Title: Quantum Insights into Optical Astronomical Interferometry

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Series: Particle Physics

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Abstract: The implementation of optical astronomical interferometry presents several technical challenges, such as establishing a shared reference frame and conducting nonlocal measurements. This presentation aims to establish a connection between the theoretical framework developed in quantum information science and the schemes employed in astronomical interferometry. Our discussion will focus on the trade-offs between required resource and operational performance. We will categorize these methods into three types: nonlocal schemes, local schemes with a reference frame, and local schemes operating without a reference frame. By using this interdisciplinary connection, we also explore a generalized intensity interferometer. This approach achieves performance levels comparable to traditional intensity interferometers but introduces interesting fundamental distinctions.

Zoom link

Quantum Insights into Optical Astronomical Interferometry

Yunkai Wang

March 8, 2024

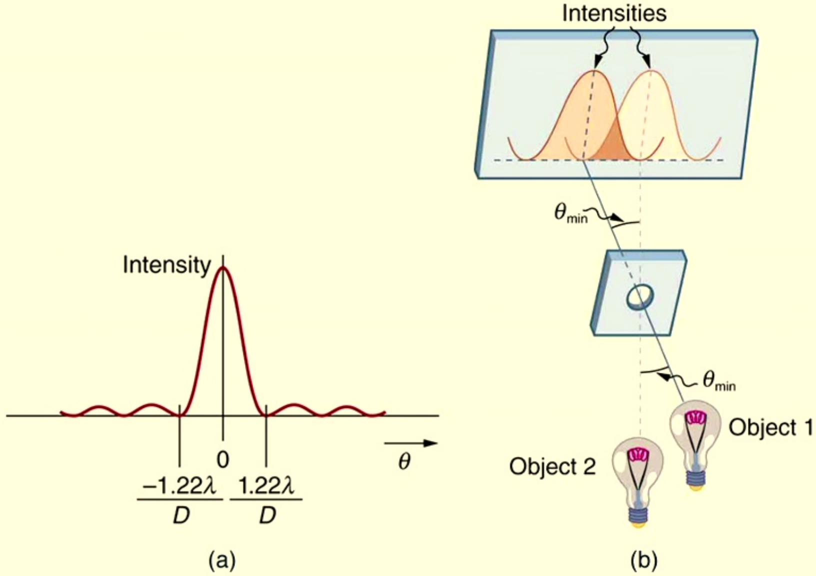
Yunkai Wang, Yujie Zhang, and Virginia O. Lorenz, manuscript under review

Outline

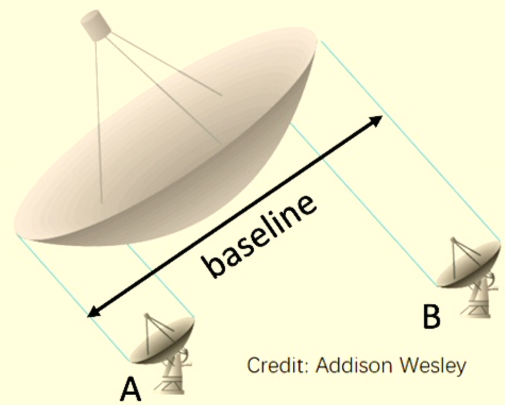
- Review of astronomical interferometer
- Two difficulties of optical astronomical interferometer and the classification
- A generalized intensity interferometer with two temporal modes

Resolution limit of imaging system

Rayleigh's limit:
The size of lens will limit the resolution of imaging



Why do we need interferometer for imaging?
Answer: Better resolution!

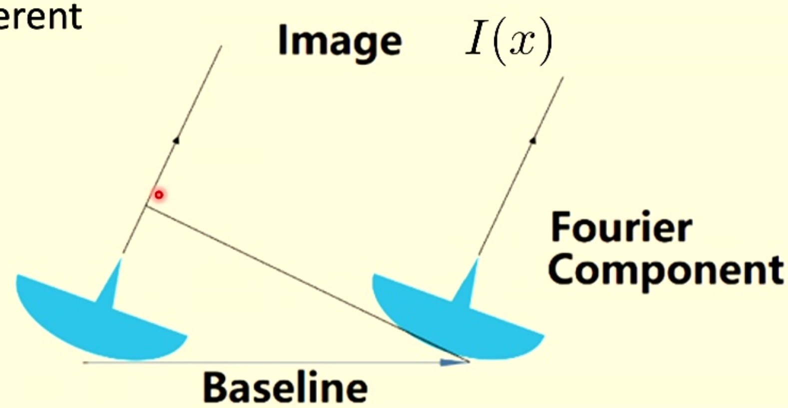


Another type of imaging method: Interferometric imaging

Physica 5, 785 (1938).

Van Cittert-Zernike theorem: coherence between signals from different telescopes is related to the Fourier components of the source.

$$g = \int dx I(x) e^{ikx}, \quad \int dx I(x) = 1$$



Reconstructed image is again the convolution between $I(x)$ and an effective PSF

$$I'(x) = (I * PSF_{\text{eff}})(x)$$

Effective PSF is determined by incomplete sampling of the Fourier components. Resolution is roughly determined by the longest baseline.

