Title: Loop corrected subleading soft graviton theorem from anomalous BRST Ward identity

Speakers: Tom Wetzstein

Series: Quantum Gravity

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Abstract: Extended BMS symmetry is believed to be a fundamental symmetry of any classical gravitational scattering process, as well as of the quantum gravity S-matrix. We explore this property using the BRST formulation of BMS symmetry, which allows to construct a non trivial solution of the Wess-Zumino consistency condition at the null boundaries of spacetime. We interpret this solution as an anomaly for asymptotic BRST Ward identities for superrotations. By relating them to the leading and subleading soft graviton theorem, we recover the well known results that the leading soft theorem is exact at all loop in perturbation theory without ever referring to Feynman diagrams, while the anomaly for superrotations is at the origin of the 1-loop correction to the subleading soft factor. This construction provides a rigorous, fully quantum origin for the invariance of the S-matrix under asymptotic symmetries.

Zoom link

Loop corrected sub-leading soft graviton theorem from anomalous BRST Ward identity

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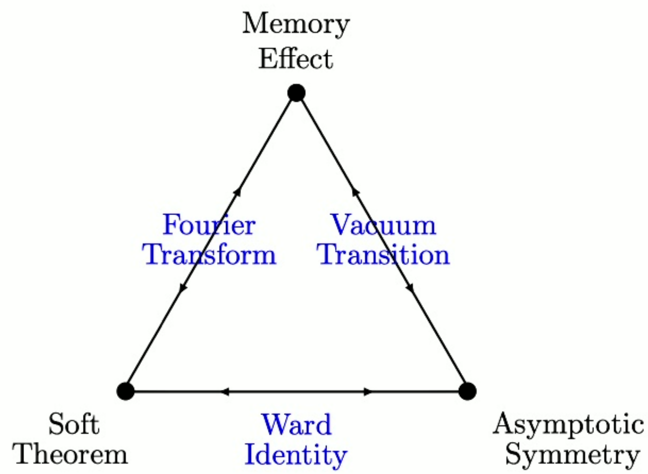
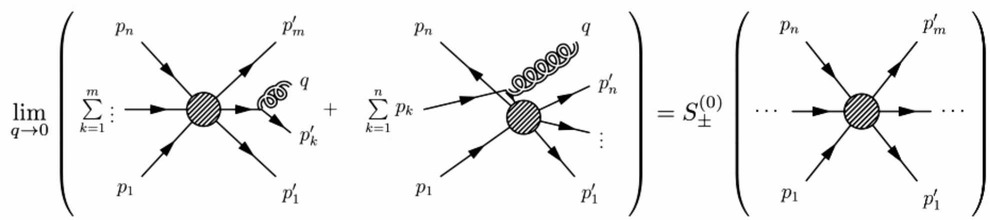
March 14, 2024

