Title: Homotopy theory and quantum field theories

Speakers: Mayuko Yamashita

Series: Colloquium

Date: March 12, 2024 - 2:00 PM

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Abstract: Recently, there has been a growing interest in the relations between homotopy theory in mathematics and theoretical physics. Homotopy theory is used to classify and study physical systems. Also, physically motivated conjectures have led to many interesting developments in homotopy theory. I have been studying this subject as a mathematician.

My recent works have been motivated by the Segal-Stolz-Teichner program, which is one of the most deep and important subjects relating homotopy theory and physics. They propose a geometric model, in terms of supersymmetric quantum field theories, of a homotopy-theoretic object "Topological Modular Forms". Based on this, we show the absence of anomaly in heterotic string theory (joint work with Yuji Tachikawa), and found a new physical and geometric understanding of duality (with Y.Tachikawa) and periodicity (with Theo Jonhson-Freyd) in homotopy theory. These works lead us to further interesting conjectures to explore. I would like to illustrate this exciting interplay between mathematics and physics.

Zoom link

13:59 3月12日(火)

2024-03 PI colloquium

Homotopy Theory

&

Quantum Field Theories

Mayuko Yamashita (Kyoto Univ)

j. w/ Yuji Tachikawa (IPMU, Univ. of Tokyo)

Theo Johnson-Freyd (Perimeter Institute & Dalhousie Univ)

Based on

[TY'21] arXiv: 2108.13542

[TY'23] arXiv: 2305.0619

[JF-Y, in preparation]

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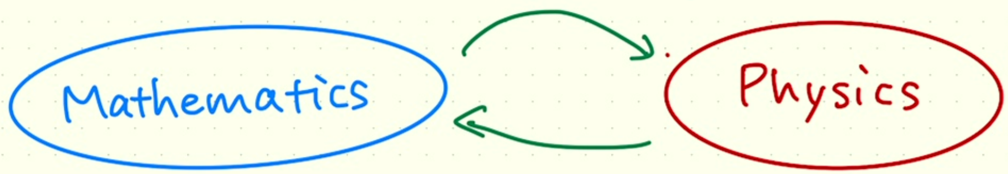
- I am Mayuko Yamashita, a mathematician in Kyoto U.
- I am working on Mathematical Physics.

Mathematical Physics



I am interested in both of

- Solving math problems motivated by physics,
- Solving physics problems using mathematics.

In particular, I use

Homotopy theory, Algebraic Topology in math.

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Conceptual Structure of my works

Start from proposals / conjectures connecting

Physics and Homotopy theory (Math):

(e.g. Segal-Stolz-Teichner proposal ('04, '11)
Kitaev ('05) / Freed-Hopkins ('16) proposal)

My works, conceptually:

(A) Use Proposal to translate
Phys. problems → Math problems
and solve it mathematically.

(B) Use Proposal to get
Math ideas from physics.

(C) Try to attack Proposal.

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Plan of the talk

(Phys ⟷ Math)

§1. Homotopy theory and Physics

(general introduction)

§2. My works on Segal-Stolz-Teichner program.

(A) ① Show Absence of anomaly in heterotic string theory
[TY21] via Homotopy theory.

(A)(B)(C) ② New interpretation of a duality in Homotopy theory
[TY23] in terms of Secondary anomalies

(A)(B) ③ Explained 576-periodicity of SUSY

+ Future directions.

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Homotopy theory. (Algebraic Topology)

Study **geometric objects**

(topological spaces, manifolds ⋯)

by extracting **algebraic** information / invariants.

X : top. sp

M : manifolds

$\Rightarrow$

- $\chi(X) \in \mathbb{Z}$ Euler number
- $H^n(X;\mathbb{Z})$ Ordinary cohomology
- $K^n(X)$ K-theory
- $H^n_{dR}(M;\mathbb{Z})$ de Rham cohomology

$\vdots$

useful for **classification problems in geometry**.