Resolution:

$$\theta \sim \lambda / D \Rightarrow \lambda / B$$

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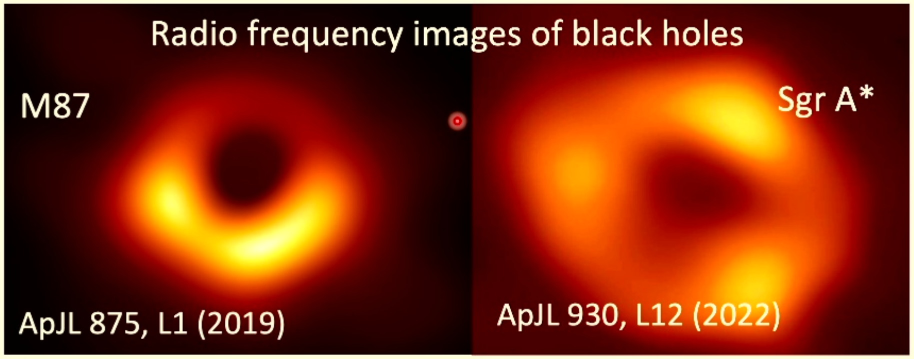
Radio interferometer array



Electromagnetic field is recorded locally.
Interference is achieved by data postprocessing on a computer.

$$R_1(t) = E\cos(\omega t) \qquad R_2(t) = E\cos(\omega(t-\tau))$$

$$|R_1(t) + R_2(t)|^2 = |R_1(t)|^2 + |R_2(t)|^2 + 2\mathrm{Re}(R_1(t)^* R_2(t))$$

$$R_1(t)R_2(t) = E^2\cos(\omega t)\cos(\omega(t-\tau)) = \frac{1}{2}E^2[\cos(\omega(2t-\tau)) + \cos(\omega\tau)]$$

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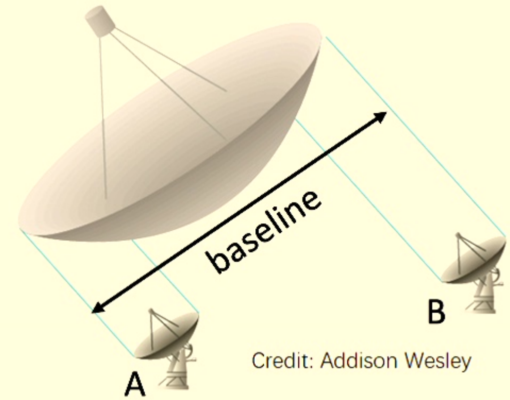
Optical Interferometers



Credit: Addison Wesley

Shorter wavelength-> Better resolution $\boldsymbol{\theta \sim \lambda / B}$

In optical wavelength, there are at least two important difference:

1. Mean photon number per mode is much smaller

State of the source:

$$\rho_s = (1-\epsilon)\rho_s^{(0)} + \epsilon\rho_s^{(1)} + O(\epsilon^2)$$

Vacuum

$$\rho_s^{(0)} = |0_A 0_B\rangle\langle 0_A 0_B|$$

No photons at telescope A

No photons at telescope B

Single photon

$$\rho_s^{(1)} = \frac{1}{2}\begin{array}{cc} \langle 0_A 1_B| \;\; \langle 1_A 0_B| & \\ \begin{bmatrix} 1 & g \\ g^* & 1 \end{bmatrix} & \begin{array}{c} |0_A 1_B\rangle \\ |1_A 0_B\rangle \end{array} \end{array}$$

Mean photon number of stellar light per temporal mode at optical wavelengths $\epsilon \ll 1$

2. It is hard to have a phase reference since the electromagnetic field oscillates too fast.

Quantum Estimation Theory

Lower bounds of estimating unknown parameters for given probes and encoding process can be calculated.

Classical Cramer-Rao bound

Quantum Cramer-Rao bound

$$\delta\phi \geq 1/\sqrt{F(\phi \mid \mathcal{P}, \hat{\rho})} \geq 1/\sqrt{K(\phi \mid \hat{\rho})}$$

Fisher information (FI)

Quantum Fisher information (QFI)

ϕ Unknown parameter

$\delta\phi$ Variance of the estimation

$\mathcal{P}$ POVM

$\hat{\rho}$ Probe state

Kay, S. M., Fundamentals of statistical signal processing. Prentice Hall PTR, 1993.
Braunstein, S. L., & Caves, C. M. Phys. Rev. Lett., 72(22), 3439. (1994).

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Local scheme for weak thermal source performs worse than nonlocal scheme

Define local scheme as the ones performed using local operations with classical communication (LOCC) without entanglement.
Define nonlocal scheme as the scheme which is not local scheme.

For the estimation of coherence function by measuring the weak thermal light

$$\rho_s = (1-\epsilon)\rho_s^{(0)} + \epsilon\rho_s^{(1)} + O(\epsilon^2)$$

Mean photon number of stellar light per temporal mode at optical wavelengths $\epsilon \ll 1$

Fisher information

Nonlocal scheme $F \sim O(\epsilon)$

Local scheme $F \sim O(\epsilon^2)$

Intuitively, this is due to the vacuum noise

Single photon

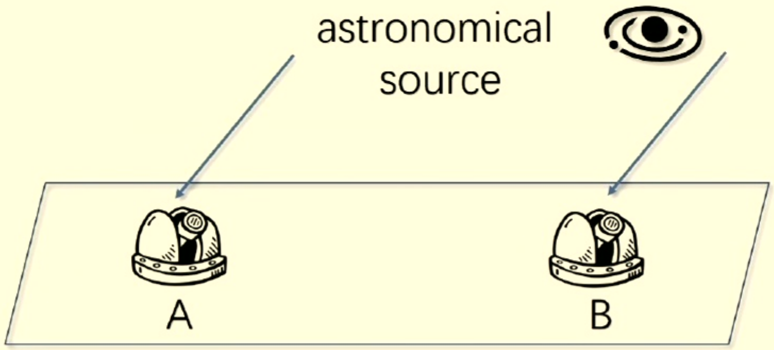
$$\begin{array}{cc} & \begin{array}{cc} \langle 0_A 1_B| & \langle 1_A 0_B| \end{array} \\ \rho_s^{(1)} = \dfrac{1}{2} & \begin{bmatrix} 1 & g \\ g^* & 1 \end{bmatrix} \end{array} \begin{array}{c} |0_A 1_B\rangle \\ |1_A 0_B\rangle \end{array}$$