Done in collaboration with Laurent Baulieu

The Infrared Triangle and the BMS group





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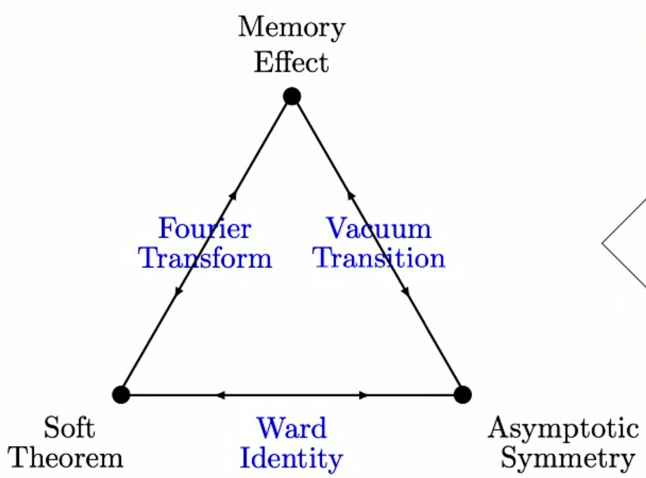
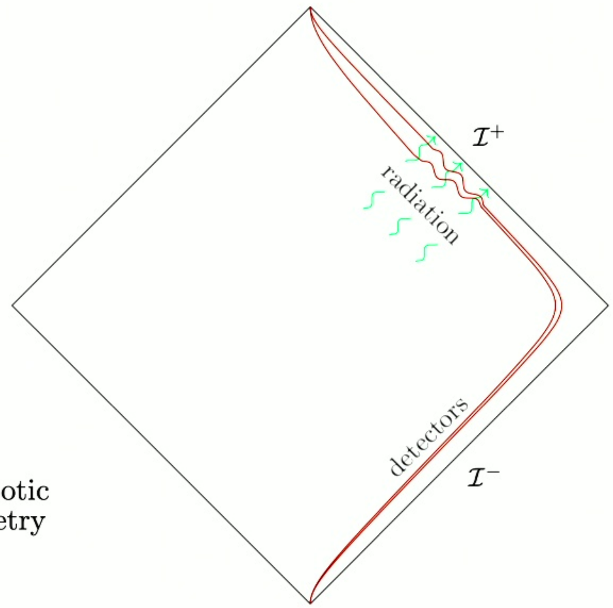
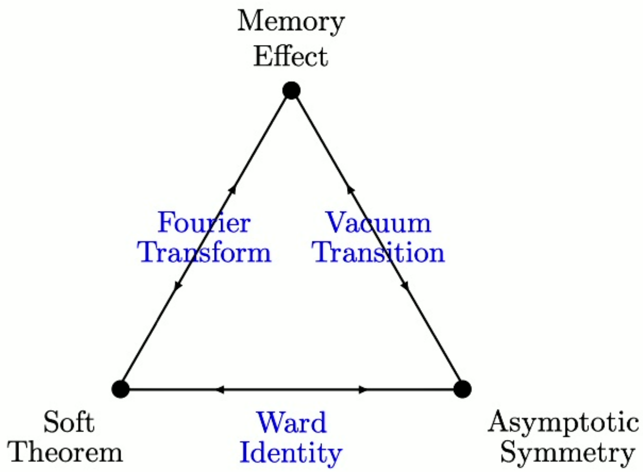




The Infrared Triangle and the BMS group



$$\textbf{Asymptotic symmetry group} \equiv \frac{\text{Allowed gauge transformations}}{\text{Trivial gauge transformations}}$$

Overview of the talk

Soft Graviton Theorem — Ward Identity — Extended BMS Symmetry

Leading soft graviton theorem

- Equivalent to $\langle \text{out}|[\mathrm{Q}_\alpha, \mathcal{S}]|\text{in}\rangle = 0$ for α the supertranslation ghosts
- Valid at all loops $\longleftrightarrow$ Non anomalous BRST Ward identity for supertranslations

Sub-leading soft graviton theorem

- Equivalent to $\langle \text{out}|[\mathrm{Q}_{\xi^A}, \mathcal{S}]|\text{in}\rangle = 0$ for ξ^A the superrotation ghosts
- Receives corrections at 1-loop $\longleftrightarrow$ Anomalous BRST Ward identity for superrotations

BRST Noether theorem / Finite charges

$\star J_{\text{BRST}} \hat{=} C + dq_\xi$ and $Q_\xi \hat{=} \int_\Sigma dq_\xi$

BRST operator

- $s = s_{\text{trivial}} + s_{\text{large}}$
- Action on phase space: $s_{\text{large}}\varphi = [\![Q_\xi, \varphi]\!]$
- Representation: $s\xi$ with $s^2 = 0$

Cocycles?

$s\Delta^1_3 = d\Delta^2_2 \longrightarrow$ anomalies?

Usual approach	**BRST approach**
$\langle \text{out}|[\mathrm{Q}_\xi, \mathcal{S}]|\text{in}\rangle = 0$	BRST formulation of this Ward identity?
• Valid at all loops? Anomalies?	• Anomalies are easy to treat
• Relation to the quantum master equation?	• Unification with the quantum master equation
• What about ghosts?	• Ghosts automatically incorporated

Extended BRST BMS4 symmetry and its cocycles

First order formulation of gravity

Einstein-Cartan-Holst gravity

Frame fields: $e^a_\mu \longrightarrow g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu$: curved metric on M_4

- enhanced symmetry: $\mathrm{Diff}_4 \times$local Lorentz $\longrightarrow \xi^\mu(x), \Lambda^{ab}(x)$
- gauge fields: e^a_μ, ω^{ab}_μ
- associated field strengths:

$$T^a = de^a + \omega^a{}_b \wedge e^b \qquad R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb}$$

