Title: Hierarchy construction for fractional quantum hall states via condensable algebras

Speakers:

Collection: Higher Categorical Tools for Quantum Phases of Matter

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Simons Collaboration on Ultra-Quantum Matter

HARVARD UNIVERSITY

Hierarchy construction of FQH states via condensable algebras

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Harvard University

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With Xiao-Gang Wen and Ashvin Vishwanath

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Outline

- Crash course on the fractional quantum Hall effect (FQHE)

  FQHE as $U(1)$ enriched PMTC

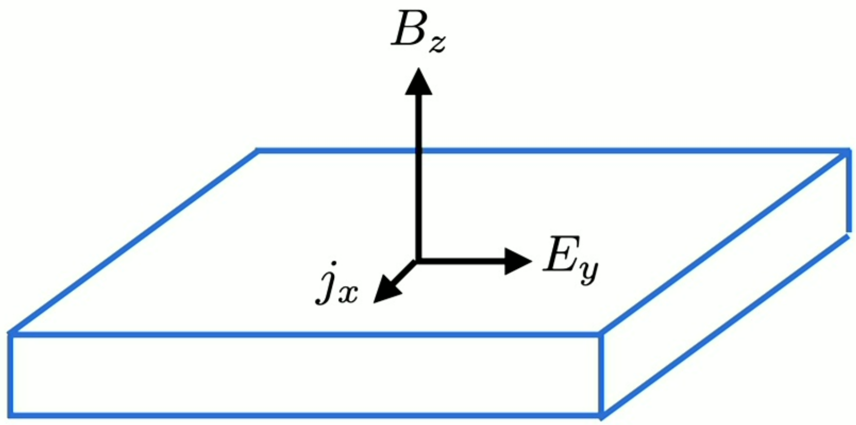
- Families of FQHE

  Wavefunction/effective field theory approach (briefly)

  Condensable subgroup/algebra approach (main)

- Examples

What is the FQHE?



$$j_x = \sigma_{xy} E_y$$

$$\sigma_{xy} = \frac{\nu}{2\pi}$$

↑

quantized!

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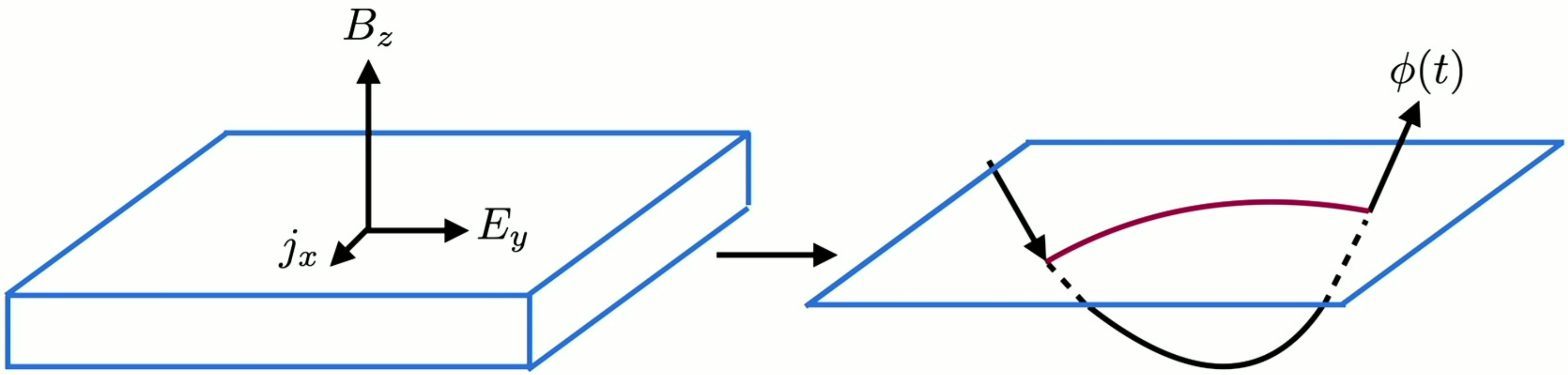
Flux insertion



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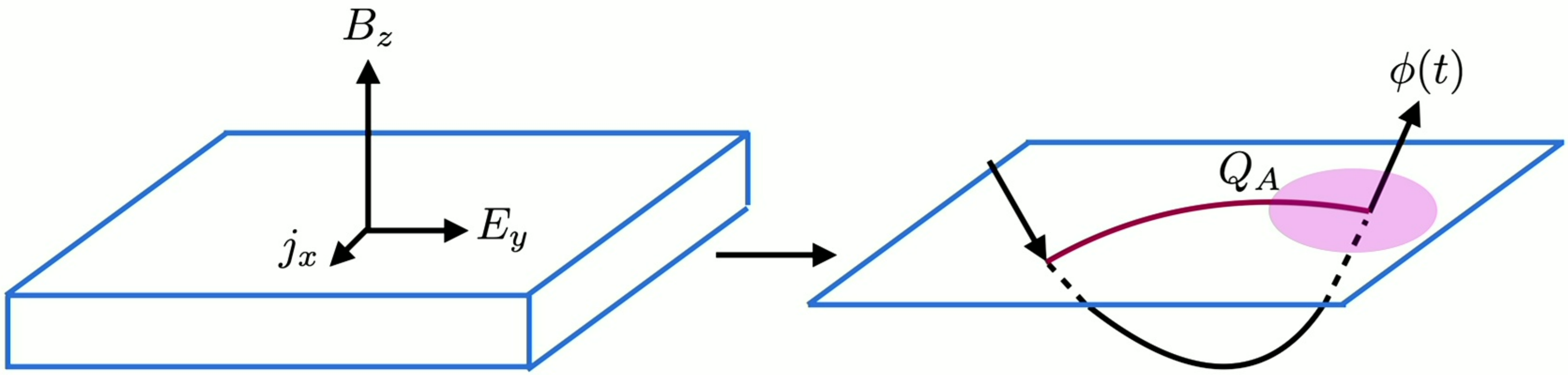
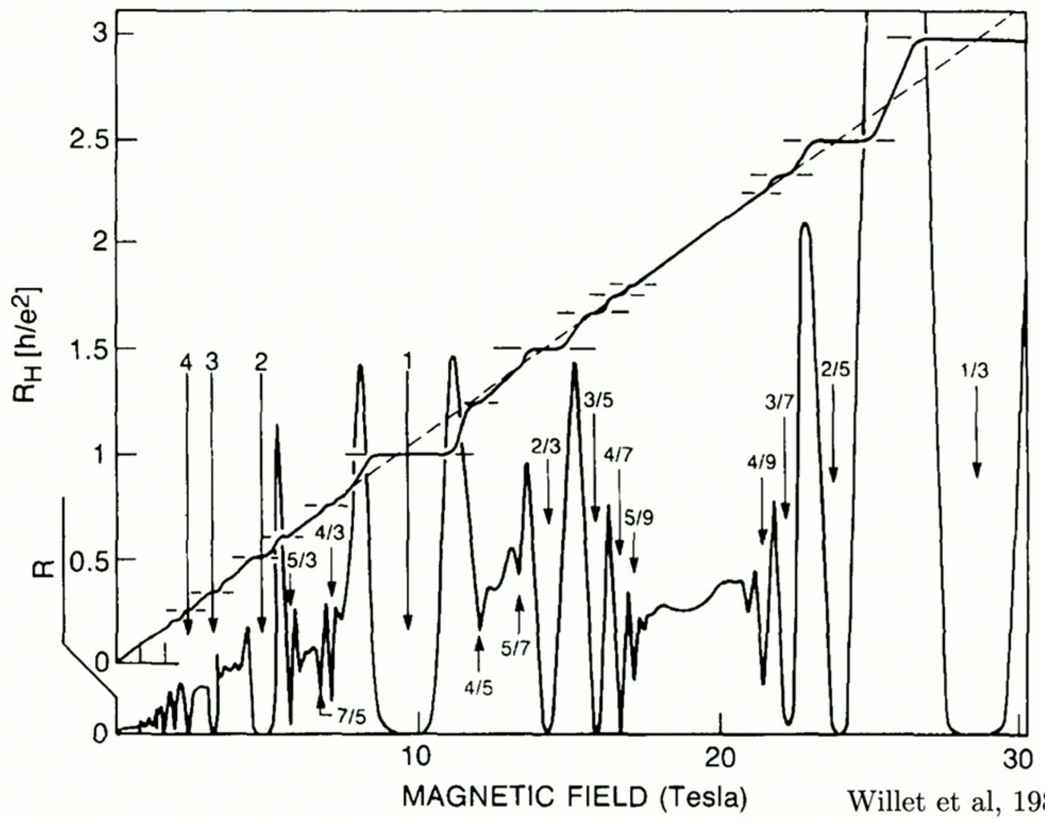
Flux insertion



$$\Delta Q_A = (\Delta\phi)\cdot\sigma_{xy}$$
$$= 2\pi\sigma_{xy} = \nu$$

Real materials!



