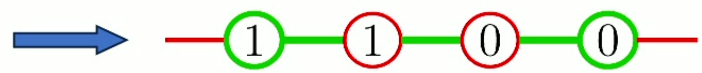
- Only mesonic excitations



Strong Coupling Vacuum (SCV)

No fermionic excitation

Low-energy boson configuration



Example of a length-3 meson

Starts at particle site

Ends at anti-particle site

Building creation operators in interacting theory

Rigobello, Notarnicola, Magnifico, and Montangero
Phys. Rev. D 104, 114501 (2021)

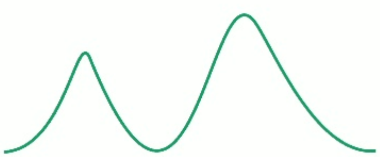
Interacting theory

Interacting wave packet

$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle$

Ansatz for $b_k^\dagger$

$b_\psi^\dagger = \sum_k \psi(k) b_k^\dagger$



$|\Omega\rangle$

Interacting ground state

Build momentum creation operator from mesonic excitations

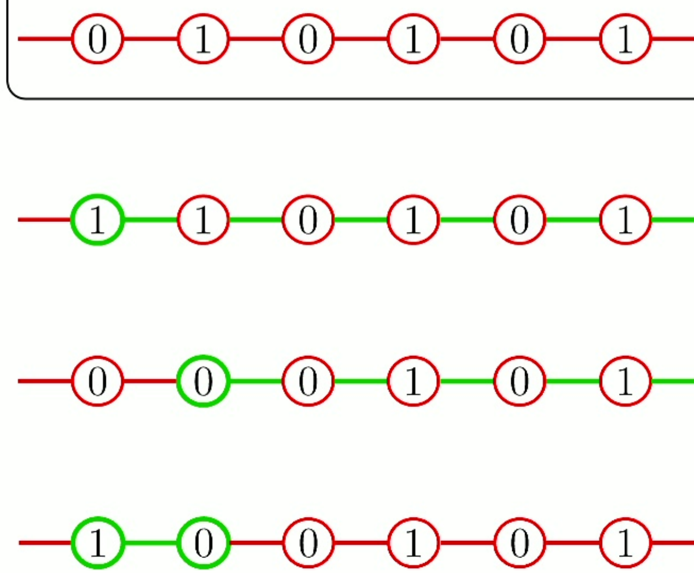
$$b_k^\dagger = \frac{2\pi}{aN} \sum_{p,q} \delta^k_{p+q}\, \eta_{pq}\, \mathcal{B}(p,q)$$

$$a\sqrt{\frac{m+\omega_p}{2\pi\omega_p}} \sum_n \left[\Pi_{n0} + \Pi_{n1} v_p\right] e^{ipna}\ \psi_n^\dagger \prod_{i\geq n} \tilde{\sigma}_i^X$$

$$a\sqrt{\frac{m+\omega_q}{2\pi\omega_q}} \sum_m \left[\Pi_{n1} - \Pi_{n0} v_q\right] e^{iqma}\ \psi_m \prod_{i\geq m} \tilde{\sigma}_i^X$$

$$\# \sum_{m,n} \left[\text{Projections}\right] e^{i(pn+qm)a}\ \psi_n^\dagger \psi_m \prod_{n\leq i\leq m} \tilde{\sigma}_i^X$$

Action on the SCV



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Building creation operators in interacting theory

Rigobello, Notarnicola, Magnifico, and Montangero
Phys. Rev. D 104, 114501 (2021)

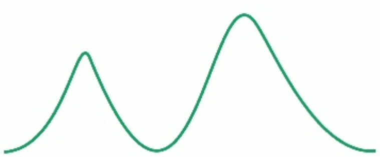
Interacting theory

Interacting wave packet

$|\Psi\rangle = b^{\dagger}_{\psi}|\Omega\rangle$

Ansatz for $b^{\dagger}_{k}$

$b^{\dagger}_{\psi} = \sum_k \psi(k) b^{\dagger}_{k}$



$|\Omega\rangle$

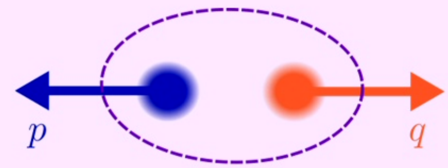
Interacting ground state

Build momentum creation operator from mesonic excitations

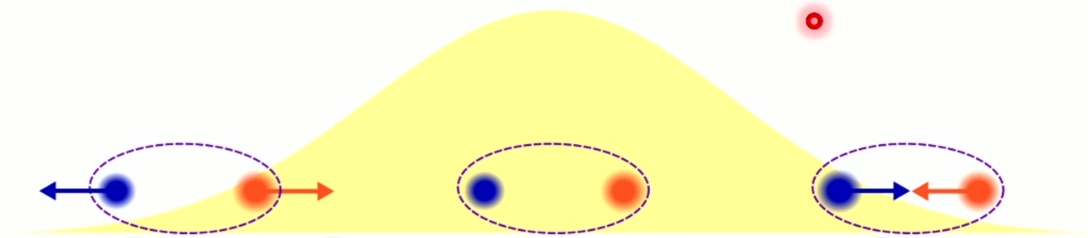
$$b^{\dagger}_{k} = \frac{2\pi}{aN} \sum_{p,q} \delta^{k}_{p+q} \, \eta_{pq} \, \mathcal{B}(p,q)$$

$$\eta_{pq} = N_{\eta} \exp\left(\frac{i(p-q)\mu^{A}_{k}}{2}\right) \exp\left(\frac{-(p-q)^2}{4\sigma^{A^2}_{k}}\right)$$

$$\mathcal{B}(p,q) = \# \sum_{m,n} [\text{Projections}] \, e^{i(pn+qm)a} \, \psi^{\dagger}_{n}\psi_{m} \prod_{n\leq i\leq m} \tilde{\sigma}^{X}_{i}$$



$k = p + q$



Example for $k = 0$

23

Building creation operators in interacting theory

Rigobello, Notarnicola, Magnifico, and Montangero
Phys. Rev. D 104, 114501 (2021)

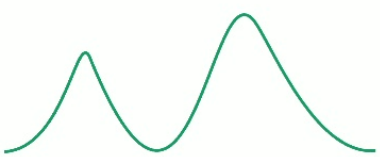
Interacting theory

Interacting wave packet

$|\Psi\rangle = b^\dagger_\psi |\Omega\rangle$

Ansatz for $b^\dagger_k$

$b^\dagger_\psi = \sum_k \psi(k) b^\dagger_k$



$|\Omega\rangle$

Interacting ground state

Build momentum creation operator from mesonic excitations

$$b^\dagger_k = \frac{2\pi}{aN} \sum_{p,q} \delta^k_{p+q}\, \eta_{pq}\, \mathcal{B}(p,q)$$

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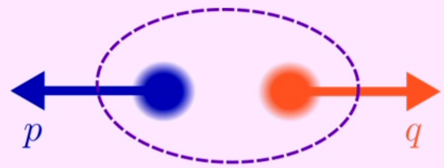
$$\mathcal{B}(p,q) = \# \sum_{m,n} \left[\text{Projections}\right] e^{i(pn+qm)a}\ \psi^\dagger_n \psi_m \prod_{n \le i \le m} \tilde{\sigma}^X_i$$



$$k = p + q$$

Optimize first excited energy eigenstates for each k

$$|k\rangle = b^\dagger_k(\mu^A_k, \sigma^A_k)\,|\Omega\rangle \qquad E^{(1)}_k = \langle k|H_{Z_2}|k\rangle$$

Building creation operators in interacting theory

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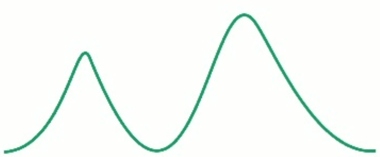

$|\Omega\rangle$

Interacting ground state

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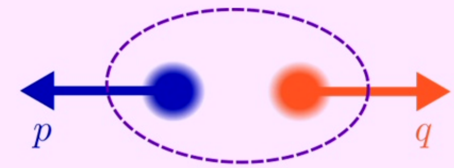
$$b^\dagger_k = \frac{2\pi}{aN} \sum_{p,q} \delta^k_{p+q} \, \eta_{pq} \, \mathcal{B}(p,q)$$

$$\mathcal{B}(p,q) = \# \sum_{m,n} [\text{Projections}] \, e^{i(pn+qm)a} \, \psi^\dagger_n \psi_m \prod_{n \leq i \leq m} \tilde{\sigma}^X_i$$



$$\eta_{pq} = N_\eta \exp\left(\frac{i(p-q)\mu^A_k}{2}\right) \exp\left(\frac{-(p-q)^2}{4\sigma^{A\,2}_k}\right)$$

$k = p + q$

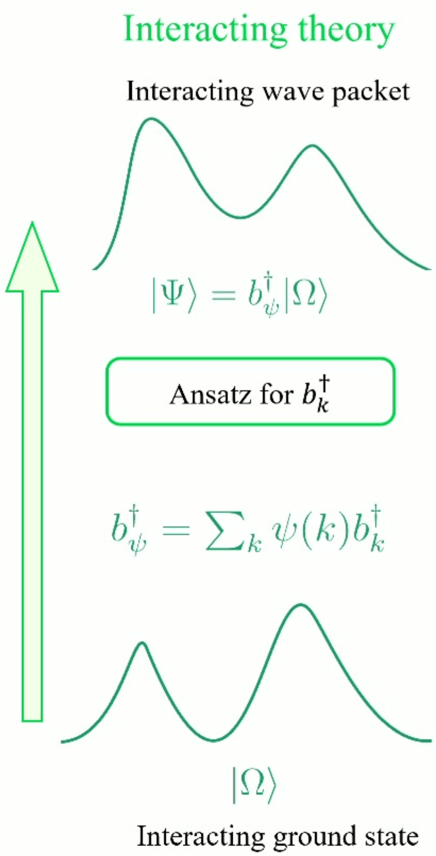
Optimize first excited energy eigenstates for each k

$$|k\rangle = b^\dagger_k(\mu^A_k, \sigma^A_k)\,|\Omega\rangle \qquad E^{(1)}_k = \langle k | H_{Z_2} | k \rangle$$

