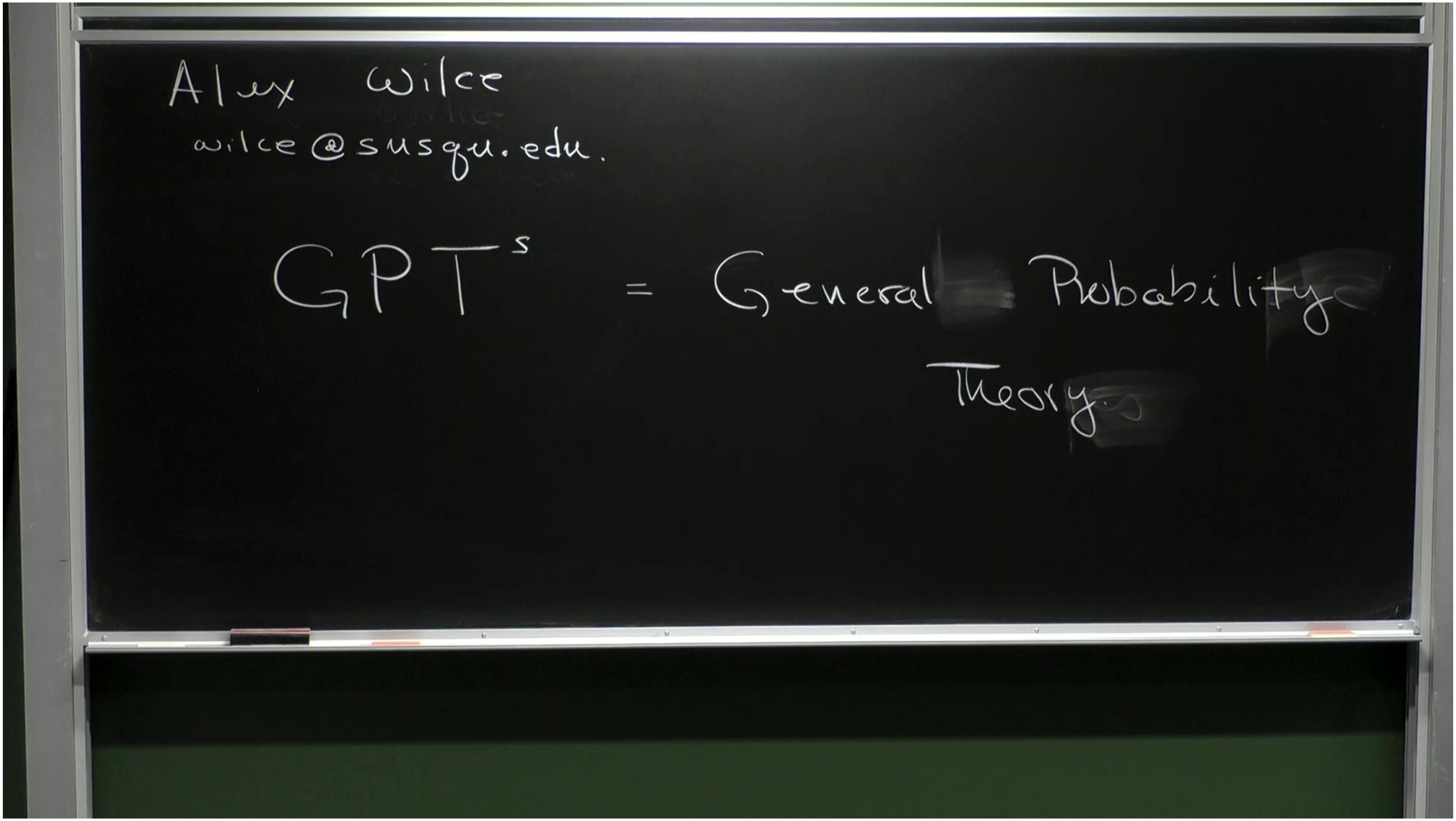
Title: GPTs and the probabilistic foundations of quantum theory - Lecture

Speakers: Alexander Wilce

Collection: GPTs and the probabilistic foundations of quantum theory - mini-course

