Title: GPTs and the probabilistic foundations of quantum theory - Lecture

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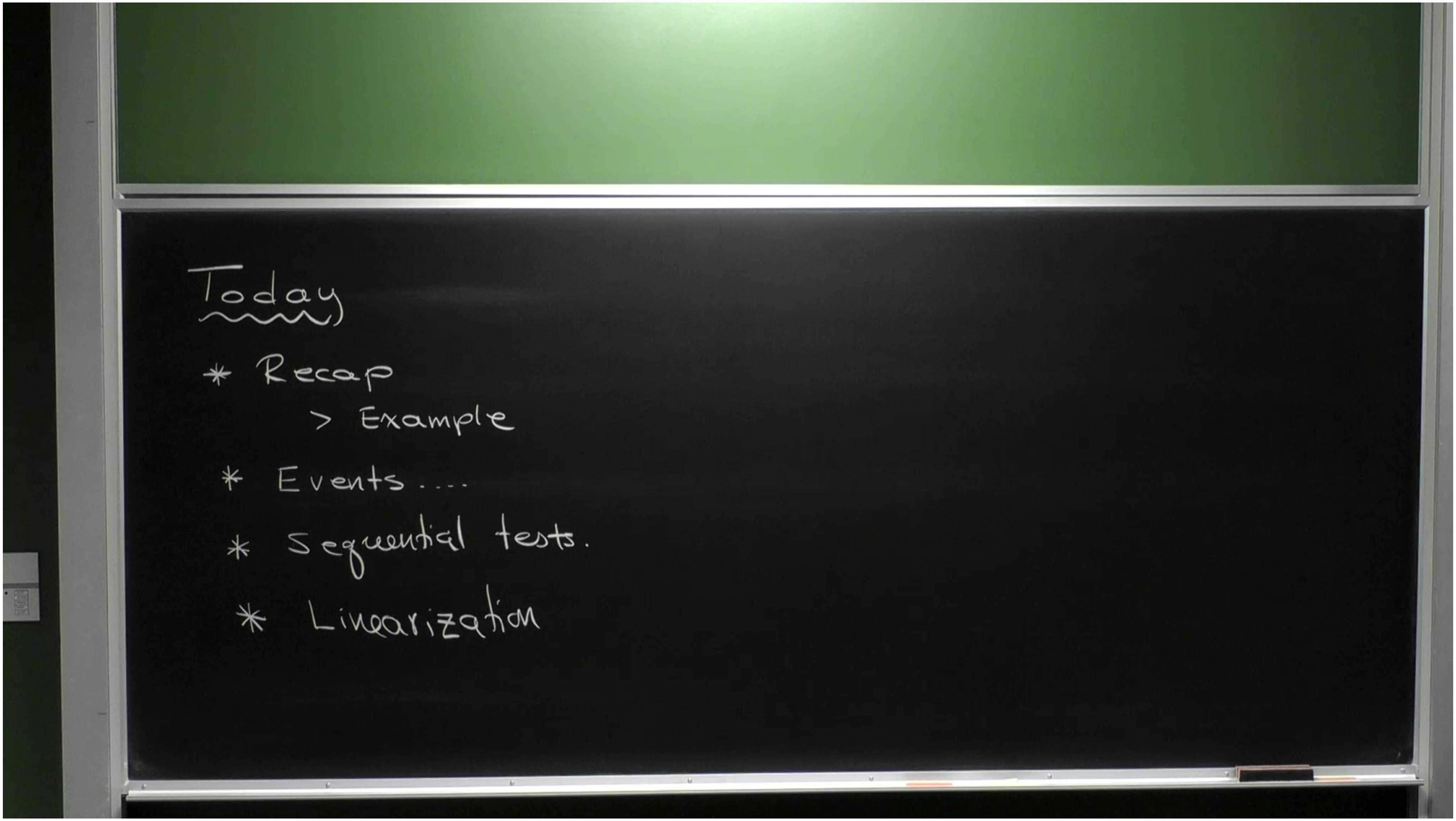
Collection: GPTs and the probabilistic foundations of quantum theory - mini-course