- first order action: $L = \frac{1}{4}\epsilon_{abcd} R^{ab} \wedge e^c \wedge e^d$

Reminders on BRST symmetry

QED: $U(1)$ gauge symmetry with parameter $\lambda(x)$

$$\delta_\lambda A_\mu = \partial_\mu \lambda$$
$$\delta_{\mathrm{BRST}} A_\mu \equiv \eta s A_\mu = \eta \partial_\mu c = \partial_\mu(\eta c)$$

The nilpotent BRST operator s

- s is a differential: $s^2 = 0$, $sd + ds = 0$, $d^2 = 0$
  $(sA = -Dc, sc = -\frac{1}{2}[c, c], s\bar{c} = b, sb = 0)$
- gauge fixing: $L_{\mathrm{GF}} = L_{\mathrm{cl}} + s\Gamma(A, c, \bar{c}, b)$
- physical states are in $H^0(s) \equiv \frac{\mathrm{Ker(s)}}{\mathrm{Im(s)}}$, anomalies in $H^1(s|d)$

Example for Yang-Mills:

$$L_{FP} = -\frac{1}{2}\,\mathrm{Tr}(F \wedge \star\, F) + s(\bar{c} d \star A) = -\frac{1}{2}\,\mathrm{Tr}(F \wedge \star\, F) + bd(\star A) - \bar{c} d(\star D)c$$

BRST symmetry of gravity

Ghosts associated to the gauge fields:

$$e^a \longrightarrow c^a = i_\xi e^a = e^a_\mu \xi^\mu$$
$$\omega^{ab} \longrightarrow \Omega^{ab}$$

Nilpotent BRST operator of first order gravity

$$s e^a_\mu = \mathcal{L}_\xi e^a_\mu - \hat{\Omega}^{ab} e^b_\mu \qquad s\omega^{ab}_\mu = \mathcal{L}_\xi \omega^{ab}_\mu - D_\mu \hat{\Omega}^{ab}$$
$$s\xi^\mu = \xi^\nu \partial_\nu \xi^\mu \qquad s\hat{\Omega}^{ab} = \mathcal{L}_\xi \hat{\Omega}^{ab} - \hat{\Omega}^{ac}\hat{\Omega}^{cb}$$

- $\hat{\Omega}^{ab} \equiv \Omega^{ab} - \xi^\nu \omega^{ab}_\nu$ and $D = d + [\omega, \ ]$
- $(s+d)^2 = 0$ ensured by construction

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Bondi gauge and falloffs

Bondi metric in its Beltrami form

$$ds^2 = -\,du^2 - 2drdu + r^2\gamma_{z\bar{z}}dzd\bar{z} + \frac{1}{r}\Big[-\mu^+_-du^2 + r^2\gamma_{z\bar{z}}(\bar{\mu}dzdz + \mu d\bar{z}d\bar{z})\Big] + \mathcal{O}(r^{-2})$$

Link with the usual variables: $\boxed{m_B = -\frac{1}{2}\mu^+_- \quad \text{and} \quad C_{zz} = \gamma_{z\bar{z}}\bar{\mu}}$

Beltrami parametrization in light cone coordinates

Choice of coordinates: $M_4 = \mathcal{S}^2_{(z,\bar{z})} \times \Sigma_2^{(\tau^\pm)}$ with $\tau^\pm \equiv \tau \pm r$

Beltrami vierbein

The 16 components of the vierbein are reduced to 10 by fixing local Lorentz symmetry:

$$\begin{pmatrix} e^z \\ e^{\bar{z}} \\ e^+ \\ e^- \end{pmatrix} = \begin{pmatrix} \exp\frac{\Phi}{2} & 0 & 0 & 0 \\ 0 & \exp\frac{\Phi}{2} & 0 & 0 \\ 0 & 0 & \mathcal{M} & 0 \\ 0 & 0 & 0 & \mathcal{M} \end{pmatrix} \begin{pmatrix} 1 & \mu_{\bar{z}}^{z} & \mu_{+}^{z} & \mu_{-}^{z} \\ \mu_{z}^{\bar{z}} & 1 & \mu_{+}^{\bar{z}} & \mu_{-}^{\bar{z}} \\ 0 & 0 & 1 & \mu_{-}^{+} \\ 0 & 0 & \mu_{+}^{-} & 1 \end{pmatrix} \begin{pmatrix} dz \\ d\bar{z} \\ d\tau^+ \\ d\tau^- \end{pmatrix}$$