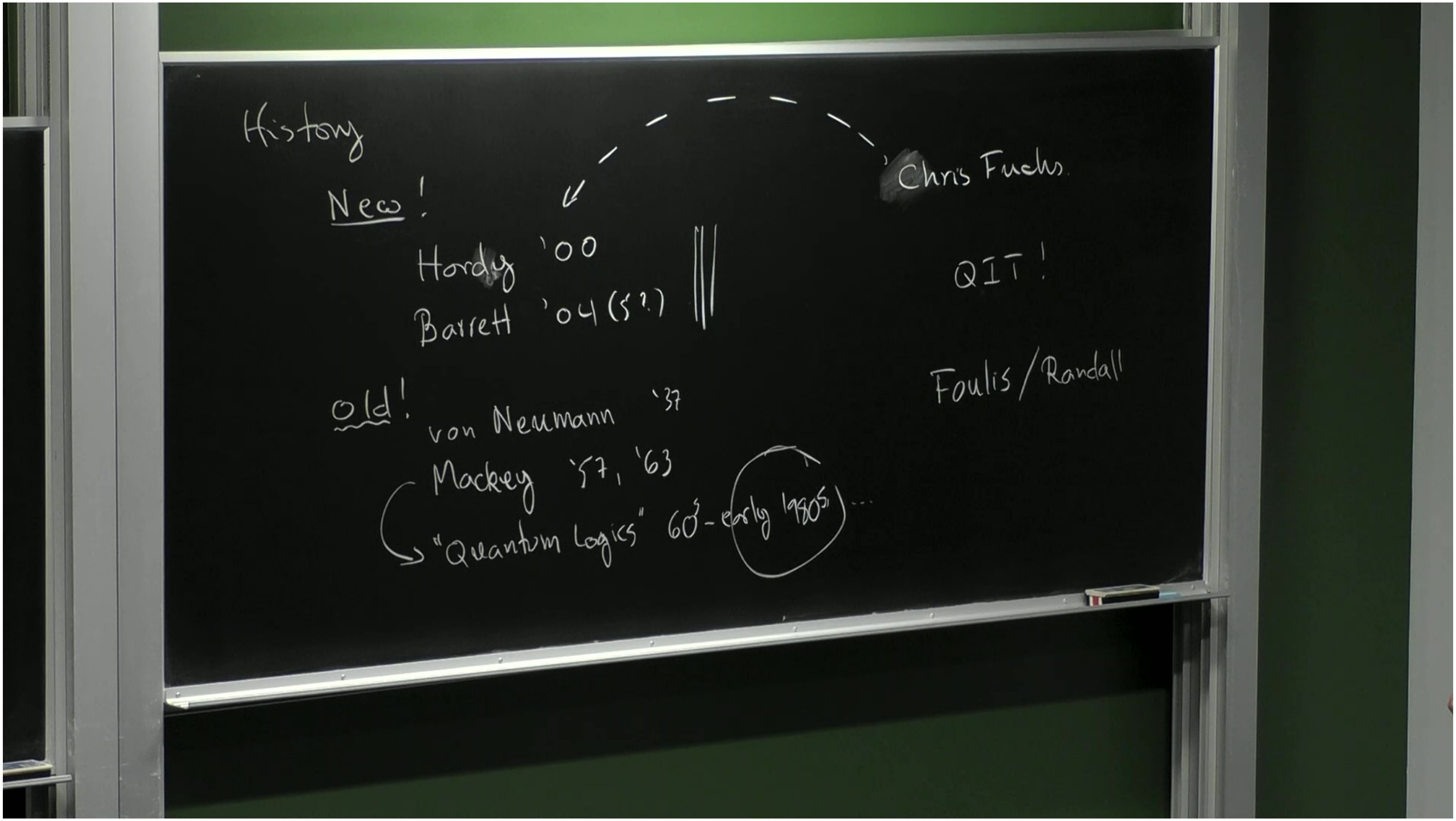
Date: March 12, 2024 - 9:00 AM

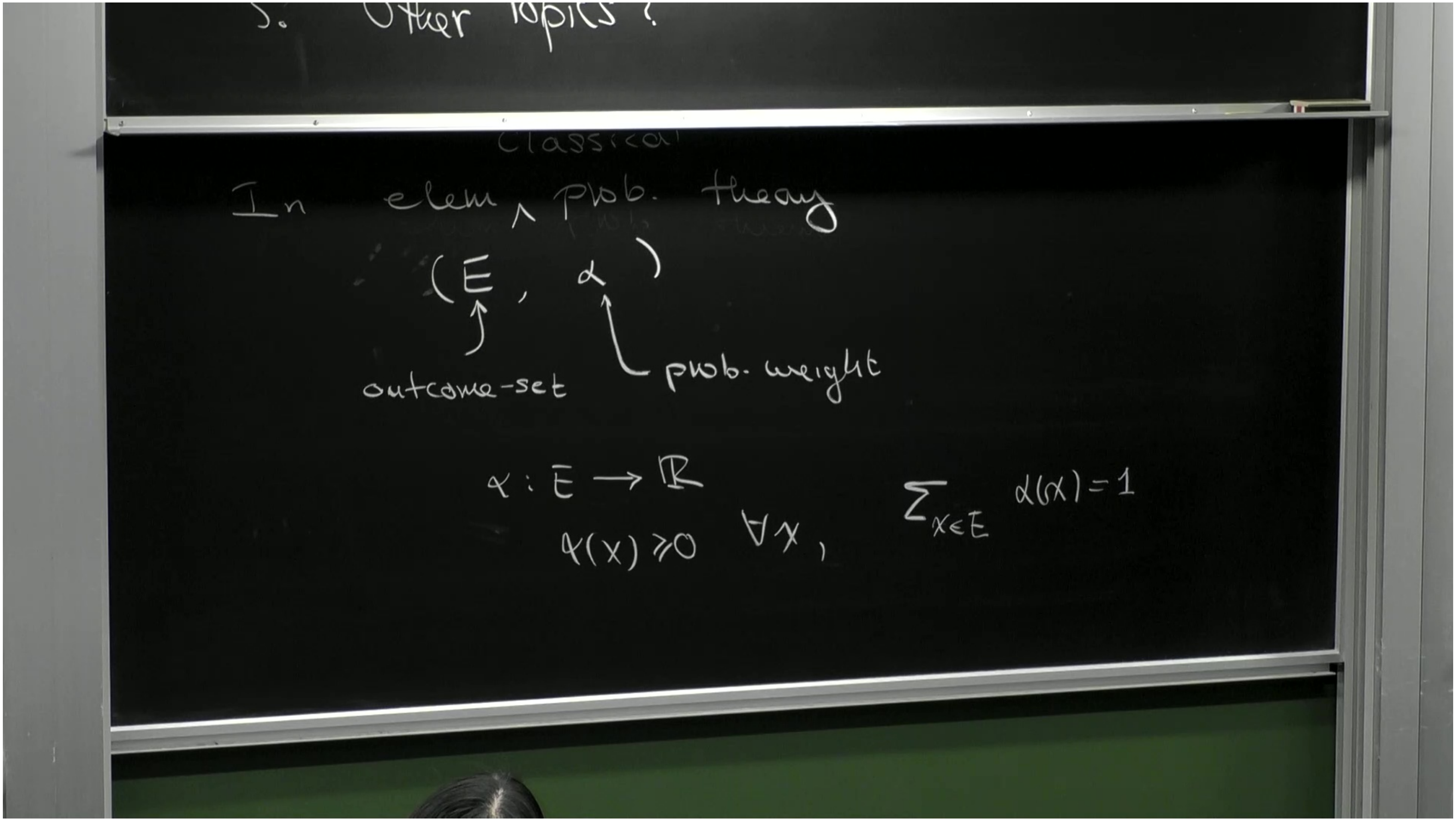
URL: https://pirsa.org/24030095


Alex Wilce
wilce@susqu.edu.
GPTs = General Probability Theory.

Plan

1. Probabilistic models ← Today
2. " " , Linearized
3. Composite models, entanglement
4. Probabilistic theories / category theory.
5. Other topics?


History
New!
Hardy '00
Barrett '04 (5?)
Chris Fuchs
QIT!
Foulis / Randall
Old!
von Neumann '37
Mackey '57, '63
"Quantum Logics" 60s - early 1980s ...


5. Other Topics ?
Classical
In elem. prob. theory
(E, α)
outcome-set
prob. weight
$\alpha : E \to \mathbb{R}$
$\alpha(x) \geq 0 \quad \forall x, \quad \sum_{x \in E} \alpha(x) = 1$

Def: A test space (or manual)

is a collection

$$\mathcal{M} = \{E, F, \ldots\}$$ of outcome-sets.

Let $X = \bigcup \mathcal{M}$; then a prob. weight

is a f'n $\alpha : X \to \mathbb{R}$

$\forall x \in X$ $\alpha(x) \geq 0$, $\sum_{x \in E} \alpha(x) = 1$ $\forall E \in \mathcal{M}$