- Classical/Quantum: We did it classically, but we checked that VQE works
- Works for U(1) LGT in 1+1D as well: We were limited by computational resources

Wave packet constructions

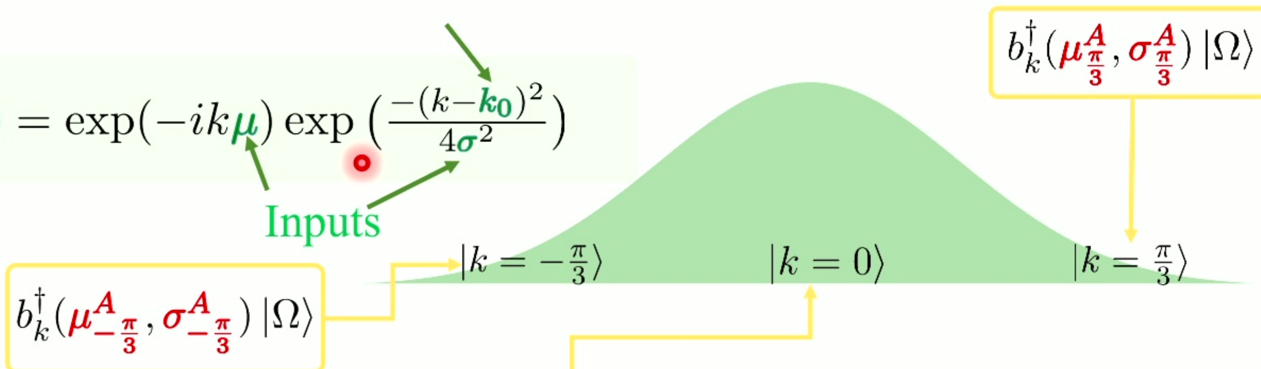
Rigobello, Notarnicola, Magnifico, and Montangero
Phys. Rev. D 104, 114501 (2021)



$$|\Psi\rangle = b^\dagger_\psi|\Omega\rangle = \sum_k \psi(k) b^\dagger_k(\mu^A_k, \sigma^A_k)|\Omega\rangle$$

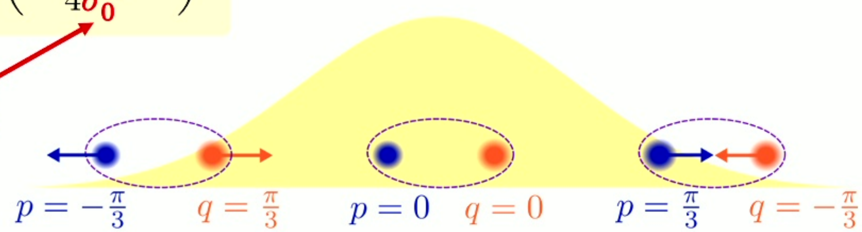
$$\psi(k) = \exp(-ik\mu)\exp\left(\frac{-(k-k_0)^2}{4\sigma^2}\right)$$

Inputs



$$\eta_{pq} = N_\eta \exp\left(\frac{i(p-q)\mu^A_0}{2}\right)\exp\left(\frac{-(p-q)^2}{4\sigma^{A\,2}_0}\right)$$

Optimized



Example for $k = 0$

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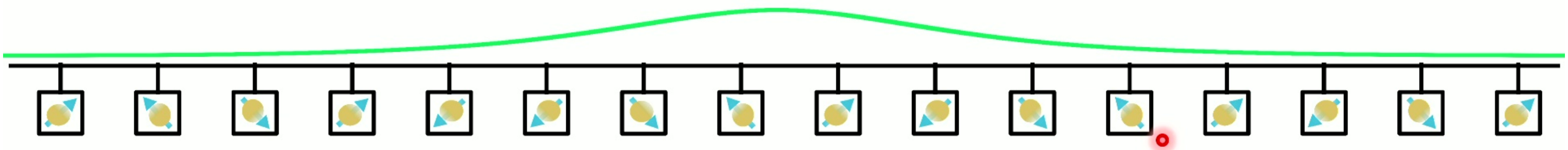
Mapping

Algorithm and circuit



Overview

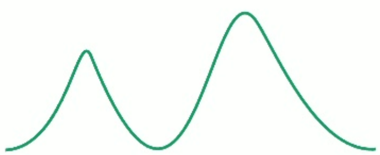
Interacting theory

Interacting wave packet

$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle$

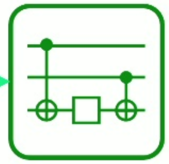
Ansatz for $b_k^\dagger$

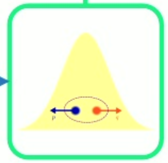
$b_\psi^\dagger = \sum_k \psi(k) b_k^\dagger$

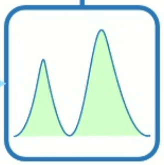


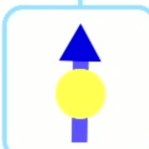
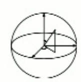
$|\Omega\rangle$

Interacting ground state

4  $|\Psi\rangle$ Circuit for the wave packet

3  $b_k^\dagger$ Optimize the momentum creation operators

2  $|\Omega\rangle$ Prepare the interacting ground state

1   Map the fermionic + bosonic DOFs to qubits

27

Overview

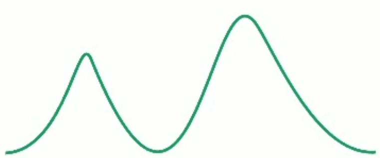
Interacting theory

Interacting wave packet

$|\Psi\rangle = b^\dagger_\psi |\Omega\rangle$

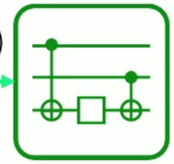
Ansatz for $b^\dagger_k$

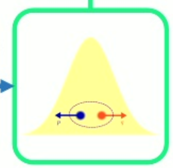
$b^\dagger_\psi = \sum_k \psi(k) b^\dagger_k$

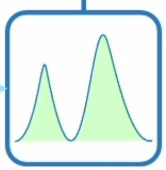


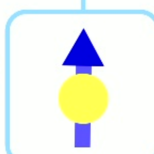
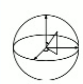
$|\Omega\rangle$

Interacting ground state

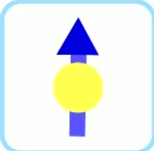
4  $|\Psi\rangle$ Circuit for the wave packet

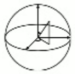
3  $b^\dagger_k$ Optimize the momentum creation operators

2  $|\Omega\rangle$ Prepare the interacting ground state

1   Map the fermionic + bosonic DOFs to qubits

1



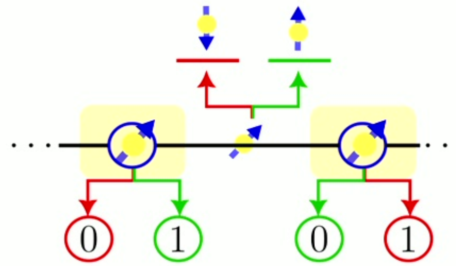


Map the fermionic + bosonic DOFs to qubits

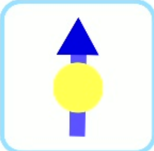
- Bosonic links: as it is
- Fermions: Jordan-Wigner transformation

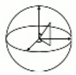
$$\psi_n^\dagger = \prod_{i<n} \sigma_i^Z \, \sigma_n^-$$

$$\psi_n = \prod_{i<n} \sigma_i^Z \, \sigma_n^+$$



1





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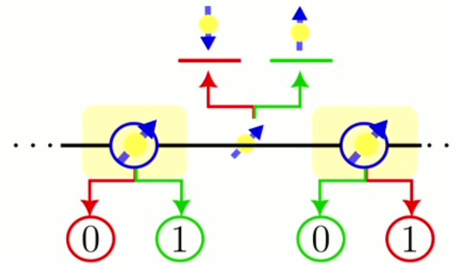


$$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle = \sum_k \psi(k) b_k^\dagger |\Omega\rangle$$

Inputs

$$\frac{2\pi}{aN} \sum_{p,q} \delta_{p+q}^k \, \eta_{pq} \, \mathcal{B}(p,q)$$

Optimized

$$\sum_{m,n} [\text{Projections}] \, e^{i(pn+qm)a} \, \psi_n^\dagger \psi_m \prod_{n \le i \le m} \tilde{\sigma}_i^X$$

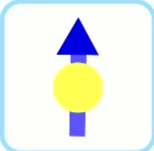
Jordan-Wigner

$$b_\psi^\dagger = \sum_{m,n} C_{m,n} \, \sigma_n^- \prod_{n<j<m} \sigma_j^Z \prod_{n \le i \le m} \tilde{\sigma}_i^X \, \sigma_m^+$$

Inputs Optimized

29

1



Map the fermionic + bosonic DOFs to qubits

- Bosonic links: as it is
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$$\psi_n^\dagger = \prod_{i<n} \sigma_i^Z \, \sigma_n^-$$