Date: March 14, 2024 - 9:00 AM

URL: https://pirsa.org/24030096


Today
* Recap
> Example
* Events ....
* Sequential tests.
* Linearization

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Last time:

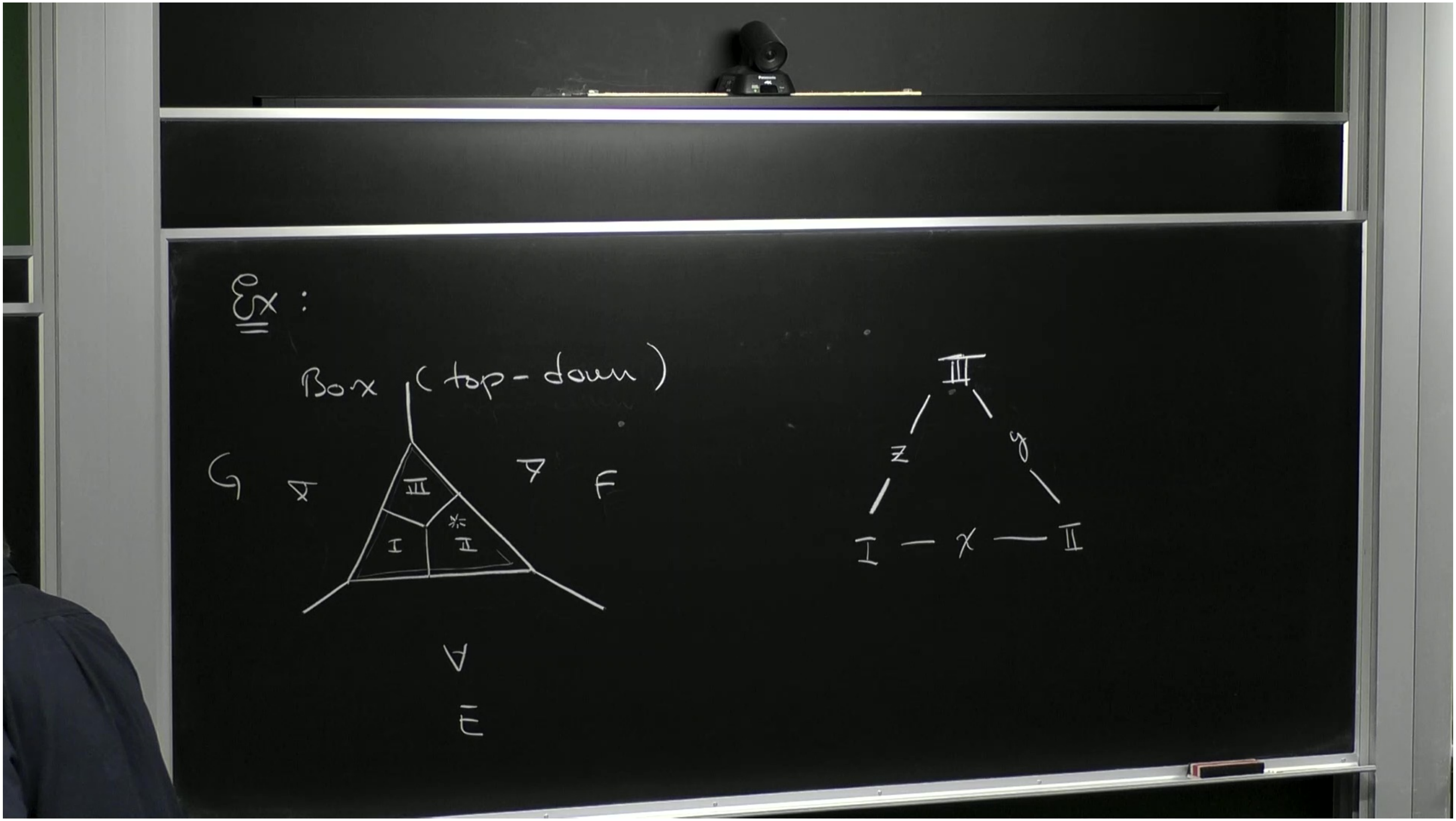
probababilistic model:

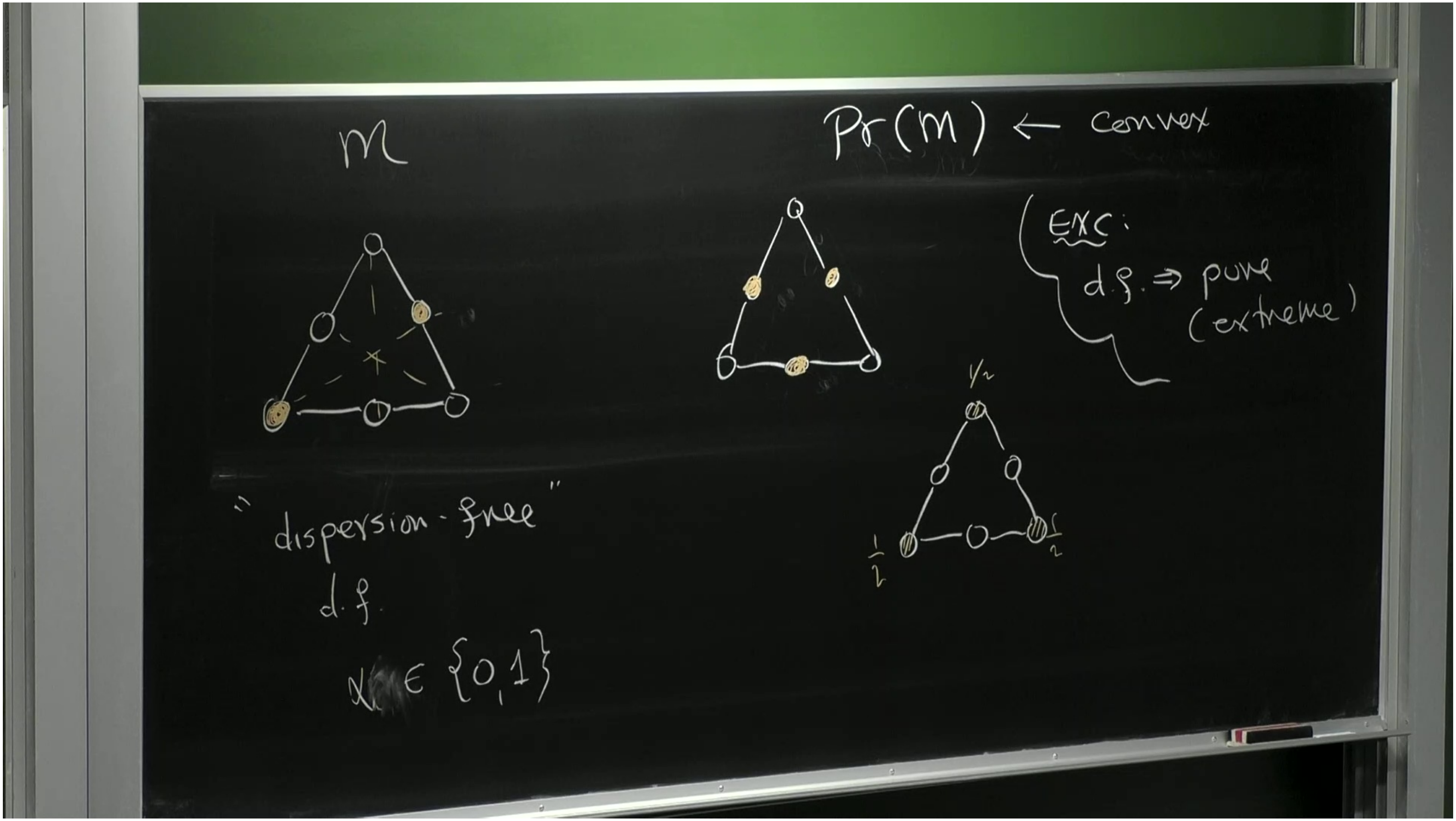
$$A = (\mathcal{M}, \Omega)$$

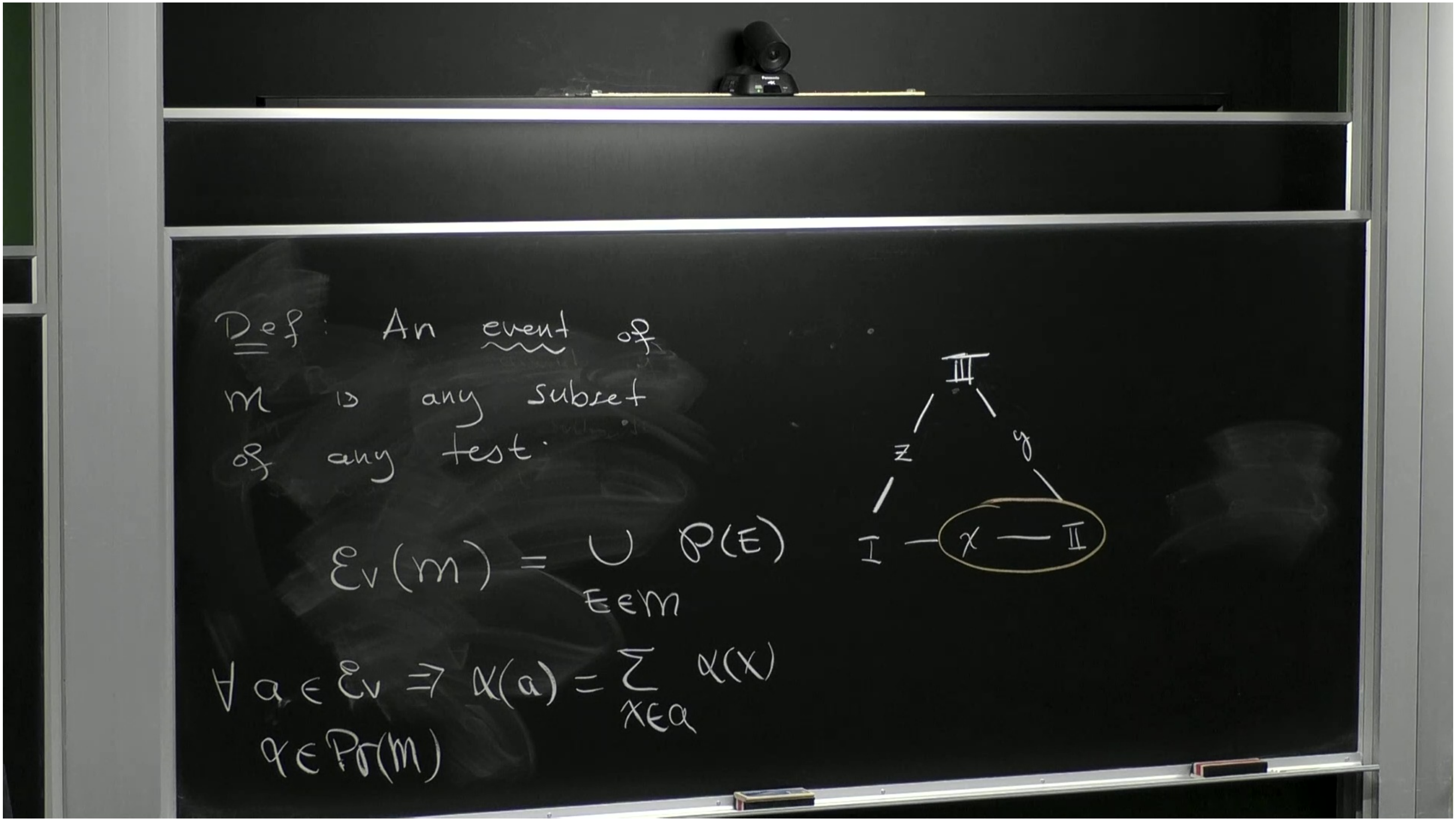
$\mathcal{M}$ — Testspace

Ω — state space - set of prob. weights on $\mathcal{M}$

$$(\mathcal{M}(A), \Omega(A))$$


Ex :
Box (top-down)
G
F
E
III
I
II
I — x — II
z
y


M
Pr(M) ← convex
EXC:
d.f. ⇒ pure
(extreme)
1/2
1/2
1/2
"dispersion-free"
d.f.
∈ {0,1}

