Title: Free-falling in Quantum Spacetime

Speakers: Maulik Parikh

Series: Quantum Gravity

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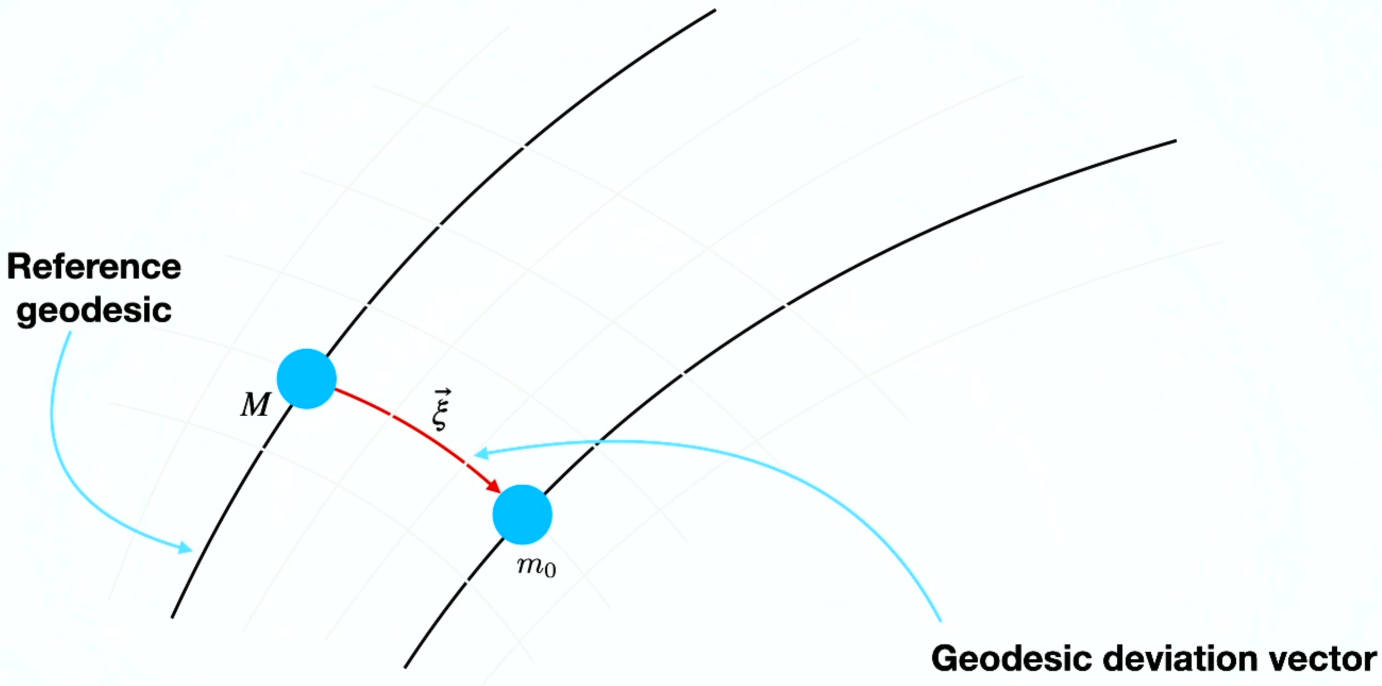
Abstract: When the spacetime metric is regarded as a quantum field, the classical trajectories of freely falling objects are subject to random fluctuations, or "noise". This fundamental noise might even be observable at gravitational wave detectors: if detected, it would provide experimental evidence for the quantization of gravity. The effect of the quantum-gravitational noise is to turn the classical geodesic deviation equation into a stochastic, Langevin-like equation. Moreover, when these results are extended to congruences of geodesics, the quantum fluctuations of spacetime give rise to an additional term in the Raychaudhuri equation with the same sign as the other terms.