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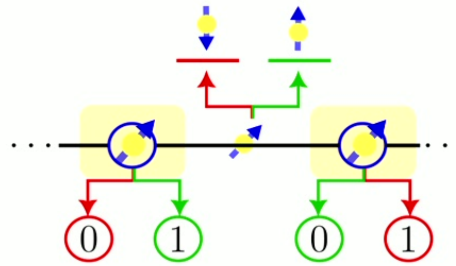


$$|\Psi\rangle = b_\psi^\dagger |\Omega\rangle = \sum_k \psi(k) b_k^\dagger |\Omega\rangle$$

Inputs

$$\frac{2\pi}{aN} \sum_{p,q} \delta_{p+q}^k \, \eta_{pq} \, \mathcal{B}(p,q)$$

Optimized

$$\sum_{m,n} [\text{Projections}] \, e^{i(pn+qm)a} \, \psi_n^\dagger \psi_m \prod_{n \le i \le m} \tilde{\sigma}_i^X$$

Jordan-Wigner

$$b_\psi^\dagger = \sum_{m,n} C_{m,n} \, \sigma_n^- \prod_{n<j<m} \sigma_j^Z \prod_{n \le i \le m} \tilde{\sigma}_i^X \, \sigma_m^+$$

Inputs → $C_{m,n}$; Optimized → $C_{m,n}$

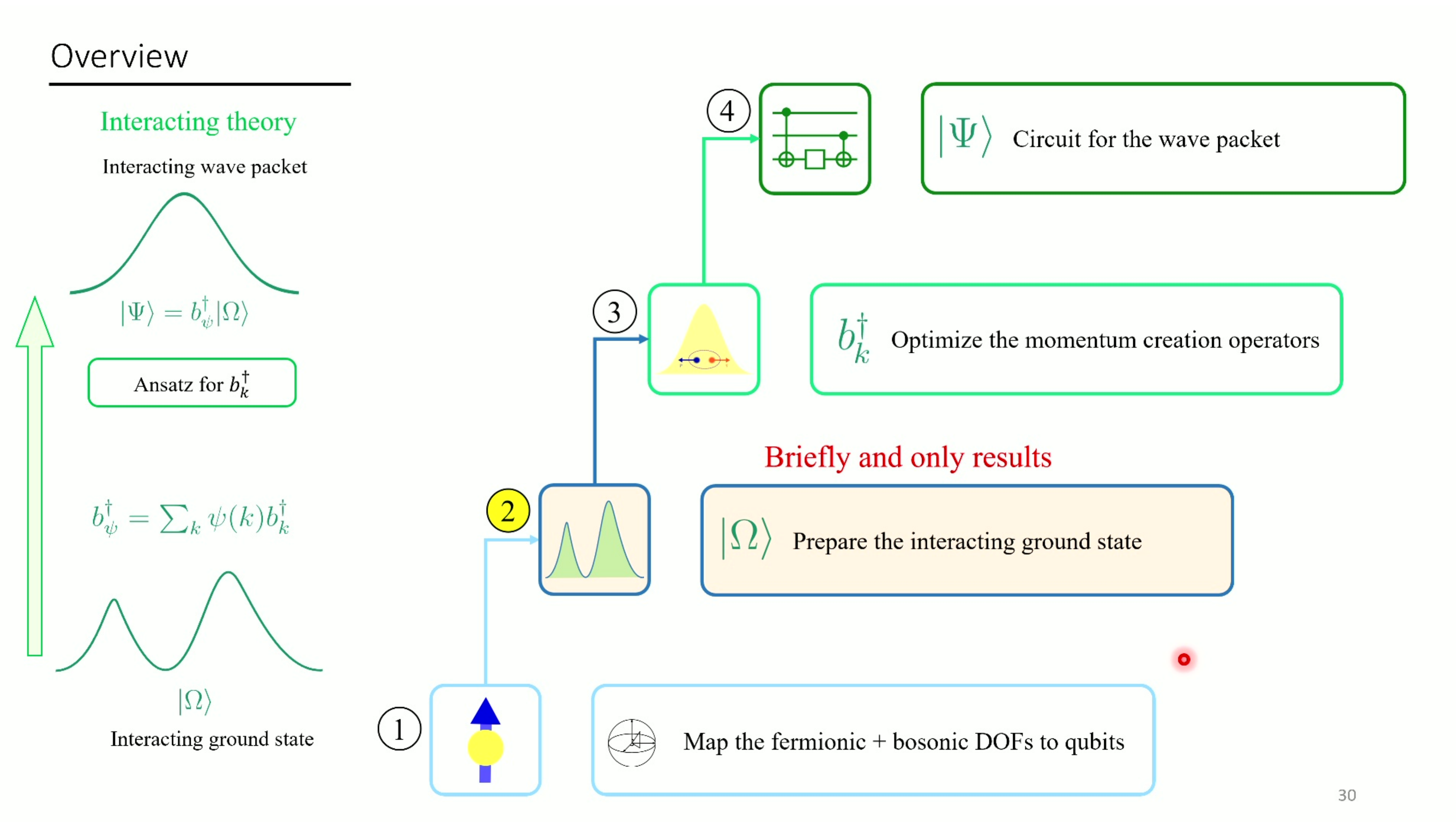
From here onwards:

- ✓ 6 staggered sites
- ✓ 3 momenta: $k = \frac{\pi}{3}, 0, -\frac{\pi}{3}$
- ✓ 12 qubits = 4096 states
- ✓ 40 Physical states

29

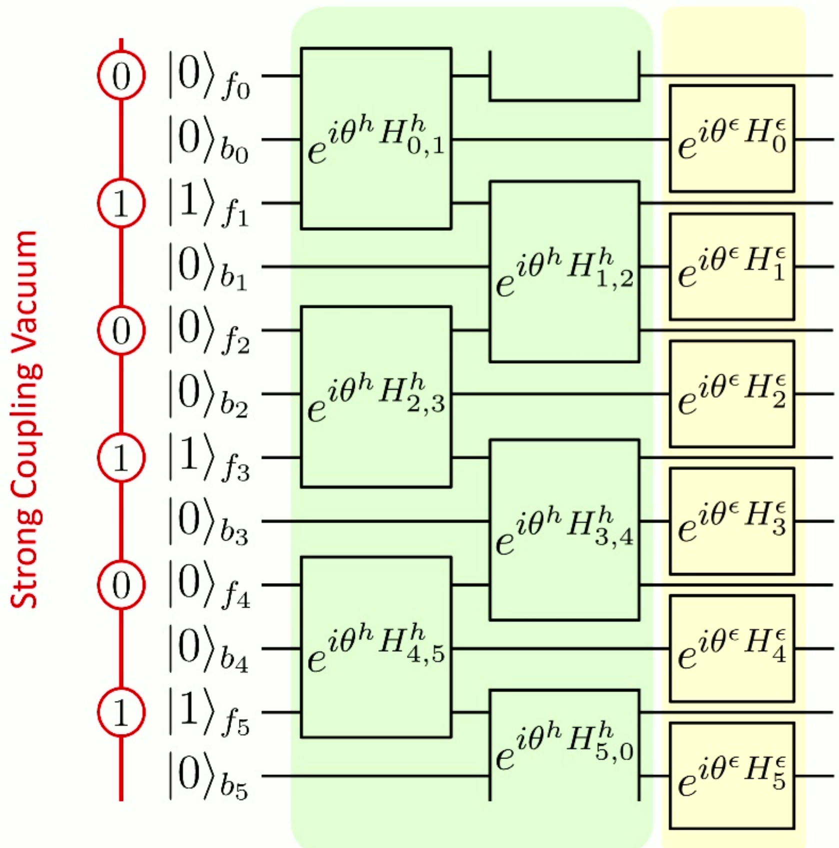
2 

$|\Omega\rangle$ Prepare the interacting ground state


Overview
Interacting theory
Interacting wave packet
$|\Psi\rangle = b_\psi^\dagger|\Omega\rangle$
Ansatz for $b_k^\dagger$
$b_\psi^\dagger = \sum_k \psi(k) b_k^\dagger$
$|\Omega\rangle$
Interacting ground state
4
$|\Psi\rangle$ Circuit for the wave packet
3
$b_k^\dagger$ Optimize the momentum creation operators
Briefly and only results
2
$|\Omega\rangle$ Prepare the interacting ground state
1
Map the fermionic + bosonic DOFs to qubits
30

2

$|\Omega\rangle$ Prepare the interacting ground state



Lumia, Torta, Mbeng, Santoro, Ercolessi, Burrello and Wauters
Phys. Rev. X Quantum 3, 020320 (2022)