Estimating the phase requires projection onto $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$

Detect whether the state is vacuum project onto $|0\rangle, |1\rangle$

[M. Tsang, PRL (2011)]

Reference frame problem from the perspective of quantum information



- If the phase reference frame of telescopes A and B are related by $\hat{U}_\phi = e^{i\phi\hat{n}}$
- A lack of shared reference frame means ϕ is unknown.
- This implies a superselection rule: coherence between bases of different photon number is not allowed. [S. D. Bartlett et al., RMP (2007)]
- **Without a shared reference frame,** if the state at telescope A is

$$|\psi\rangle = |+\rangle_A = (\underbrace{|0\rangle_A}_{\text{No photon at A}} + \underbrace{|1\rangle_A}_{\text{1 photon at A}})/\sqrt{2}$$

it is described at telescope B as $\int \frac{d\phi}{2\pi}\hat{U}(\phi)\,|\psi\rangle\langle\psi|\,\hat{U}(\phi)^\dagger = (|0\rangle\langle 0|_A + |1\rangle\langle 1|_A)/2$

→ decoherence effect

→ Operations can only be done within a **decoherence-free subspace**

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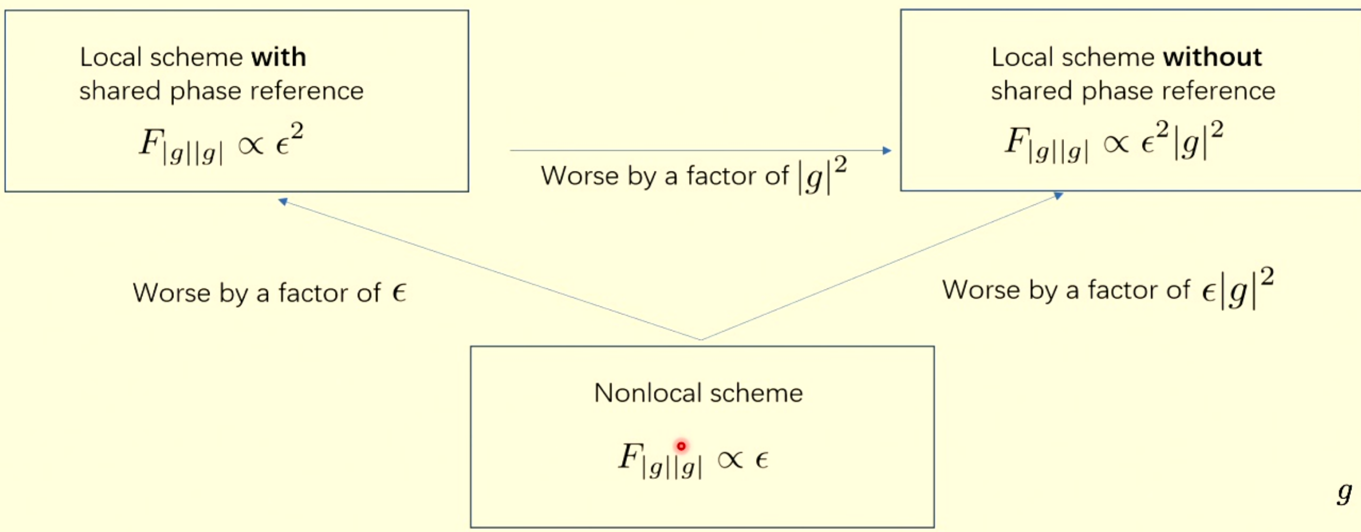
Three types of astronomical interferometer

We introduced the perspective of reference frame of these existing schemes in additional to local and nonlocal scheme.

$$\rho_s = (1-\epsilon)\rho_s^{(0)} + \epsilon\rho_s^{(1)} + O(\epsilon^2)$$

$$\rho_s^{(1)} = \frac{1}{2} \begin{array}{cc} \begin{array}{cc} \langle 0_A 1_B| & \langle 1_A 0_B| \end{array} & \\ \begin{bmatrix} 1 & g \\ g^* & 1 \end{bmatrix} & \begin{array}{c} |0_A 1_B\rangle \\ |1_A 0_B\rangle \end{array} \end{array}$$

$$\epsilon \ll 1$$



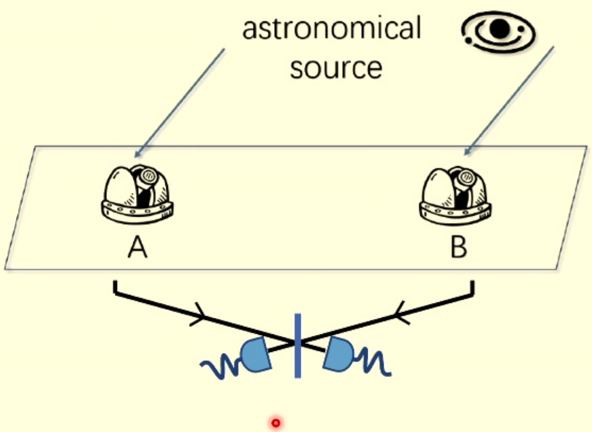
$$g = \int dx I(x) e^{ikx}, \quad \int dx I(x) = 1$$

For general extended sources $|g| \ll 1$

We will discuss each of these three types and see how each scheme solves the problem of reference frame.