Zoom link

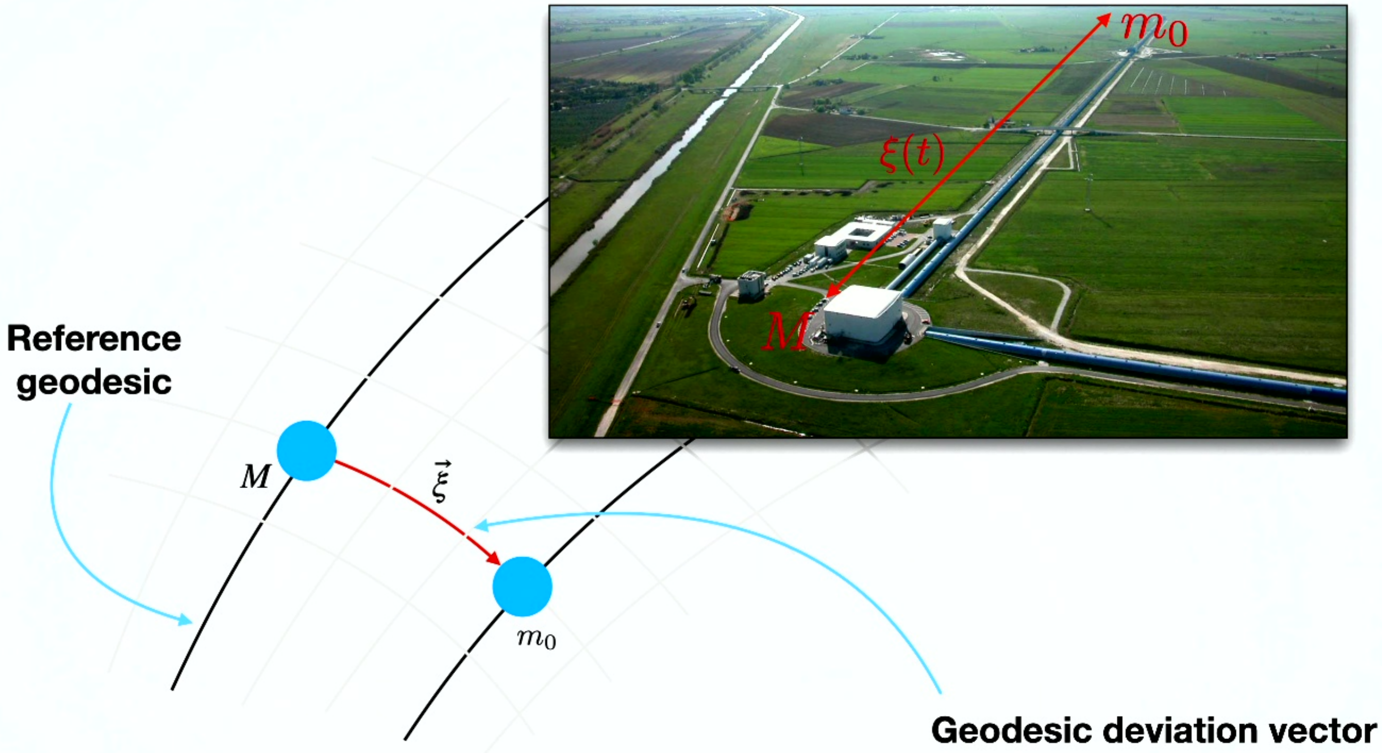
Free-Falling in Quantum Spacetime

Maulik Parikh — Arizona State University
Perimeter Institute

Geodesic Deviation



Geodesic Deviation



Geodesic Deviation Equation

geodesic deviation

$$\ddot{\xi}^{\mu} = -R^{\mu}_{0\nu 0}\xi^{\nu}$$

Riemann tensor

$$R_{i0j0}(t,0) = -\frac{1}{2}\ddot{h}_{ij}(t,0)$$

gravitational wave

$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$

This is the geodesic deviation equation in the presence of a **classical** gravitational wave

Quantum Geodesic Deviation Equation?

$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$

What is the generalization of this equation when the spacetime metric is treated as a **quantum** field?

"The Noise of Gravitons," arXiv:2005.07211

The Noise of Gravitons

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Abstract

We show that when the gravitational field is treated quantum-mechanically, it induces fluctuations – noise – in the lengths of the arms of gravitational wave detectors. The characteristics of the noise depend on the quantum state of the gravitational field, and can be calculated exactly in several interesting cases. For coherent states the noise is very small, but it can be greatly enhanced in thermal and (especially) squeezed states. Detection of this fundamental noise would constitute direct evidence for the quantization of gravity and the existence of gravitons.

This essay was awarded First Prize in the 2020 Essay Competition of the Gravity Research Foundation.

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"Quantum Mechanics of Gravitational Waves," PRL, arXiv:2010.08205

"Signatures of the Quantization of Gravity at Gravitational Wave Detectors," PRD, arXiv:2010.08208

Frank Wilczek

George Zahariade

Action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - M\int d\lambda\sqrt{-g_{\mu\nu}\frac{dX^\mu}{d\lambda}\frac{dX^\nu}{d\lambda}} - m_0\int d\tau\sqrt{-g_{\mu\nu}\frac{dY^\mu}{d\tau}\frac{dY^\nu}{d\tau}}$$

Einstein-Hilbert action + action for two free-falling particles

Use Fermi normal coordinates, putting mass M on classical trajectory

$$X^\mu = (t, \vec{0})$$

Let the other particle be at

$$Y^\mu = (t, \vec{\xi})$$

Action

Next, insert metric in Fermi normal coordinates into particle action:

$$g_{00}(t,\xi) = -1 - R_{i0j0}(t,0)\xi^i\xi^j + O(\xi^3)$$
$$g_{0i}(t,\xi) = -\frac{2}{3}R_{0jik}(t,0)\xi^j\xi^k + O(\xi^3)$$
$$g_{ij}(t,\xi) = \delta_{ij} - \frac{1}{3}R_{ikjl}(t,0)\xi^k\xi^l + O(\xi^3)\ .$$