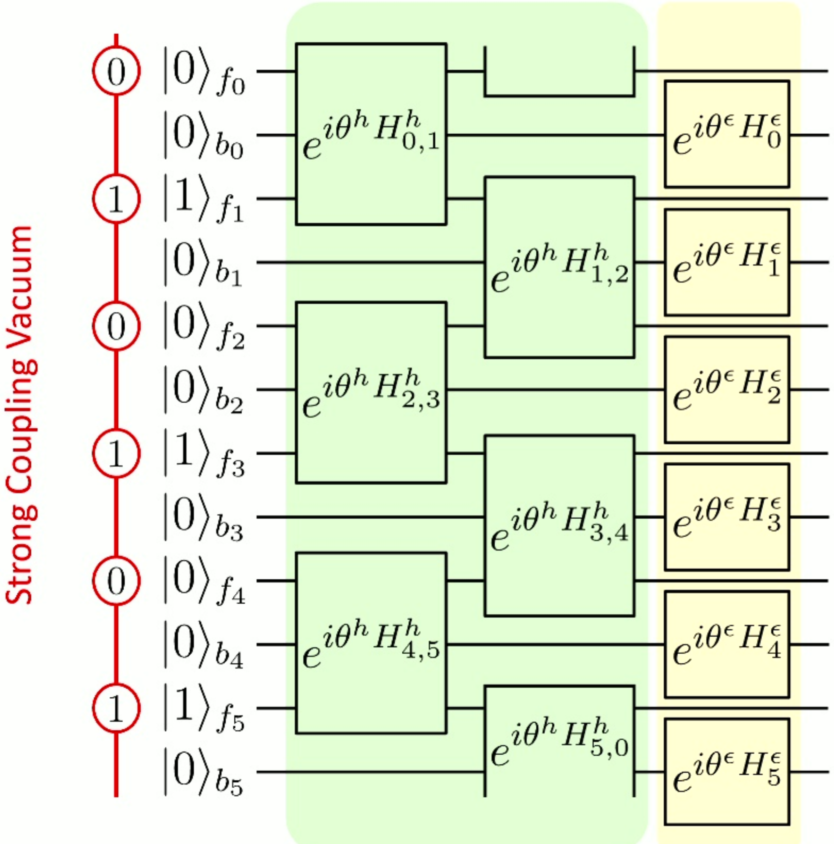
Variational Quantum Eigensolver (VQE) for the GS preparation:

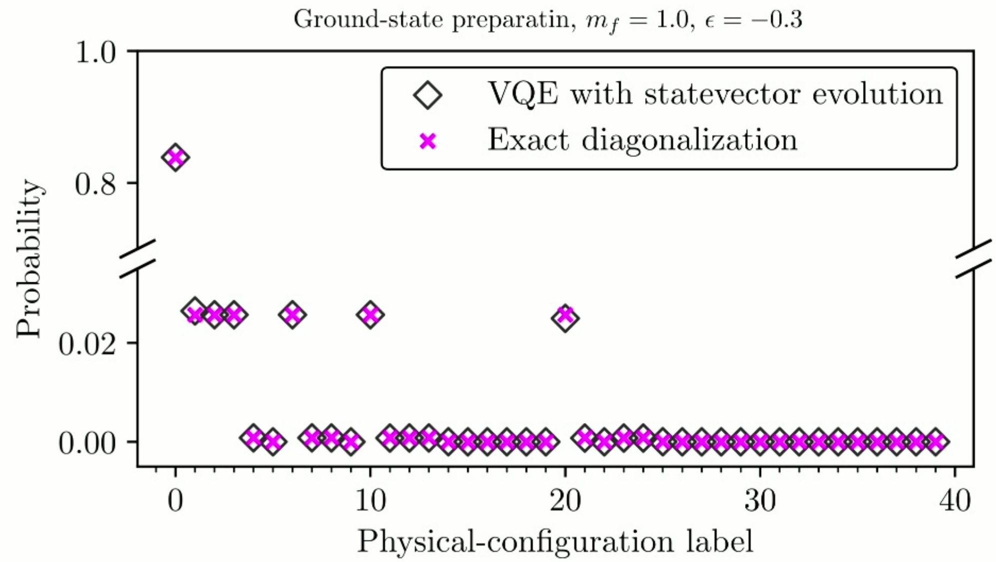
- Parameterized circuit with 2 parameters
  - ✓ Inspired from the Hamiltonian
  - ✓ Gauge invariant by construction
- Calculate energy with the Quantum circuit
- Optimize the parameters classically

31

2

$|\Omega\rangle$ Prepare the interacting ground state



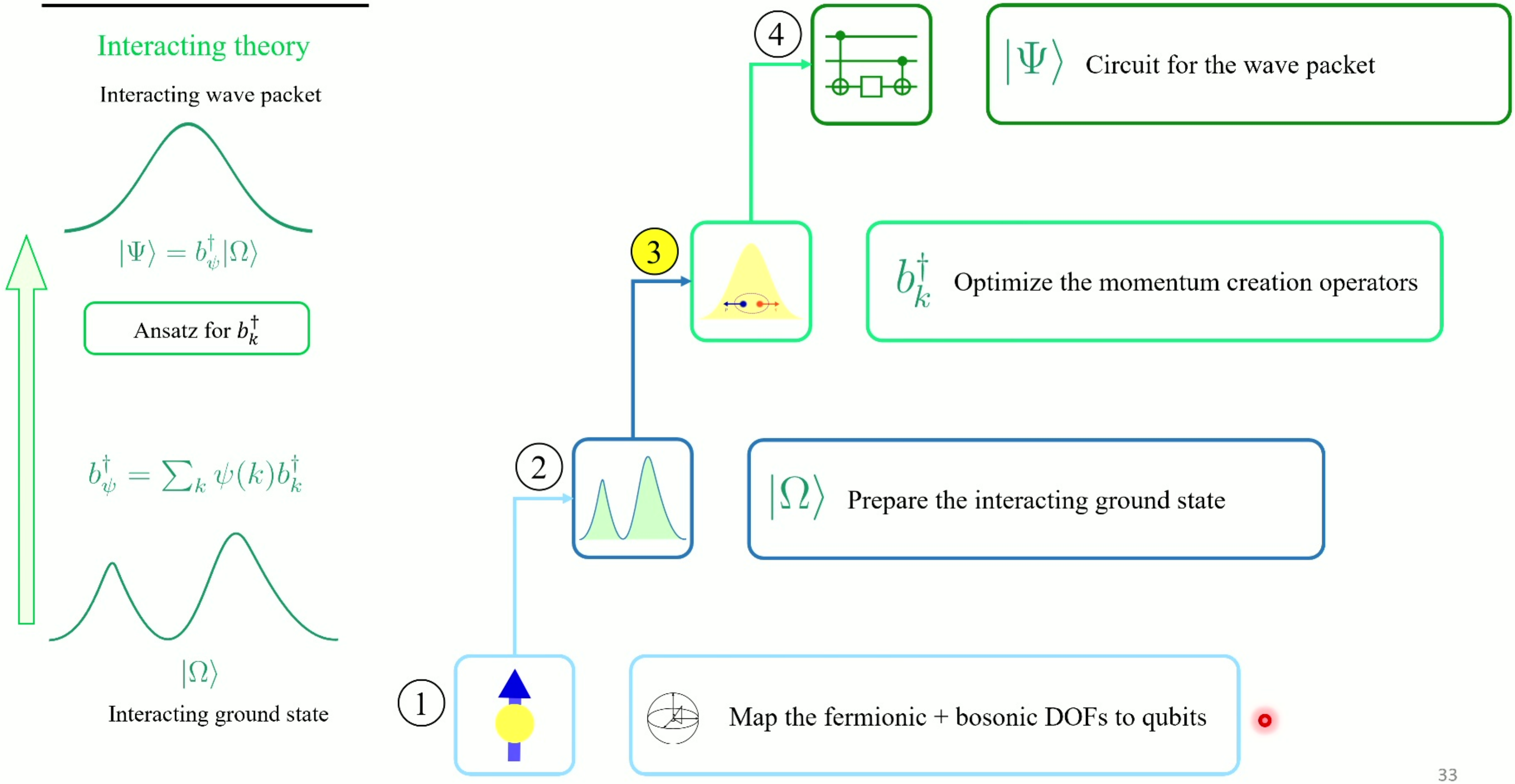


$$E_\Omega^{\text{Exact}} = -5.32483 \qquad \mathcal{F} = |\langle \Omega_{\text{Exact}} | \Omega_{\text{VQE}} \rangle|^2$$

$$E_\Omega^{\text{VQE}} = -5.32452 \qquad = 0.99992$$

32

Overview



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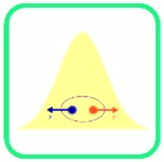
Interacting ground state
1
2
3
4
$|\Psi\rangle$ Circuit for the wave packet
$b_k^\dagger$ Optimize the momentum creation operators
$|\Omega\rangle$ Prepare the interacting ground state
Map the fermionic + bosonic DOFs to qubits

