Title: Topological sectors in quantum lattice models

Speakers: Clement Delcamp

Collection: Higher Categorical Tools for Quantum Phases of Matter

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Rationale

[Gaiotto, Kapustin, Seiberg, Willett '14]

"Quantum theories assign topological operators to global symmetries"

Generalised global symmetry

- Higher-codimensional symmetry operators
- Non-invertible
- ...and combinations thereof

Intrinsic definition

- Higher category theory
- Abstract $\neq$ concrete symmetry

$\rightsquigarrow$ Morita theory: gauging

Symmetry TQFT

- Symmetric theory as boundary of a TQFT
- Calculus of topological defects leverages results from TQFTs

$\rightsquigarrow$ Topological sectors $\leftrightarrow$ topological excitations

$\rightsquigarrow$ Gapped symmetric phases $\leftrightarrow$ gapped boundaries

RATIONALE: (1+1)D

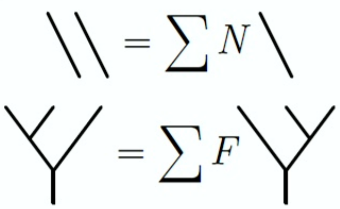
[Verlinde '88] [Oshikawa, Affleck '96] [Petkova, Zuber '01] [Bachas, Gaberdiel '04] [Fröhlich et al. '09] [Fuchs, Runkel, Schweigert '02 '04]...

[Bhardwaj, Tachikawa '17] [Thorngren, Wang '19] [Komargodski, Ohmori, Roumpedakis, Seifnashri '20]...

[Ji, Wen '19] [Apruzzi et al. '19] [Gaiotto, Kulp '21] [Kaidi, Ohmori, Zheng '22] [Freed, Moore, Teleman '22]...

Generalised global symmetry

- Non-invertible topological operators and defects



Intrinsic definition

- Finite semisimple: fusion category $\mathcal{C}$
- Morita representatives labelled by indecomposable $\mathcal{C}$-module categories $\mathcal{M}$

Symmetry TQFT

- $\mathcal{C}$-symmetric theory as boundary theory of $\mathrm{TV}_{\mathcal{C}}$
- Gapped $\mathcal{C}$-symmetric phases $\leftrightarrow$ indecomposable $\mathcal{C}$-module categories
- Topological sectors $\leftrightarrow$ Simple objects in $\mathcal{Z}(\mathcal{C})$

[Aasen, Fendley, Mong '16 '20] [Ji, Wen '19] [Lootens, CD, Verstraete '21 '22] [Lin et al. '22] [Moradi, Moosavian, Tiwari '22] [Inamura '22]...

One-dimensional quantum lattice models

Outline

(1+1)d Hamiltonians with anomalous invertible symmetry

Closed/open boundary conditions

Topological sectors

Gauging and mapping of topological sectors

Higher-dimensional generalisation

arXiv:2112.09091 (PRXQ) w/ L. Lootens, G. Ortiz, F. Verstraete

arXiv:2211.03777 (PRXQ) w/ L. Lootens, F. Verstraete

arXiv:2301.01259 w/ A. Tiwari

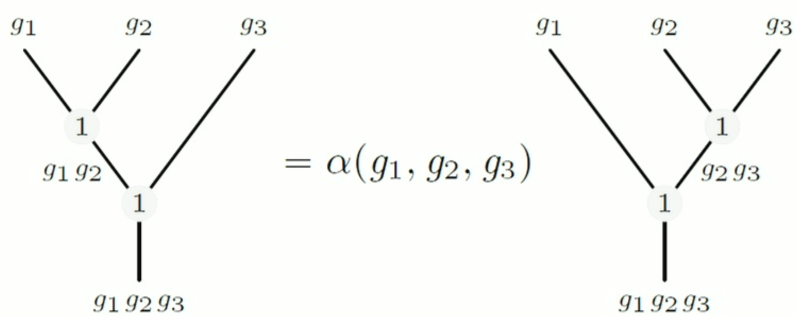
arXiv:2306.?????