Homotopy theory and physics

There has been growing interest in the relation

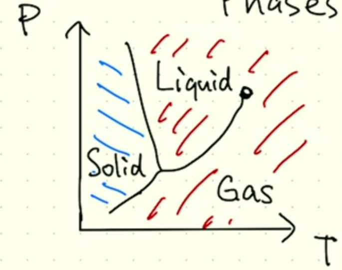
Homotopy theory $\leftrightarrow$ physics.

In particular, algebraic topology is useful for

classification problems in physics.

classifications of physical systems are important :

Phases of water



Quantum Field Theories (QFT)

- describes physical systems.
- $\exists$ various formulations.

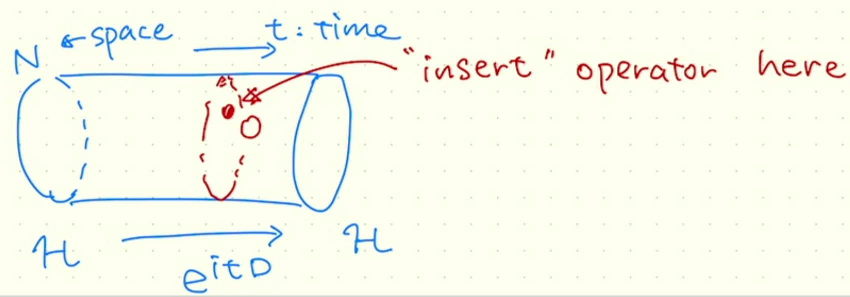
Recall : **Quantum Mechanics**

A physical system : $(\mathcal{H}, D)$

$\mathcal{H}$: Hilbert space $\quad \mathcal{H} \ni |\psi\rangle$: "physical state"

$D : \mathcal{H} \to \mathcal{H}$ Hamiltonian " time evolution"

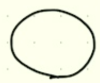
Observables : $\mathcal{O} : \mathcal{H} \to \mathcal{H}$. operators



Cobordism Picture for QFTs :

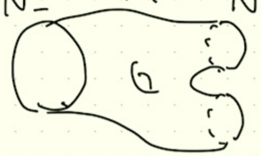
A $(d-1,d)$-dim QFT T is an assigment

- N^{d-1} closed $(d-1)$-dim mfd $\overset{T}{\longmapsto} T(N^{d-1}) \in \mathrm{Vect}_{\mathbb{C}}$



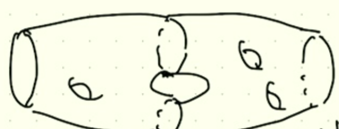
$\mathbb{C}$-vector space

- N_-^{d-1} M^d N_+^{d-1} d-dim compact mfd with boundary $\overset{T}{\longmapsto} T(M^d) : T(N_-) \to T(N_+)$



linear map

which is functorial :



N_1 M N_2 M' N_3

$$\begin{tikzcd} T(N_1) \arrow[rr, "T(M \cup M')"] \arrow[dr, "T(M)"'] & \circlearrowright & T(N_3) \\ & T(N_2) \arrow[ur, "T(M')"'] & \end{tikzcd}$$

Example : Holonomy theory

Fix $(L,\nabla) \leftarrow$ Line bundle + connection

$\quad\ \downarrow \mathbb{C}$

$\quad\ X \leftarrow$ a manifold

$\to$ (0,1)-dim QFT $\mathrm{Hol}_{(X,\nabla)}$

with structure = orientation + map to X :

- $\left(\overset{+/-}{\underset{N}{\bullet}} \xrightarrow{f} X \right) \mapsto L_{f(N)}$
- $\left(\underset{-\qquad +}{\overset{M}{\bullet \to \bullet}} \xrightarrow{f} X \right) \mapsto \mathrm{Hol}_\nabla : L_{f(\partial_- M)} \to L_{f(\partial_+ M)}$

$\rightsquigarrow Z_{\mathrm{Hol}}(f \ X) = \mathrm{Hol}_f(\nabla) \in U(1)$ Holonomy along a loop

Classification of QFTs

$\Leftarrow$ up to "**deformation equivalence**"
(変形同値)

QFT's I_0 & I_1 are **deformation equivalent** $I_0 \sim I_1$

"$\underset{\text{def}}{\Leftrightarrow}$" $\exists \{I_t\}_{t\in[0,1]}$ conti. path of QFT's from I_0 to I_1

Ex Holonomy theory with target X.