Nonlocal scheme

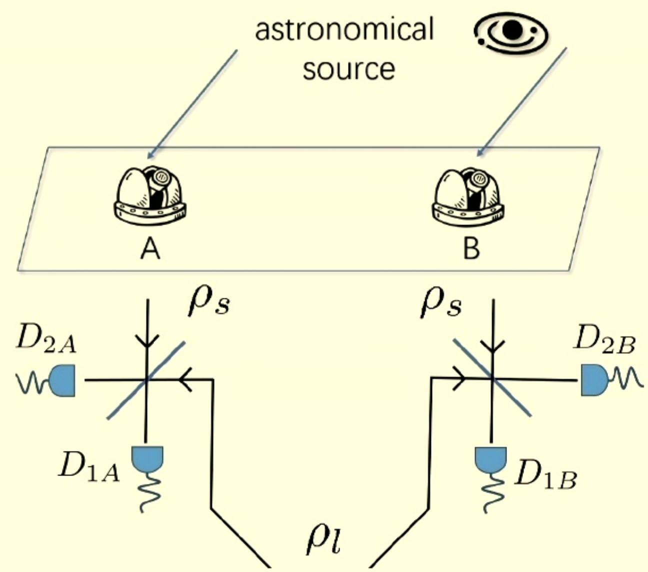
Bring the photons together (conventional method)



No reference state needed

[J. D. Monnier, Rep. Prog. Phys. (2003)]

Use entanglement



Uses entanglement
Requires shared reference state

[D. Gottesman et al., PRL (2012)]

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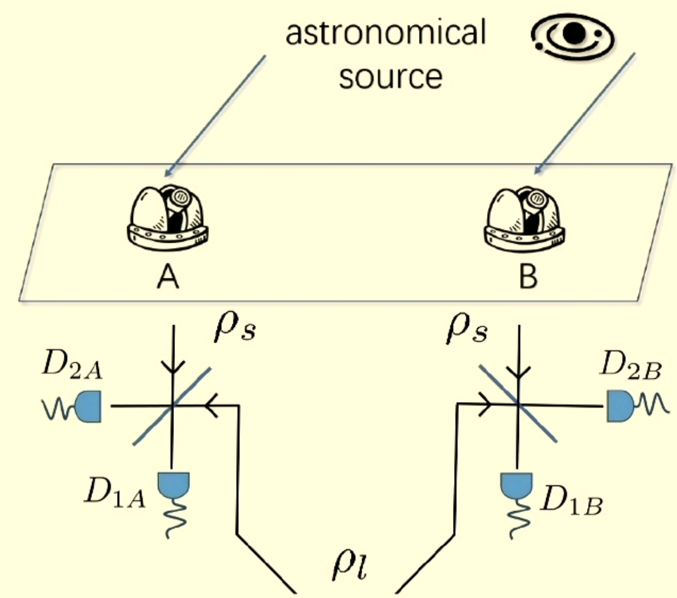
Quantum-network-based scheme



Reference state

Single photons generated by a lab ρ_l are distributed to the two telescopes as a reference state (uses entanglement resources)

$$\rho_l = |\psi_l\rangle\langle\psi_l| \qquad |\psi_l\rangle = (|0_l\rangle_A |1_l\rangle_B + |1_l\rangle_A |0_l\rangle_B)/\sqrt{2}$$

[D. Gottesman et al., PRL (2012)]

Composite system

Lab mode Stellar mode

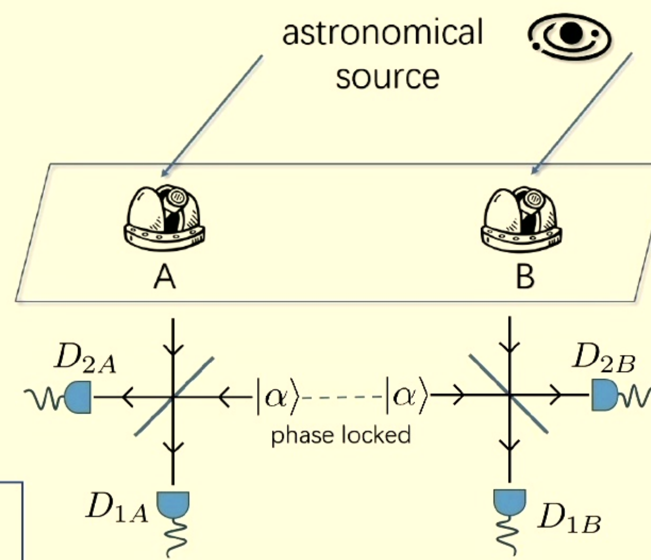
$$\rho_s^{(1)} \otimes \rho_l = \frac{1}{4} \begin{array}{cl} \begin{bmatrix} 1 & 1 & ge^{-i\delta} & ge^{-i\delta} \\ 1 & 1 & ge^{-i\delta} & ge^{-i\delta} \\ g^* e^{i\delta} & g^* e^{i\delta} & 1 & 1 \\ g^* e^{i\delta} & g^* e^{i\delta} & 1 & 1 \end{bmatrix} & \begin{matrix} |0_l 0_s\rangle_A |1_l 1_s\rangle_B \\ |0_l 1_s\rangle_A |1_l 0_s\rangle_B \\ |1_l 0_s\rangle_A |0_l 1_s\rangle_B \\ |1_l 1_s\rangle_A |0_l 0_s\rangle_B \end{matrix} \end{array}$$

Within a decoherence-free subspace

Cite eric's paper for this extension?