Willet et al, 1987

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FQH states as U(1) enriched PMTC

- PMTC/fermionic anyon theory

  Simple objects $\{a\}$, fusion coefficients $N^c_{a,b}$, F and R symbols

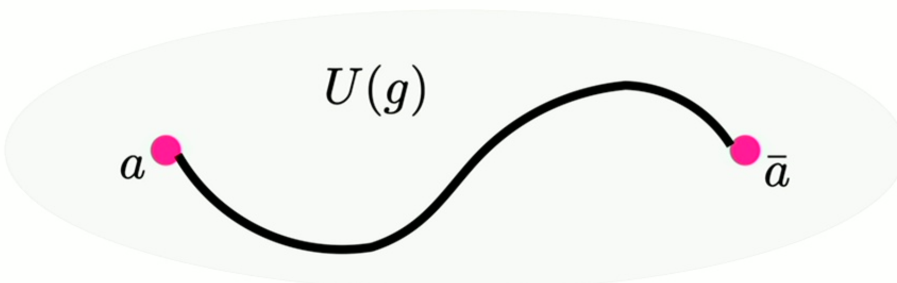
FQH states as U(1) enriched PMTC

- PMTC/fermionic anyon theory

  Simple objects $\{a\}$, fusion coefficients $N_{a,b}^{c}$, F and R symbols

- $U(1)$ symmetry fractionalization data

  Symmetry fractionalization: $U_a(g_1)U_a(g_2) = \omega_a(g_1, g_2)U_a(g_1 g_2)$



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FQH states as U(1) enriched PMTC

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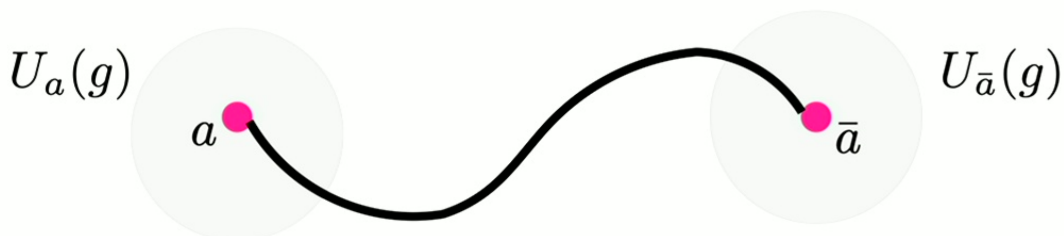

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FQH states as U(1) enriched PMTC

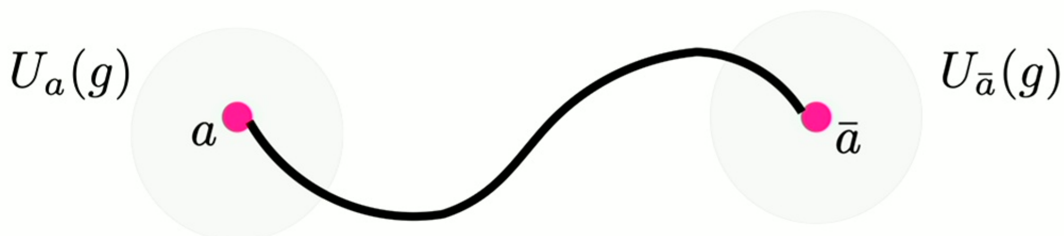
- PMTC/fermionic anyon theory

  Simple objects $\{a\}$, fusion coefficients $N^c_{a,b}$, F and R symbols

- $U(1)$ symmetry fractionalization data

  Symmetry fractionalization: $U_a(g_1)U_a(g_2) = \omega_a(g_1, g_2)U_a(g_1 g_2)$

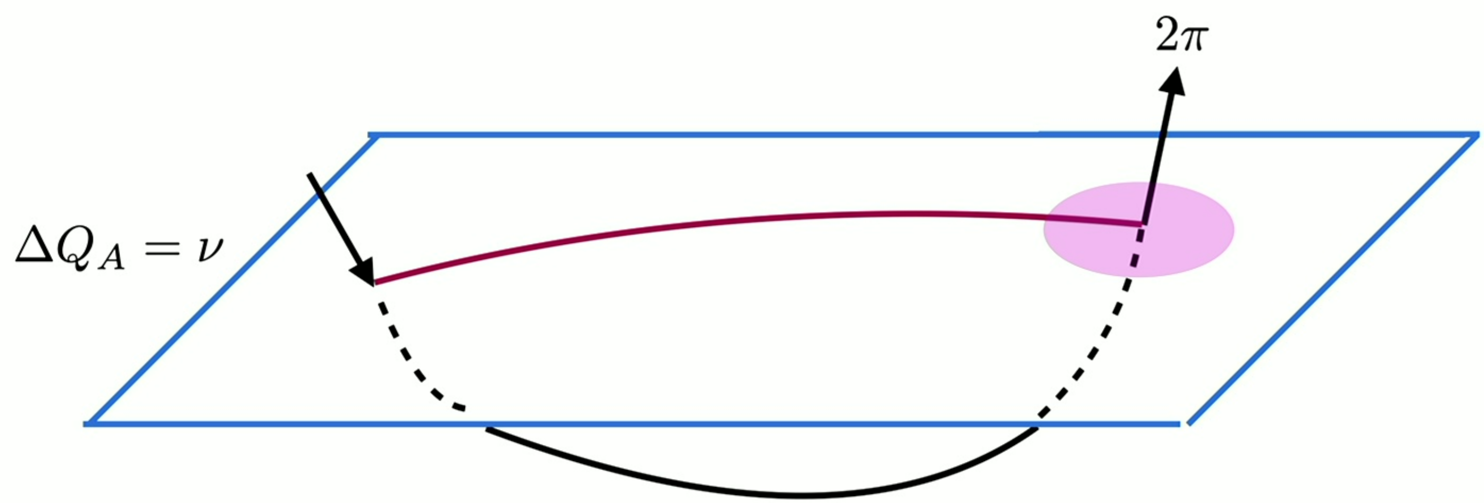
  $$U_a(\pi)U_a(\pi) = e^{2\pi i q_a}U_a(0) = B(a,v)U_a(0)$$



Meaning of U(1) frac data

$$U_a(\pi)U_a(\pi) = e^{2\pi i q_a} U_a(0) = B(a,v)U_a(0)$$

Filling fraction: $\theta_v = e^{\pi i \nu}$



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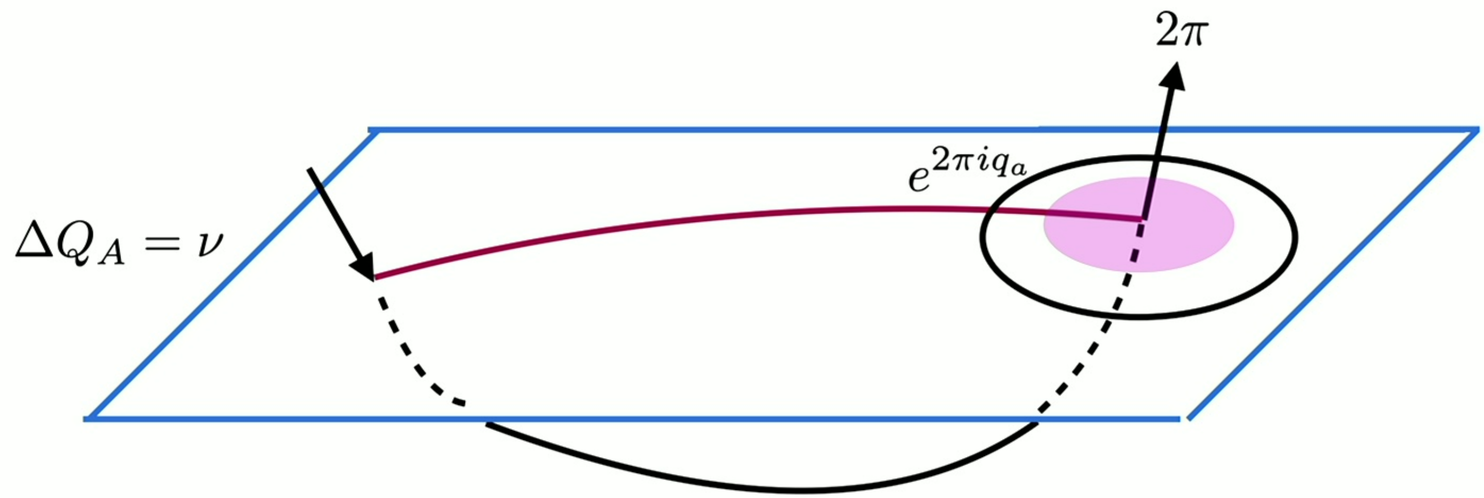