Anomalous Invertible Symmetry

[Dijkgraaf, Witten '90]

Fusion category Vec_G^α

- Objects: G-graded vector spaces $V = \bigoplus_{g\in G} V_g$
  $\rightsquigarrow$ Simple objects: $\mathbb{C}_g \equiv g$, $\forall g \in G$
- Morphisms: Grading preserving maps
  $\rightsquigarrow$ $\mathrm{Hom}_{\mathrm{Vec}_G^\alpha}(\mathbb{C}_{g_1}, \mathbb{C}_{g_2}) \cong \delta_{g_1,g_2}\mathbb{C}$
- Associator: $\alpha_{\mathbb{C}_{g_1},\mathbb{C}_{g_2},\mathbb{C}_{g_3}} = \alpha(g_1,g_2,g_3)\cdot(\mathring{\alpha}_{\mathbb{C},\mathbb{C},\mathbb{C}})_{g_1 g_2 g_3}$ s.t. $[\alpha] \in H^3(G, \mathrm{U}(1))$



Invertible symmetry G with 't Hooft anomaly α

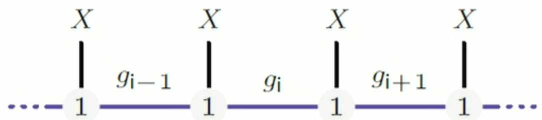
Anyonic Chains

[Feiguin et al. '07][Gils et al. '13][Buican et al. '17]...[Lootens, CD, Verstraete '22]

Input

- Infinite chain
- Object $X = \bigoplus_{g\in G} n_g \mathbb{C}_g$ in Vec_G^α with $n_g \in \{0,1\}$
- Abstract local operators

Microscopic Hilbert space

$\mathcal{H} = \mathsf{Span}_{\mathbb{C}} \Big\{$  $\Big\}$

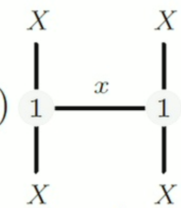
$$\simeq \bigoplus_{\{g\}} \bigotimes_{i} \underbrace{\mathsf{Hom}_{\mathsf{Vec}_G^\alpha}(\mathbb{C}_{g_{i-1}}, X \otimes \mathbb{C}_{g_i})}_{\cong \mathbb{C}}$$

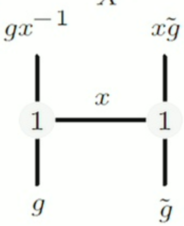
e.g. $X = \bigoplus_{g\in G} \mathbb{C}_g \rightsquigarrow \mathcal{H} \simeq \bigotimes_i \mathbb{C}[G]$

$X = \{\mathbb{1}\} \rightsquigarrow$ SSB

Abstract Hamiltonian

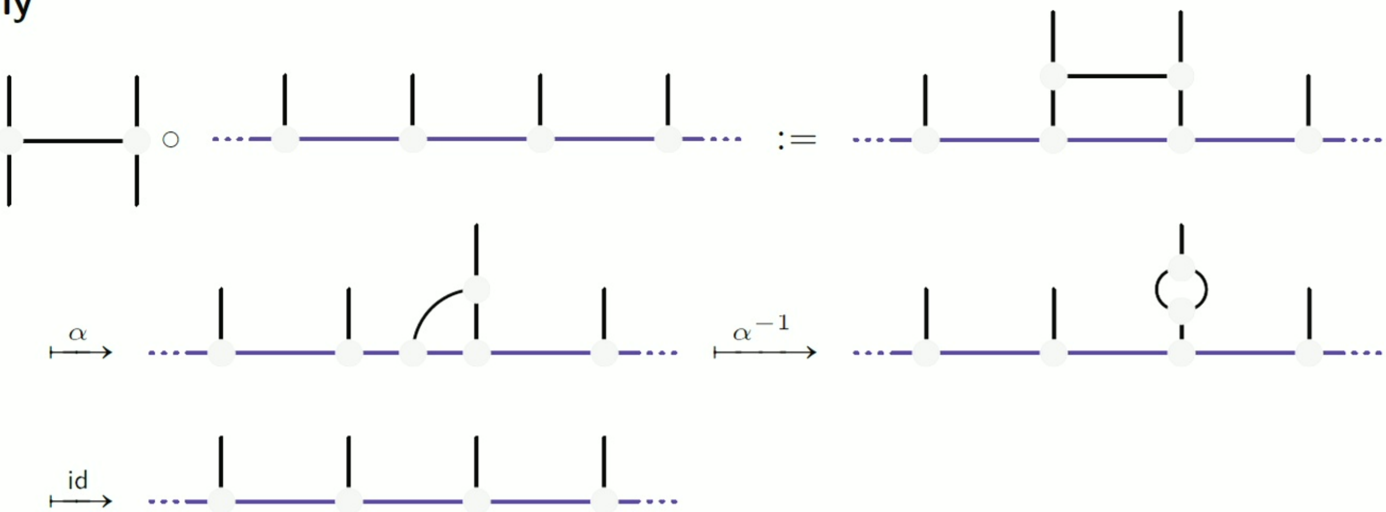
$$\mathbb{H} = \sum_{i} \sum_{n} \mathbb{h}_{i,n}(X)$$