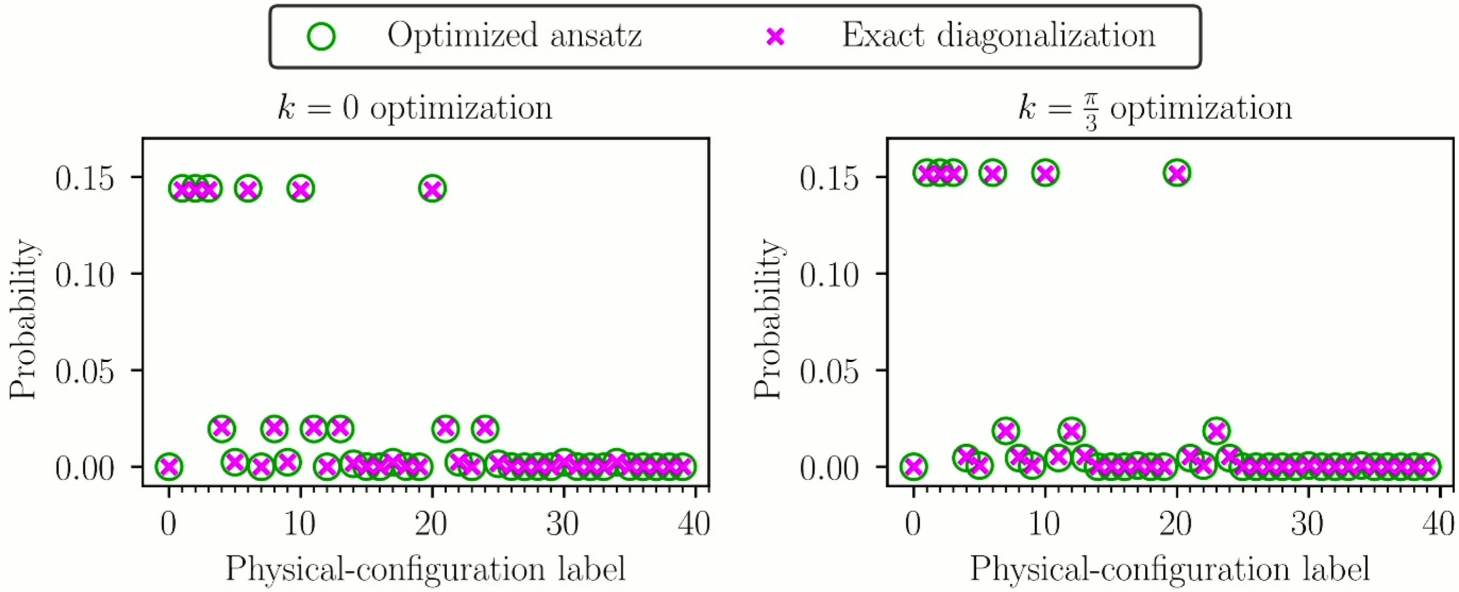
33

3



$b_k^\dagger$ Optimize the momentum creation operators

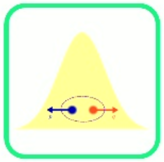
$$\eta_{pq} = N_\eta \exp\left(\frac{i(p-q)\mu_k^A}{2}\right)\exp\left(\frac{-(p-q)^2}{4\sigma_k^{A2}}\right)$$



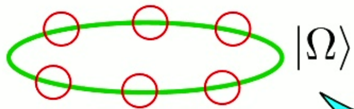
	$\mathcal{F} = \lvert\langle k_{\text{Exact}} \vert k_{\text{Optimized}}\rangle\rvert^2$	$E_k^{\text{Optimized}}$	E_k^{Exact}
$k = 0$	0.98756	-2.45688	-2.46734
$k = \pm\frac{\pi}{3}$	0.99977	-2.57561	-2.57613

34

3



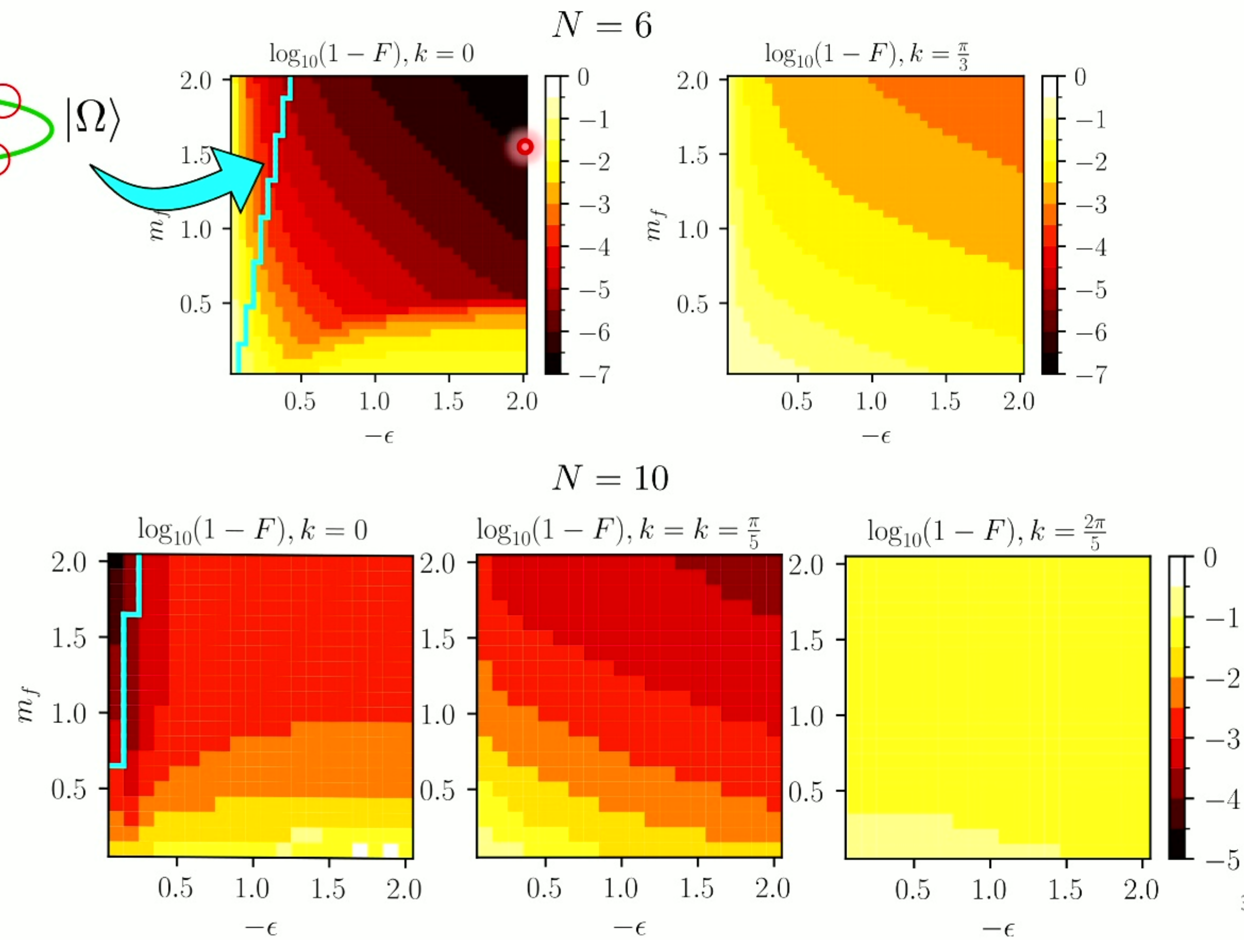
$b_k^\dagger$ Optimize the momentum creation operators



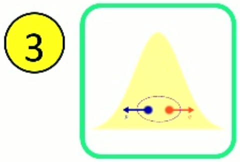
Z_2 LGT

How well does the ansatz work for different Hamiltonian parameters?

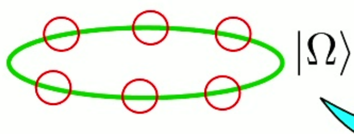
$$\mathcal{F} = |\langle k_{\text{Exact}} | k_{\text{Optimized}} \rangle|^2$$