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Meaning of U(1) frac data

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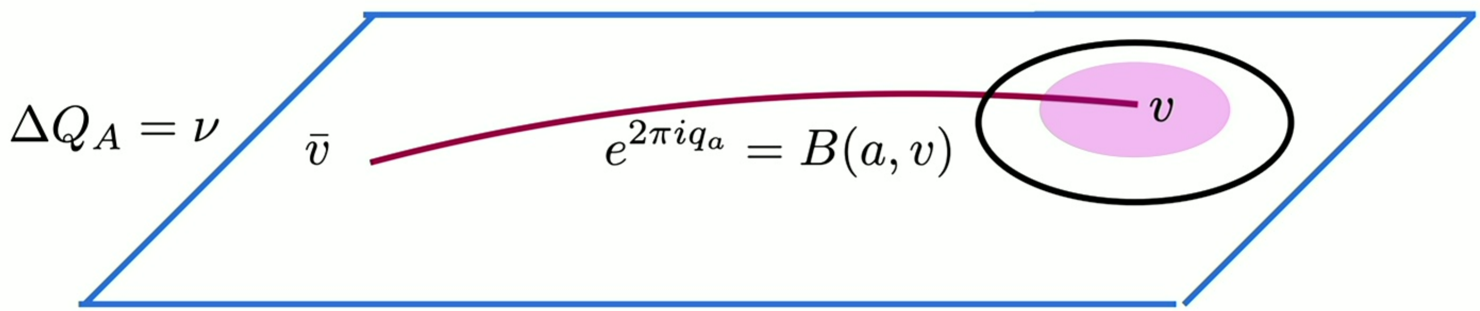

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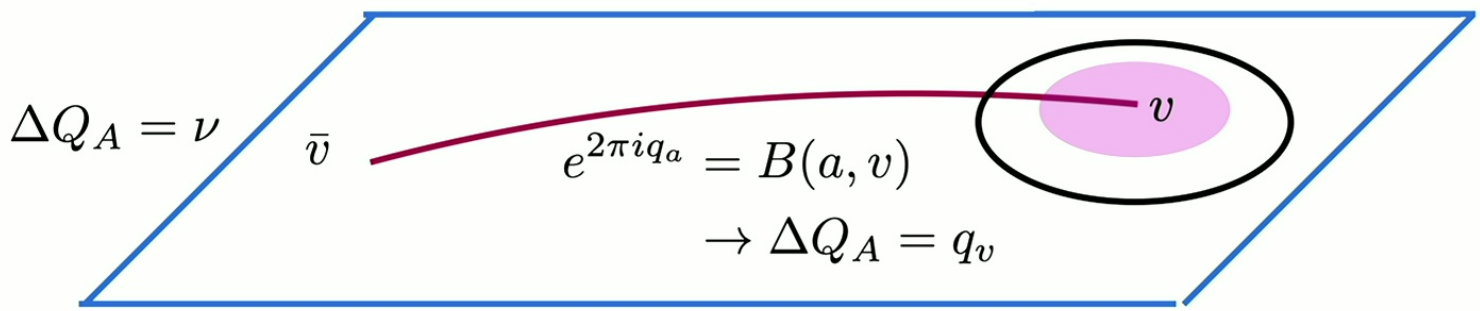
Filling fraction: $\theta_v = e^{\pi i \nu}$

Minimal quasihole: anyon with smallest positive charge
Minimal quasiparticle: anyon with smallest negative charge



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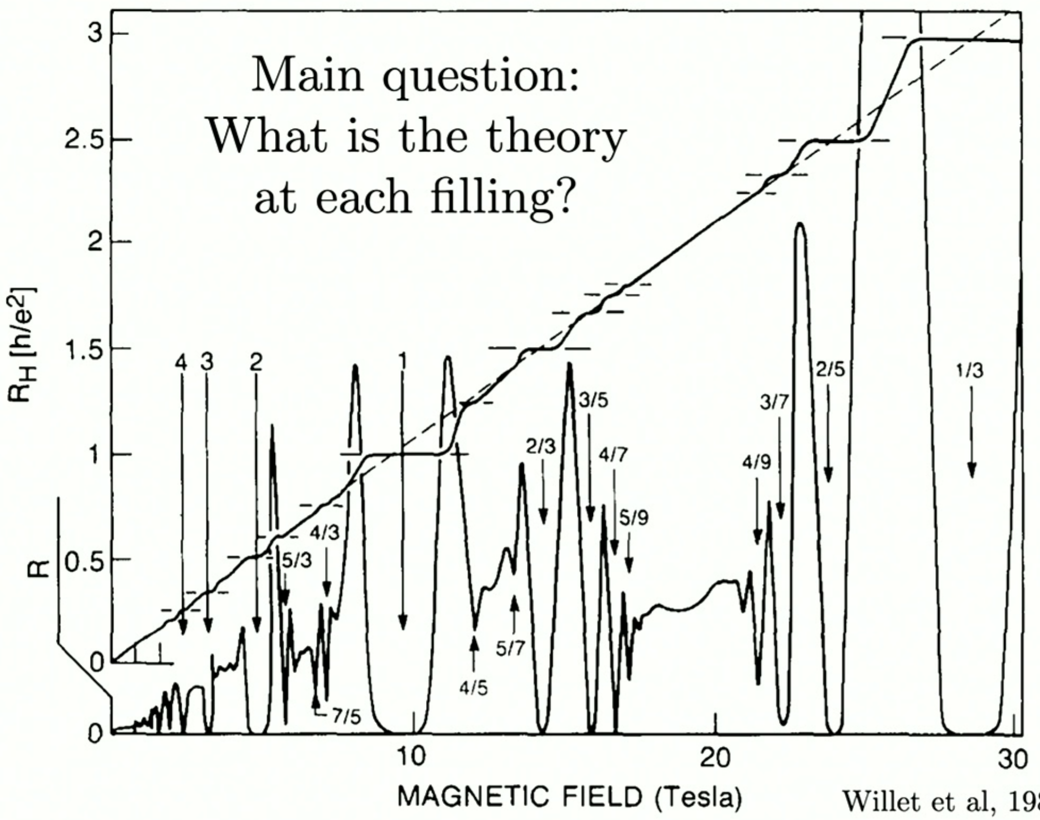
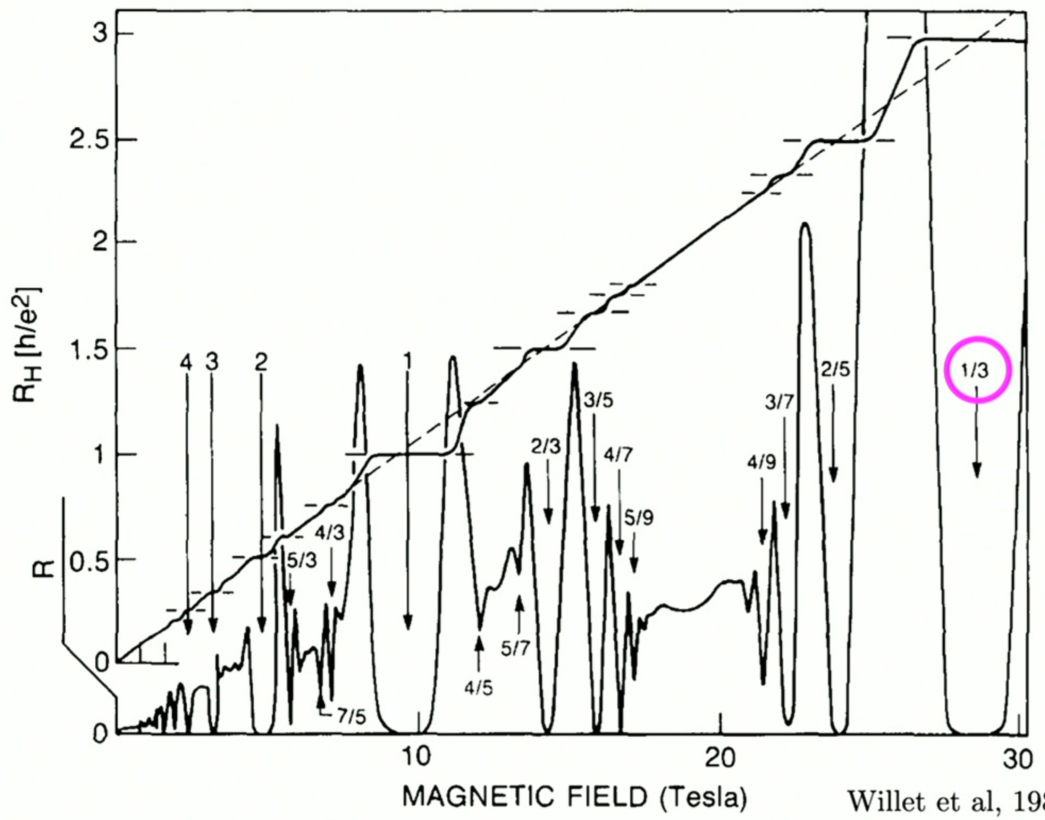
What is realized in experiment?