Consistency of the gauge fixing: $\Omega^{ab} \to \Omega^{ab}(\xi)$

- $g_{\mu\nu} = e^a_\mu \eta_{ab} e^b_\nu$ leads to

$$ds^2 = -\mathcal{M}^2(d\tau^+ + \mu_-^+ d\tau^-)(d\tau^- + \mu_+^- d\tau^+) + r^2\gamma_{z\bar{z}} \exp\Phi ||dz + \mu_{\bar{z}}^{z} d\bar{z} + \mu_+^z d\tau^+ + \mu_-^z d\tau^-||^2$$

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Link with the usual variables: $m_B = -\frac{1}{2}\mu_-^+$ and $C_{zz} = \gamma_{z\bar{z}}\bar{\mu}$

- **Diff$_4$ gauge fixing:** $\mu_+^- = \mu_+^z = \mu_+^{\bar{z}} = 0$, $\frac{\det g_{AB}}{r^4\gamma_{z\bar{z}}^2} = \varphi(u, z, \bar{z})$
- **Boundary conditions:** $\mu_{\bar{A}}^A = \mathcal{O}(r^{-1})$, $\mu_-^A = \mathcal{O}(r^{-2})$, $\mu_-^+ = \mathcal{O}(r^{-1})$, $\mathcal{M} = 1 + \mathcal{O}(r^{-2})$, $\exp\Phi = \sqrt{\varphi} + \mathcal{O}(r^{-2})$
- **Consistency:** $s(\text{gauge fixing}) = 0$
- **Allowed gauge transformations:** $s(\text{BC}) =$ same order in $1/r$

Extended BRST BMS4 symmetry

Allowed gauge transformations:

$$\xi^u = \alpha(z,\bar{z}) + \frac{u}{2}\partial_A \xi^A$$

$$\xi^z = \xi(z) - \frac{1}{r}\partial^z \xi^u + \mathcal{O}(r^{-2})$$

$$\xi^{\bar{z}} = \bar{\xi}(\bar{z}) - \frac{1}{r}\partial^{\bar{z}} \xi^u + \mathcal{O}(r^{-2})$$

$$\xi^r = -\frac{r}{2}\partial_A \xi^A + \frac{1}{2}\partial_A \partial^A \xi^u + \mathcal{O}(r^{-1})$$

/trivial at $\mathcal{I}^+$ $\longrightarrow$

ASG:

$$\xi^u_{\rm BMS} = \alpha(z,\bar{z}) + \frac{u}{2}\partial_A \xi^A$$

$$\xi^z_{\rm BMS} = \xi(z)$$

$$\xi^{\bar{z}}_{\rm BMS} = \bar{\xi}(\bar{z})$$

eBMS = Superrotations $\ltimes$ Supertranslations

Bondi gauge and falloffs

Bondi metric in its Beltrami form

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Link with the usual variables: $m_B = -\frac{1}{2}\mu_-^+$ and $C_{zz} = \gamma_{z\bar{z}}\bar{\mu}$

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Extended BRST BMS4 symmetry

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$$\xi^u = \alpha(z,\bar z) + \frac{u}{2}\partial_A\xi^A$$
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$$\xi^{\bar z} = \bar\xi(\bar z) - \frac{1}{r}\partial^{\bar z}\xi^u + \mathcal{O}(r^{-2})$$
$$\xi^r = -\frac{r}{2}\partial_A\xi^A + \frac{1}{2}\partial_A\partial^A\xi^u + \mathcal{O}(r^{-1})$$

$\xrightarrow{/\text{trivial at } \mathcal{I}^+}$

ASG:

$$\xi^u_{\rm BMS} = \alpha(z,\bar z) + \frac{u}{2}\partial_A\xi^A$$
$$\xi^z_{\rm BMS} = \xi(z)$$
$$\xi^{\bar z}_{\rm BMS} = \bar\xi(\bar z)$$

eBMS = Superrotations ⋉ Supertranslations

Let $(\xi(z), \bar\xi(\bar z), \alpha(z,\bar z)) \in$ eBMS, then:

Nilpotent BRST BMS4 operator s

$$\begin{cases} s\mu &= \left(\xi^u\partial_u + \xi^A\partial_A + \frac{3}{2}\bar\partial\bar\xi - \frac{1}{2}\partial\xi\right)\mu - \bar\partial^2\xi^u \\ s\alpha &= \xi^A\partial_A\alpha + \frac{\alpha}{2}\partial_A\xi^A \\ s\xi &= \xi\partial\xi \end{cases}$$

A possible anomaly for the extended BMS4 symmetry

Non trivial BMS4 cocycles in $\mathcal{I}^+$

$$\Delta_3^1 = du \wedge dz \wedge d\bar{z}\ (\mu \partial^3 \xi) + \text{c.c.}$$

- Wess-Zumino consistency condition: $s\Delta_3^1 = d\Delta_2^2$
- Non trivial solution: $\Delta_3^1 \neq s\Delta_3^0$ and $\Delta_3^1 \neq d\Delta_2^1$

Question: What does this anomaly possibly break? It must be a 3D Ward identity.

Link with the double soft theorem: Field dependent central extension

- $\Delta_2^2 = \ dz \wedge d\bar{z}\ \mu \xi^u \partial^3 \xi - du \wedge d\bar{z}\ \left(\xi(\mu \partial^3 \xi + \bar{\mu} \bar{\partial}^3 \bar{\xi}) + \partial \xi^u \bar{\partial}^3 \bar{\xi}\right) - (c.c.)$
- $\langle \mathrm{out} | \left[[\![Q_{\xi,\bar{\xi}}, Q_\alpha]\!] + \int_{\mathcal{S}^2} \Delta_2^2, \mathcal{S} \right] | \mathrm{in} \rangle = S(k_1, k_2) \langle \mathrm{out} | \mathcal{S} | \mathrm{in} \rangle$

BRST Ward identities for asymptotic symmetries

Splitting of the BRST operator

Vague idea: $s = s_{\text{trivial}} + s_{\text{large}}$

1. $s_{\text{trivial}} \longrightarrow$ Ward identity in the bulk: $\boxed{\{\Gamma, \Gamma\} = 0}$
   Quantum master equation for the connected 1PI Green functions Γ
2. $s_{\text{large}} \longrightarrow$ Boundary Ward identity? $\boxed{[Q, \mathcal{S}] = 0}$

Goal: Unify these two Ward identities

$\longrightarrow$ turn $[Q, \mathcal{S}] = 0$ into an equation on Γ

$\longrightarrow$ couple L to $\star J_{\text{BRST}}$ by gauging BRST symmetry

Generating functional

Gauged BRST: $\boxed{\delta_{\mathrm{BRST}} = \eta(x)s}$

$\longrightarrow$ associated commuting ghost and 1-form: $C_\eta, \beta = \beta_\mu^{-1} dx^\mu$

Minimal coupling: $\partial_\mu \longrightarrow \mathcal{D}_{\beta_\mu} \equiv \partial_\mu - \beta_\mu s$

Correlation functions of the fields and BRST Noether current

$$\exp W_c \equiv \int_{\varphi_0}^{\varphi_f} [\delta\varphi] \exp \int_M \Big(L(\varphi, \mathcal{D}_\beta \varphi) + j_\varphi \varphi + v_\varphi s\varphi(\varphi, \mathcal{D}_\beta\varphi) + j_B B(\varphi, \mathcal{D}_\beta\varphi) + v_B sB(\varphi, \mathcal{D}_\beta\varphi) \Big)$$

- β is a source for $I_V \theta^\mu = \frac{\delta L(\varphi, \partial_\mu \varphi)}{\delta \partial_\mu \varphi} s\varphi$
- j_B is a source for B, coming from $sL = dB$

Generating functional

Gauged BRST: $\boxed{\delta_{\mathrm{BRST}} = \eta(x)s}$

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- β is a source for $I_V\theta^\mu = \frac{\delta L(\varphi, \partial_\mu\varphi)}{\delta \partial_\mu \varphi} s\varphi$
- j_B is a source for B, coming from $sL = dB$
- functional differentiation by β and j_B can insert a Noether current $\star J = I_V\theta - B$ into correlation functions