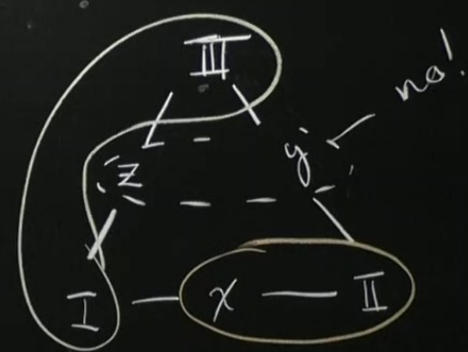

Def: An event of M is any subset of any test.
$\mathcal{E}v(M) = \bigcup_{E \in M} \mathcal{P}(E)$
$\forall\, a \in \mathcal{E}v \;\exists\; \alpha(a) = \sum_{x \in a} \alpha(x)$
$\alpha \in Pr(M)$
III
z
y
I
x
II

Def: An event of $\mathcal{M}$ is any subset of any test.

$$\mathcal{E}v(\mathcal{M}) = \bigcup_{E \in \mathcal{M}} \mathcal{P}(E)$$

$$\forall a \in \mathcal{E}v \Rightarrow \alpha(a) := \sum_{x \in a} \alpha(x)$$

$$\alpha \in Pr(\mathcal{M})$$



Jargon:

$\forall\, a, b \in \mathcal{E}v(M)$

a, b are

* compatible

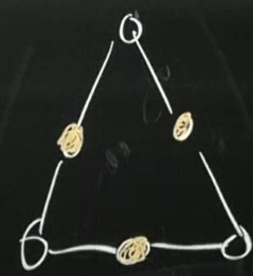
$\Leftrightarrow a \cup b \in \mathcal{E}v.$

* "orthogonal" $(a \perp b)$

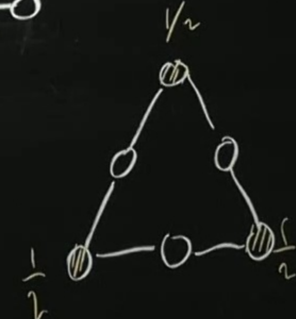
$\Leftrightarrow$ compatible, and

$a \cap b = \emptyset$

$Pr(M) \leftarrow$ convex



(Exc: d.ρ. ⇒ pure (extreme))



← pure.

Jargon:

$\forall\, a, b \in \mathcal{E}v(\mathcal{M})$

a, b are

* compatible
$\iff a \cup b \in \mathcal{E}v.$

* "orthogonal" $(a \perp b)$
$\iff$ compatible, and
$a \cap b = \emptyset$

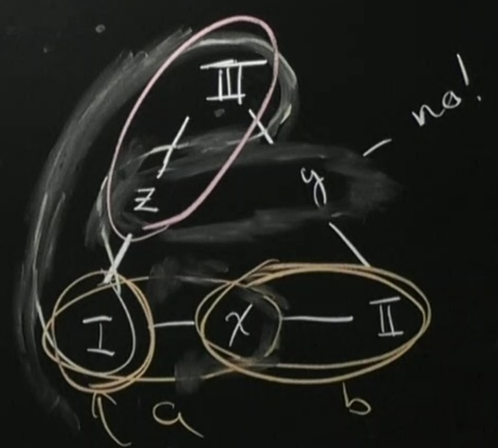
* complementary $(a \text{ co } b)$
$\iff a \perp b,$
$a \cup b \in \mathcal{M}.$