- two connections ∇_0, ∇_1 on $L \to X$ can always deformed to each other: $\nabla_t = t\nabla_0 + (1-t)\nabla_1$

  $\rightsquigarrow \mathrm{Hol}(L,\nabla_0) \sim \mathrm{Hol}(L,\nabla_1)$

- two line bundles L, L' on X cannot necessarily deformed to each other...

  $\rightsquigarrow \mathrm{Hol}(L,\nabla) \not\sim \mathrm{Hol}(L',\nabla')$ in general.

Algebraic topology can be useful!

Ex. • $H^2(X;\mathbb{Z}) = \{ L \to X \text{ line bundles over } X \} / \underset{\text{isom}}{\cong}$

$\therefore [L] = [L'] \in H^2(X;\mathbb{Z}) \Rightarrow \mathrm{Hol}_{(L,\nabla)} \sim \mathrm{Hol}_{(L',\nabla')}$

Moreover, the partition function $Z_{\mathrm{Hol}(L,\nabla)}$ recovers the isom class of (L,∇), since

$\mathrm{Hol}_f(\nabla) = \mathrm{Hol}_f(\nabla') \ \forall f: S^1 \to X \iff (L,\nabla) \cong (L',\nabla')$

$\left(Z_{\mathrm{Hol}(X,\nabla)}(f \quad X) = \underset{\in U(1)}{\mathrm{Hol}_f(\nabla)} \quad \text{Holonomy along a loop} \right)$

$\therefore \underbrace{[\mathrm{Hol}_{(X,\nabla)}]}_{\text{a deformation class}} \in H^2(X;\mathbb{Z})$.

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A naive expectation:

- $H^2(X;\mathbb{Z})$ classifies (a subclass of) 1-dim QFTs for oriented manifolds with map to X ?
- $H^d(X;\mathbb{Z})$ classifies higher-dim QFTs ?

→ False.

We need to work more !!

- we should fix nice "subclass" to be classified.
- we should use generalized cohomology theory.

(These data gives a functor

$$T : \mathrm{Bord}_{\langle d-1, d\rangle} \to \mathrm{Vect}_{\mathbb{C}} .$$
)

In particular, for **closed d-dim manifolds** ...

M^d

ϕ  ϕ

closed d-dim mfd

$$\xmapsto{T}$$

$$T(\phi) \longrightarrow T(\phi)$$

$T(\phi) = \mathbb{C}$, $T(\phi) = \mathbb{C}$

$T(M^d)$

$=$

multiplication by

$Z_T(M^d) \in \mathbb{C}$

partition function for M

Z_T can reflect

geometry & topology of manifolds.

※ We can also put **Structures** on manifolds.

(orientation, metric, map to X ...)

14:26 3月12日(火) 2024-03 PI colloquium

§2 My works : I explain our works on

Segal-Stolz-Teichner proposal. ('04, '11)

... a deep "conjecture" connecting

Homotopy theory (Math) and Physics.

My works, conceptually :

(A) Use Proposal to translate

Phys. problems $\rightarrow$ Math problems

and solve it mathematically.

(B) Use Proposal to get

Math ideas from physics.

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14:27 3月12日(火)

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① Main Result of [TY '21] (Physical statement)

Heterotic String theories are anomaly-free.

Strategy

We use Proposals relating Homotopy theory & Physics :

Physics $\longleftrightarrow$ Homotopy theory

Stolz - Teichner proposal

Freed - Hopkins proposal

to translate Phys. Q to Math. Q.

and solve it mathematically.