Willet et al, 1987

1/3 filling in experiment



Willet et al, 1987

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Example: 1/3 filling

- Anyon theory: fermionic abelian anyons

$$a^n \times a^m = a^{n+m}, \theta_{a^n} = e^{2\pi i n^2/6}, B(a^n, a^m) = e^{2\pi i nm/3}$$

$$a^3 = \text{transparent fermion}$$

Example: 1/3 filling

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  $a^3 =$ transparent fermion

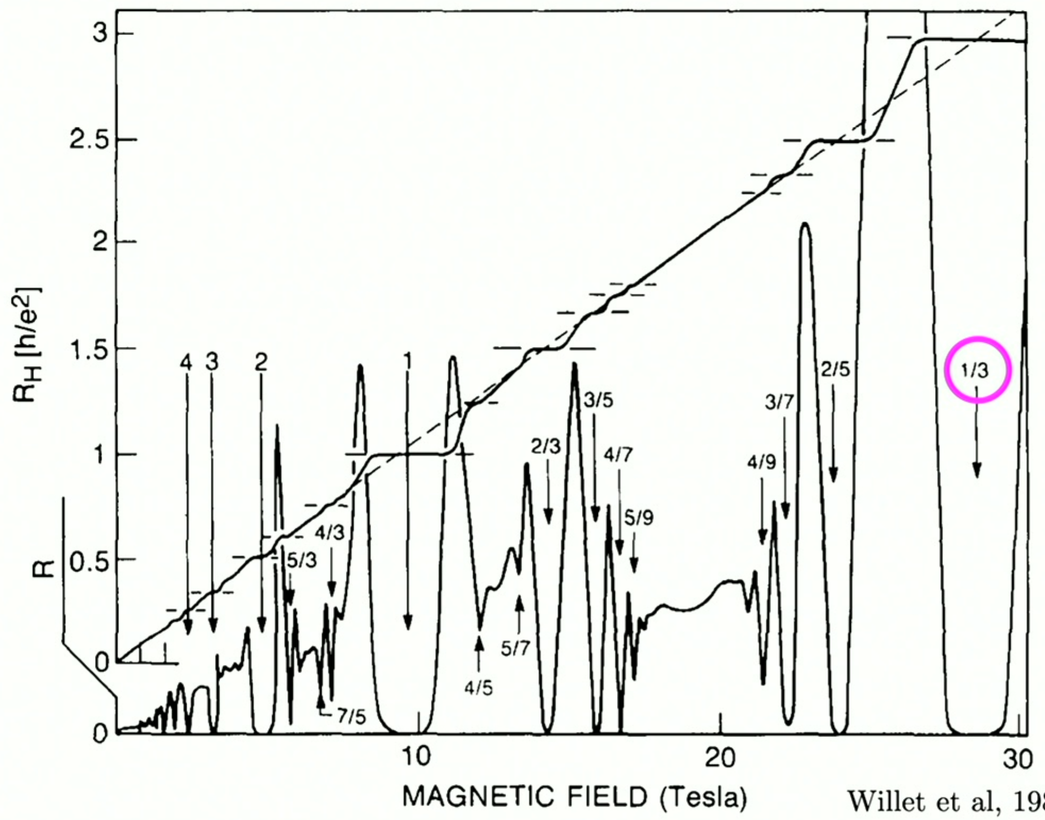
  $U(1)_3$ CS theory $\mathcal{L} = -\frac{3}{4\pi} a_\mu \partial_\nu a_\lambda \epsilon^{\mu\nu\lambda}$

- $U(1)$ fractionalization

  $v = a \to q_{a^n} = \frac{n}{3}$

  $\theta_a = e^{2\pi i/6} = e^{\pi i \nu} \to \nu = \frac{1}{3}$

1/3 filling in experiment



Willet et al, 1987

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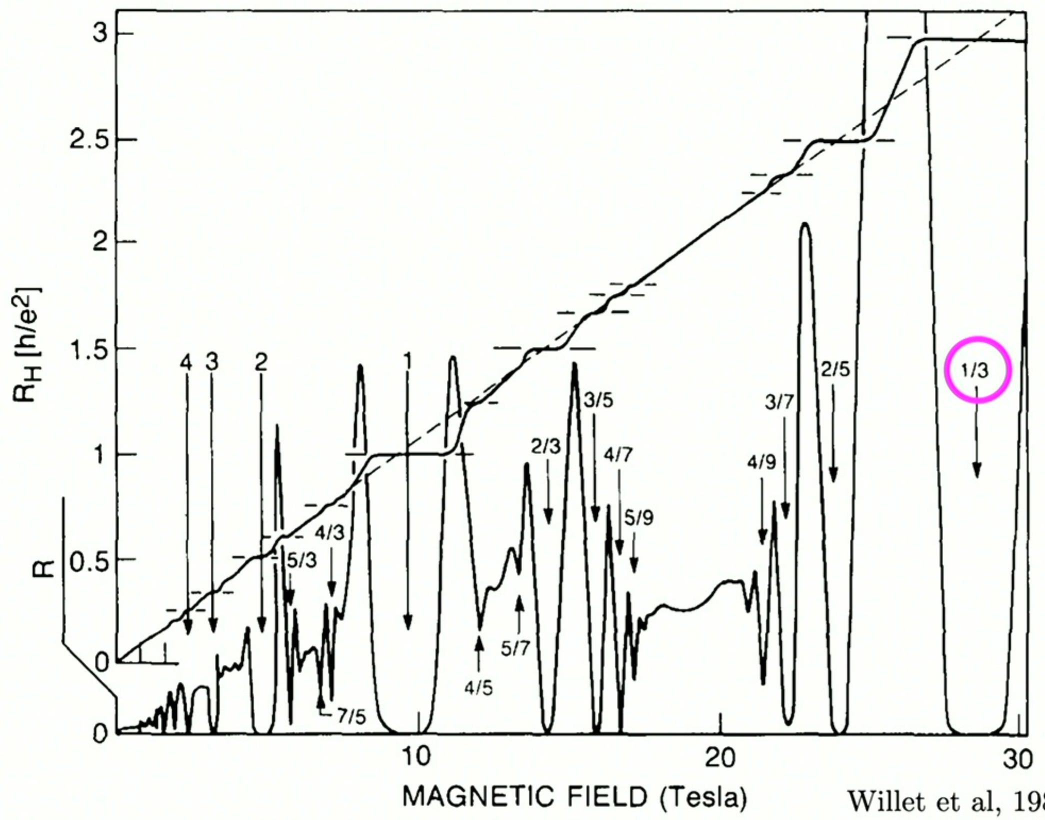