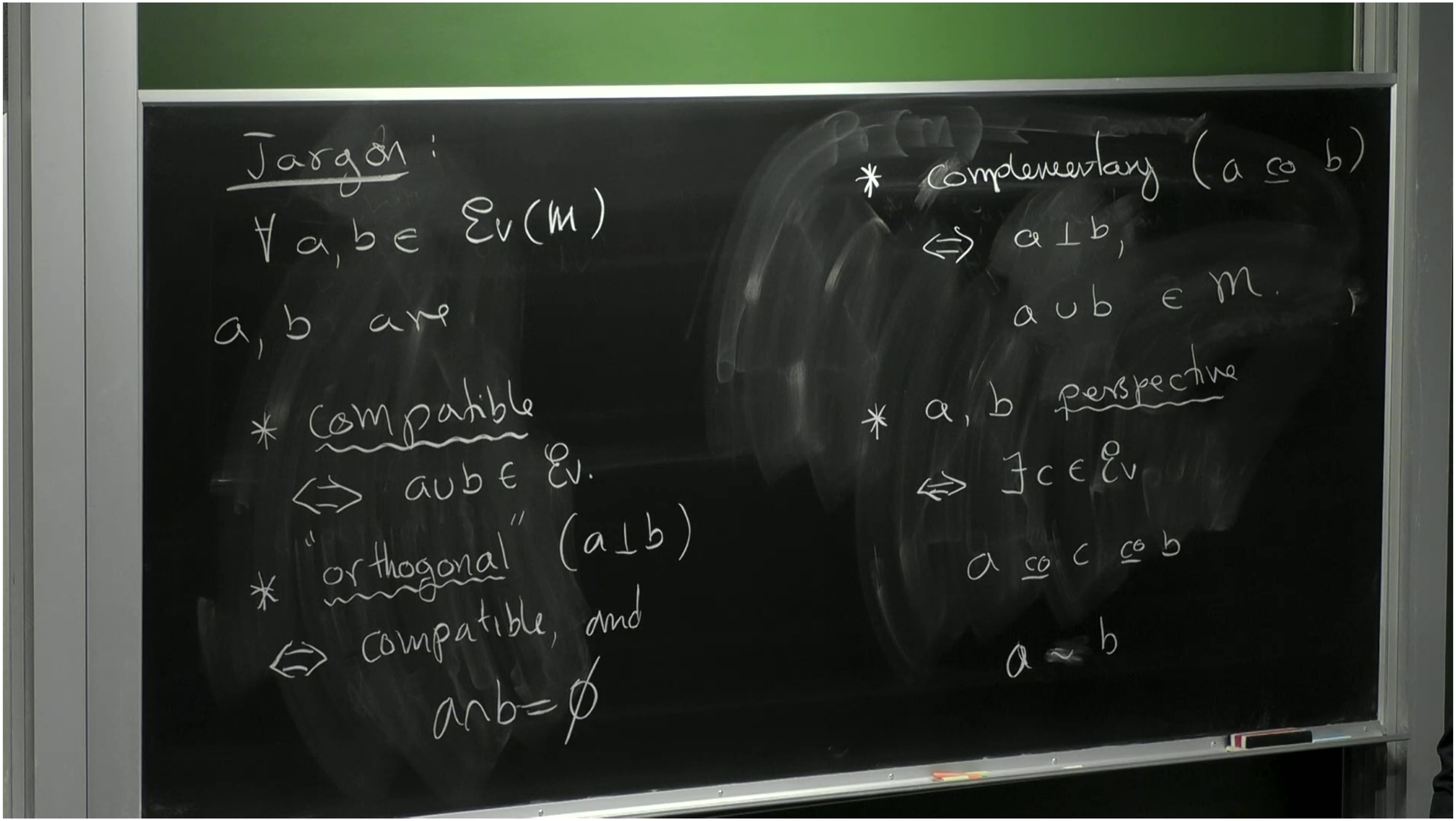
<u>Def</u>: An event of M is any subset of any test.

$$\mathcal{E}v(M) = \bigcup_{E \in M} \mathcal{P}(E)$$

$$\forall\, a \in \mathcal{E}v \Rightarrow \alpha(a) := \sum_{x \in a} \alpha(x)$$

$$\alpha \in Pr(M)$$




Jargon:
∀ a, b ∈ Ev(M)
a, b are
* compatible
⟺ a∪b ∈ Ev.
* "orthogonal" (a⊥b)
⟺ compatible, and
a∩b = ∅
* complementary (a co b)
⟺ a⊥b,
a∪b ∈ M.
* a, b perspective
⟺ ∃c ∈ Ev
a co c co b
a ~ b

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$\underline{\underline{Nr}}$:

$$a \sim b \Rightarrow \forall \alpha \in Pr$$

$$\alpha(a) = \alpha(b)$$

$\forall E, F \in m, \quad E \sim F$

* Linearization

Def: $\mathcal{M}$ is algebraic iff

$\forall\, a, b, c \in \mathcal{E}v(\mathcal{M})$,

$$a \sim b,\quad b \; \underline{\text{co}} \; c \Rightarrow a \; \underline{\text{co}} \; c$$

☆ Exercise:

$\mathcal{M}$ algebraic $\Rightarrow$ $\sim$ is an equiv. rel'n on $\mathcal{E}v$.