Performance is comparable to the conventional method. $F = O(\epsilon)$ [M. Tsang, PRL (2011)]
But half of the stellar photons are wasted because they are not brought into the decoherence-free subspace.
It is possible to expand the decoherence-free subspace with more lab photons. [R. Czupryniak, PRA (2022)]

Local scheme with shared reference state



Reference state

A separable reference state (no entanglement) is distributed to the two telescopes

$$|\alpha\rangle_A = \gamma_0 |0\rangle_A + \gamma_1 |1\rangle_A + \gamma_2 |2\rangle_A + \cdots$$

[D. D., Hale, et al. ApJ (2000).]

Composite system

$$(a|0\rangle_A + b|1\rangle_A) \otimes |\alpha\rangle_A = a\gamma_0 |00\rangle_A + \overbrace{a\gamma_1 |01\rangle_A + b\gamma_0 |10\rangle_A}^{\text{within a decoherence-free subspace}} + \cdots$$

$$\hat{U}_\phi (a\gamma_1 |01\rangle_A + b\gamma_0 |10\rangle_A) = \underbrace{e^{i\phi}}_{\text{global phase}} (a\gamma_1 |01\rangle_A + b\gamma_0 |10\rangle_A)$$

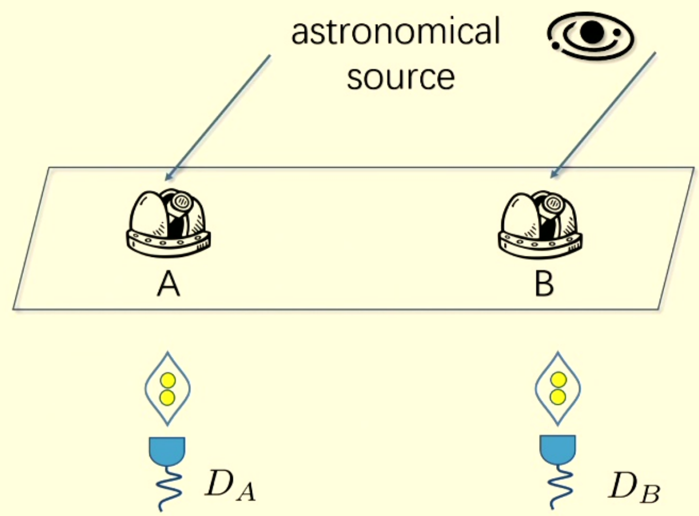
Performance is worse than the conventional method in the lossless case because we cannot distinguish vacuum terms in astronomical light without using entanglement $F = O(\epsilon^2)$

[M. Tsang, PRL (2011)]

Local scheme without shared reference frame: Intensity interferometer

For the weak thermal state,

$$\rho_{AB} = (1 - \epsilon - \epsilon^2) \left|0_A 0_B\right\rangle \left\langle 0_A 0_B\right| + \frac{\epsilon}{2}(\left|1_A 0_B\right\rangle \left\langle 1_A 0_B\right| + g \left|1_A 0_B\right\rangle \left\langle 0_A 1_B\right| + g^* \left|0_A 1_B\right\rangle \left\langle 1_A 0_B\right| + \left|0_A 1_B\right\rangle \left\langle 0_A 1_B\right|) + \frac{\epsilon^2}{4}(1 + |g|^2) \left|1_A 1_B\right\rangle \left\langle 1_A 1_B\right| + \cdots .$$



The intensity interferometer (in the weak limit) is described by the projection onto $|1_A 1_B\rangle$

$$P = \frac{\epsilon^2}{4}(1 + |g|^2) \qquad F = O(\epsilon^2 |g|^2)$$

The probability distribution depends on the absolute value of coherence function.

Since the POVM has only one term, it naturally stay in the decoherence-free subspace and can be implemented locally.

Quantum communication without shared reference state

In quantum communication, without a reference frame, information cannot be encoded in the subspace spanned by $\{|0\rangle, |1\rangle\}$

If Alice prepares

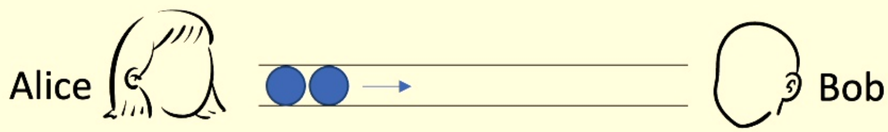
$$|\psi\rangle = a|0\rangle + b|1\rangle$$

Bob will describe the state as

$$\rho = |a|^2 |0\rangle\langle 0| + |b|^2 |1\rangle\langle 1|$$

→ Quantum communication is not possible using a single mode with at most one photon.

If we encode the information with **two modes**



If Alice prepares

$$|\psi\rangle = a|10\rangle + b|01\rangle$$

Bob will describe the state as

$$\hat{U}_\phi(a|10\rangle + b|01\rangle) = \boxed{e^{i\phi}}(a|10\rangle + b|01\rangle)$$

Global phase

→ The state is within a decoherence-free subspace

S. D. Bartlett et al., RMP, (2007)

A new astronomical interferometer scheme without shared reference state

Reference state

- No distributed reference states. Identical stellar photons are used as reference states for each other.