Ward identity for gauged BRST symmetry

1. **Change of field variables:**

$$\varphi(x) \longrightarrow \varphi'(x) \equiv \varphi(x) + \eta(x) s\varphi[\varphi, \mathcal{D}_\beta \varphi](x)$$

2. **Legendre transform:** $Z_c = \exp W_c$

$$\Gamma[\varphi, v_\varphi, \alpha, j_B, v_B] = -\int J(x)\varphi(x) + Z_c[J, v_\varphi, \alpha, j_B, v_B]$$

$$\longrightarrow j_\varphi = \frac{\delta \Gamma[\varphi, v_\varphi, \alpha, j_B, v_B]}{\delta \varphi}$$

3. **Integration by parts:**

Bulk + boundary Ward identity for connected 1PI Green functions

$$0 = \int_M \eta(x)\left(\frac{\delta \Gamma}{\delta \varphi(x)} \frac{\delta \Gamma}{\delta v_\varphi(x)}\right) - d\left(\eta(x) \frac{\delta \Gamma}{\delta j_B(x)}\right) + d\eta(x)\left(\frac{\delta \Gamma}{\delta \beta(x)} - \frac{\delta \Gamma}{\delta j_B(x)}\right)$$

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Quantum master equation: constant η

Global BRST $\iff$ **constant** η: *Quantum master equation for a semi symmetry*

$$\frac{1}{2}\{\Gamma,\Gamma\} - \int_M d\left(\frac{\delta\Gamma}{\delta j_B(x)}\right) = 0$$

- Symplectic bracket: $\{\Gamma,\bullet\} \equiv \int_M \frac{\delta\Gamma}{\delta\varphi(x)}\frac{\delta(\bullet)}{\delta v_\varphi(x)} + \frac{\delta\Gamma}{\delta v_\varphi(x)}\frac{\delta(\bullet)}{\delta\varphi(x)}$
- It renormalizes L order by order in perturbation theory while maintaining $sL = dB$

Anomalies: Assume it holds true at $n-1$ loops

$$\frac{1}{2}\{\Gamma,\Gamma\}_n - \int_M d\left(\frac{\delta\Gamma_n}{\delta j_B(x)}\right) = \hbar^n\int_M \Delta_4^1(\varphi) + \mathcal{O}(\hbar^{n+1})$$

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Jacobi identity:

$$\{\Gamma_0, \textstyle\int_M \Delta_4^1(\varphi)\} = 0 \quad \Longleftrightarrow \quad s\displaystyle\int_M \Delta_4^1(\varphi) = 0$$

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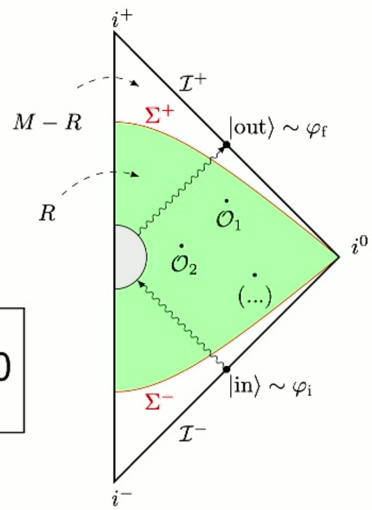
Asymptotic BRST Ward identity

Boundary aspects: indicator function

$$\eta(x) = \begin{cases} 1, & x \in R \\ 0, & x \in M - R \end{cases}$$

$\longrightarrow$ Asymptotic BRST Ward identity:

$$\boxed{\int_{\mathcal{I}^+ - \mathcal{I}^-} \left(\frac{\delta\Gamma}{\delta\beta(x)} - \frac{\delta\Gamma}{\delta j_B(x)} \right) = 0}$$



As $\star J_{\mathrm{BRST}} = I_V\theta - B = \left(\frac{\delta\Gamma}{\delta\beta(x)} - \frac{\delta\Gamma}{\delta j_B(x)} \right)\Big|_{\text{sources}=0}$, we get

$$\left\langle \left(\int_{\mathcal{I}^+} \star J \right) \mathcal{T}(\mathcal{O}_1 \ldots \mathcal{O}_n) \right\rangle - \left\langle \mathcal{T}(\mathcal{O}_1 \ldots \mathcal{O}_n) \left(\int_{\mathcal{I}^-} \star J \right) \right\rangle = 0$$