Def: If $\mathcal{M}$ is algebraic, its "logic" is

$$\Pi(\mathcal{M}) = \mathcal{E}_V/\sim$$

$\Rightarrow$ an orthoalgebra

Def: An orthoalgebra:

$(L, \perp, \oplus, 0, 1)$

$\oplus : \perp \to L$

* $a \perp b \Rightarrow a \oplus b = b \oplus a$
* $a \perp (b \oplus c)$
  $\Rightarrow a \perp b, \quad a \oplus b \perp c,$
  $a \oplus (b \oplus c) = (a \oplus b) \oplus c$
* $\forall a \; \exists! \, a' \quad a \oplus a' = 1$
* $\forall a \quad a \perp a \Rightarrow a = 0$

$\forall a, b \in L$

$a \leq b \iff \exists c,$

$a \perp c,$

$a \oplus c = b$

Fact: any orthomodular lattice/poset is an OA

Exc: for $a, b \in \mathcal{E}_V$,

$[a] = [a]_\sim \in \Pi(m)$

show:

$[a] \perp [b] \iff a \perp b$

$[a] \oplus [b] = [a \cup b]$

are well-def'd.

also: $0 = [\emptyset]$

$1 = [E] \ \forall E \in m$

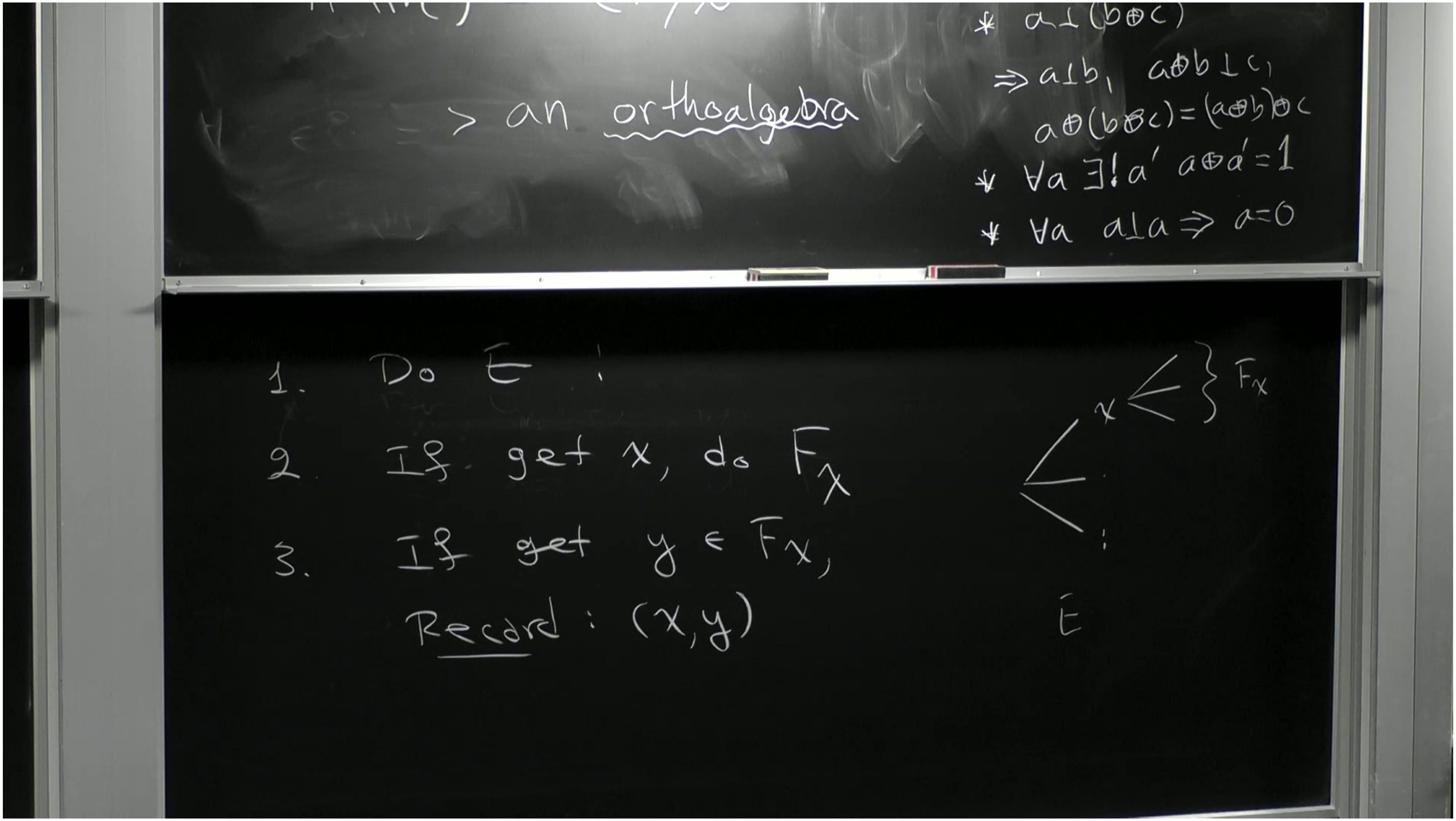
* Linearization

Two-stage measurements

$\mathcal{M}$ a test space

$E \in \mathcal{M}$, $F : E \longrightarrow \mathcal{M}$: $\forall x \in E$

$F_x \in \mathcal{M}$


* a⊥(b⊕c)
⇒ a⊥b, a⊕b⊥c,
a⊕(b⊕c) = (a⊕b)⊕c
* ∀a ∃! a' a⊕a' = 1
* ∀a a⊥a ⇒ a=0
> an orthoalgebra
1. Do E
2. If get x, do F_x
3. If get y ∈ F_x,
Record: (x,y)
x
F_x
E

$1 = [E] \ \forall E \in m$