w/ $\mathbb{h}_{i,n}(X) = \sum_{x\in G} h_n(X,x)$ 

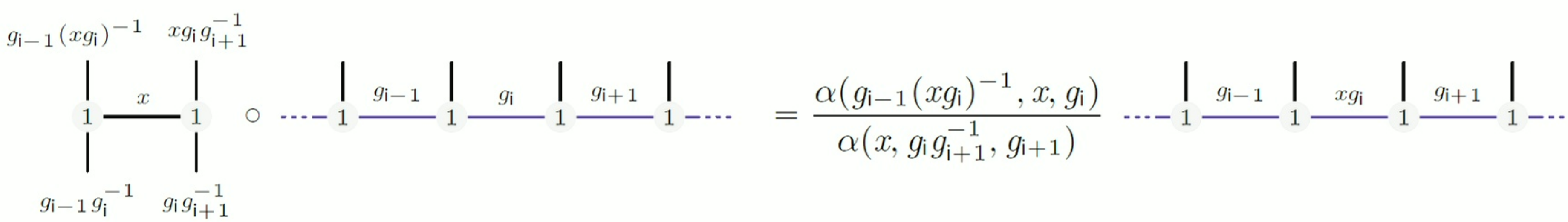
$\equiv \sum_{g,\tilde{g},x} \underbrace{h_n(g,\tilde{g},x)}_{\in \mathbb{C}}$ 

Hamiltonian Action

Heuristically



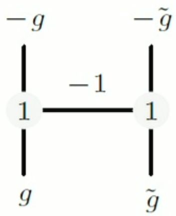
Explicitly

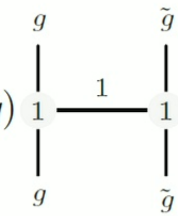


EXAMPLES

$G = \mathbb{Z}/2\mathbb{Z} = \{\pm 1\}$ and $\alpha = 1$:

- Hamiltonian: $\mathbb{H} = -\sum_{\mathsf{i}}(\mathbb{h}_{\mathsf{i},1} + g\,\mathbb{h}_{\mathsf{i},-1})$
- Local operators

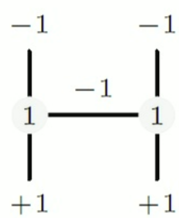
$\mathbb{h}_{\mathsf{i},-1} = \sum_{g,\tilde{g}}$ 

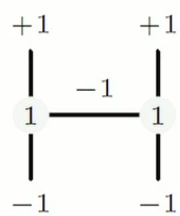
and $\mathbb{h}_{\mathsf{i},1} = \sum_{g,\tilde{g}} \mathsf{sgn}(g)$ 

$\rightsquigarrow \quad \mathbb{H} = -\sum_{\mathsf{i}}(Z_{\mathsf{i}}Z_{\mathsf{i}+1} + gX_{\mathsf{i}})$

$G = \mathbb{Z}/2\mathbb{Z} = \{\pm 1\}$ and non-trivial α in $H^3(\mathbb{Z}/2\mathbb{Z}, \mathsf{U}(1)) \cong \mathbb{Z}/2\mathbb{Z}$:

- Hamiltonian: $\mathbb{H} = -\sum_{\mathsf{i}} \mathbb{h}_{\mathsf{i},-1}$
- Local operators

$\mathbb{h}_{\mathsf{i},-1} =$ 

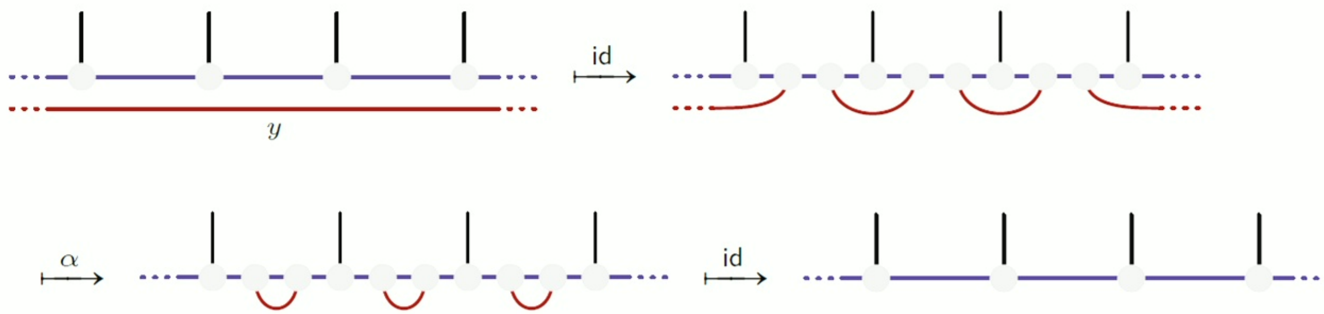
$+$ 

$\rightsquigarrow \quad \mathbb{H} = -\sum_{\mathsf{i}} Z_{\mathsf{i}-1}(X_{\mathsf{i}} + Z_{\mathsf{i}-1}X_{\mathsf{i}}Z_{\mathsf{i}+1})$