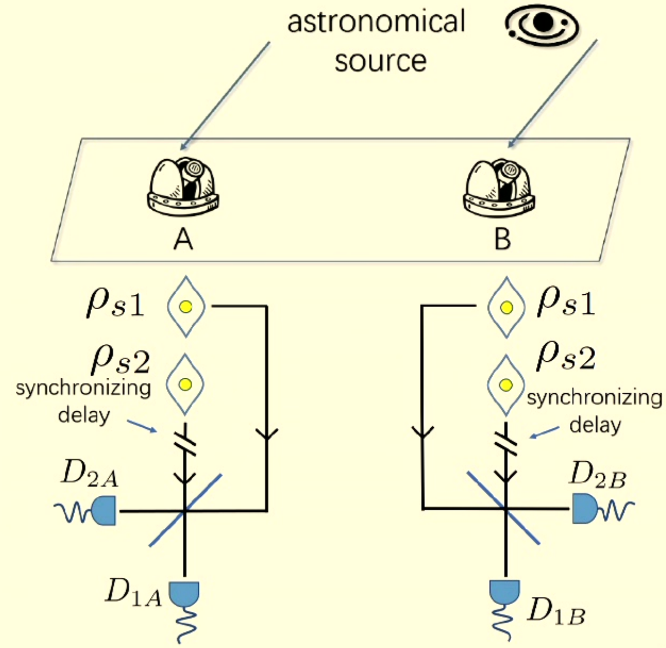
Composite system

First temporal mode

Second temporal mode

$$\rho_{s1} \otimes \rho_{s2} = \frac{1}{4} \begin{bmatrix} 1 & g & g & g^2 \\ g^* & 1 & |g|^2 & g \\ g^* & |g|^2 & 1 & g \\ g^{*2} & g^* & g^* & 1 \end{bmatrix} \begin{array}{l} |0_1 0_2\rangle_A |1_1 1_2\rangle_B \\ |0_1 1_2\rangle_A |1_1 0_2\rangle_B \\ |1_1 0_2\rangle_A |0_1 1_2\rangle_B \\ |1_1 1_2\rangle_A |0_1 0_2\rangle_B \end{array}$$

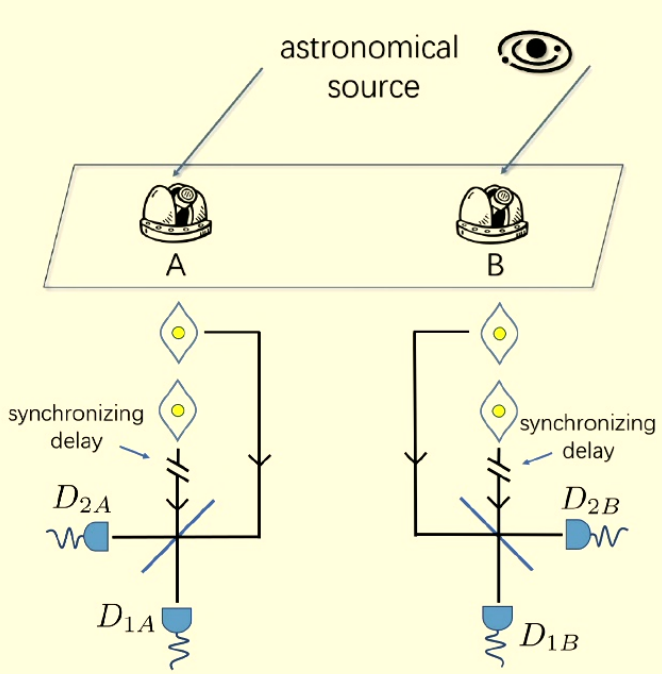
Within a decoherence-free subspace

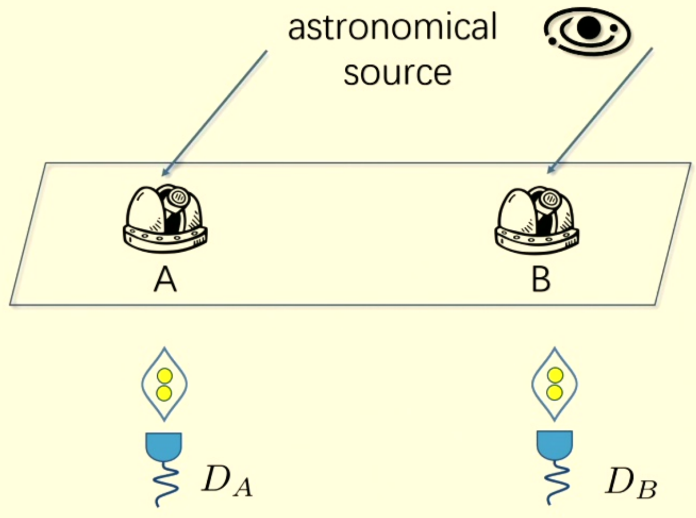


Performance is worse than schemes with shared reference states or entanglement resources.
Stellar photons do not have full coherence between two telescopes. They are imperfect reference states for each other, which degrades the sensitivity. (Fisher information has additional factor of $|g|^2$.)

To measure the **phase** of the coherence function, use three identical stellar photons $\rho_s^{\otimes 3}$ and three telescopes.
The **closure phase** can be measured in the same fashion.

Comparison with the conventional intensity interferometer





$$\rho_{AB} = (1-\epsilon-\epsilon^2)\,|0_A 0_B\rangle\langle 0_A 0_B| + \frac{\epsilon}{2}(|1_A 0_B\rangle\langle 1_A 0_B| + g\,|1_A 0_B\rangle\langle 0_A 1_B| + g^*\,|0_A 1_B\rangle\langle 1_A 0_B| + |0_A 1_B\rangle\langle 0_A 1_B|) + \frac{\epsilon^2}{4}(1+|g|^2)\,|1_A 1_B\rangle\langle 1_A 1_B| + \cdots .$$

The information for our scheme comes from the one photon terms for each of two temporal modes.

$$P = \frac{\epsilon^2}{2}(1+|g|^2)$$

Improve by a factor of 2 but use two copies

The information for a conventional intensity interferometer comes from the two photon terms within one temporal mode.