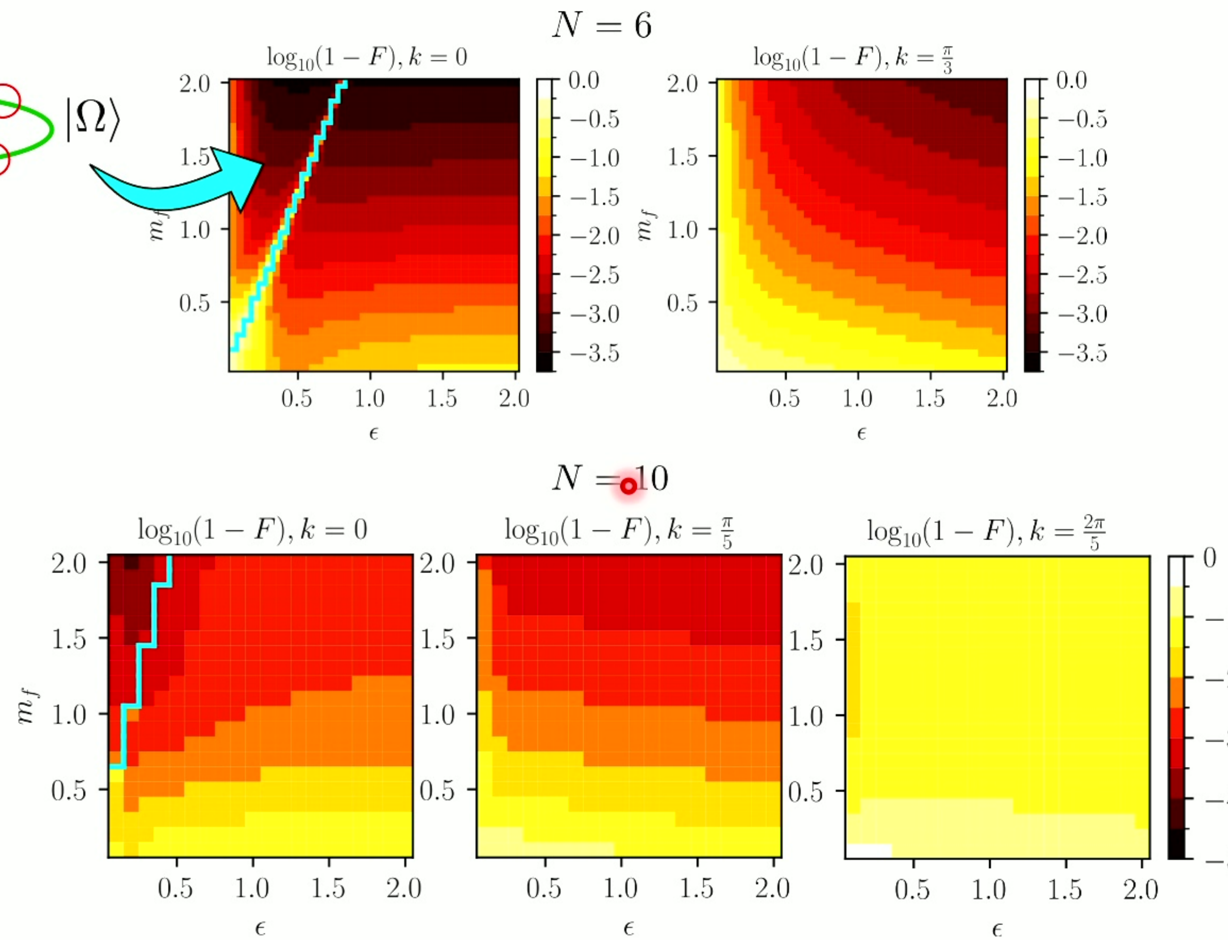


35



$b_k^\dagger$ Optimize the momentum creation operators





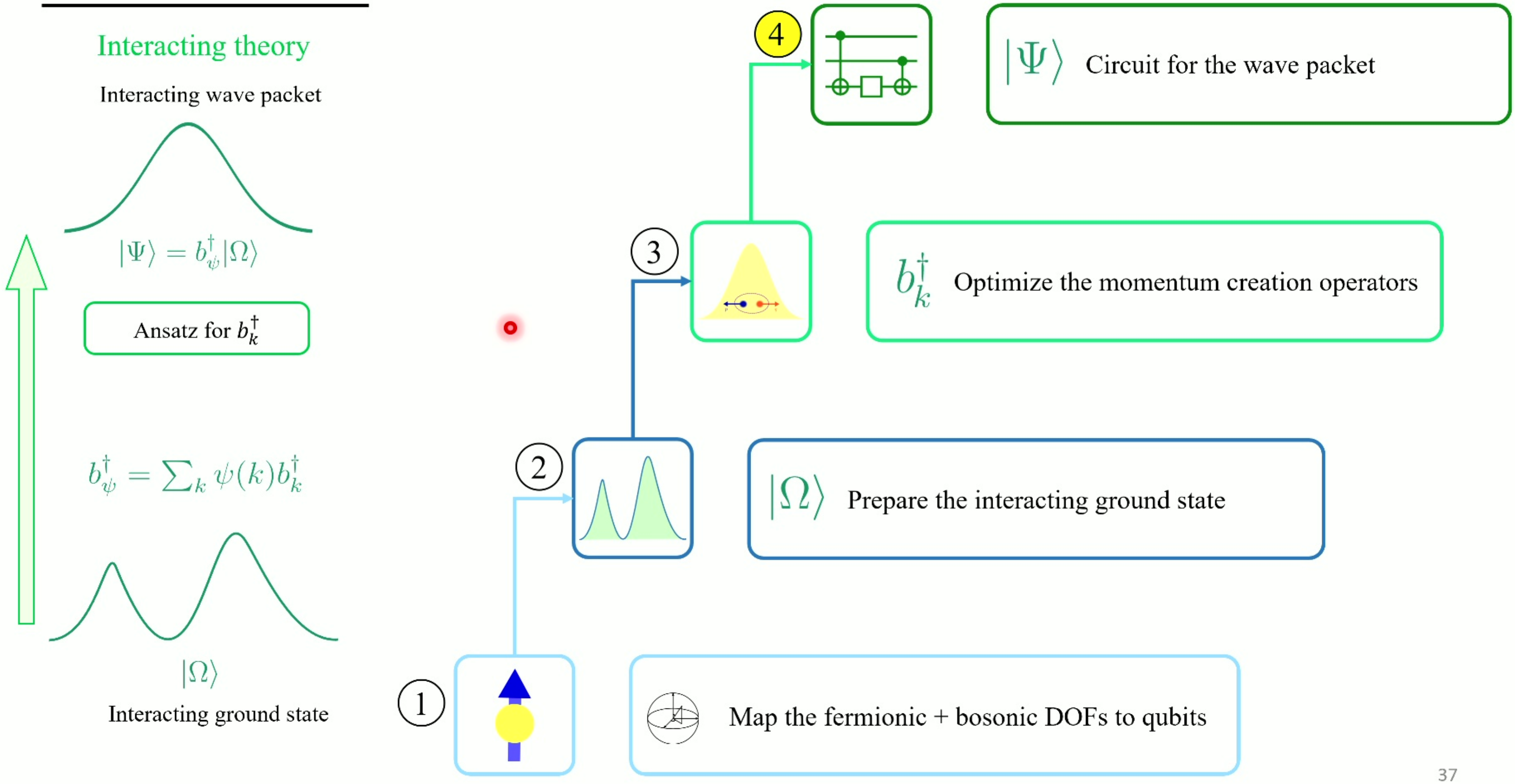
U(1) LGT

How well does the ansatz work for different Hamiltonian parameters?

$$\mathcal{F} = |\langle k_{\text{Exact}} | k_{\text{Optimized}} \rangle|^2$$

36

Overview



Interacting theory
Interacting wave packet
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Ansatz for $b_k^\dagger$
$b_\psi^\dagger = \sum_k \psi(k) b_k^\dagger$
$|\Omega\rangle$
Interacting ground state
4
$|\Psi\rangle$ Circuit for the wave packet
3
$b_k^\dagger$ Optimize the momentum creation operators
2
$|\Omega\rangle$ Prepare the interacting ground state
1
Map the fermionic + bosonic DOFs to qubits

37

4

$|\Psi\rangle$ Circuit for the wave packet

$$b^{\dagger}_{\psi} = \sum_{m,n} C_{m,n}\ \sigma^{-}_{n} \prod_{n<j<m} \sigma^{Z}_{j} \prod_{n\leq i\leq m} \tilde{\sigma}^{X}_{i}\ \sigma^{+}_{m}$$

Inputs Optimized

$$\psi(k) = \exp(-ik\mu) \exp\left(\frac{-(k-k_0)^2}{4\sigma^2}\right)$$

Issues

- Non-Unitary operator
- Needs efficient circuit design

4

$|\Psi\rangle$ Circuit for the wave packet

$$b_\psi^\dagger = \sum_{m,n} C_{m,n}\ \sigma_n^- \prod_{n<j<m} \sigma_j^Z \prod_{n\le i\le m} \tilde{\sigma}_i^X\ \sigma_m^+$$

❖ Non-Unitary operator

❖ Needs efficient circuit design

➢ Ancilla encoding:

Jordan, Lee, and Preskill (JLP)
Quantum Info. and Comp. 14, 1014-80

Embed the state into a larger Hilbert space using an ancilla qubit

If $b_\psi|\Omega\rangle = 0$ $\quad [b_\psi, b_\psi^\dagger] = \hat{1}$

Then

$$\Theta = b_\psi^\dagger \otimes |1\rangle\langle 0|_a + b_\psi \otimes |0\rangle\langle 1|_a$$

$$e^{-i\frac{\pi}{2}\Theta}|\Omega\rangle \otimes |0\rangle_a = -i\, b_\psi^\dagger|\Omega\rangle \otimes |1\rangle_a$$

➢ Applicable for Trotterization $\Theta = \sum_{\{m,n\}} \Theta_{m,n}$ upon

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4

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$$b_\psi^\dagger = \sum_{m,n} C_{m,n} \; \sigma_n^- \prod_{n<j<m} \sigma_j^Z \prod_{n\le i\le m} \tilde{\sigma}_i^X \; \sigma_m^+$$

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- Applicable for $\Theta = \sum_{\{m,n\}} \Theta_{m,n}$ upon Trotterization

- Singular Value Decomposition (SVD)

Find a basis that diagonalizes $\Theta_{m,n}$

Davoudi, Shaw and Stryker
Quantum 7, 1213 (2023)

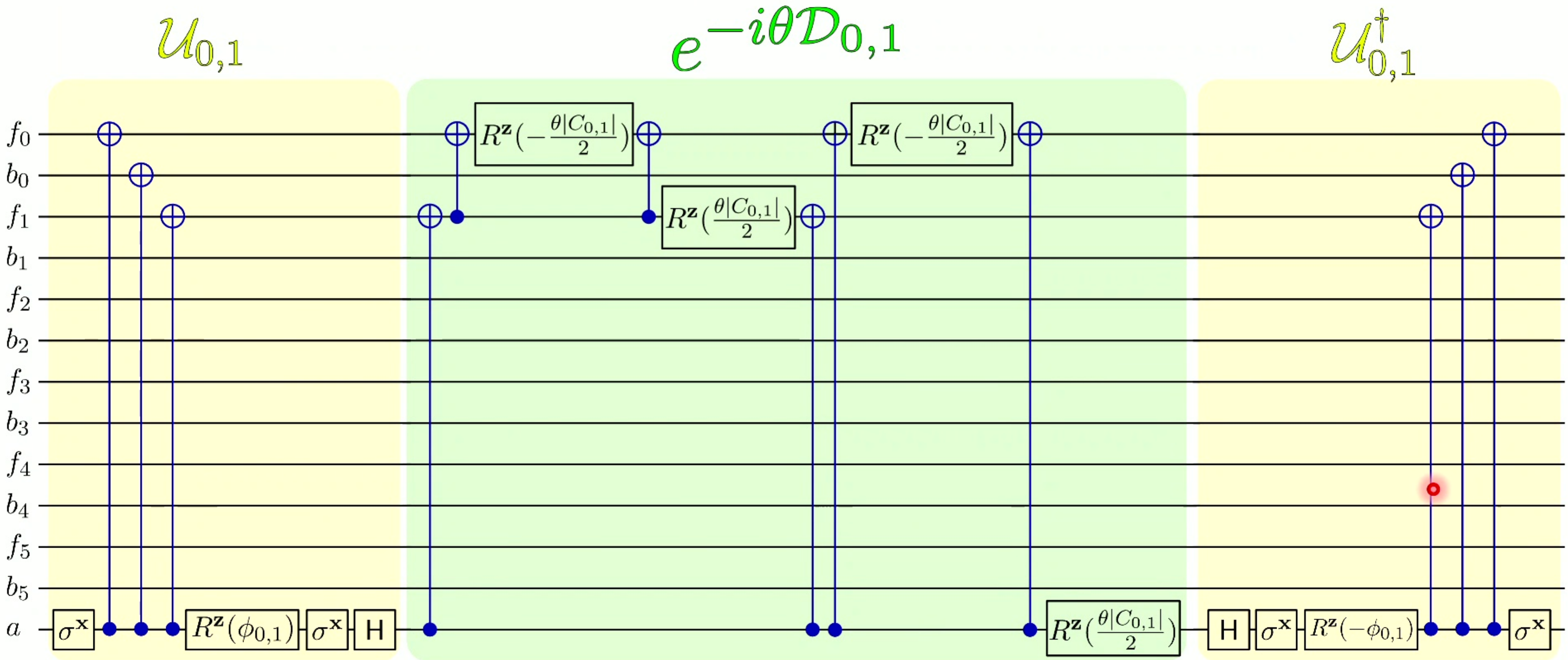
If $b_{m,n}^{\dagger\,2} = b_{m,n}^2 = 0 \quad \& \quad b_{m,n} = VSW^\dagger$