We use the following diagram:

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\text{Phys} & \left\{\begin{matrix}\text{2d SCFTs}\\ \mathcal{N}=(0,1)\end{matrix}\right\} \arrow[r, "\text{heterotic}", "\text{compactification}"'] \arrow[d, "\begin{matrix}\text{Stolz-Teichner}\\ \text{conjecture}\\ \text{['04, '11]}\end{matrix}"] & \left\{\begin{matrix}\text{QFTs on}\\ \text{String manifolds}\end{matrix}\right\} \arrow[d, "\begin{matrix}\text{extract}\\ \text{anomalies}\end{matrix}", "\begin{matrix}\text{Freed-Hopkins}\\ \text{conjecture}\\ \text{['16]}\end{matrix}"'] \\
\text{Math} & TMF^{d+22} \arrow[r, "\alpha"] & I_{\mathbb{Z}} MString^{d+2}
\end{tikzcd}$$

Topological Modular Forms $\qquad$ Anderson dual to String bordism

$\rightsquigarrow$ Phys. Problem Show that heterotic anomaly vanishes.

$\Leftrightarrow$ Math Problem (roughly): Show $\alpha = 0$.
under some conditions on α implied by Physics.

Segal-Stolz-Teichner proposal ('04, '11)

$$\left\{\begin{array}{c}\text{2-dim } \mathcal{N}=(0,1) \text{ SUSY} \\ \text{unitary QFTs}\end{array}\right\} \simeq \mathrm{TMF}.$$

Topological Modular Forms.

a Spectrum (generalized cohomology) which is a "topological" version of Modular Forms.

※ SST proposal is a refinement of :

$$\left\{\begin{array}{c}\text{2d } \mathcal{N}=(0,1) \text{ unitary} \\ \text{SQFTs}\end{array}\right\} \longrightarrow MF[\Delta^{-1}] \simeq \mathbb{Z}[c_4, c_6, \Delta, \Delta^{-1}] \Big/ c_4^3 - c_6^2 = 1728\Delta$$

$$\mathcal{T} \longmapsto Z_{\mathcal{T}}(T^2, RR)$$

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Why is SST-proposal interesting / important?

- Mathematically,

TMF itself has been deep & important.
('90s ~ now)

"topological ver." of Modular Forms.

mixture of homotopy theory & algebraic geometry...

"Spectral Algebraic Geometry"

- Physically,

SST-proposal has a lot of implications for SQFTs.

such as torsion phenomena, duality, periodicity...
(the topic of this talk.)

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  ('90s ~ now)

  "topological ver." of Modular Forms.
  mixture of homotopy theory & algebraic geometry...
  "Spectral Algebraic Geometry"

- Physically,

  SST-proposal has a lot of implications for SQFTs.

  Such as torsion phenomena, duality, periodicity...
  (the topic of this talk.)

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Math $\underline{TMF}$ is a spectrum which is a "topological" version of $MF[\Delta^{-1}]$ ...

- Defined as a global section of an E_∞-sheaf on $M_{ell}/\mathbb{Z}$

- $\pi_* TMF \xrightarrow{\exists} (MF[\Delta^{-1}])_{*/2} = \mathbb{Z}[c_4, c_6, \Delta, \Delta^{-1}]/(c_4^3 - c_6^2 - 1728\Delta)$

  inducing $\pi_* TMF \otimes \mathbb{Q} \cong MF[\Delta^{-1}]_{*/2} \otimes \mathbb{Q}$.

  but $\exists$ nontrivial cokernels.

  e.g. $\pi_{\pm 24} TMF \to MF[\Delta^{-1}]_{\pm 12}$

  $\nexists \mapsto \Delta, \Delta^{-1}$ (with $\Delta \in MF[\Delta^{-1}]_{\pm 12}$)