Inserting this into the particle action gives

$$-m_0 \int d\tau \sqrt{-g_{\mu\nu}\frac{dY^\mu}{d\tau}\frac{dY^\nu}{d\tau}} \approx -m_0 \int dt \left(\frac{1}{2}R_{i0j0}(t,0)\xi^i\xi^j - \frac{1}{2}\delta_{ij}\dot{\xi}^i\dot{\xi}^j\right)$$

Action

Expanding action to lowest order in metric perturbation, we have:

$$S = -\frac{1}{64\pi G}\int d^4x\ \partial_\mu h_{ij}\partial^\mu h^{ij} + \int dt \frac{1}{2} m_0 \left(\delta_{ij}\dot{\xi}^i\dot{\xi}^j - \dot{h}_{ij}\dot{\xi}^i\xi^j\right)$$

Write metric perturbation in Fourier modes:

$$h_{ij}(t,\vec{x}) = \frac{1}{\sqrt{\hbar G}}\sum_{\vec{k},s} q_{\vec{k}}(t) e^{i\vec{k}\cdot\vec{x}}\epsilon^s_{ij}(\vec{k})$$

Then

$$S = \int dt \frac{1}{2} m_0 \dot{\xi}^2 + \int dt \sum_{\vec{k},s} \frac{1}{2} m \left(\dot{q}^2_{\vec{k},s} - \omega^2_{\vec{k}} q^2_{\vec{k}}\right) - g\int dt \sum_{\vec{k},s} \dot{q}_{\vec{k},s}\epsilon^s_{ij}(\vec{k})\dot{\xi}^i\xi^j$$

Geodesic Deviation in the Presence of a Graviton Mode

free particle

$$S_\omega = \int dt \left(\frac{1}{2} m(\dot{q}^2 - \omega^2 q^2) + \frac{1}{2} m_0 \dot{\xi}^2 - g \dot{q} \dot{\xi} \xi \right)$$

simple harmonic oscillator

cubic interaction term

where $g \equiv \frac{m_0}{2\sqrt{\hbar G}}$

Quantization Strategy

We will treat **both** the deviation/second particle/mirror and gravity **quantum mechanically.**

We will then **integrate out gravity,** giving the effective dynamics of the geodesic deviation in the presence of quantized gravity.

Quantum Mechanics

Suppose the gravitational field is initially in state $|\Psi\rangle$

We don't know what the final state of the field is.

Formally, we calculate the transition probability of the particle, or second mirror, to go from state A to state B in time T in the presence of a gravitational field that is initially in state $|\Psi\rangle$

$$P_\Psi(A \to B) = \sum_{|f\rangle} |\langle f, B|\hat{U}(T)|\Psi, A\rangle|^2$$

The relatively simple form of the action allows the calculation to be performed **exactly**

Transition Probability

wave-function dependent term

noise

auto-correlation function

$$P_\Psi = \mathcal{I}_{A,B} \int \tilde{\mathcal{D}}\xi \tilde{\mathcal{D}}\xi' \mathcal{D}\mathcal{N}_\Psi \exp\left[-\frac{1}{2}\int_0^T \int_0^T dt\, dt'\, A_\Psi^{-1}(t-t')\mathcal{N}_\Psi(t)\mathcal{N}_\Psi(t')\right] \times$$

$$\exp\left[\frac{i}{\hbar}\int_0^T dt \left\{\frac{1}{2} m_0 \left(\dot\xi^2 - \dot\xi'^2\right) + \frac{1}{4} m_0 \left(h(t) + \mathcal{N}_\Psi(t)\right)\left(X(t) - X'(t)\right)\right\} - \frac{i m_0^2 G}{8\hbar}\int_0^T dt\, \left(X(t) - X'(t)\right)\left(\dot X(t) + \dot X'(t)\right)\right]$$

fluctuation term

classical wave

dissipation term

where $X(t) = \frac{d^2}{dt^2}(\xi^2)$

"Signatures of the Quantization of Gravity at Gravitational Wave Detectors," arXiv:2010.08208

Langevin Equation

Taking the classical limit for the geodesic deviation, we find

$$\ddot{\xi} = \frac{1}{2}\left(\ddot{h} + \ddot{N}_\Psi - \frac{m_0 G}{c^5}\frac{d^5}{dt^5}\xi^2\right)\xi$$