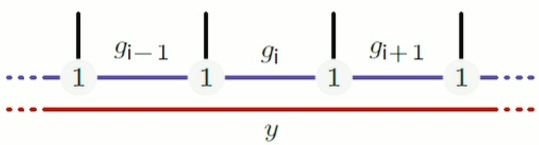
Symmetry Operators

"Topological line whose support extends over the whole Hilbert space at a given time"

Heuristically



Explicitly



$$= \left(\prod_i \alpha(g_{i-1} g_i^{-1}, g_i, y) \right)$$

⇝ Commutation with Hamiltonian follows from $d\alpha = 1$

Examples

$G = \mathbb{Z}/2\mathbb{Z} = \{\pm 1\}$ and $\alpha = 1$:

- Hamiltonian: $\mathbb{H} = -\sum_i (Z_i Z_{i+1} + gX_i)$
- $\mathbb{Z}/2\mathbb{Z}$ symmetry generated by

$$\prod_i X_i$$

[Levin, Gu '12]

$G = \mathbb{Z}/2\mathbb{Z} = \{\pm 1\}$ and non-trivial α in $H^2(\mathbb{Z}/2\mathbb{Z}, \mathrm{U}(1)) \cong \mathbb{Z}/2\mathbb{Z}$:

- Hamiltonian $\mathbb{H} = -\sum_i Z_{i-1}(X_i + Z_{i-1} X_i Z_{i+1})$
- $\mathbb{Z}/2\mathbb{Z}$ symmetry generated by

$$\prod_i X_i \prod_i i^{\frac{-Z_i + Z_i Z_{i+1} + Z_{i+1} - 1}{2}}$$

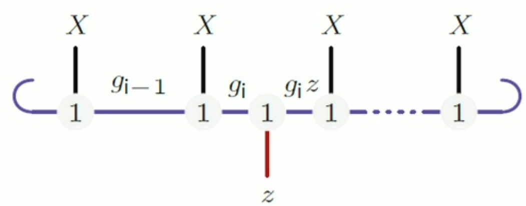
Closed Boundary

"Topological line localised at one point whose support extends in time direction"

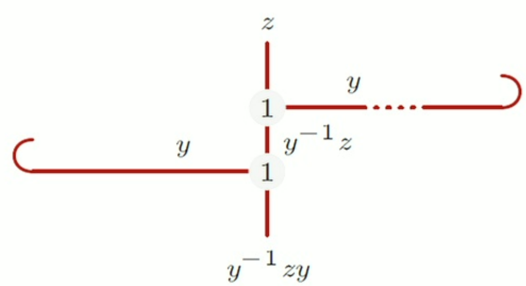
[Aasen, Fendley, Mong '20] [Lin et al. '22] [Lootens, CD, Verstraete '22]

Symmetry-twisted boundary conditions

- Hilbert space



- Symmetry operators

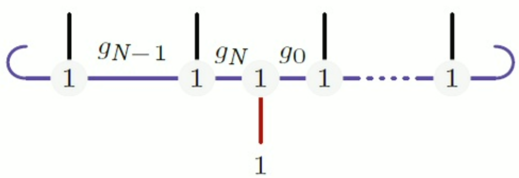


$\rightsquigarrow$ Only symmetry operators labelled by $y \in Z_z$ preserve boundary conditions

Transverse-Field Ising Model

Periodic boundary conditions

- Hilbert space



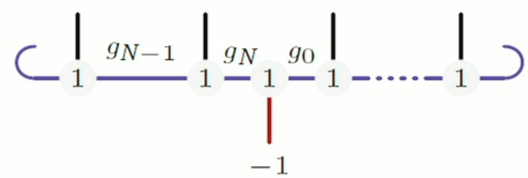
- Hamiltonian

$$\mathbb{H} = -\sum_{i=1}^{N}(Z_i Z_{i+1} + gX_i)$$

- $\mathbb{Z}/2\mathbb{Z}$ symmetry generated by $\prod_i X_i$
  $\rightsquigarrow$ 2 charge sectors: odd & even