Current algebra anomaly

Boundary anomalies $\longleftrightarrow$ non conservation of the Noether current:

$$\int_{\mathcal{I}^+ - \mathcal{I}^-} \left(\frac{\delta \Gamma_n}{\delta \beta(x)} - \frac{\delta \Gamma_n}{\delta j_B(x)} \right) = \hbar^n \int_{\mathcal{I}^+ - \mathcal{I}^-} \Delta_3^1(\varphi) + \mathcal{O}(\hbar^{n+1})$$

- Anticommuting nature of $\beta_\mu(x)$ and $j_{B_\mu}(x)$

$$\{\Gamma_0, \int_{\mathcal{I}^\pm} \Delta_3^1(\varphi)\} = 0$$

Boundary cocycle as a possible current algebra anomaly

$$\boxed{s\Delta_3^1 = d\Delta_2^2}$$

1. Δ_2^2 vanishes at the boundaries of $\mathcal{I}^\pm$
2. the cocycle is non trivial

Application to gravity: loop corrected sub-leading soft graviton theorem

BRST Noether second?

- **Gauge fixed Lagrangian:**

$$L = \frac{1}{4}\epsilon_{abcd}R^{ab}e^c e^d + s\Big((\bar{\Omega}^I \star M^I_a + \bar{\xi}^i\delta^i_a \star \delta_+)e^a\Big)$$

- **BRST Noether current:** $\delta L = E + d\theta$ and $\star J_{\mathrm{BRST}} = I_V\theta - B$

$$\boxed{\star J_{\mathrm{BRST}} \hat{=} \; - s\Big((\bar{\Omega}^I \star M^I_a + \bar{\xi}^i\delta^i_a \star \delta_+)i_\xi e^a\Big) + dq_\Omega}$$

with

$$\begin{cases} \Omega^{ab} = \hat{\Omega}^{ab} + i_\xi \omega^{ab} \\ q_\Omega = \frac{1}{4}\epsilon_{abcd}\Omega^{ab}e^c e^d \end{cases}$$

- **Holographic "renormalization":** finite and non zero charges

$$Q^\pm = Q^\pm_\alpha + Q^\pm_\xi \equiv \int_{\mathcal{I}^\pm} dq^{\mathrm{r}}_\Omega$$

Soft graviton theorem

- **Properties of correlation functions:**

$$\langle (s\mathcal{G})\mathcal{T}(\mathcal{O}_1 \dots \mathcal{O}_n)\rangle = 0 \quad \text{and} \quad \langle f(\text{EoM})\mathcal{T}(\mathcal{O}_1 \dots \mathcal{O}_n)\rangle = 0$$

Current Ward identity

$$\left\langle \left[\int_{\mathcal{I}^\pm} \star J, \mathcal{T}(\mathcal{O}_1 \dots \mathcal{O}_n) \right] \right\rangle \Big|_{\text{LSZ}} = 0 \quad \Longleftrightarrow \quad \langle \text{out}| \left[Q, \mathcal{S}\right] |\text{in}\rangle = 0$$

- Leading soft theorem $\longleftrightarrow$ supertranslation WI $[Q_\alpha, \mathcal{S}] = 0$:

$$\lim_{\omega \to 0} \omega \, \langle \text{out}| \, a_\pm^{\text{out}}(\omega \hat{q}) \mathcal{S} \, |\text{in}\rangle \big|_{\text{tree}} = S_\pm^{(0)} \, \langle \text{out}| \, \mathcal{S} \, |\text{in}\rangle \big|_{\text{tree}}$$

- Sub-leading soft theorem $\longleftrightarrow$ superrotation WI $[Q_\xi, \mathcal{S}] = 0$:

$$\lim_{\omega \to 0} (1 + \omega \partial_\omega) \, \langle \text{out}| \, a_\pm^{\text{out}}(\omega \hat{q}) \mathcal{S} \, |\text{in}\rangle \big|_{\text{tree}} = S_\pm^{(1)} \, \langle \text{out}| \, \mathcal{S} \, |\text{in}\rangle \big|_{\text{tree}}$$