  $\exists$ lift $\longmapsto 24\Delta, 24\Delta^{-1}$

- $\exists$ many 2,3-power torsions in $\pi_* TMF$

- 576-periodic $\pi_* TMF \cong \pi_{*+576} TMF$

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	0	1	2	3	4	5	6	7	8	9	10	11	12	1
$f)_{(2)}$	$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$						$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$	
				$\mathbb{Z}_8$			$\mathbb{Z}_2$		$\mathbb{Z}_2$	$\mathbb{Z}_2$				

	16	17	18	19	20	21	22	23	24	25	26	27	28	2
$f)_{(2)}$	$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$				$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$	
		$\mathbb{Z}_2$			$\mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_2$	

	32	33	34	35	36	37	38	39	40	41	42	43	44	4
$f)_{(2)}$	$\mathbb{Z}^2_{(2)}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$		$\mathbb{Z}^2_{(2)}$				$\mathbb{Z}^2_{(2)}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$		$\mathbb{Z}^2_{(2)}$	
	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$				$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_2$	$\mathbb{Z}_2$			$\mathbb{Z}$

	48	49	50	51	52	53	54	55	56	57	58	59	60	6
$f)_{(2)}$	$\mathbb{Z}^2_{(2)}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$		$\mathbb{Z}^2_{(2)}$				$\mathbb{Z}^3_{(2)}$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$		$\mathbb{Z}^3_{(2)}$	
	$\mathbb{Z}_{(2)}$		$\mathbb{Z}_2$	$\mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_4$			$\mathbb{Z}_2$		$\mathbb{Z}_2$	$\mathbb{Z}_4$	

	64	65	66	67	68	69	70	71	72	73	74	75	76	7
$f)^3_{(2)}$	$\mathbb{Z}^3_{(2)}$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$		$\mathbb{Z}^3_{(2)}$				$\mathbb{Z}^3_{(2)}$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^3$		$\mathbb{Z}^3_{(2)}$	
21 / 31		$\mathbb{Z}_2^2$	$\mathbb{Z}_2$		$\mathbb{Z}_2$		$\mathbb{Z}_2$		$\mathbb{Z}_{(2)}$			$\mathbb{Z}_2$		

Our Result [TY21]

- Using the diagram, we translated

Phys. Problem Show that heterotic anomaly vanishes

into Math Problem : Show that a morphism

$$\alpha: TMF \to \Sigma^{-20} I_{\mathbb{Z}} MString$$

vanishes, under some conditions

translated from physics

- Then Solved it mathematically.

Proof turns out to be very simple.

(ingredients : • $MF[\Delta^{-1}]_2 \ni {}^{\forall}\phi(q)$, q^0-coeff of $\phi(q)$ is 0.
• $TMF_{-21} = 0$.)

14:44 3月12日(火) 2024-03 PI colloquium 97%

② Secondary anomaly

& Anderson self-duality of TMF [TY'23]

[TY'23] + ε implies ∃! canonical lift

$$\begin{tikzcd}
 & \mathrm{TMF} \arrow[dl, dashed, "\exists!\,\tilde{\alpha}"'] \arrow[d, "\alpha_{\mathrm{spin}}"] \arrow[dr, "0 \text{ by [TY21]}"] & \\
\Sigma^{-20} I_{\mathbb{Z}} \mathrm{MSpin}/\mathrm{MString} \arrow[r] & \Sigma^{-20} I_{\mathbb{Z}} \mathrm{MSpin} \arrow[r] & \Sigma^{-20} I_{\mathbb{Z}} \mathrm{MString}
\end{tikzcd}$$

which we call secondary morphism.

Main Result in [TY'23]

$\tilde{\alpha}$ is directly related to Anderson self-duality of Tmf:

Fact (Stojanoska '13) $\mathrm{Tmf} \simeq \Sigma^{-21} I_{\mathbb{Z}} \mathrm{Tmf}$.