Def: A *test space* (or *manual*) is a collection

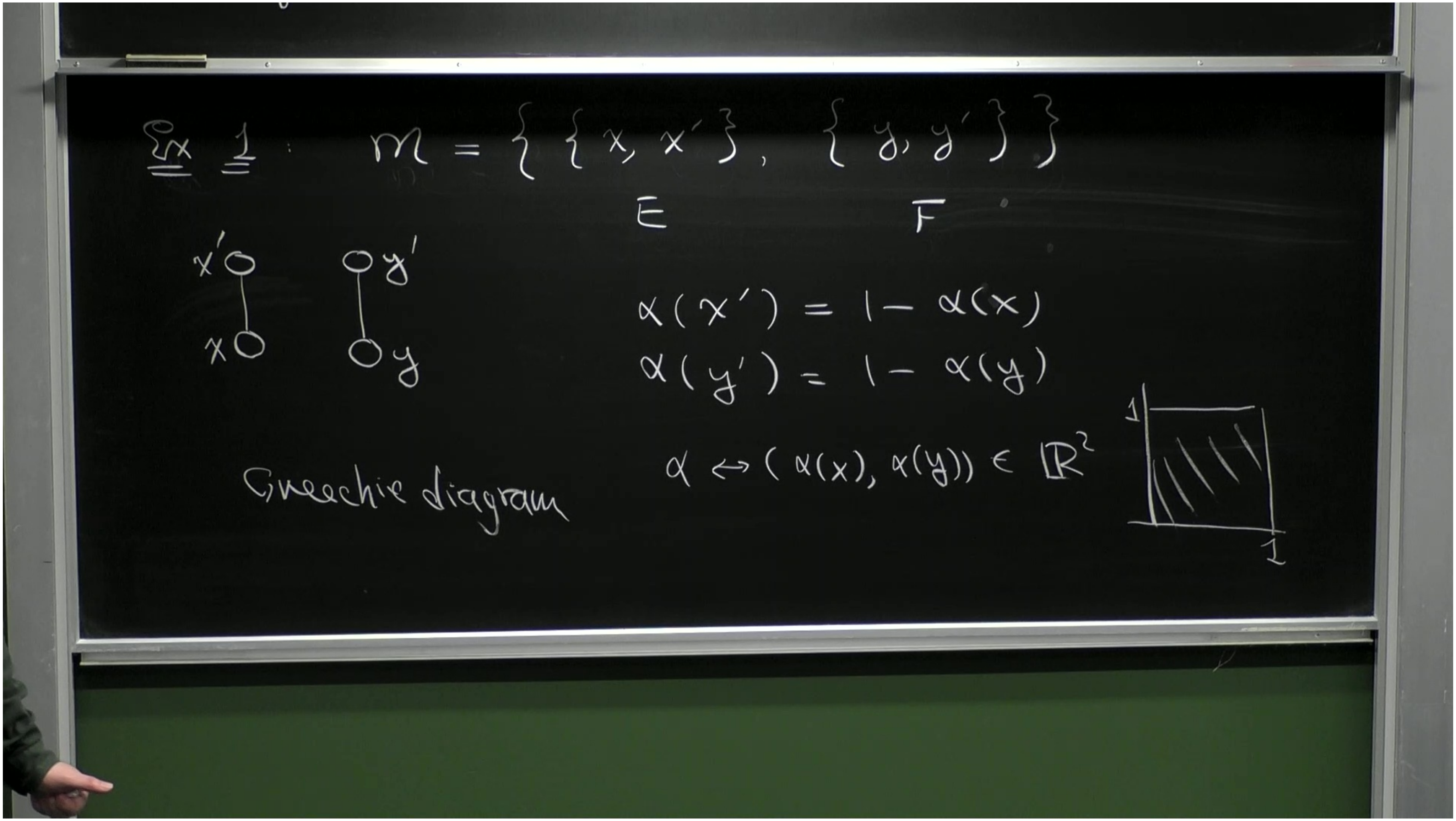
$$\mathcal{M} = \{E, F, \ldots\} \text{ of outcome-sets.}$$

Let $X = \bigcup \mathcal{M}$; then a prob. weight is a f'n $\alpha : X \to \mathbb{R}$

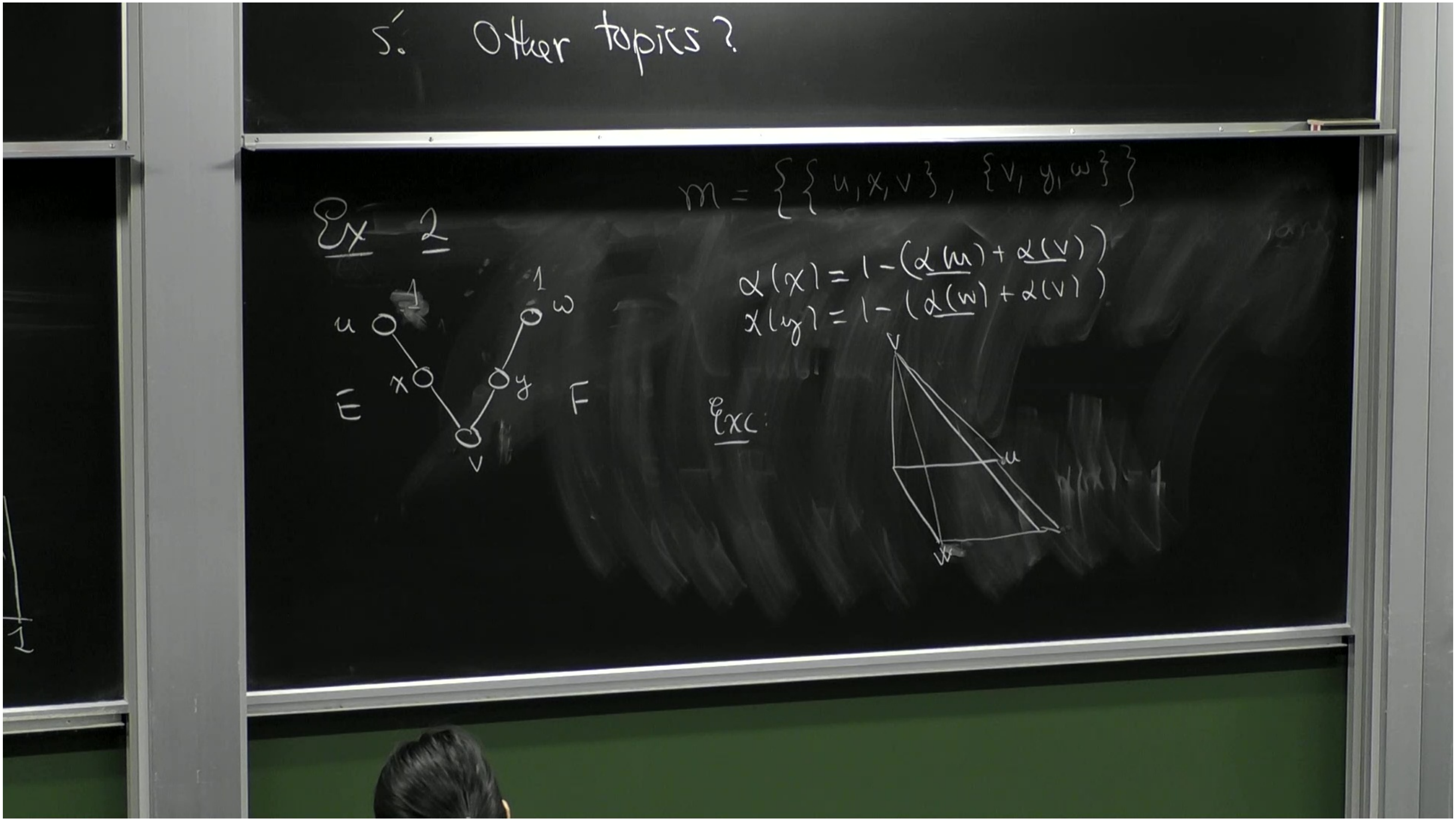
$$\forall x \in X \;\; \alpha(x) \geq 0, \quad \sum_{x \in E} \alpha(x) = 1 \quad \forall E \in \mathcal{M}$$