Non anomalous supertranslation symmetry or infrared finiteness

Feynman diagrammatics

- $\mathcal{S}$-matrix factorization: $\mathcal{M} = \langle \text{out} | \mathcal{S} | \text{in} \rangle = \mathcal{M}_{\text{soft}} \mathcal{M}_{\text{finite}}$
- Pole in $1/\omega$: $S^{(0)}_{\pm} = \frac{\kappa}{2} \sum_{k=1}^{n} \frac{p_k^\mu p_k^\nu \epsilon^{\pm}_{\mu\nu}(\hat{q})}{p_k \cdot \hat{q}}$ exact at all loops
- Add more soft gravitons: these poles exponentiate and cancel $\mathcal{M}_{\text{soft}} \longrightarrow$ infrared finite inclusive cross sections

A cohomological take:

$$S^{(0)}_{\pm} \text{exact at all loops} \iff \langle \text{out} | [Q_\alpha, \mathcal{S}] | \text{in} \rangle = 0 \text{ is non anomalous}$$
$$\iff H^1(s^{\alpha}_{\text{large}} | d) = 0$$

$\longrightarrow$ This is the case! $\Delta^1_3 = 0$ for supertranslations

Anomalous superrotation symmetry

Superrotation cocycle : $\begin{cases} \int_{\mathcal{I}^+} \Delta^1_3 = \int_{\mathcal{I}^+} du \wedge dz \wedge d\bar{z} \; (\mu \partial^3 \xi + \text{c.c.}) \\ \int_{\mathcal{I}^+_\pm} \Delta^2_2 = \int_{\mathcal{I}^+_\pm} dz \wedge d\bar{z} \; \frac{u}{2} (\mu \partial \xi \partial^3 \xi + \text{c.c.}) \end{cases}$

- Δ^1_3 integrable $\Longleftrightarrow$ hard part of the shear $\tilde{\mu}$ in Δ^1_3
- Falloffs of $\tilde{\mu}$: $\tilde{\mu}|_{\mathcal{I}^+_\pm} = \mp 2 \partial^2 N^{(0)}$
- $N^{(0)}$ measures the memory, same value at $\mathcal{I}^+_\pm$

$$s \int_{\mathcal{I}^+} \Delta^1_3 = \int_{\mathcal{I}^+} d\Delta^2_2 = \Delta^2_2 \Big|_{\mathcal{I}^+_+} - \Delta^2_2 \Big|_{\mathcal{I}^+_-} = 0$$

$\Longrightarrow$ $\boxed{H^1(s^{\xi,\bar{\xi}}_{\text{large}} | d) \neq 0}$

LPTHE

Loop corrected sub-leading soft graviton theorem

Feynman diagrammatics

- Sub-leading soft factor at tree level:

$$S_{\pm}^{(1)} = -\frac{i\kappa}{2}\sum_{k=1}^{n}\frac{p_k^{\mu}\epsilon_{\mu\nu}^{\pm}(\hat{q})q_{\lambda}}{p_k\cdot\hat{q}}J_k^{\lambda\nu}$$

- Corrections at 1-loop:

$$S_{\pm}^{(1)} \longrightarrow S_{\pm}^{(1)} + \frac{\kappa^2}{\epsilon}\left(\hat{\sigma}'_{n+1}S_{\pm}^{(0)} - S_{\pm}^{(1)}\sigma_n\right)$$

$$\Longleftrightarrow \int_{\mathcal{I}^+-\mathcal{I}^-}\left(\frac{\delta\Gamma_1}{\delta\beta(x)} - \frac{\delta\Gamma_1}{\delta j_B(x)}\right) = \mathfrak{a}\hbar\int_{\mathcal{I}^+-\mathcal{I}^-}\Delta_3^1(\varphi)$$

$$\Longleftrightarrow H^1(s_{\text{large}}^{\xi,\bar{\xi}}|d) \neq 0 \quad \text{and} \quad \mathfrak{a}\neq 0$$

$\longrightarrow$ This is the case! $\Delta_3^1 = du\wedge dz\wedge d\bar{z}\ (\mu\partial^3\xi) + \text{c.c.} \neq 0$ for superrotations

Thank you for your attention!