classical gravitational wave

quantum noise

radiation reaction

This is the geodesic deviation equation in **quantum** spacetime

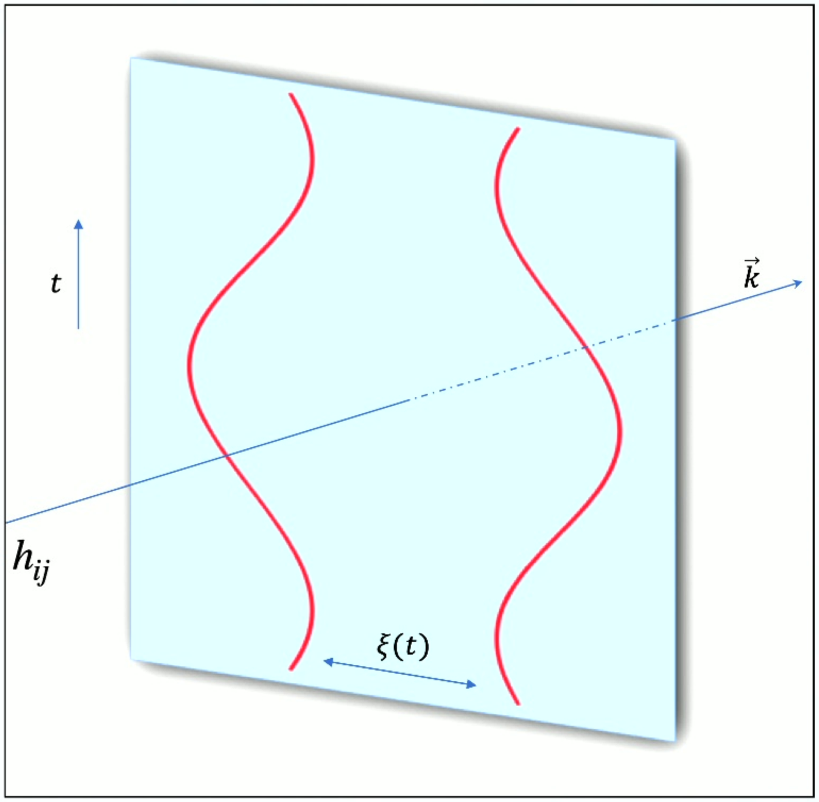
Compare classical geodesic deviation: $\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$

Because of the noise term, the new equation is a **stochastic** equation

Classical Geodesic Separation by Gravitational Waves

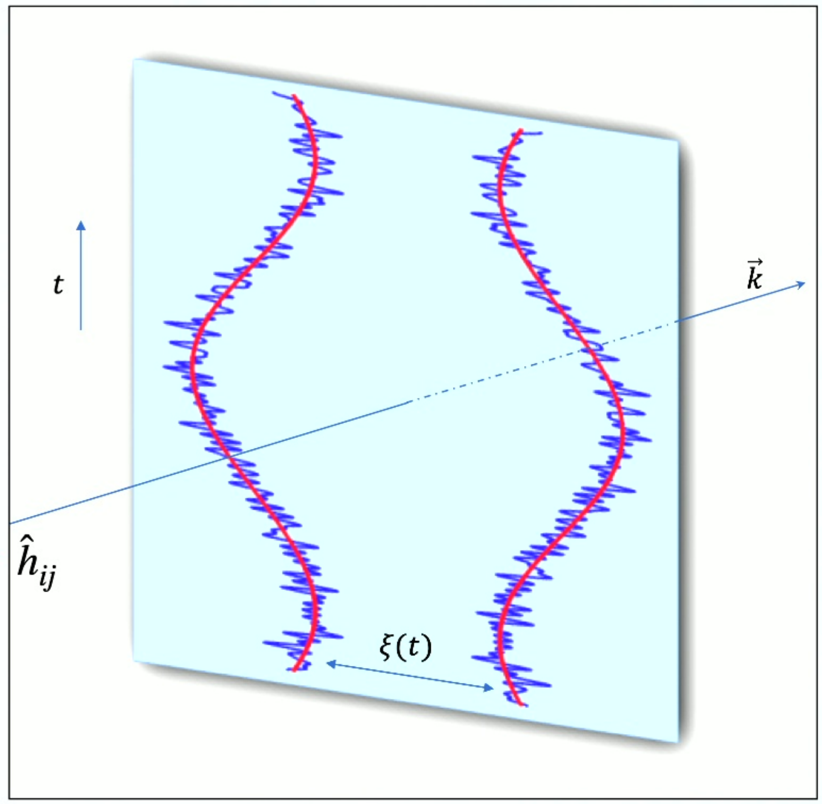
$$\ddot{\xi} = \frac{1}{2}\ddot{h}\xi$$

classical gravitational wave



The Noise of Gravitons

$$\ddot{\xi} \approx \frac{1}{2}\left(\ddot{h} + \ddot{N}_{\Psi}\right)\xi$$



quantized gravitational wave

Main Message

The signal of **quantum gravity** is in the **noise.**

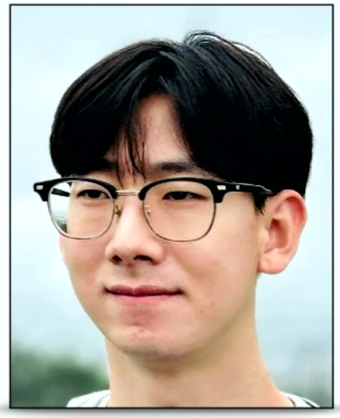
Is The Noise Detectable?