Antiperiodic boundary conditions

- Hilbert space



- Hamiltonian

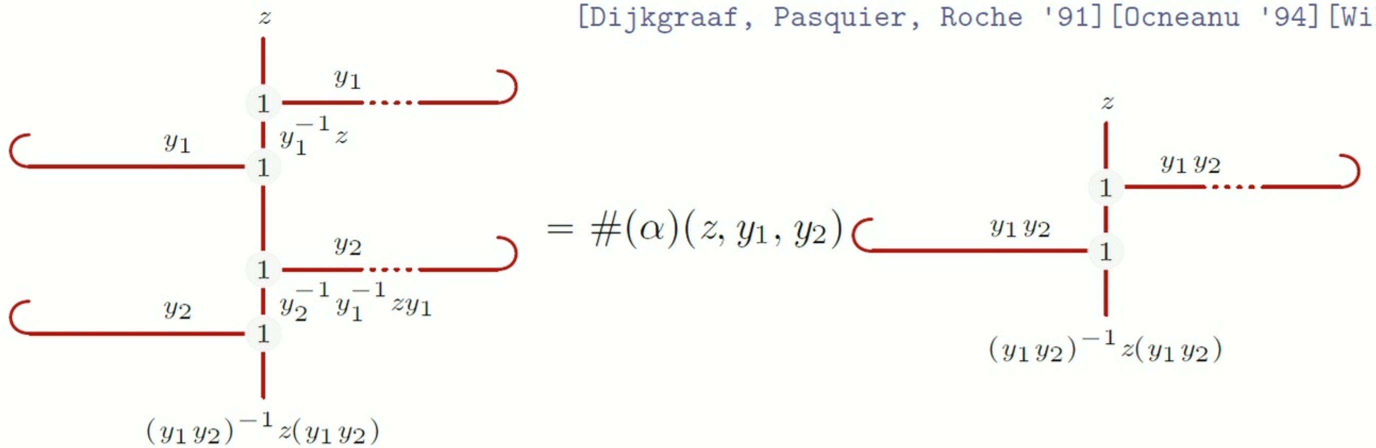
$$\mathbb{H} = -\sum_{i=1}^{N-1}(Z_i Z_{i+1} + gX_i) - (-Z_N Z_1 + gX_N)$$

- $\mathbb{Z}/2\mathbb{Z}$ symmetry generated by $\prod_i X_i$
  $\rightsquigarrow$ 2 charge sectors: odd & even

Four topological sectors: (P,even), (P,odd), (AP,even) & (AP,odd)

Tube Algebra

[Dijkgraaf, Pasquier, Roche '91] [Ocneanu '94] [Willerton '05]



$\rightsquigarrow$ Isomorphic to twisted Drinfel'd double $\mathcal{D}^\alpha(G) \cong \mathbb{C}[\Lambda \overline{G}]^{t(\alpha)}$

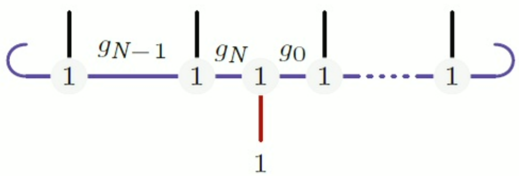
Topological sectors:

- Symmetry-twisted boundary conditions (orbits) + charge sectors (irreps centraliser)
- Category $\mathsf{Mod}(\mathbb{C}[\Lambda \overline{G}]^{t(\alpha)}) \cong \mathcal{Z}(\mathsf{Vec}_G^\alpha) \cong \boxplus_{[c] \in \mathsf{Cl}(G)} \mathsf{Rep}^{t_c(\alpha)}(Z_c)$
- Quantum invariant $\mathsf{DW}_G^\alpha(\mathbb{S}^1)$ Dijkgraaf-Witten theory assigns to circle $\rightsquigarrow$ Point-like excitations

Transverse-Field Ising Model

Periodic boundary conditions

- Hilbert space



- Hamiltonian

$$\mathbb{H} = -\sum_{i=1}^{N}(Z_i Z_{i+1} + gX_i)$$

- $\mathbb{Z}/2\mathbb{Z}$ symmetry generated by $\prod_i X_i$
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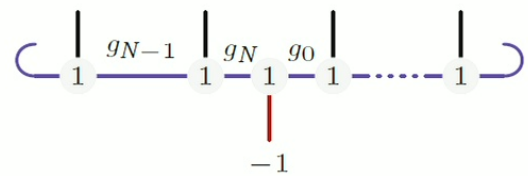
Antiperiodic boundary conditions

- Hilbert space



- Hamiltonian

$$\mathbb{H} = -\sum_{i=1}^{N-1}(Z_i Z_{i+1} + gX_i) - (-Z_N Z_1 + gX_N)$$

- $\mathbb{Z}/2\mathbb{Z}$ symmetry generated by $\prod_i X_i$
  $\rightsquigarrow$ 2 charge sectors: odd & even

Four topological sectors: (P,even), (P,odd), (AP,even) & (AP,odd)