Willet et al, 1987

- Small rank
- Small c_-
- Abelian

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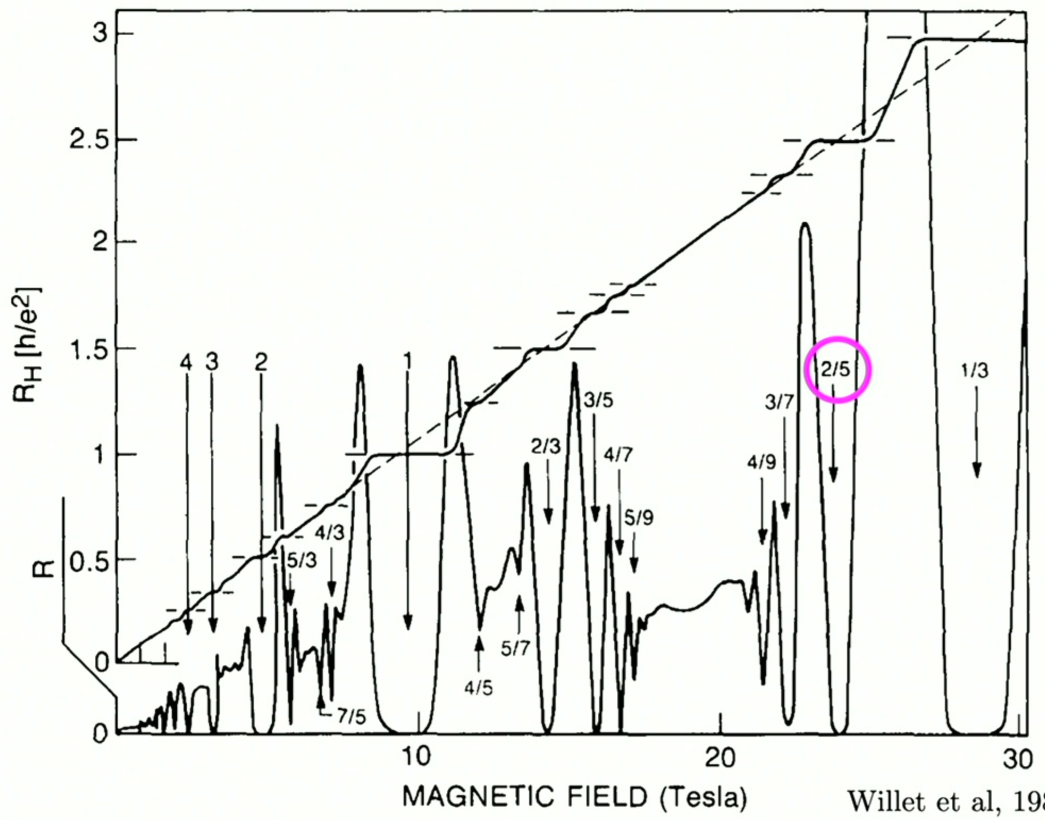
Other simple FQH states



Willet et al, 1987

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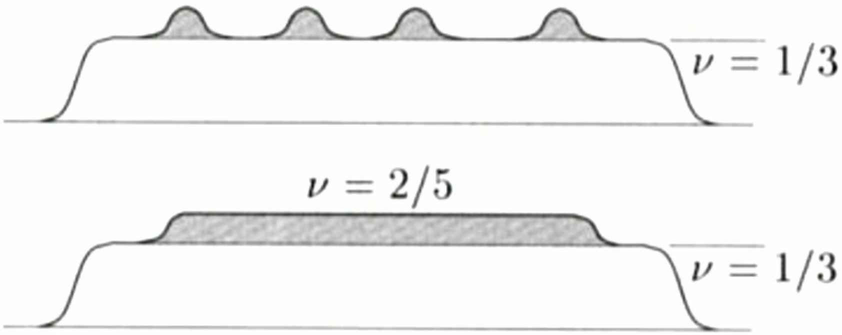
FQH states come in families



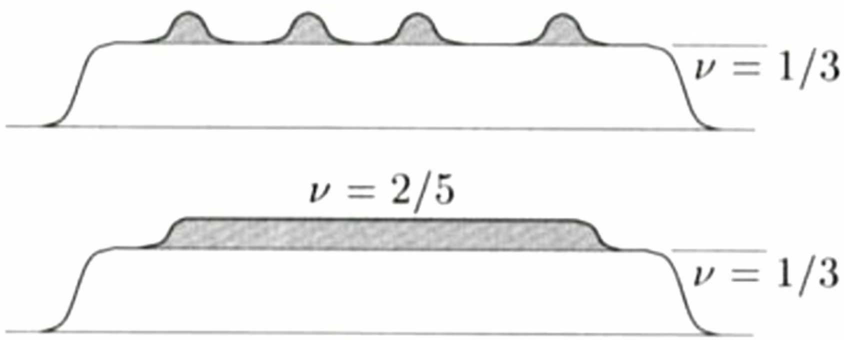
Wen, 2004

Haldane, 1983
Halperin, 1984
...
Hansson et al, RMP 2017

$$\mathcal{L} = -\frac{m}{4\pi} a_\mu \partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{4\pi} A_\mu \partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} + n a_\mu j^\mu + \cdots$$

charge $-\frac{n}{3}$ excitation ($n = 1$: minimal quasiparticle)

FQH states come in families



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charge $-\frac{n}{3}$ excitation ($n = 1$: minimal quasiparticle)

$$\mathcal{L} = -\frac{1}{4\pi} K_{IJ} a_{I\mu} \partial_\nu a_{J\lambda} \epsilon^{\mu\nu\lambda} + \frac{q_I}{2\pi} A_\mu \partial_\nu a_I \epsilon^{\mu\nu\lambda} \qquad K = \begin{pmatrix} 3 & -1 \\ -1 & p_2 \end{pmatrix}$$

FQH states come in families

$$\mathcal{L} = -\frac{1}{4\pi} K_{IJ} a_{I\mu} \partial_\nu a_{J\lambda} \epsilon^{\mu\nu\lambda} + \frac{q_I}{2\pi} A_\mu \partial_\nu a_I \epsilon^{\mu\nu\lambda} \qquad K = \begin{pmatrix} 3 & -1 \\ -1 & p_2 \end{pmatrix}$$

$$\nu = \frac{p_2}{3p_2 - 1}$$

$p_2 = 1$: $\nu = \frac{2}{5}$

abelian anyon theory generated by anyon with spins $e^{2\pi i 3n^2/10}$ and charge $-\frac{n}{5}$

FQH states come in families

$$\mathcal{L} = -\frac{1}{4\pi} K_{IJ} a_{I\mu} \partial_\nu a_{J\lambda} \epsilon^{\mu\nu\lambda} + \frac{q_I}{2\pi} A_\mu \partial_\nu a_I \epsilon^{\mu\nu\lambda} \qquad K = \begin{pmatrix} 3 & -1 \\ -1 & p_2 \end{pmatrix}$$

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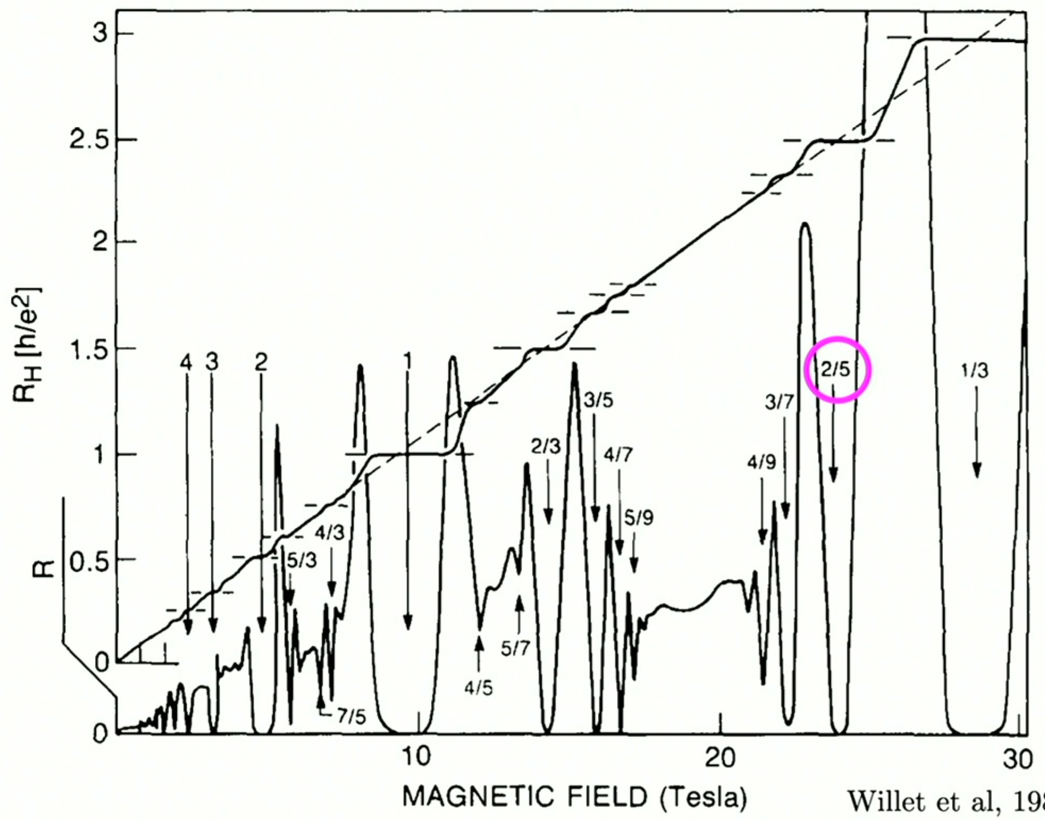
More generally: $\frac{1}{p_1} \rightarrow \frac{p_2}{p_1 p_2 - 1}$

