Title: GPTs and the probabilistic foundations of quantum theory - Lecture

Speakers: Alexander Wilce

Collection: GPTs and the probabilistic foundations of quantum theory - mini-course

Date: March 21, 2024 - 9:00 AM

URL: https://pirsa.org/24030098

Today

* Recap +
* Joint Probability, No-signaling, & Entanglement
* Composites

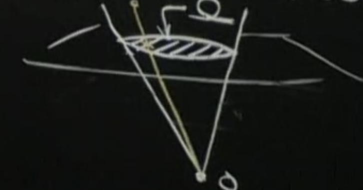
Last time

$$A = (\mathcal{M}(A), \Omega(A))$$

$\mathcal{M}(A)$: locally finite

$\Omega(A)$: closed convex (compact)

Ω



$$\mathbb{V}(A)$$

$$\mathbb{V}^*(A) \cong \mathrm{Aff}_b(\Omega), \quad u(\alpha) \equiv 1 \quad \forall \alpha \in \Omega$$

$$\mathbb{V}^*(A) = \mathbb{V}(A)^*$$

effects: $a \in \mathbb{V}^*$

$0 \leq a \leq u$

$\forall \alpha \in \Omega$, think of

$a(\alpha)$ = "prob. of a in state α".

$\forall a \in \mathcal{E}(M)$, $\hat{a} \in [0,u]$

$$\hat{a}(\alpha) := \alpha(a) = \sum_{x \in a} \alpha(x)$$

Have:

$$\mathbb{X}(A) \longrightarrow [0,u]$$

$$x \longmapsto \hat{x}$$

$$\sum_{x \in E} \hat{x} = u$$

"Observable": $\{a_i\} \subseteq [0,u]$

$$\sum_i a_i = u$$

Why not just use

$\mathcal{Q}$, $[0,u]$, observables...

(operational)

A1: Hard to find physical justification for arbitrary effects, observables

(NR hypoth for effects)

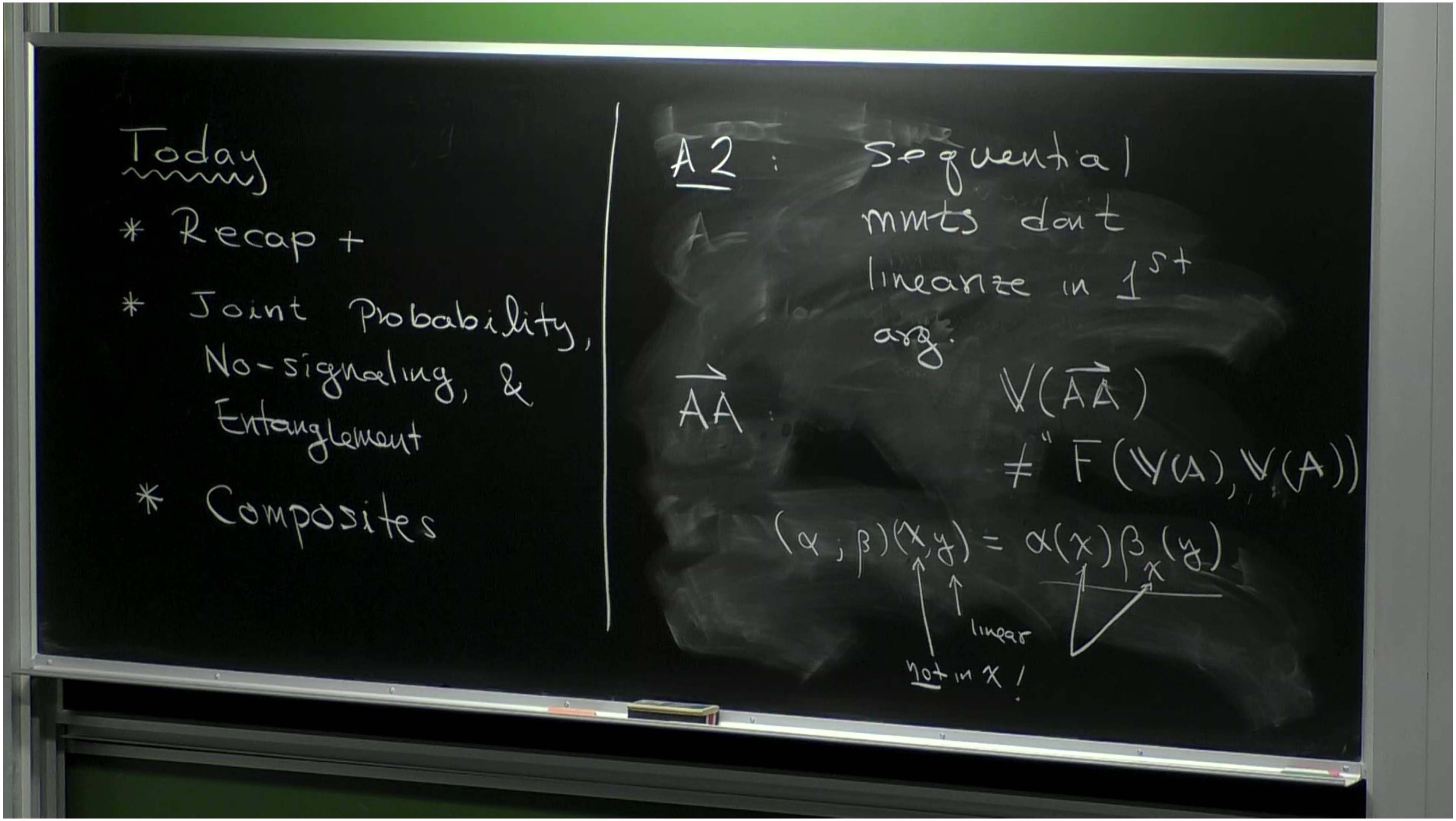
can form effect from events:

$a_1, a_2 \ldots a_n \in \mathcal{E}_V$

$0 \le t_1 \ldots t_n$, $\sum t_i = 1$

$a = \sum t_i \hat{a}_i \in [0,u]$

$E_1, E_2 \ldots$?


Today
* Recap +
* Joint Probability, No-signaling, & Entanglement
* Composites
A2 : Sequential mmts don't linearize in 1st arg.
$\overrightarrow{AA}$:
$\mathbb{V}(\overrightarrow{AA}) \neq$ " $F(\mathbb{V}(A), \mathbb{V}(A))$
$(\alpha ; \beta)(x,y) = \alpha(x)\beta_x(y)$
linear
not in x !

Today

* Recap +
* Joint Probability, No-signaling, & Entanglement
* Composites

Two test spaces

$$\mathcal{A}, \quad \mathcal{B}$$

Define:

$$\mathcal{A} \times \mathcal{B} = \{ E \times F \mid E \in \mathcal{A}, F \in \mathcal{B} \}$$

Joint P. weight on $\mathcal{A}, \mathcal{B}$:

$$\omega \in Pr(\mathcal{A} \times \mathcal{B})$$

marginals? $\underline{\text{Sure}}$!

jt state

$$\omega_1^F(x) = \sum_{y \in F}{}' \; \omega(x,y) \in \Pr(\mathcal{A}) \rightsquigarrow \Omega(\mathcal{A})$$

$$\omega_2^E(y) = \sum_{x \in E} \omega(x,y) \in \Pr(\mathcal{B}) \qquad \Omega(\mathcal{B})$$

marginals? Sure!

Jt state

$$\omega_1^F(x) = \sum_{y \in F} \omega(x,y) \in Pr(\mathcal{A}) \rightsquigarrow \Omega(A)$$

$$\omega_2^E(y) = \sum_{x \in E} \omega(x,y) \in Pr(\mathcal{B}) \quad \Omega(B)$$

If A, B are "far apart", these shouldn't depend on E, F!

Def: $\omega \in \mathrm{Pr}(\mathcal{A} \times \mathcal{B})$

is non-signaling (NS)

$\Leftrightarrow$ $\omega_1^F = \omega_1^{F'}$ $\forall F, F' \in \mathcal{B}$

$\omega_2^E = \omega_2^{E'}$ $\forall E, E' \in \mathcal{A}$

Pr_{NS} convex

If A, B are "far apart",
These shouldn't depend on E, F!