A prob model is a pair $(\mathcal{M}, \Omega)$

$\Omega \subseteq \Pr(\mathcal{M})$ = all prob. weights.



Ex 1 : M = { {x, x'}, {y, y'} }
E
F
x'
x
y'
y
α(x') = 1 − α(x)
α(y') = 1 − α(y)
Greechie diagram
α ↔ (α(x), α(y)) ∈ ℝ²
1
1


5. Other topics?
m = {{u, x, v}, {v, y, w}}
Ex 2
α(x) = 1 − (α(u) + α(v))
α(y) = 1 − (α(w) + α(v))
Exc:
u
x
v
w
y
E
F
1
1

$\underline{N}$:

* If $E, F \in \mathcal{M}$, $E \subseteq F$

  If $y \in F \setminus E$, $\alpha(y) = 0$ $\forall \alpha \in Pr(\mathcal{M})$

$\therefore$ assume $\mathcal{M}$ "irredundant": $E \subseteq F \Rightarrow E = F$ $\forall E, F \in \mathcal{M}$.

$Pr(\mathcal{M})$ always convex in $\mathbb{R}^{X}$

* Usual to assume for a model $(\mathcal{M}, \Omega)$ that Ω is convex.

Ex 3. Let (S, Σ) be a measurable space

S: set

Σ: a σ-boolean algebra of subsets of S, "events"

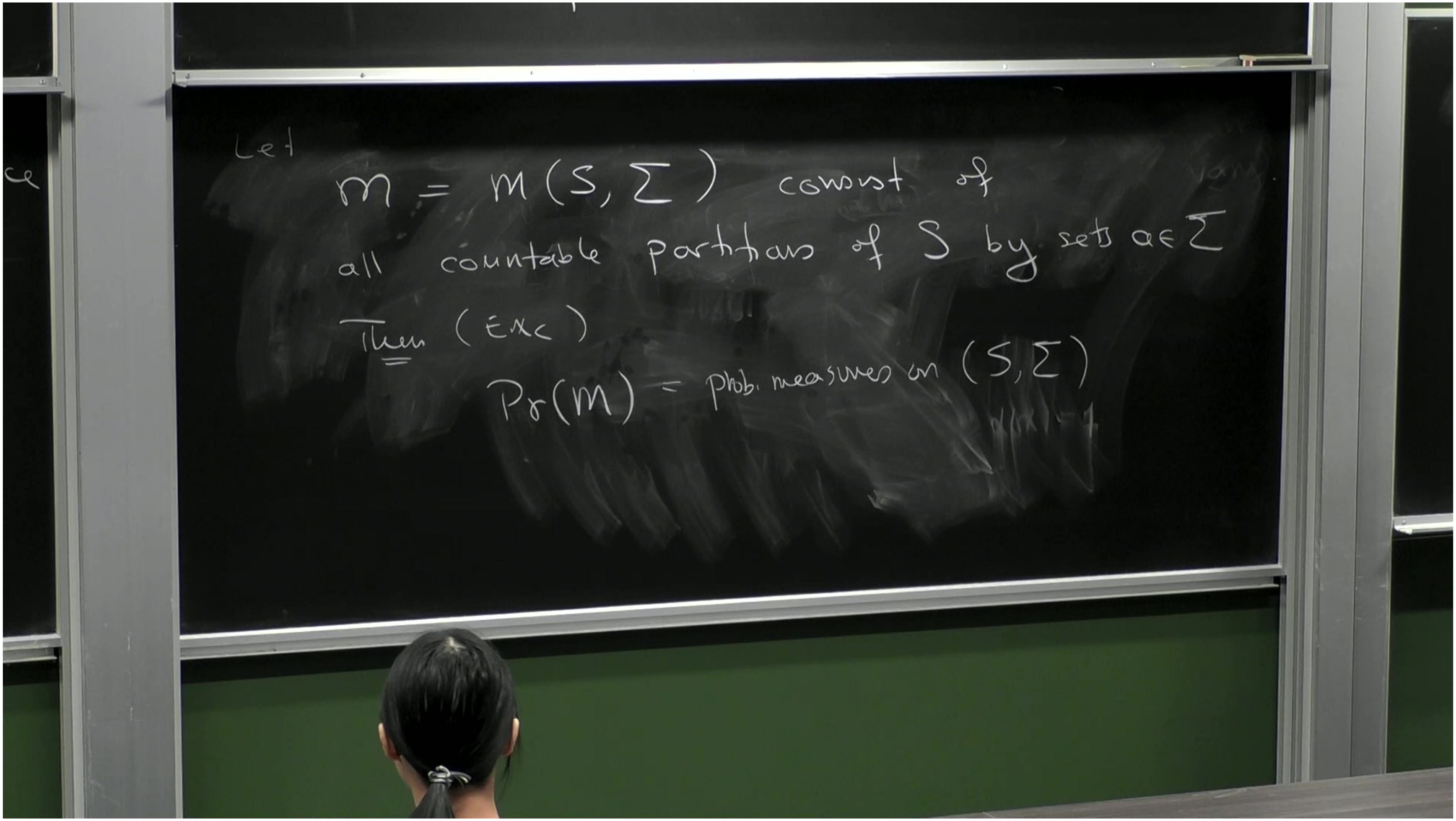
* $a_1, a_2, \ldots \in \Sigma$
  $\bigcup_{i=1}^{\infty} a_i \in \Sigma$
* $\emptyset, S \in \Sigma$
* $a \in \Sigma \Rightarrow a^c = S - a \in \Sigma$.