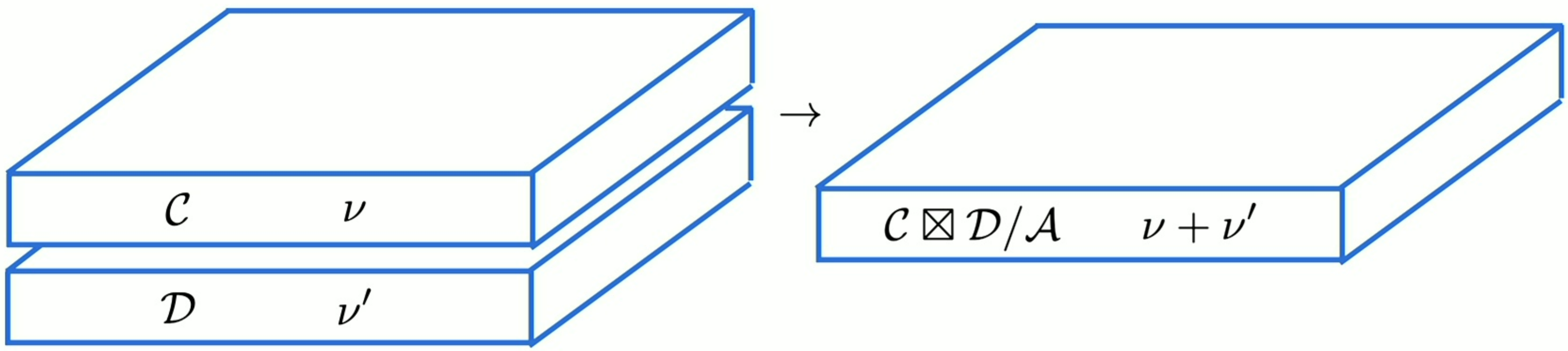
Hierarchy state in experiment



Willet et al, 1987

- Small rank
- Small(ish) c_-
- Abelian
- Closely related to strong plateau

A new approach



Hierarchy states: $\mathcal{C} \boxtimes \mathcal{D}/\mathcal{A}$

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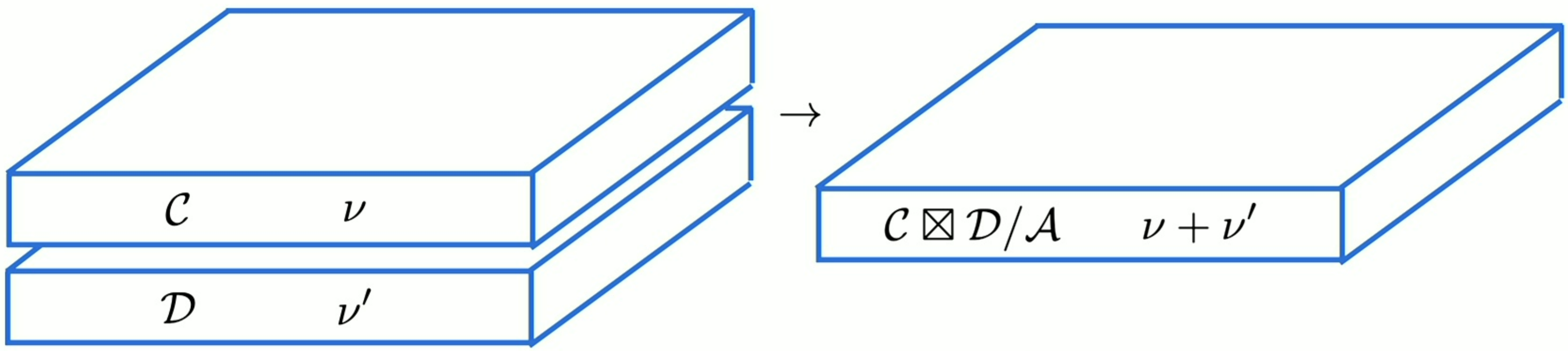
A new approach

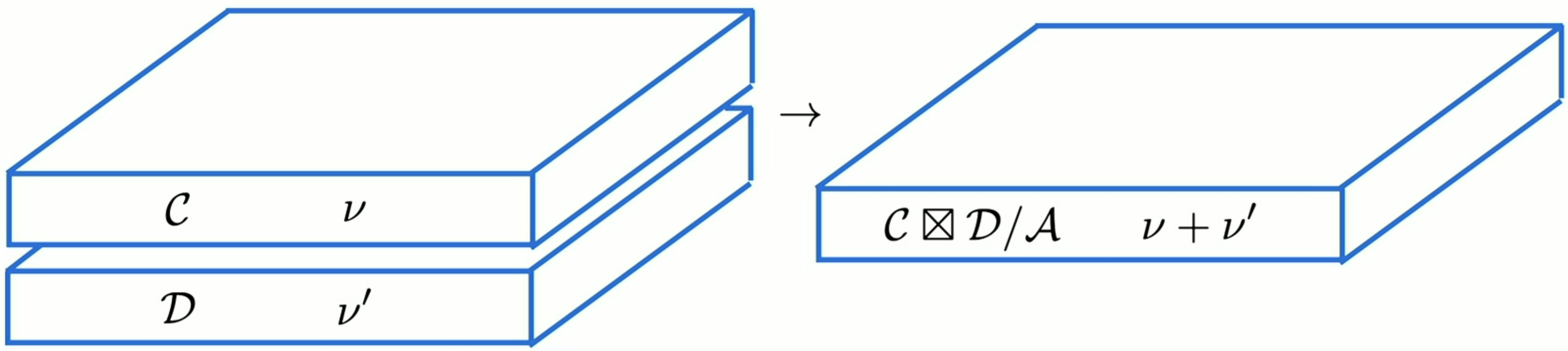


Hierarchy states: $\mathcal{C} \boxtimes \mathcal{D}/\mathcal{A}$

*We will be dealing with fermionic theories, where there isn't as much work done on condensable algebras.

*$\mathcal{A}$ is generally *not* Lagrangian

A new approach



Hierarchy states: $\mathcal{C} \boxtimes \mathcal{D}/\mathcal{A}$

Advantages:

(1) Relatively easy to generalize to nonabelian FQH

(2) Does not use wavefunction/effective field theory

The math

- Find $\mathcal{D}$ such that $\mathcal{C} \boxtimes \mathcal{D}$ has a condensable algebra including a_h or a_p
- Condensed anyons must carry zero electric charge
- $\mathcal{D}$ gives ν' of the correct sign for quasihole/quasiparticle hierarchy

The physics

- $\mathcal{C} \boxtimes \mathcal{D}/\mathcal{A}$ has small(ish) rank
- $\mathcal{D}$ has small c_-
- ν_{tot} is close to ν

Warm-up: condensable subgroups

Def: a subgroup of the abelian anyons with trivial mutual braiding

$\mathcal{C} \boxtimes \mathcal{D}/\mathcal{A}$: group of anyons with trivial braiding with every anyon in $\mathcal{A}$

Example: 1/3 filling

- Anyon theory: fermionic abelian anyons

$$a^n \times a^m = a^{n+m}, \theta_{a^n} = e^{2\pi i n^2/6}, B(a^n, a^m) = e^{2\pi i nm/3}$$

- $U(1)$ fractionalization

$$v = a \to q_{a^n} = \frac{n}{3}$$

$$\theta_a = e^{2\pi i/6} = e^{\pi i \nu} \to \nu = \frac{1}{3}$$

Task: condense minimal quasiparticle a^{-1}

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Try stacking with $U(1)_m$, anyons b^l have spin $e^{2\pi i l^2/2m}$

Example: 1/3 filling

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Condense subgroup generated by $a^{-1}b^l$

$$\left.\frac{1}{6} + \frac{l^2}{2m} = n \qquad n \in \mathbb{Z} \;\right\} a^{-1}b^l \text{ is a boson}$$