S'pose ω is NS. Can define
conditional states:

$$\omega_{2|x} = \frac{\omega(x,y)}{\omega_1(x)} \neq 0$$

$$\omega_{1|y} = \frac{\omega(x,y)}{\omega_2(y)}$$

Today

- ✓ * Recap +
- * Joint Probability, No-signaling, & Entanglement
- * Composites

$$\omega(x,y) = \omega_1(x)\,\omega_{2|x}(y)$$

$$\Downarrow$$

convex combo of $\omega_{2|x}$

$$\boxed{\omega_2 = \sum_{x\in E} \omega_1(x)\,\omega_{2|x}}$$

Law of Total Prob

LOTP

Def: If $\alpha \in \Pr(\mathcal{A})$, $\beta \in \Pr(\mathcal{B})$

$$(\alpha \otimes \beta)(x, y) := \alpha(x)\beta(y)$$

$\alpha \otimes \beta \in \Pr_{NS} \leftarrow$ convex, so

$$\sum_{i=1}^{n} t_i (\alpha_i \otimes \beta_i) \in \Pr_{NS}$$

"separable"

<u>FACT</u>: Unless $Pr(\mathcal{A})$ or $Pr(\mathcal{B})$ is a <u>simplex</u>

$$Pr_{NS} \supsetneq Pr_{sep}.$$

ω is <u>entangled</u> iff

$$\omega \in Pr_{NS} \setminus Pr_{sep}$$

$$\sum_{i=1} t_i (\alpha_i \otimes \beta_i) \in \ldots_{NS}$$

"Separable"

Lemma: S'pose ω is NS.
If ω_2 is pure (extreme),
Then $\omega = \omega_1 \otimes \omega_2$

Pf

$\sum_{i=1} t_i (\alpha_i \otimes \beta_i) \in \mathcal{P}_{NS}$

"Separable"

Lemma: S'pose ω is NS.

If ω_2 is pure (extreme),

Then $\omega = \omega_1 \otimes \omega_2$

Pf LoTP:

$$\underset{\substack{\uparrow \\ \text{pure}}}{\omega_2} = \sum_{x \in E} \omega_1(x)\, \omega_{2|x} \Rightarrow \quad \forall x \ \omega_1(x) \neq 0, \ \omega_{2|x} = \omega_2.$$

so $\forall y,\ \overbrace{\forall x \in E,\ \forall E}^{\forall x} \in \mathcal{B}(\mathcal{B})$

$$\omega_{2|x}(y) = \frac{\omega(x,y)}{\omega_1(x)} = \omega_2(y)$$

$$\Rightarrow \omega(x,y) = \omega_1(x)\,\omega_2(y)$$

✓

Today

* Recap +
* Joint Probability, No-signaling, & Entanglement
* Composites

Cor: If ω is entangled
Then ω_1, ω_2 are mixed.

F R – 1981

Kläy, F, R – 1986

$\boxed{\text{Kläy – 1988}}$

Today

- * Recap +
- * Joint Probability, No-signaling, & Entanglement
- * Composites

Cor: If ω is entangled
Then ω_1, ω_2 are mixed.

F. R. – 1981

Kläy, F., R – 1986

$\boxed{\text{Kläy – 1988}}$ ← (Nobody cared)

$$A \underset{NS}{\times} B = \left(m(A) \times m(B) \,,\, \Omega_{NS} \right)$$

$$\overleftrightarrow{AB} = \left(\overrightarrow{m(A)\,m(B)} \cup \overleftarrow{m(A)\,m(B)} \right)$$

$$\left\{ \omega \in Pr_{NS} \;\middle|\; \underset{\in \Omega(A)}{\omega_{1|y_1}} \;\; \underset{\in \Omega(B)}{\omega_{2|x}} \right\}$$

bi-lateral

$$\left\{ \omega \in Pr \;\middle|\; \begin{matrix} \omega_{1|y} \\ \omega_{2|x} \end{matrix} \;\dots \right\}$$