Def:

$$\overrightarrow{m} = \left\{ \bigcup_{x \in E} \underbrace{\{x\} \times F_x}_{x F_x} \right\}$$

$\Rightarrow$ an orthoalgebra

* $a \perp (b \oplus c)$

  $\Rightarrow a \perp b,\ a \oplus b \perp c,$

  $a \oplus (b \oplus c) = (a \oplus b) \oplus c$

* $\forall a\ \exists!\, a'\ \ a \oplus a' = 1$

* $\forall a\ \ a \perp a \Rightarrow a = 0$

s'pose $E, F, E', F' \in \mathcal{M}$

$\forall\, x, y \in \bigcup \mathcal{M},$

Exc $\begin{cases} xE \sim xF \\ Ex \not\sim Fx \end{cases}$

$$\bigcup_{x \in E} \{x\} \times F_x$$

$1 = [E] \ \forall E \in m$

$\omega \in Pr(\overrightarrow{mm})$

$$\sum_{y \in F} \omega(x, y) = \sum_{z \in F'} \omega(x, z)$$

$$\Downarrow$$

$$\omega(x, \cdot) = \underline{\alpha(x)\beta_x}(\cdot)$$

$\beta_x \in Pr(m)$

$\ast\ a \perp (b \oplus c)$

$\Rightarrow a \perp b,\ a \oplus b \perp c,$

$a \oplus (b \oplus c) = (a \oplus b) \oplus c$

$\ast\ \forall a\ \exists!\, a'\ \ a \oplus a' = 1$

$\ast\ \forall a\ \ a \perp a \Rightarrow a = 0$

$\Rightarrow$ an orthoalgebra

$\alpha \in Pr(M),\ \beta : X \to Pr(M)$

$\mathrm{Pr}(\overrightarrow{MM})$

$\simeq Pr(M) \times Pr(M)^X$

$(\alpha ; \beta)(x, y)$

$= \alpha(x)\beta_x(y)$

$\in Pr(\overrightarrow{MM})$