Open Boundary

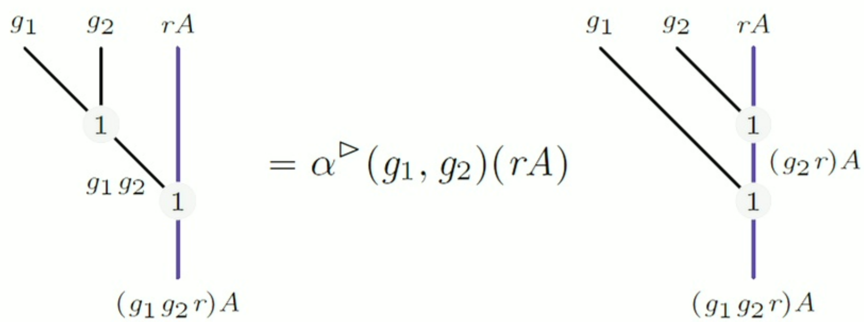
[Kong, Kitaev '11] [Fuchs, Schweigert, Valentino '13]

"Topological boundary conditions are labelled by module categories over Vec_G^α"

[Ostrik '01 '03]

Indecomposable module category $\mathcal{M}(A,\psi)$

- Data: Subgroup $A \subseteq G$ w/ trivialising 2-cochain ψ s.t. $\alpha^{-1}|_{A\times A\times A} = \mathrm{d}\psi$
- Set of simple objects $\leftrightarrow$ Transitive G-set G/A
- Module associator $\alpha^{\triangleright}_{\mathbb{C}_{g_1}\mathbb{C}_{g_2},\mathbb{C}_{rA}} = \alpha^{\triangleright}(g_1,g_2)(rA)\cdot(\mathring{\alpha}_{\mathbb{C},\mathbb{C},\mathbb{C}})_{g_1 g_2 g_3}$ s.t. $\alpha^{\triangleright} \in C^2(G, \mathrm{Hom}(G/A, \mathrm{U}(1)))$

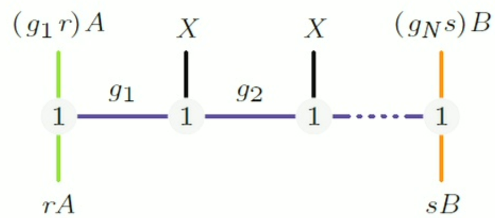


Open Boundary

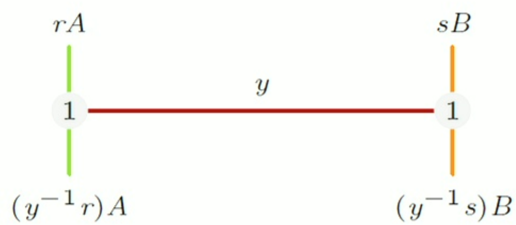
[Lootens, CD, Verstraete '22]

Symmetry-twisted boundary conditions

- Choose two indecomposable module categories $\mathcal{M}(A,\psi)$ and $\mathcal{M}(B,\phi)$
- Hilbert space



- Symmetry operators



⇝ Only symmetry operators stabilising (rA, sB) preserve boundary conditions

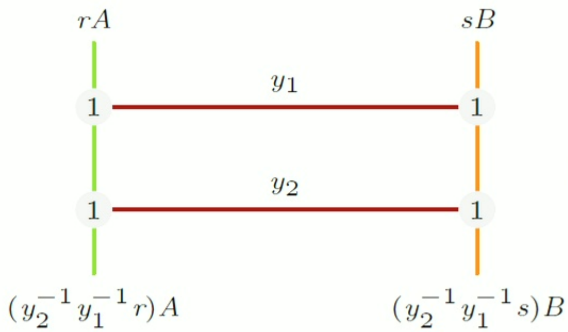
⇝ Commutation with Hamiltonian $\Rightarrow [\alpha|_{A\times A\times A}]$ and $[\alpha|_{B\times B\times B}]$ trivial

[Thorngren, Wang '20] [Choi et al. '23]

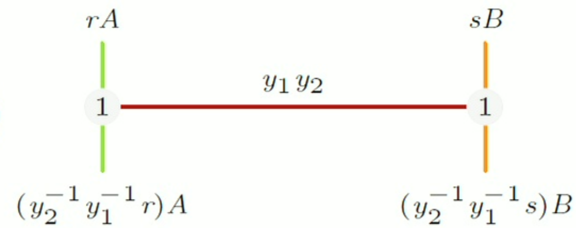
"Anomalous symmetries end at the boundary"