For the noise to be detectable:

1. Its **spectrum** should be distinguishable from other sources of noise

2. Its **amplitude** should not be too small

Collaborators



Sang-Eon Bak

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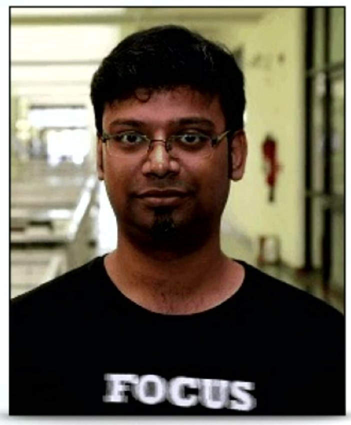


Sudipta Sarkar

Francesco Setti

Raphaela Wutte

Noise Correlations

We can consider **two** separated detector arms

Since our noise originates directly from the source, both arms will detect the **same** noise

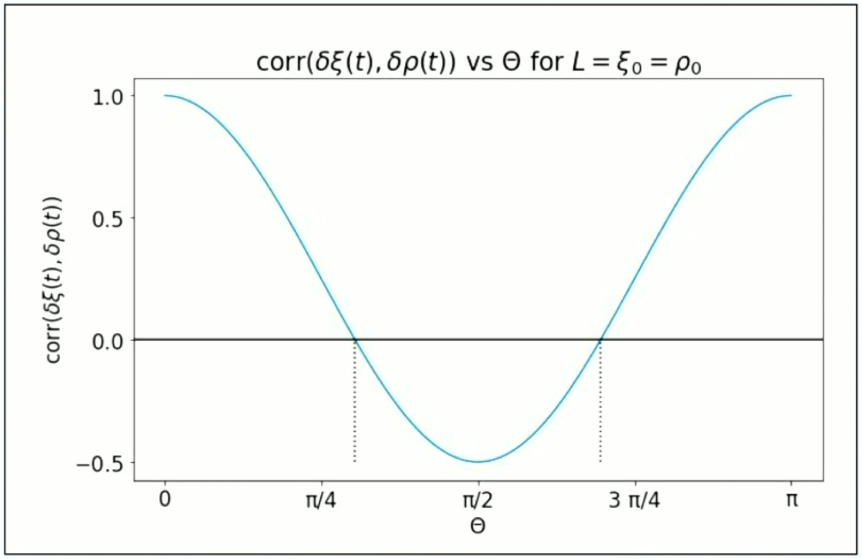
Quantum-gravitational noise is **correlated** across detectors

"Graviton Noise Correlation in Nearby Detectors," MP, F. Setti, arXiv:2312.17335

Angular Dependence of Noise Correlations

$$\mathrm{corr}(\delta\xi, \delta\rho) = \frac{\mathrm{cov}(\delta\xi, \delta\rho)}{\sqrt{\mathrm{Var}(\delta\xi)}\sqrt{\mathrm{Var}(\delta\rho)}} \approx \frac{\xi_0^2 \rho_0^2}{L^4} \left[\frac{1}{2}(3\cos^2\theta - 1)\right]$$

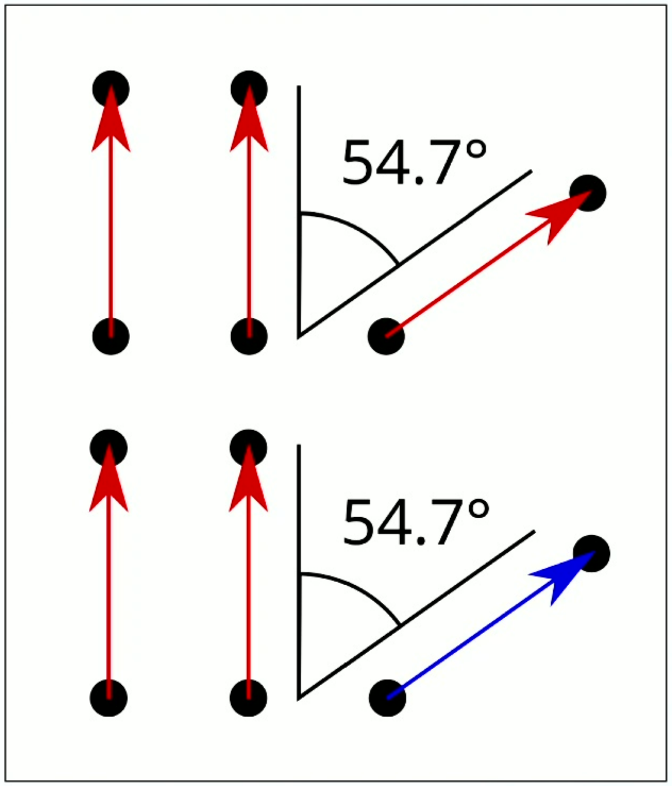
The correlation for quantum-gravitational noise is maximal when the detectors are aligned and zero when they are at an angle of 54.7 degrees