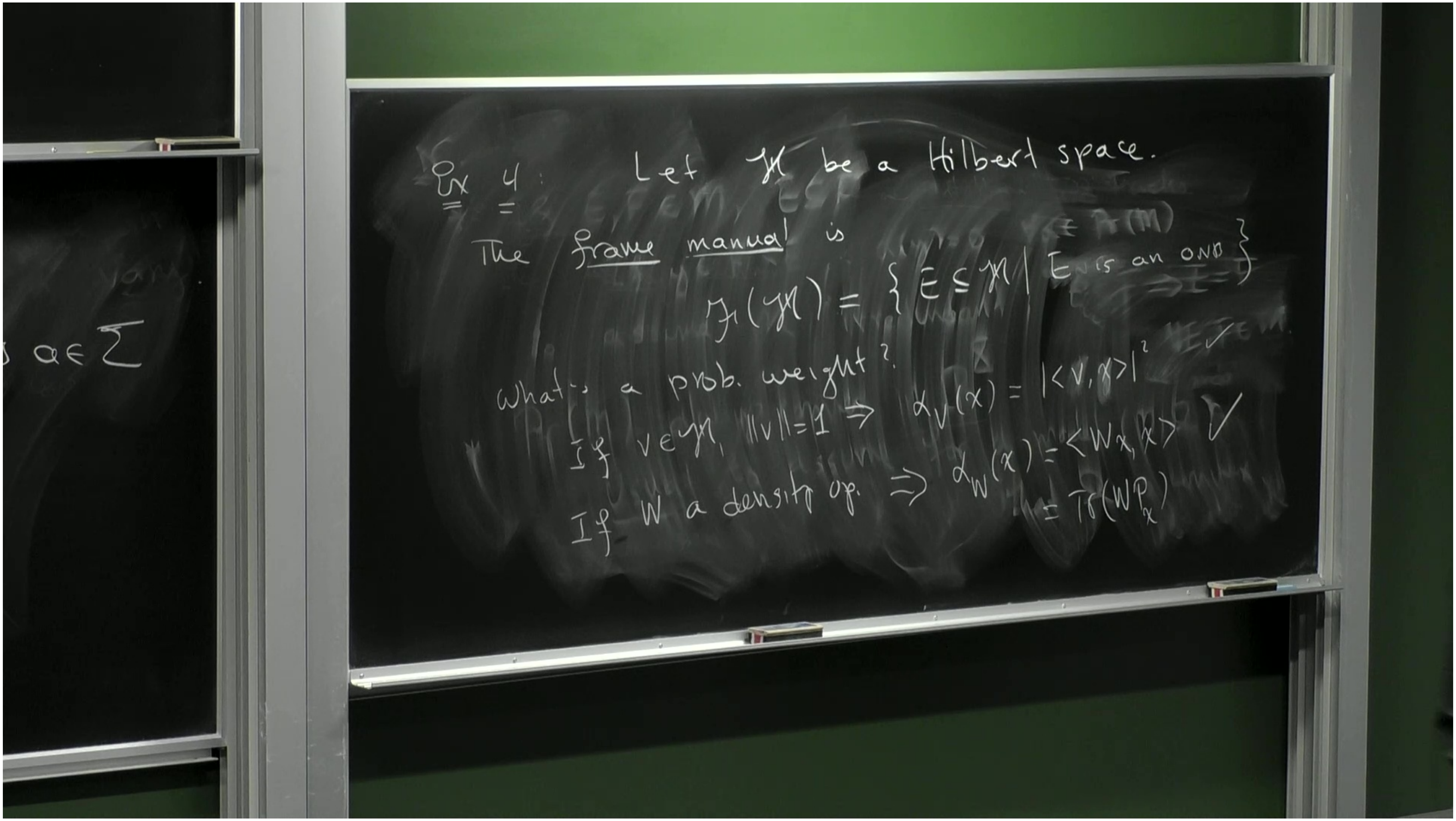
prob. measure: $0 \le \mu(a)$, $\mu(S) = 1$