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Example: 1/3 filling

$$\left.\frac{1}{6}+\frac{l^2}{2m}=n \qquad n\in\mathbb{Z}\ \right\}a^{-1}b^l \text{ is a boson}$$

$$\frac{l^2}{m}=\frac{6n-1}{3}\to l=\pm(6n-1), m=3(6n-1)$$

$$\left.-\frac{1}{3}+\frac{l}{m}=0\ \right\}a^{-1}b^l \text{ has zero charge}$$

$$\to l=6n-1$$

n=1 hierarchy state

Condense $a^{-1}b^5$ in $U(1)_3 \times U(1)_{15}$

Filling fraction: $\frac{1}{3} + \frac{1}{15} = \frac{2}{5}$

Resulting theory generated by b^{-3}, with spins $e^{2\pi i 3n^2/10}$ and charge $-\frac{n}{5}$

n=1 hierarchy state

Condense $a^{-1}b^5$ in $U(1)_3 \times U(1)_{15}$

Filling fraction: $\frac{1}{3} + \frac{1}{15} = \frac{2}{5}$

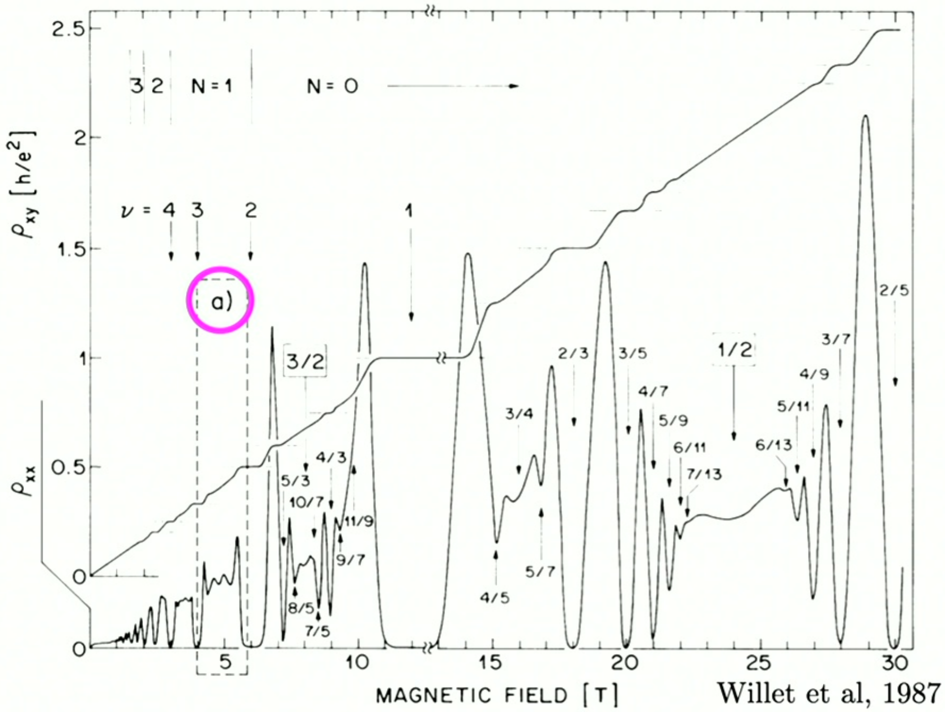
Resulting theory generated by b^{-3}, with spins $e^{2\pi i 3n^2/10}$ and charge $-\frac{n}{5}$

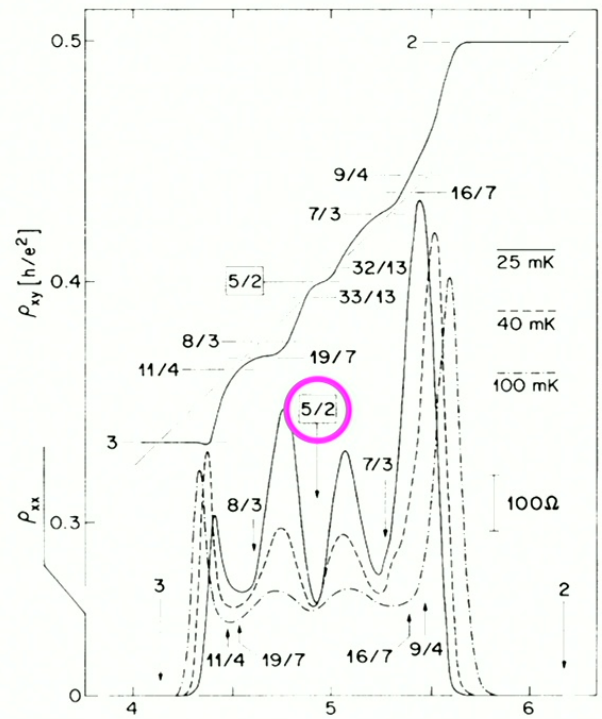
More generally, with $U(1)_{p_1}$ and define $2n = p_2 \rightarrow$

$$\nu_{\text{tot}} = \frac{p_2}{p_1 p_2 - 1}$$

Can perform the same exercise to get quasihole hierarchy $\nu_{\text{tot}} = \frac{p_2}{p_1 p_2 + 1}$

Nonabelian FQH states





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Three candidates for 1/2 filling

Pfaffian: $\text{Ising} \times U(1)_8/\mathbb{Z}_2$

Anti-Pfaffian: $\overline{\text{Ising}} \times U(1)_{-8}/\mathbb{Z}_2$

PH-Pfaffian: $\text{Ising} \times U(1)_{-8}/\mathbb{Z}_2$

Which theory is actually realized in a given experiment?

Three candidates for 1/2 filling

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Anti-Pfaffian: $\overline{\text{Ising}} \times U(1)_{-8}/\mathbb{Z}_2$

PH-Pfaffian: Ising $\times U(1)_{-8}/\mathbb{Z}_2$

Which theory is actually realized in a given experiment?

Solution: measure the hierarchy fillings.

Pfaffian anyon theory

Pfaffian: Ising $\times U(1)_8/\mathbb{Z}_2$

$$\{1, a^2, \psi, \psi a^2, \sigma a, \sigma a^3\} \times \{1, \psi a^4\}$$

Fusion rules: $\psi \times \psi = 1 \qquad (\sigma a) \times (\sigma a) = (1 + \psi) a^2 \qquad \psi \times (\sigma a) = \sigma a$

Spins: $\theta_\sigma = e^{2\pi i/16} \qquad \theta_\psi = -1 \qquad \theta_{a^n} = e^{2\pi i n^2/16}$

Pfaffian U(1) symmetry fractionalization

Pfaffian: Ising $\times U(1)_8/\mathbb{Z}_2$

$$\{1, a^2, \psi, \psi a^2, \sigma a, \sigma a^3\} \times \{1, \psi a^4\}$$

Fractional charge given by braiding with a^2: $B(a^n, a^2) = e^{2\pi i n/4} \to q_{a^n} = \frac{n}{4}$

Minimal quasihole: $q_{\sigma a} = \frac{1}{4}$
Minimal quasiparticle: $q_{\sigma a^{-1}} = -\frac{1}{4}$

$$\theta_{a^2} = e^{2\pi i/4} \to \nu = \frac{1}{2}$$

Pfaffian quasihole hierarchy: first try

$$\{1, a^2, \psi, \psi a^2, \sigma a, \sigma a^3\} \times \{1, \psi a^4\}$$

Task: condense σa

Try stacking with $\mathcal{D} = U(1)_{-m}$

$$\left.\frac{1}{16} + \frac{1}{16} - \frac{l^2}{2m} = n \qquad n \in \mathbb{Z}\ \right\} \sigma a \bar{b}^l \text{ is a boson}$$

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Pfaffian quasihole hierarchy: first try

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$$\frac{l^2}{2m} = n + \frac{1}{8} = \frac{8n+1}{8} \rightarrow l = \pm(8n+1), m = 4(8n+1)$$

Pfaffian quasihole hierarchy: first try

$$\{1, a^2, \psi, \psi a^2, \sigma a, \sigma a^3\} \times \{1, \psi a^4\}$$

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$$\left.\frac{1}{16} + \frac{1}{16} - \frac{l^2}{2m} = n \qquad n \in \mathbb{Z}\ \right\} \sigma a \bar{b}^l \text{ is a boson}$$

$$\frac{l^2}{2m} = n + \frac{1}{8} = \frac{8n+1}{8} \to l = \pm(8n+1), m = 4(8n+1)$$

$$\left.\sigma a \bar{b}^l \times \sigma a \bar{b}^l = (1+\psi) a^2 \bar{b}^{2l}\ \right\} a^2 \bar{b}^{2l} \text{ has spin } -i!$$

None of these can condense!