An Experiment to Isolate Quantum-Gravitational Noise

If the noise in all 3 detectors is correlated, it is **not** quantum-gravitational noise

If the noise in the parallel detectors is correlated, and the noise in the third detector is not correlated, it **is** likely quantum-gravitational noise



Noise for Quantum States of Gravitational Field

The magnitude of the noise depends on the quantum state as well as on the detector

$$\sigma^2 \sim \xi_0^2 \int_0^{\omega_{\max}} d\omega S(\omega)$$

variance of fluctuation

arm length

detector sensitivity

noise power spectrum

For the vacuum state and for coherent states (classical gravitational waves from weak sources)

$$\left(\frac{\Delta L}{L}\right)_{\text{vac}} \sim l_P \omega_{\max}/c = \frac{\omega_{\max}}{10^{44}\text{Hz}}$$

For squeezed states (cosmology, non-linear gravitational waves)

$$\left(\frac{\Delta L}{L}\right)_{\text{sq}} \sim e^{r/2} l_P \omega_{\max}/c = e^{r/2} \frac{\omega_{\max}}{10^{44}\text{Hz}}$$

exponential enhancement in squeezing parameter

Summary

There could potentially be **observable quantum-gravitational noise** provided the quantum state of the gravitational field is far from a coherent state e.g. a squeezed state

Lessons from EM: Why Quantum Gravity could be Observable

Many **observed phenomena** show that the **electromagnetic field** is **quantized:**

Photon anti-bunching, entangled photons, sub-Poissonian statistics,
Compton effect, Lamb shift, …

Most of these are ***tree-level*** effects in a **state** that has no classical counterpart

The same is true for gravity: there can be potentially observable effects
if the quantum state of the gravitational field is **not** a coherent state

Squeezed States from Nonlinear Couplings

If we have a field coupled to a classical source $H_{\text{int}} = J\varphi$

where $\varphi \sim a + a^\dagger$

then turning on the coupling naturally produces a **coherent state**:

$$|0\rangle \to e^{-i\int H_{\text{int}}dt}|0\rangle \sim e^{-i\int Ja^\dagger dt}|0\rangle$$

However if the coupling is **non-linear** $H_{\text{int}} = J\varphi^2$

we produce roughly a **squeezed state**: $|0\rangle \to e^{-i\int Ja^\dagger a^\dagger dt}|0\rangle$

Gravity Naturally Produces Squeezed States

So we need nonlinear couplings to produce squeezed states $H_{\text{int}} = J\varphi^2$

But gravity naturally has such nonlinear couplings!

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$S_{\text{E-H}} = \int d^4x\sqrt{-g}R = \int d^4x\sqrt{-\bar{g}}\left(\bar{R} + \frac{1}{2}(\bar{\nabla}h)^2 + \bar{R}_{abcd}h^{ac}h^{bd} + \dots\right)$$

This should produce squeezed states during the **merger phase** of black hole collisions

A. Das, MP, F. Wilczek, R. Wutte, in progress.

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Extensions

Quantum gravity corrections to **Newtonian free-fall**

Quantum Raychaudhuri equation

Newton's Law, Corrected

A similar calculation can be performed for a free-falling body in the presence of the quantum fluctuations about a weak static classical gravitational field

$$m\frac{d^2\vec{x}}{dt^2} = -m\vec{\nabla}\phi - mv^2\vec{\nabla}\phi + 4m\vec{v}(\vec{v}\cdot\vec{\nabla})\phi - 4m\phi\vec{\nabla}\phi + m\ddot{N}\vec{x}$$

$$F = m\ddot{z} = -mg + 2mg\dot{z}^2 - \frac{1}{2}mg^2z + m\ddot{N}z$$

"Quantum Gravity Corrections to the Fall of the Apple," S. Chawla, MP, arXiv:2212.14010

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Newton 1666

SR Einstein 1905

GR Einstein 1916

QG our result 2022

"Quantum Gravity Corrections to the Fall of the Apple," S. Chawla, MP, arXiv:2212.14010

Quantum Raychaudhuri Equation

Before we can work with null congruences, we will have to re-derive the quantum geodesic deviation for **null** geodesics

"Quantum Gravity Fluctuations in the Timelike Raychaudhuri Equation,"
S.-E. Bak, MP, S. Sarkar, F. Setti, arXiv:2212.14010

"Quantum-Gravitational Null Raychaudhuri Equation,"
S.-E. Bak, MP, S. Sarkar, F. Setti, arXiv:2312.17214