Then
$$e^{-i\frac{\pi}{2}\Theta_{m,n}} = \mathcal{U}_{m,n}^\dagger e^{-i\frac{\pi}{2}\mathcal{D}_{m,n}} \mathcal{U}_{m,n}$$
$$\mathcal{U}_{m,n} = \mathrm{Had}_a\left(V^\dagger \otimes |0\rangle\langle 0|_a + W^\dagger \otimes |1\rangle\langle 1|_a\right)$$
$$\mathcal{D}_{m,n} = S_{m,n} \otimes Z_a$$

4

$|\Psi\rangle$ Circuit for the wave packet

$$b^{\dagger}_{0,1} = e^{i\phi_{0,1}} |C_{0,1}| \, \sigma^{-}_{0} \tilde{\sigma}^{X}_{0} \sigma^{+}_{1}$$



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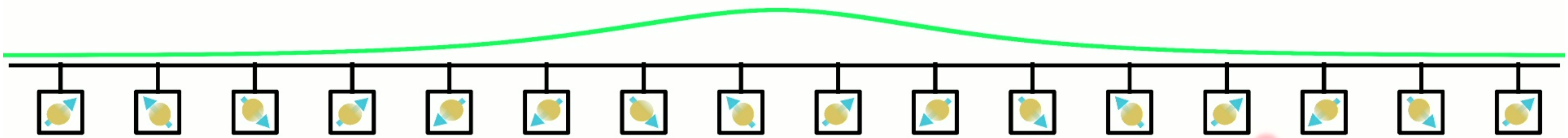
Measurements

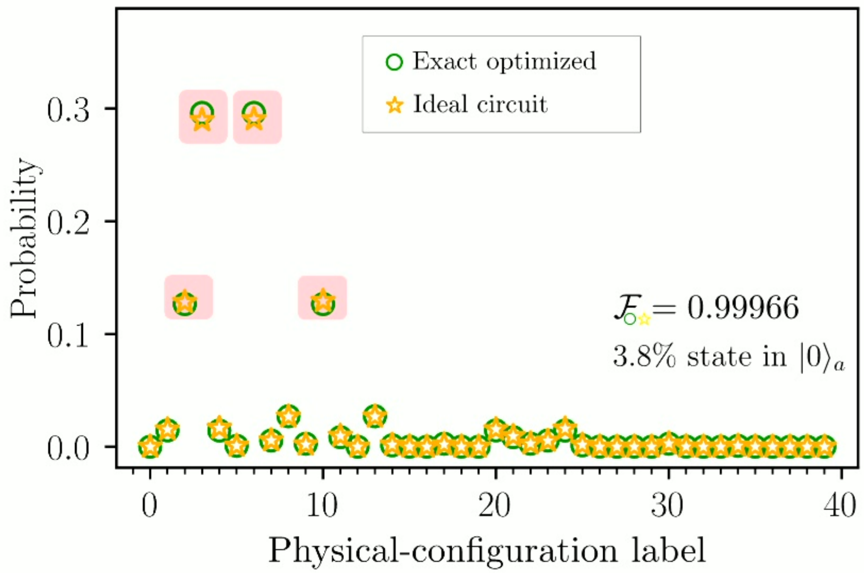
Hardware results



Results

The ideal quantum circuit

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$



☆ Ideal quantum circuit

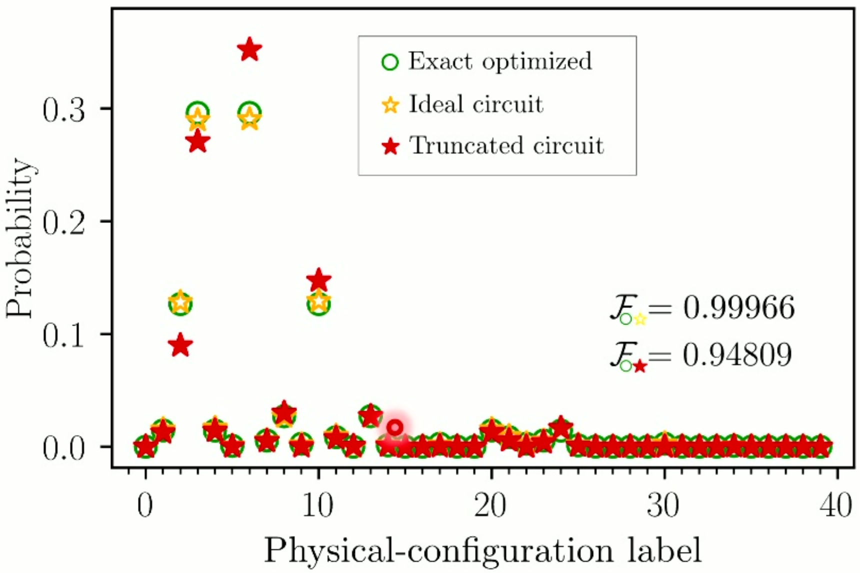
- Circuit statevector method & many Trotter steps
- Systematic error from:

$$b_\psi|\Omega\rangle \simeq 0 \qquad [b_\psi, b_\psi^\dagger] \simeq \hat{1}$$

Results

Truncated quantum circuit

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$



☆ Ideal quantum circuit

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★ Truncated quantum circuit

- Resource limitation:

  Only $|C_{m,n}| \geq 0.1$ terms were implemented

- 2nd order Trotter with 1 Trotter step

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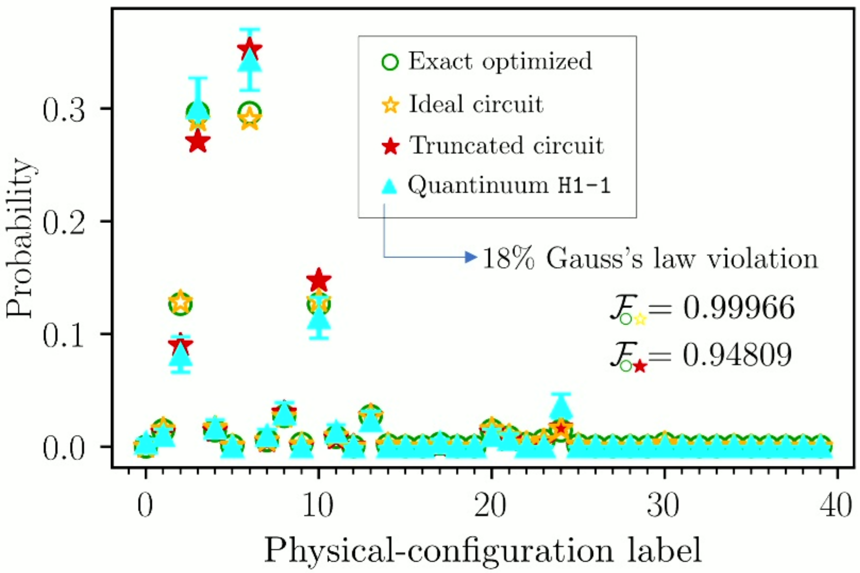
Results

Quantinuum Hardware results

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$



QUANTINUUM

H1-1

- Trapped ion with 20 qubits
- all-to-all connectivity
- $\sim 10^{-5}$ single-qubit gate infidelity
- $\sim 10^{-3}$ two-qubit gate infidelity

Ideal quantum circuit

- Circuit statevector method & many Trotter steps
- Systematic error from:

$$b_\psi|\Omega\rangle \simeq 0 \qquad [b_\psi, b_\psi^\dagger] \simeq \hat{1}$$

Truncated quantum circuit

- Resource limitation:

  Only $|C_{m,n}| \geq 0.1$ terms were implemented
- 2^{nd} order Trotter with 1 Trotter step

Quantinuum H1-1

- ~300 (350) two- (single-) qubit gates with 500 shots
- Error mitigation using the gauge invariant nature of our method