Pfaffian quasihole hierarchy: second try

$$\{1, a^2, \psi, \psi a^2, \sigma a, \sigma a^3\} \times \{1, \psi a^4\}$$

Task: condense σa

Try stacking with $\mathcal{D} = \overline{\text{Ising}} \times U(1)_{-m}/\mathbb{Z}_2$

Pfaffian quasihole hierarchy: second try

$$\{1, a^2, \psi, \psi a^2, \sigma a, \sigma a^3\} \times \{1, \psi a^4\}$$

Task: condense σa

Try stacking with $\mathcal{D} = \overline{\text{Ising}} \times U(1)_{-m}/\mathbb{Z}_2$

Solutions: $\mathcal{D} = \overline{\text{Ising}} \times U(1)_{-8(16n+1)}/\mathbb{Z}_2$

$n = 1$: $\mathcal{D} = \overline{\text{Ising}} \times U(1)_{-136}/\mathbb{Z}_2$

$$\mathcal{A} = 1 + \psi\bar{\psi} + a^2\bar{b}^3 4 + a^2\bar{b}^{34}\psi\bar{\psi} + \cdots + \sigma\bar{\sigma}a\bar{b}^{17} + \cdots$$

$$\nu_{\text{tot}} = \frac{1}{2} - \frac{1}{2(16n+1)} = \frac{8n}{16n+1}$$

Resulting theory generated by $\bar{\psi}\bar{b}^4$ with spin $e^{15\pi i/17}$ and charge $\frac{1}{17}$

Filling 1/2 hierarchies

Pfaffian: $\text{Ising} \times U(1)_8/\mathbb{Z}_2$ Levin & Halperin 2009

$$\text{quasihole: } \frac{8}{17}, \text{ quasiparticle: } \frac{7}{13}$$

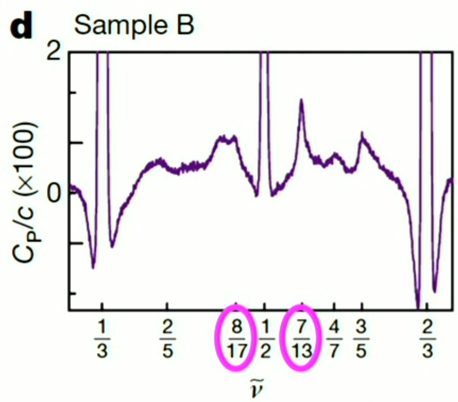
Anti-Pfaffian: $\overline{\text{Ising}} \times U(1)_{-8}/\mathbb{Z}_2$ Levin & Halperin 2009

$$\text{quasihole: } \frac{9}{17}, \text{ quasiparticle: } \frac{6}{13}$$

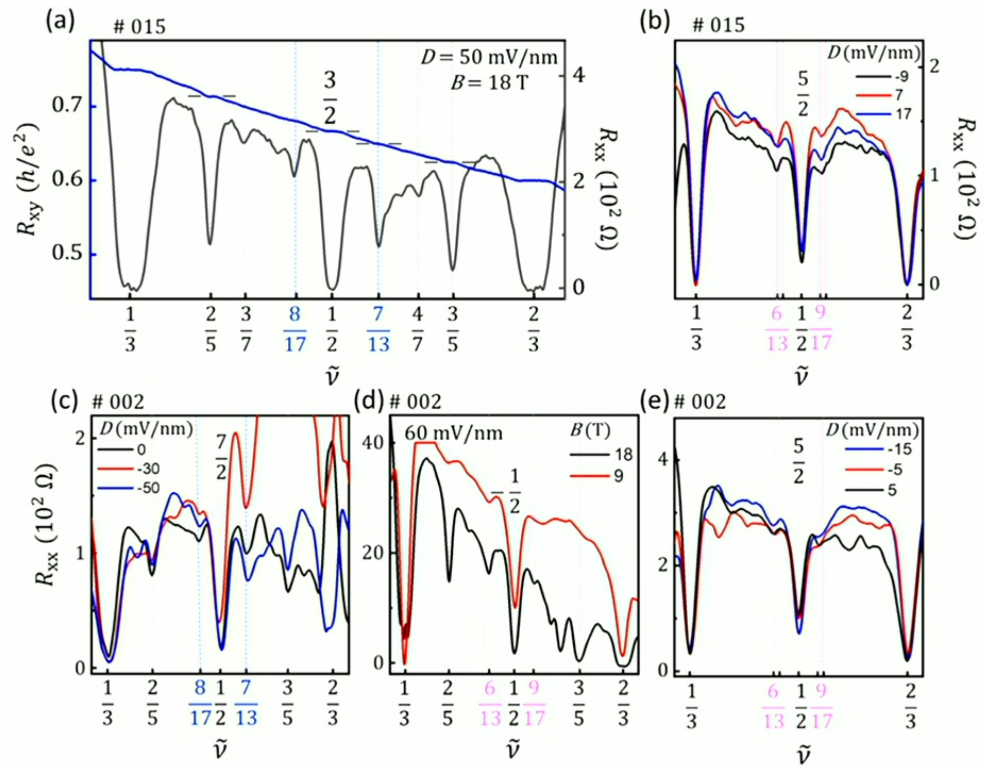
PH-Pfaffian: $\text{Ising} \times U(1)_{-8}/\mathbb{Z}_2$

$$\text{quasihole: } \frac{9}{17}, \text{ quasiparticle: } \frac{8}{17}$$

Pfaffian and anti-pfaffian experiments!

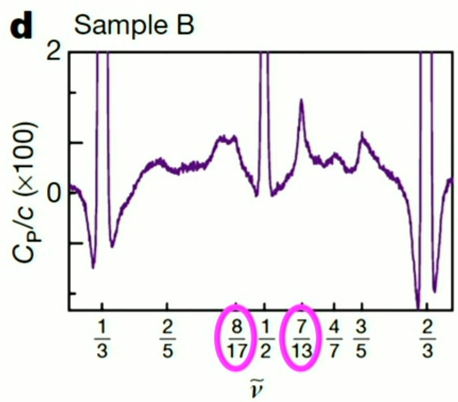


Zibrov et al, 2017



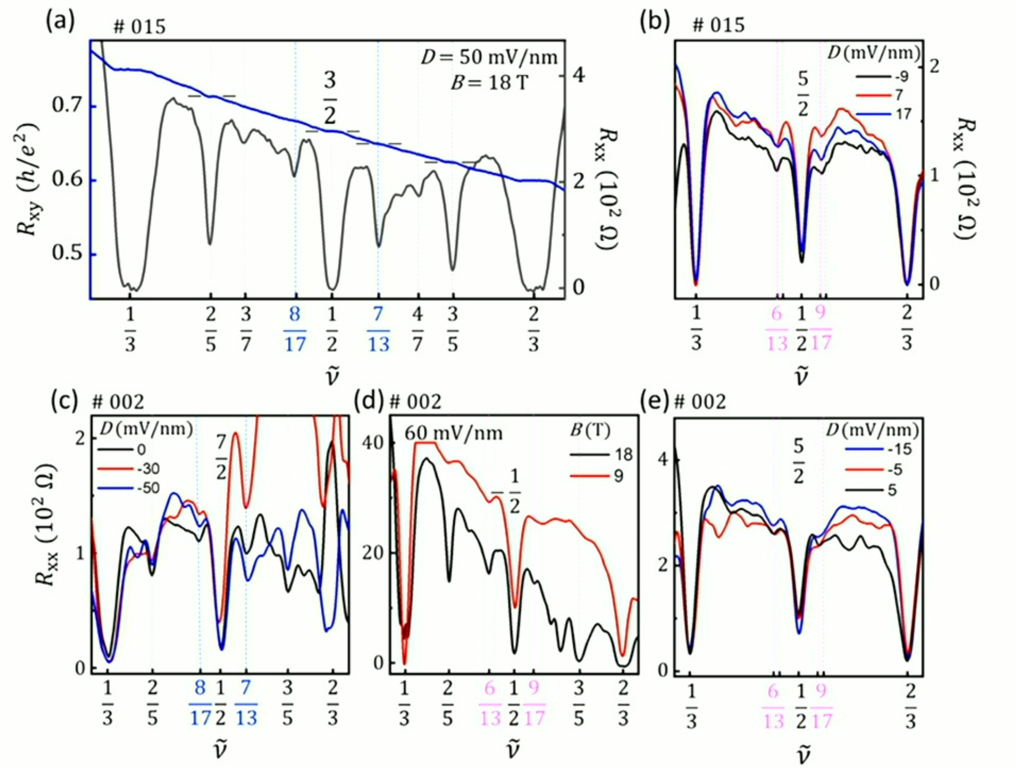
Huang et al, 2022

Pfaffian and anti-pfaffian experiments!



Zibrov et al, 2017

Thanks :)



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