TUBE ALGEBRA

[Ostrik '01 '03] [Kitaev, Kong '12] [Lan, Wen '13] [Williamson, Bultinck, Verstraete '17]
[Bridgeman, Barter '19] [Bullivant ,CD '20]



$$= \#(\alpha,\psi,\phi)(r,s,y_1,y_2)$$



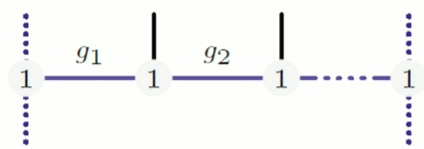
Topological sectors:

- Twisted sectors: Orbits of $(rA, sB) \leftrightarrow A\backslash G/B \ni [c]$ with representative (A, cB)
- Charge sectors are $\omega_c(\alpha,\psi,\phi)$-projective irreps of stabiliser group $A \cap cBc^{-1}$
- Category $\boxplus_{[c]\in A\backslash G/B} \mathsf{Rep}^{\omega_c}(A\cap cBc^{-1}) \simeq \mathsf{Fun}_{\mathsf{Vec}_G^\alpha}(\mathcal{M}(A,\psi),\mathcal{M}(B,\phi))$
- 2-category $\mathsf{Mod}(\mathsf{Vec}_G^\alpha) \simeq \mathsf{DW}_G^\alpha(\mathsf{pt}) \rightsquigarrow$ Domain-wall point-like excitations

TRANSVERSE-FIELD ISING MODEL

Free boundary cond. $\mathcal{M}(\mathbb{Z}/2\mathbb{Z}, 1) \simeq \mathsf{Vec}$

- Hilbert space



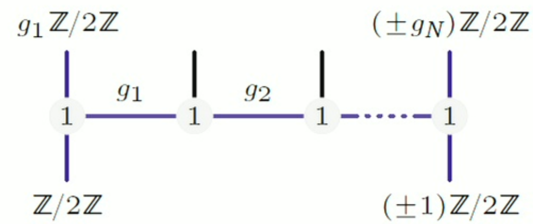
- Hamiltonian

$$\mathbb{H} = -\sum_{i=1}^{N-1} Z_i Z_{i+1} - g \sum_{i=1}^{N} X_i$$

- $\mathbb{Z}/2\mathbb{Z}$ symmetry geneated by $\prod_{i=1}^{N} X_i$
- Charge sectors labelled by simple objects in $\mathsf{Fun}_{\mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}}(\mathsf{Vec}, \mathsf{Vec}) \simeq \mathsf{Rep}(\mathbb{Z}/2\mathbb{Z})$

Fixed boundary cond. $\mathcal{M}(\{\mathbb{1}\}, 1) \simeq \mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}$

- Hilbert space



- Hamiltonian

$$\mathbb{H} = \mu Z_1 \pm \mu Z_N - \sum_{i=1}^{N-1} Z_i Z_{i+1} - g \sum_{i=1}^{N} X_i$$

- Twist sectors labelled by simple objects in $\mathsf{End}_{\mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}}(\mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}) \simeq \mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}$

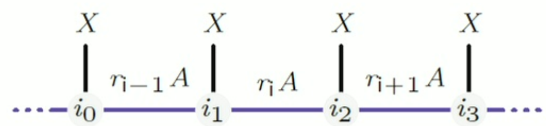
GENERALISATION / MOTIVATION

[Lootens, CD, Verstraete '21 '22]

Input

- Infinite chain
- Object $X = \bigoplus_{g \in G} n_g \mathbb{C}_g$ in Vec_G^α with $n_g \in \{0, 1\}$
- Abstract local operators
- Indecomposable Vec_G^α-module category $\mathcal{M}(A, \psi)$

Microscopic Hilbert space

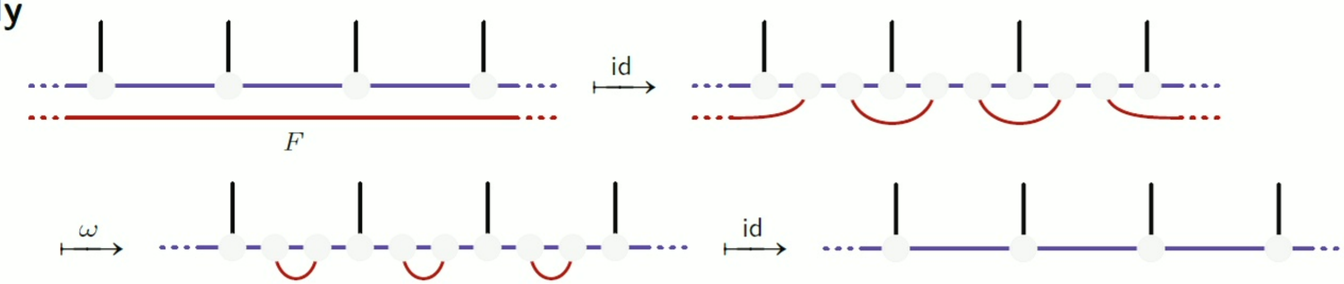
$\mathcal{H} = \mathsf{Span}_{\mathbb{C}} \Big\{$  $\Big\}$