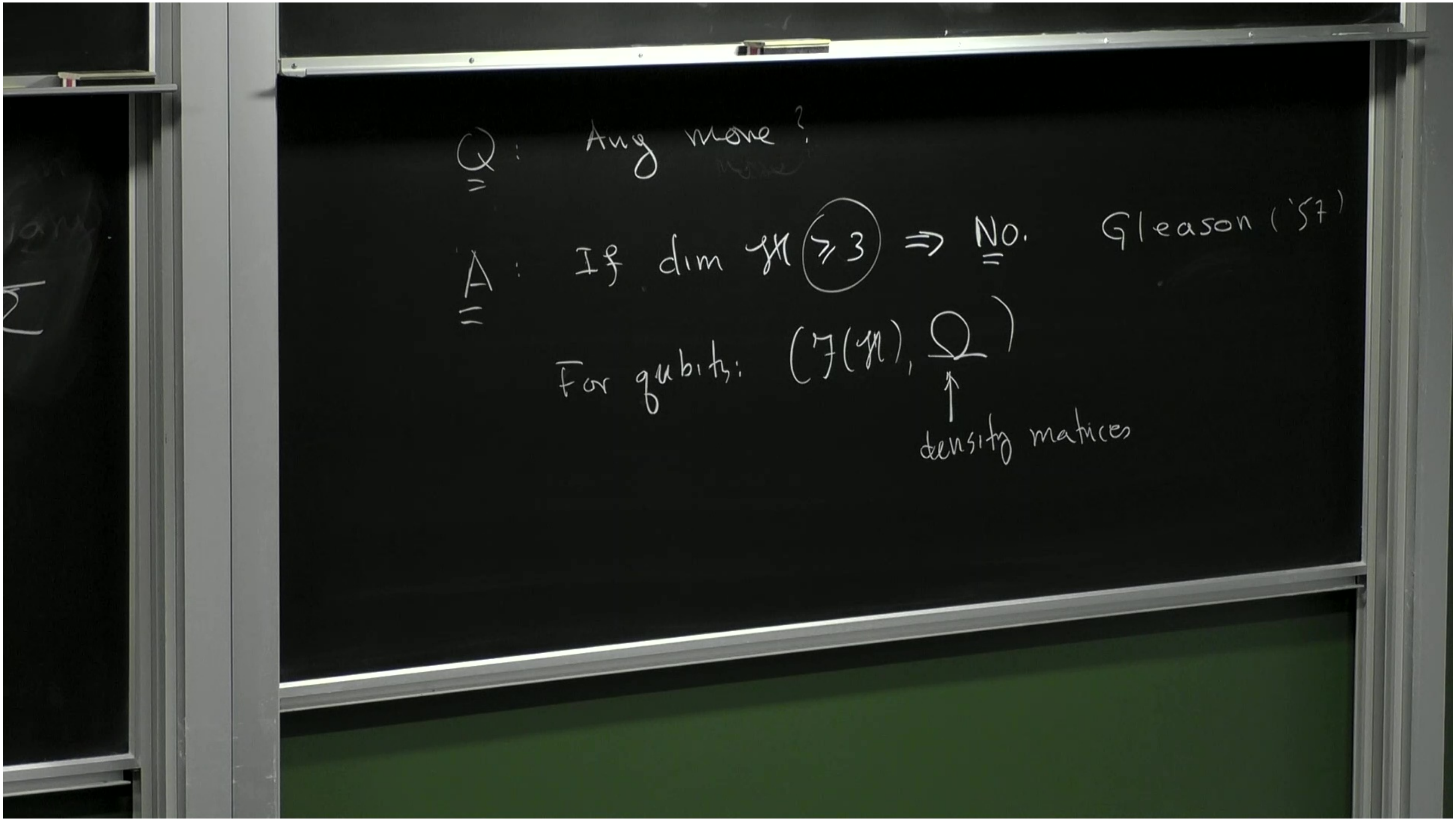
$\mu : \Sigma \to \mathbb{R}$

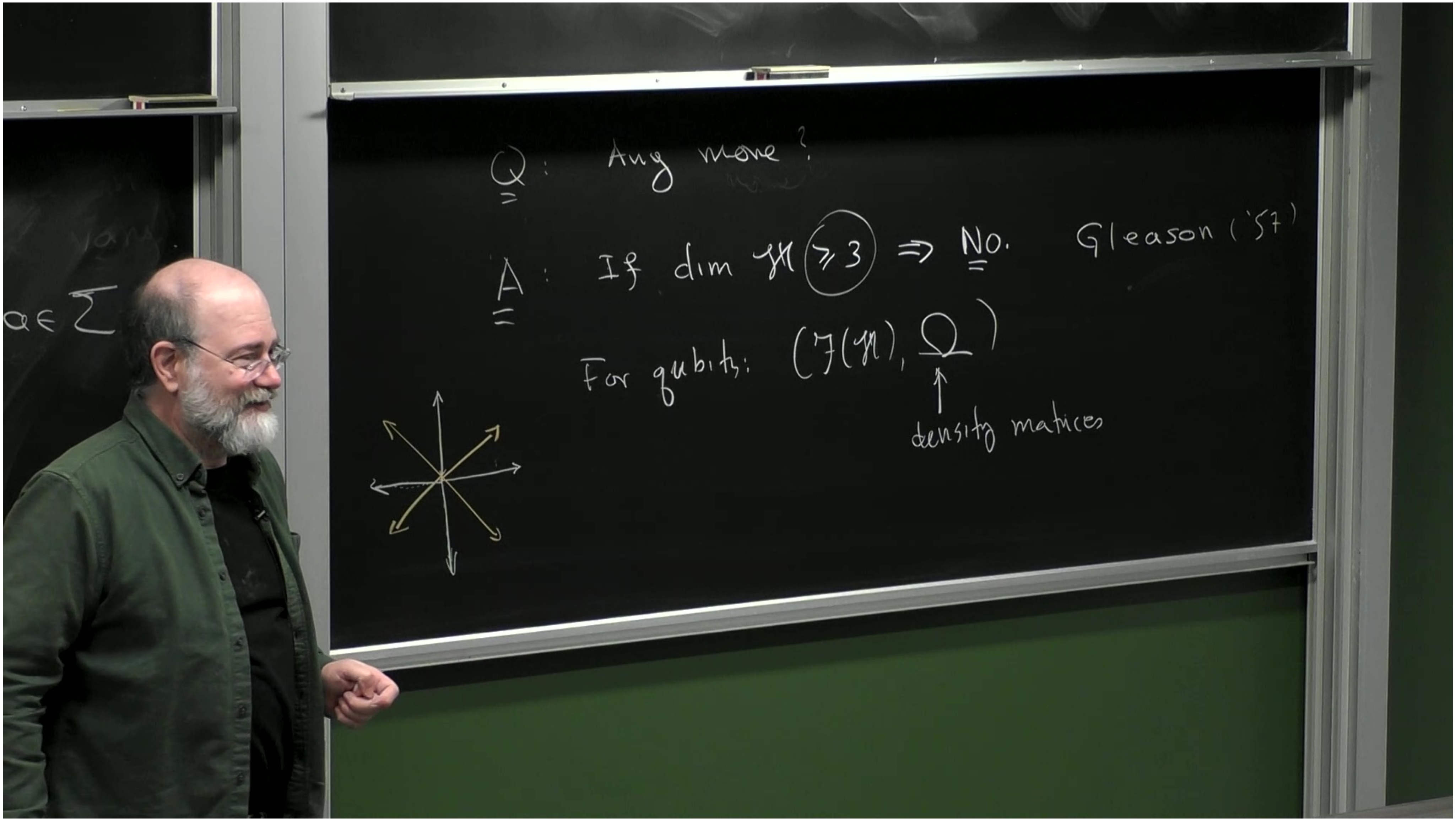
$\{a_i\}$ pairwise disjoint

$\Rightarrow \sum_{i=1}^{\infty} \mu(a_i) = \mu\left(\bigcup_{i=1}^{\infty} a_i\right)$


Let
$\mathcal{M} = \mathcal{M}(S, \Sigma)$ consist of
all countable partitions of S by sets $a \in \Sigma$
Then (Exc)
$Pr(\mathcal{M})$ = prob. measures on (S, Σ)


a ∈ Σ
Ex 4: Let ℋ be a Hilbert space.
The frame manual is
𝓕(ℋ) = { E ⊆ ℋ | E is an ONB }
What's a prob. weight?
If v ∈ ℋ, ||v|| = 1 ⇒ α_v(x) = |⟨v, x⟩|²
If W a density op. ⇒ α_W(x) = ⟨Wx, x⟩
= Tr(WP_x)


Q: Any more ?
A: If dim $\mathcal{H} \geq 3 \Rightarrow$ No. Gleason ('57)
For qubits: $(\mathcal{P}(\mathcal{H}), \Omega)$
density matrices


Q: Any more?
A: If dim $\mathcal{H}$ ≥ 3 $\Rightarrow$ No. Gleason ('57)
For qubits: $(\mathcal{T}(\mathcal{H}), \Omega)$
density matrices