$$P = \frac{\epsilon^2}{4}(1+|g|^2)$$

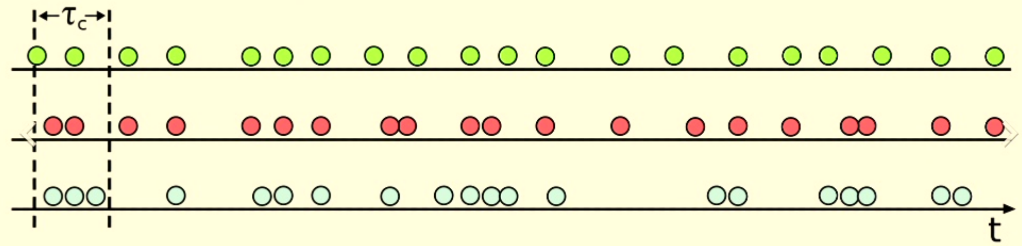
$$F = O(\epsilon^2|g|^2)$$

In the weak source limit, the two scheme has the same performance for imaging thermal sources.

An artificial example: imaging of antibunching source

The performance of conventional and generalized intensity interferometer can be different when the source is antibunching.

$$\begin{aligned}\rho_{AB} = &(1-\epsilon-\epsilon^2)\,|0_A0_B\rangle\langle 0_A0_B| \\ &+\frac{\epsilon}{2}(|1_A0_B\rangle\langle 1_A0_B| + g\,|1_A0_B\rangle\langle 0_A1_B| \\ &\quad + g^*\,|0_A1_B\rangle\langle 1_A0_B| + |0_A1_B\rangle\langle 0_A1_B|) \\ &\cancel{+\frac{\epsilon^2}{4}(1+|g|^2)\,|1_A1_B\rangle\langle 1_A1_B| + \cdots}\,.\end{aligned}$$



Photon detections as function of time for a) antibunched, b) random, and c) bunched light

Conventional intensity interferometer

POVM $|1_A1_B\rangle$ Probability $P=o(\epsilon^2)$ Fisher information $F=o(\epsilon^2)$

Generalized intensity interferometer

POVM

$$|\psi_{1,2}\rangle = \frac{1}{2}(|0_11_2\rangle \pm |1_10_2\rangle)_A \otimes (|0_11_2\rangle \pm |1_10_2\rangle)_B,$$
$$\Pi_1 = |\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|,$$
$$|\psi_{3,4}\rangle = \frac{1}{2}(|0_11_2\rangle \pm |1_10_2\rangle)_A \otimes (|0_11_2\rangle \mp |1_10_2\rangle)_B,$$
$$\Pi_2 = |\psi_3\rangle\langle\psi_3| + |\psi_4\rangle\langle\psi_4|,$$

Probability $P_{1,2} = \frac{1}{4}\epsilon^2(1 \pm |g|^2)$

Fisher information $F_{|g||g|} = \frac{2\epsilon^2|g|^2}{1-|g|^4}$

An artificial example: imaging of antibunching source

It is even possible to observe a superresolution for the generalized interferometer while imaging antibunching source

Probability $$P_{1,2} = \frac{1}{4}\epsilon^2(1 \pm |g|^2)$$

Separation between two point sources

If we are imaging two antibunching point sources. $g = e^{i\theta}\cos\phi$, $\phi = kX$

In the case two point sources are close to each other, we have $X \to 0$ $|g| \to 1$

The Fisher information $$F = \frac{4\epsilon^2 k^2 \cos^2(kX)}{3+\cos(2kX)} \to \frac{4\epsilon^2 k^2}{4}$$

$X \to 0$

If we consider the conventional nonlocal scheme,

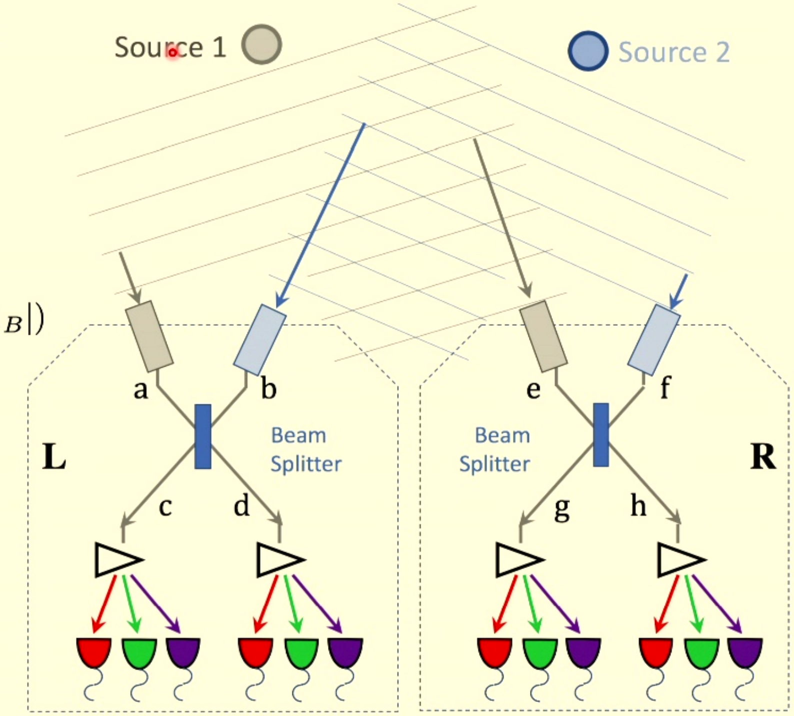
$$P_{1,2} = \frac{\epsilon}{2}(1 \pm |g|\cos(\theta+\delta)) \qquad F = \frac{\epsilon k^2 \cos^2(\delta+\theta)\sin^2(kX)}{-1+\cos^2(\delta+\theta)\cos^2(kX)} \to 0$$

If these three conditions are satisfied: (1) source is antibunching (2) Separation approaches zero faster than ϵ (3) The nonlocal measurement cannot have prior knowledge of the phase of coherence function, our scheme performs even better than a nonlocal scheme.