Null Fermi Normal Coordinates

$$g_{uv} = g_{vu} = 1 + \mathcal{O}(x^3)$$

$$g_{uu} = -R_{uaub}(u)x^a x^b + \mathcal{O}(x^3)$$

$$g_{ua} = -\frac{2}{3}R_{ubac}(u)x^b x^c + \mathcal{O}(x^3)$$

$$g_{ab} = \delta_{ab} - \frac{1}{3}R_{abcd}(u)x^c x^d + \mathcal{O}(x^3)$$

Inserting this into the action for two null rays we have

$$S = \int d\lambda \left[\frac{1}{2e}\left(\delta_{ab}\frac{d\xi^a}{d\lambda}\frac{d\xi^b}{d\lambda} - R_{uaub}\xi^a\xi^b\right)\right]$$

where now the separation vector lives in a codimension-2 hypersurface

Quantize

Now we quantize the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$$

And we promote the geodesic deviation equation to an operator equation

$$\ddot{\hat{\xi}}^a = -\hat{R}^a_{ubu}\hat{\xi}^b = \delta^{ab}\hat{N}_{bc}\hat{\xi}^c$$

where the noise tensor is

$$\hat{N}_{bc}(t) = -\frac{1}{2}\sum_s \sum_{\vec{k}} k^2 \epsilon^s_{bc} \hat{h}_s(\vec{k}, t)$$

Solving the Equations

We want to solve this equation:

$$\ddot{\hat{\xi}}^a = -\hat{R}^a_{ubu}\hat{\xi}^b = \delta^{ab}\hat{N}_{bc}\hat{\xi}^c$$

We can do it perturbatively, where each term has an extra power of hbar:

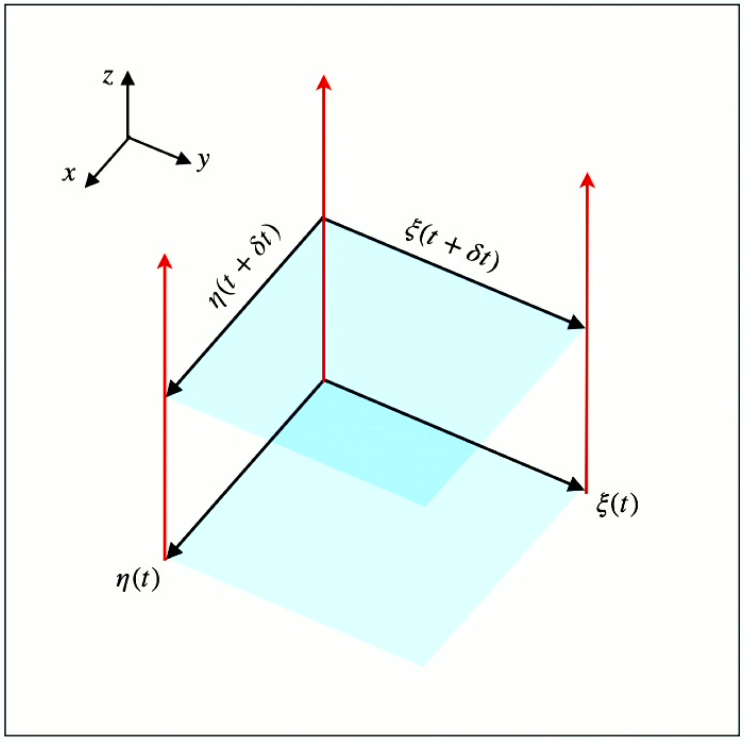
$$\hat{\xi}^a(t) = \xi_0(t) + \xi^a_1(t) + \xi^a_2(t) + ...$$

Congruences in Quantum Spacetime

Consider now a rectangular congruence of null geodesics

Taking the geodesic at one corner to be the reference geodesic, the area of the congruence is the **product** of the geodesic deviations to the adjacent corner geodesics:

$$A(t) = \xi(t)\eta(t)$$



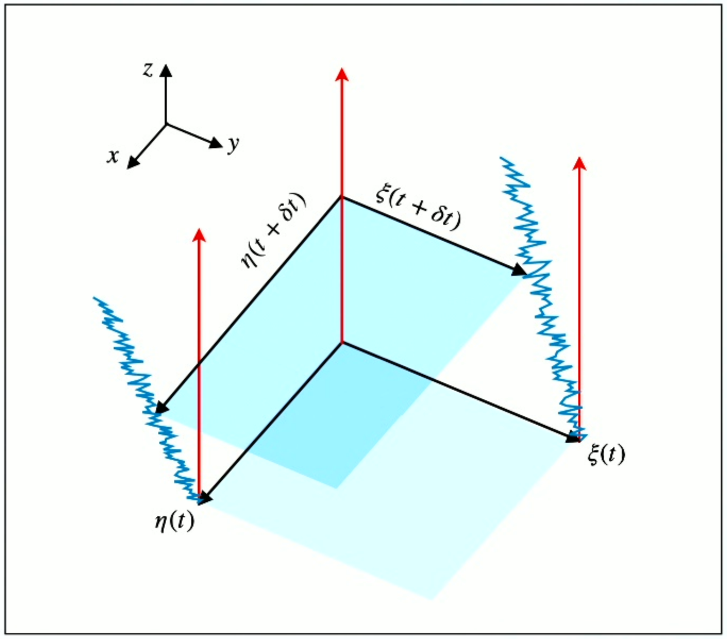
Quantum Area of a Null Congruence

The geodesic deviations are again stochastic but **correlated**:

$$\xi(t) = \xi_0(1 + N(t))$$

$$\eta(t) = \eta_0(1 - N(t))$$

Then the ***expectation*** of the area picks up the ***variance*** of the geodesic deviations:



$$\langle A(t)\rangle = \langle \xi(t)\eta(t)\rangle = \xi_0\eta_0(1 - \langle N^2\rangle) \equiv A_c + A_q$$

Quantum Area

From the two-point function

$$\langle 0|\hat{\xi}_1(t)\hat{\eta}_1(t)|0\rangle \approx -\frac{\kappa^2\xi_0\eta_0}{120\pi^2}\Lambda^4 t^2$$

we find (for the usual Minkowski vacuum state) that the quantum contribution to the area is

$$A_q(t) \approx -\frac{\kappa^2}{20\pi^2}\xi_0\eta_0\Lambda^4 t^2$$

S.-E. Bak, MP, S. Sarkar, F. Setti, arXiv:2312.17214

Quantum Raychaudhuri Equation

Define the congruence expansion

$$\theta \equiv \frac{\frac{d}{dt}\langle A(t)\rangle}{\langle A\rangle} \approx \frac{\dot{A}_c}{A_c} + \frac{d}{dt}\left(\frac{A_q}{A_c}\right) \equiv \theta_c + \theta_q$$

The **quantum** expansion depends on the state and is generically **negative**

The **classical** expansion obeys the classical Raychaudhuri equation

We therefore find a **quantum Raychaudhuri equation**

$$\dot{\theta} = -\frac{1}{2}\theta_c^2 - 2\sigma_c^2 + 2\omega_c^2 - R_{ab}u^a u^b - \frac{8G\hbar}{5\pi c^4}\Lambda^4$$

Comments

$$\dot{\theta} = -\frac{1}{2}\theta_c^2 - 2\sigma_c^2 + 2\omega_c^2 - R_{ab}u^a u^b - \frac{8G\hbar}{5\pi c^4}\Lambda^4$$

The sign of the quantum-gravitational term is the same as the sign of the classical terms. This suggests we still have a focusing theorem, which is a key ingredient in the prediction of singularities

If Lambda is related to the size of the congruence, it means that the fluctuations are greater for sub-congruences. The area is different depending on what scale it is measured on: fractal spacetime?

Summary

Quantum fluctuations of spacetime produce **noise** in the deviation of particles

$$\ddot{\xi} \approx \frac{1}{2}\left(\ddot{h} + \ddot{N}_\Psi\right)\xi$$

The characteristics of the noise depend on the **quantum state** of gravity

For a **squeezed** state, the enhanced noise might even be **observable**

$$S_{\text{squeezed}}(\omega) \approx \cosh(2r) 4G\hbar\omega$$

Detectors with very high frequency sensitivity might be able to detect **enhanced correlated noise with a linear power spectrum** during the **merger phase** of black holes

The noise adds a **quantum** term to the **Raychaudhuri equation**

$$\dot{\theta} = -\frac{1}{2}\theta_c^2 - 2\sigma_c^2 + 2\omega_c^2 - R_{ab}u^a u^b + \dot{\theta}_q$$

Transition Probability

wave-function dependent term

noise

auto-correlation function

$$P_\Psi = \mathcal{I}_{A,B} \int \tilde{\mathcal{D}}\xi \tilde{\mathcal{D}}\xi' \mathcal{D}\mathcal{N}_\Psi \exp\left[-\frac{1}{2}\int_0^T\int_0^T dt\, dt'\, A_\Psi^{-1}(t-t')\mathcal{N}_\Psi(t)\mathcal{N}_\Psi(t')\right] \times$$

$$\exp\left[\frac{i}{\hbar}\int_0^T dt \left\{\frac{1}{2}m_0\left(\dot{\xi}^2 - \dot{\xi}'^2\right) + \frac{1}{4}m_0\left(h(t) + \mathcal{N}_\Psi(t)\right)\left(X(t) - X'(t)\right)\right\} - \frac{i m_0^2 G}{8\hbar}\int_0^T dt\, \left(X(t) - X'(t)\right)\left(\dot{X}(t) + \dot{X}'(t)\right)\right]$$

fluctuation term

classical wave

dissipation term

where $X(t) = \frac{d^2}{dt^2}(\xi^2)$

"Signatures of the Quantization of Gravity at Gravitational Wave Detectors," arXiv:2010.08208