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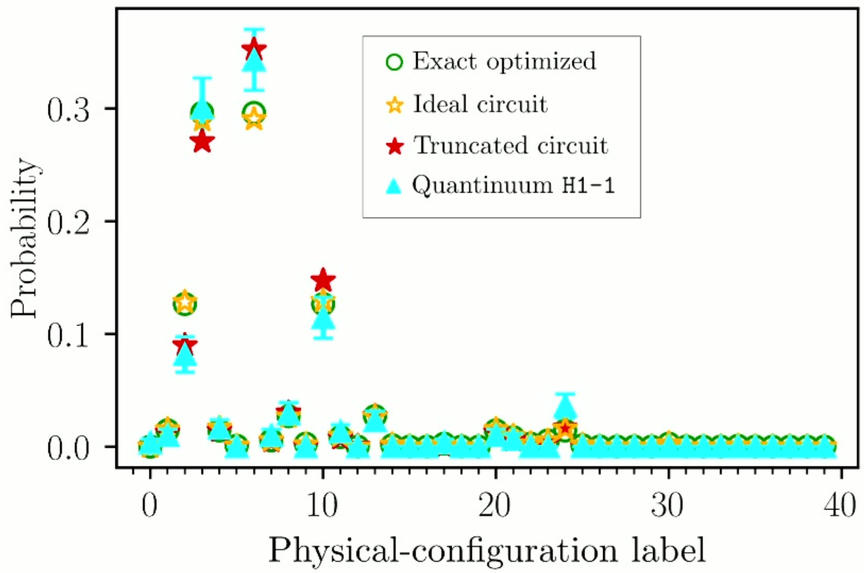
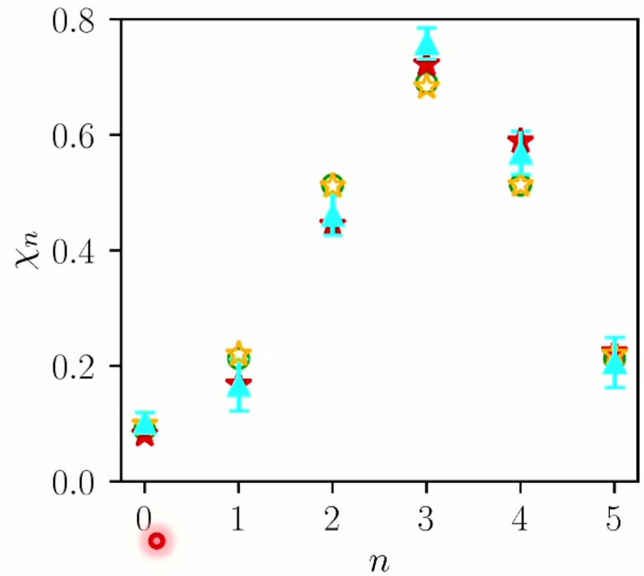
Results

Staggered number density

$$\chi(n) = \begin{cases} \langle \psi^\dagger(n)\psi(n) \rangle & n \text{ even} \\ 1 - \langle \psi^\dagger(n)\psi(n) \rangle & n \text{ odd} \end{cases}$$

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$





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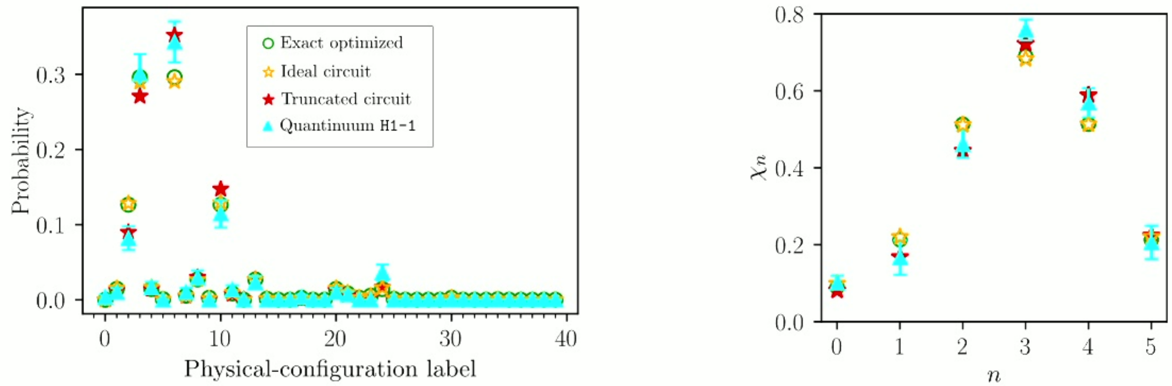
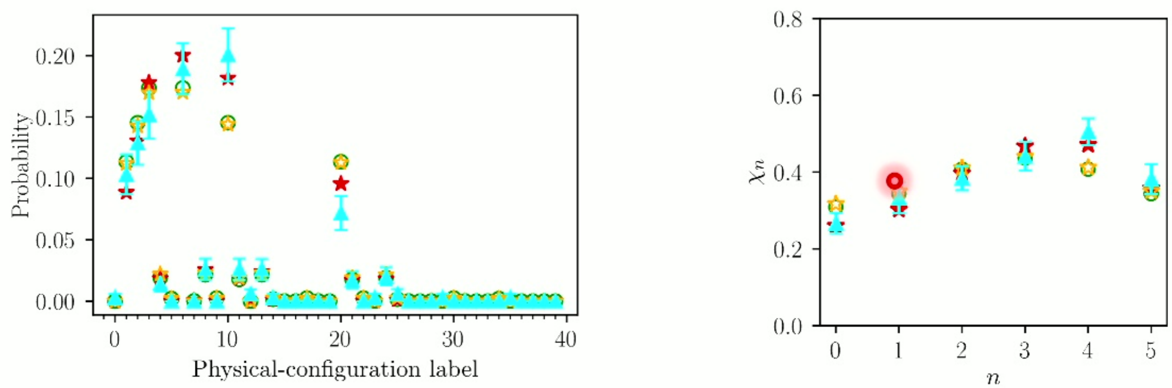
Results

Different WP widths

6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{6}$, $\mu = 3$, and $k_0 = 0$



6-site, $m_f = 1$, $\epsilon = -0.3$ with WP details: $\sigma = \frac{\pi}{10}$, $\mu = 3$, and $k_0 = 0$



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Summary

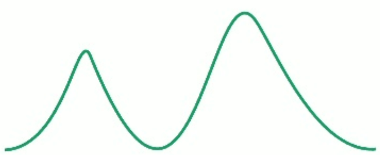
Interacting theory

Interacting wave packet

$|\Psi\rangle = b^\dagger_\psi |\Omega\rangle$

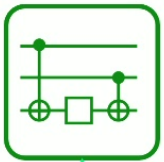
Ansatz for $b^\dagger_k$

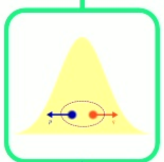
$b^\dagger_\psi = \sum_k \psi(k) b^\dagger_k$

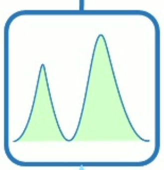


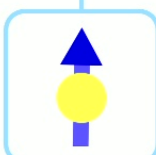
$|\Omega\rangle$

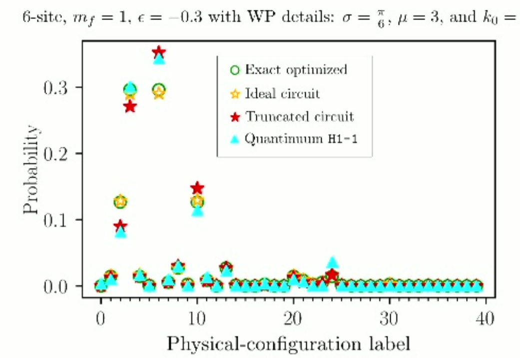
Interacting ground state

4  $|\Psi\rangle$ Circuit for the wave packet

3  $b^\dagger_k$ Optimize the momentum creation operators

2  $|\Omega\rangle$ Prepare the interacting ground state

1  Map the fermionic + bosonic DOFs to qubits

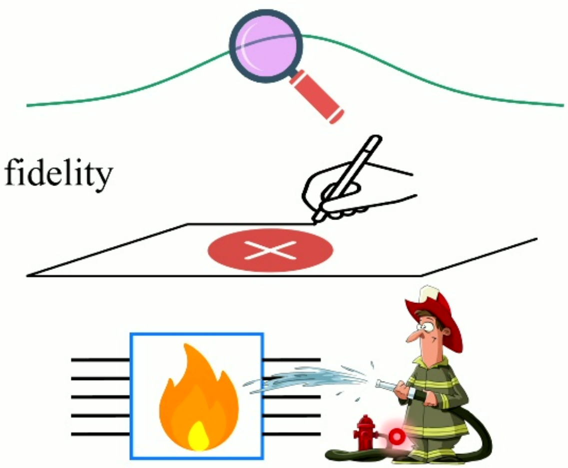


- ✓ Gauge invariant construction
- ✓ VQE based method
- ✓ Systematic error has been identified
- ✓ Works for the U(1) LGT in 1+1D
- ✓ Can be implemented on NISQ devices

Outlook

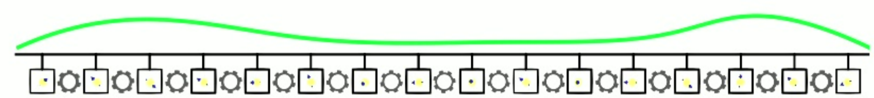
❖ What’s more?

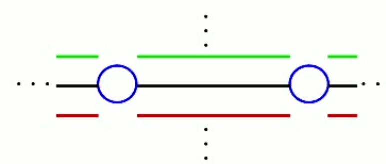
- ➢ More observables for measuring the wave packet fidelity
- ➢ Analytical bounds on systematic errors
- ➢ Advanced noise mitigation techniques



❖ What’s next?

- ➢ Prepare two wave packets and perform scattering
- ➢ Wave packet in the U(1) LGT on larger devices
- ➢ Ansatz for non-Abelian theories





GS on 100+ qubits:
Farrell, Illa, Ciavarella, and Savage
arXiv:2308.04481(2023)