Compare with another generalized of intensity interferometer

P Stankus et al. arXiv:2010.09100v6

They consider a generalization with **spatially** separated two sources while our generalization considers **temporally** separated modes.



They are considering the thermal states $\rho_1 \otimes \rho_2$

$$\rho_{1,2} = (1-\epsilon-\epsilon^2)\,|0_A 0_B\rangle\langle 0_A 0_B| + \frac{\epsilon}{2}\left(|1_A 0_B\rangle\langle 1_A 0_B| + g_{1,2}\,|1_A 0_B\rangle\langle 0_A 1_B| + g_{1,2}^*\,|0_A 1_B\rangle\langle 1_A 0_B| + |0_A 1_B\rangle\langle 0_A 1_B|\right) + \frac{\epsilon^2}{4}(1+|g_{1,2}|^2)\,|1_A 1_B\rangle\langle 1_A 1_B| + \cdots .$$

POVM $|\pm\pm\rangle = (|0_{1A}1_{2A}\rangle \pm |1_{1A}0_{2A}\rangle)(|0_{1B}1_{2B}\rangle \pm |1_{1B}0_{2B}\rangle)/2$

Probability
$$P(++) = P(--) = \frac{\epsilon^2}{16}(4+|g_1|^2+|g_2|^2+g_1 g_2^* + g_1^* g_2)$$
$$P(+-) = P(-+) = \frac{\epsilon^2}{16}(4+|g_1|^2+|g_2|^2-g_1 g_2^* - g_1^* g_2)$$

In comparison, our scheme considers $\rho^{\otimes 2}$

The label 1,2 in our POVM means temporal modes

$$|\pm\pm\rangle = (|0_{1A}1_{2A}\rangle \pm |1_{1A}0_{2A}\rangle)(|0_{1B}1_{2B}\rangle \pm |1_{1B}0_{2B}\rangle)/2 \qquad P(++) = P(--) = \frac{\epsilon^2}{4}(1+|g|^2) \qquad P(+-) = P(-+) = \frac{\epsilon^2}{4}$$

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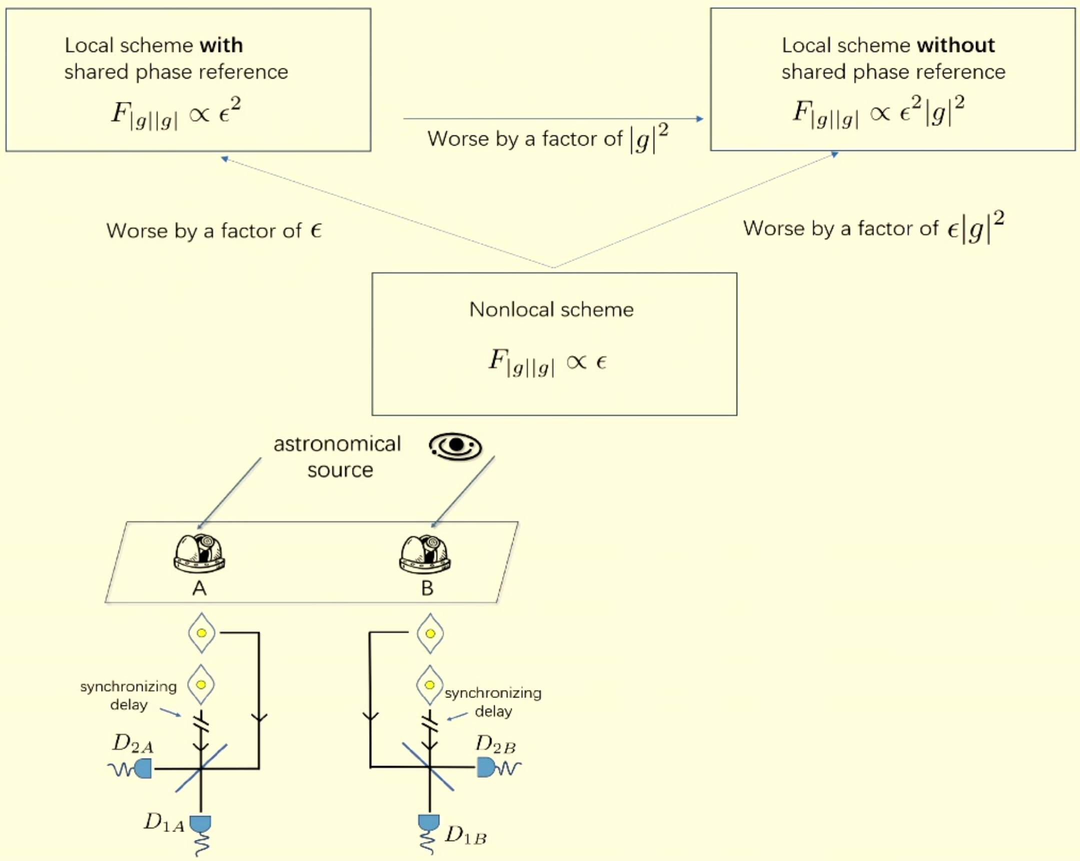
Summary

We introduce a new perspective of reference frame and classify the astronomical interferometer accordingly. The Fisher information of local scheme with/without reference frame can differ by a factor of |g|^2.

We introduce a generalized intensity interferometer, which uses the single photon term of two temporal modes. The performance can be superior for antibunching sources.



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