Main Result in [TY'23]

$\tilde{\alpha} : TMF \to \Sigma^{-20} I_{\mathbb{Z}} MSpin / MString$

is directly related to Anderson self-duality of Tmf.

→ : New physical understanding of
purely homotopy-theoretic result !

: $\tilde{\alpha}$ is very nontrivial !

$\tilde{\alpha}$ induces pairings

$$\underset{\text{SQFTs}}{\underset{\Updownarrow}{\pi_* TMF}} \times \underset{\text{manifolds}}{\underset{\Updownarrow}{\Omega_\bullet^{Spin/string}}} \to \begin{cases} \mathbb{Z} & * + \bullet = -20 \\ \mathbb{Q}/\mathbb{Z} & * + \bullet = -21 \end{cases}$$

Spin & String

Which is very nontrivial & computable by differential geometry !

14:48 3月12日(火)

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Applications

Math detect power operations in TMF via differential-geometric methods.

Phys extract "chromatic height-2" information from 2d SQFTs, leading to new conjectures...

Lattice	$n=2c$	in $\mathrm{KO}((q))^n(\mathrm{pt})$	we conjecture...
E_8	$n=16$	c_4/Δ	
$\widetilde{D_{12}}$	$n=24$	$24/\Delta$	
$\widetilde{E_7\times E_7}$	$n=28$	0	nontrivial in $A^{28}\simeq\mathbb{Z}/2$ $\subset \mathrm{TMF}^{28}$
$\widetilde{A_{15}}$	$n=30$	0	nontrivial in $A^{30}\simeq\mathbb{Z}/2$ $\subset \mathrm{TMF}^{30}$
$\widetilde{D_8\times D_8}$	$n=32$	0	nontrivial in $A^{32}\simeq\mathbb{Z}/3$ $\subset \mathrm{TMF}^{31}$
$\widetilde{D_{16}}$	$n=32$	$(c_4/\Delta)^2$	

TABLE 1. Examples of lattice SVOAs and conjectured image in TMF.

"difficult" torsions in $\pi_*\mathrm{TMF}$

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③ Proof of 576-periodicity in SQFTs.

WITHOUT using TMF.

(j. w/ Theo Johnson-Freyd, in preparation.)

Math Fact : • TMF is exactly 576-periodic.

($TMF^n \cong TMF^{n+576}$)

• periodicity element $\longmapsto \Delta^{24}$.

$$\begin{array}{ccc} \text{periodicity element} & \longmapsto & \Delta^{24} \\ \cap & & \cap \\ \pi_{576} TMF & \to & MF[\Delta^{-1}]_{24\times 12} \end{array}$$

⇝ If you believe in SST proposal,

SQFT should be exactly 576-periodic.

However, there had been NO physical explanation

SQFT should be exactly 576-periodic.

To prove, we should do two things:

- give upper bound 576.

  $\Leftrightarrow$ Verify $\pi_{-576}\mathrm{SQFT} \to \mathrm{MF}[\Delta^{-1}]_{24\times 12}$

  $\exists\, T \longmapsto \Delta^{24}$

  "Existence" result. Done by Gaiotto et al.

  essentally

- ☆ give lower bound 576.

  "Non-Existence" result. Difficult!

  e.g. need to show $\pi_{-576/2}\mathrm{SQFT} \to \mathrm{MF}[\Delta^{-1}]_{12\times 12}$

  $\nexists \longmapsto \Delta^{12}$

We settle this! $(\exists\, T_{288} \longmapsto 2\Delta^{12})$

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Formulation of the problem.

Recall: Segal-Stolz-Teichner proposal ('04, '11)

(1) The "Space"

$\{$2-dim, $\mathcal{N}=(0,1)$ SUSY unitary QFTs$\}$

forms a spectrum "SQFT".

(2) We have SQFT $\cong$ TMF.

In this work, we only use (1) of the Proposal

We do NOT use TMF. To do:

- Start from a spectrum SQFT
- Put assumptions for SQFT implied by physics.
- Show periodicity of SQFT ≥ 576.

(Mathematical) assumptions for SQFT (roughly)

- $$\begin{tikzcd} \mathrm{MString} \arrow[r, "\exists"] \arrow[r, swap, "\sigma\text{-model}"] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{SQFT} \arrow[r, "\exists"] \arrow[r, swap, "ev_{T^2}"] \arrow[d, "ev_{S^1}"] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{MF}[\Delta^{-1}] \arrow[d] \\ \mathrm{MSpin} \arrow[r, swap, "\mathrm{Wit}"] & KO((q)) \arrow[r] & \mathbb{Z}((q)) \end{tikzcd}$$

- $\pi_{-21}\,\mathrm{SQFT} = 0$ $\rightsquigarrow$ Heterotic anomaly vanishing.

<u>Thm</u> (JF - Y, in preparation)

- $k\Delta^{-12} \in \mathrm{Im}(\mathrm{SQFT} \to \mathrm{MF}[\Delta^{-1}]) \Rightarrow 2 \mid k.$
- $k\Delta^{-16} \in \mathrm{Im}(\mathrm{SQFT} \to \mathrm{MF}[\Delta^{-1}]) \Rightarrow 3 \mid k.$

<u>Cor</u> periodicity of SQFT ≥ 576.

Strategy : use **Anderson duality pairings.**

Assumptions allows us to do **analogy of** §2. [TY23]

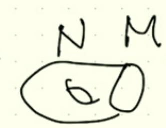
we get $\mathbb{Z}$-valued pairing

integrality is the Key!

$$\langle\, ,\, \rangle : \pi_{-d}\, SQFT \otimes \Omega^{Spin/String}_{d-20} \to \mathbb{Z}.$$

with explicit formula.

e.g. we can find



$$[\ \cdot\ ] \in \Omega^{Spin/String}_{24\times 12-20}$$

such that

$$\langle k\Delta^{-12}, [N,M] \rangle = \frac{3k}{2}$$

$\Rightarrow$ $2 | k$, as desired.

Future directions

- Attack Segal-Stolz-Teichner proposal.

  $\exists$ various subproblems,

  e.g. developing equivariant refinements of SST.

  take Symmetries of SQFTs into account.

  need Modular-Tensor-Category equivariant TMF

  which is Not mathematically established...

  (partly in progress with Ying Hsuang-Lin)

- Establish relations among

  different pictures for QFTs.

  Lagrangians, AQFTs, lattice models, cobordisms...

- Find more physical applications of

  Homotopy theories and more!

15:02 3月12日(火) 2024-03 PI colloquium 100%

Applications

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TABLE 1. Examples of lattice SVOAs and conjectured image in TMF.

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Future directions

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  ∃ various subproblems,

  e.g. developing equivariant refinements of SST.

  take Symmetries of SQFTs into account

  need Modular-Tensor-Category equivariant TMF

  which is Not mathematically established ...

  (partly in progress with Ying Hsuang-Lin)

- Establish relations among

  different pictures for QFTs.

  Lagrangians, AQFTs, lattice models, cobordisms ...

- Find more physical applications of

  Homotopy theories and more!

Main Result in [TY'23]

$\tilde{\alpha} : TMF \to \Sigma^{-20} I_{\mathbb{Z}} MSpin/MString$

is directly related to Anderson self-duality of Tmf:

→ : New physical understanding of
purely homotopy-theoretic result !

: $\tilde{\alpha}$ is very nontrivial !

$\tilde{\alpha}$ induces pairings

$$\pi_* TMF \times \Omega_\bullet^{Spin/string} \to \begin{cases} \mathbb{Z} & * + \bullet = -20 \\ \mathbb{Q}/\mathbb{Z} & * + \bullet = -21 \end{cases}$$

$\pi_* TMF$ ↕ SQFTs

$\Omega_\bullet$ ↕ manifolds

Spin
String

Which is very nontrivial & computable by differential geometry !