$\rightsquigarrow$ Changing from $\mathsf{Vec}_G^\alpha \circlearrowleft \mathsf{Vec}_G^\alpha$ to $\mathcal{M}(A, \psi)$: ψ-twisted gauging of A

$\rightsquigarrow$ Different realisation of the same symmetry

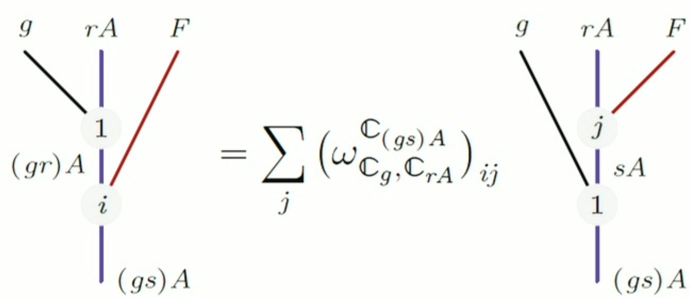
Dual Symmetry

Heuristically



Symmetry operators

- Action on degrees of freedom $F : \mathcal{M}(A,\psi) \to \mathcal{M}(A,\psi)$
- Commute w/ G-action $\omega_{\mathbb{C}_g,\mathbb{C}_{rA}} : F(\mathbb{C}_g \triangleright \mathbb{C}_{rA}) \xrightarrow{\sim} \mathbb{C}_g \triangleright F(\mathbb{C}_{rA})$



- Pentagon satisfied by ω involving $\alpha^{\triangleright} \rightsquigarrow$ Commute w/ $\mathbb{H}$

Morita equivalence

Dual symmetry encoded into $(\mathsf{Vec}^{\alpha}_{G})^{\star}_{\mathcal{M}(A,\psi)} = \mathsf{End}_{\mathsf{Vec}^{\alpha}_{G}}(\mathcal{M}(A,\psi))$

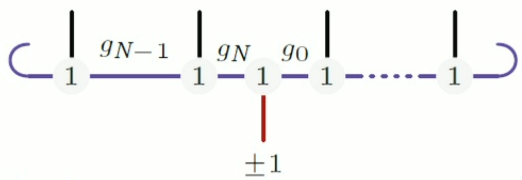
$\mathcal{Z}((\mathsf{Vec}^{\alpha}_{G})^{\star}_{\mathcal{M}(A,\psi)}) \simeq \mathcal{Z}((\mathsf{Vec}^{\alpha}_{G})^{\star}_{\mathcal{M}(B,\phi)})$

Duality operator encoded into $\mathsf{Fun}_{\mathsf{Vec}^{\alpha}_{G}}(\mathcal{M}(A,\psi),\mathcal{M}(B,\phi))$

TRANSVERSE-FIELD ISING MODEL

$\mathcal{M}(\{\mathbb{1}\}, 1) \simeq \mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}$

- Hilbert space



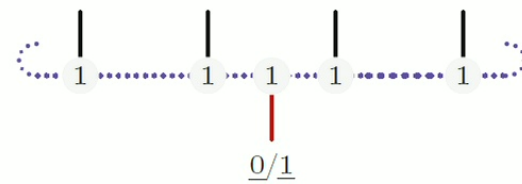
- Hamiltonian

$$\mathbb{H} = -\sum_{\mathsf{i}=1}^{N-1} (Z_{\mathsf{i}} Z_{\mathsf{i}+1} + g X_{\mathsf{i}}) - (\pm Z_N Z_1 + g X_N)$$

- $\mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}$ symmetry generated by $\prod_{\mathsf{i}=1}^{N} X_{\mathsf{i}}$

$\mathcal{M}(\mathbb{Z}/2\mathbb{Z}, 1) \simeq \mathsf{Vec}$

- Hilbert space



- Hamiltonian

$$\mathbb{H} = -\sum_{\mathsf{i}=1}^{N-1} (Z_{\mathsf{i}+\frac{1}{2}} + g X_{\mathsf{i}-\frac{1}{2}} X_{\mathsf{i}+\frac{1}{2}}) - (Z_{\frac{1}{2}} + \chi^{\underline{0}/\underline{1}}(-1) g X_{N-\frac{1}{2}} X_{\frac{1}{2}})$$

- $\mathsf{Rep}(\mathbb{Z}/2\mathbb{Z})$ symmetry generated by $\prod_{\mathsf{i}=1}^{N} Z_{\mathsf{i}+\frac{1}{2}}$

Non-unitary operation on the total Hilbert space

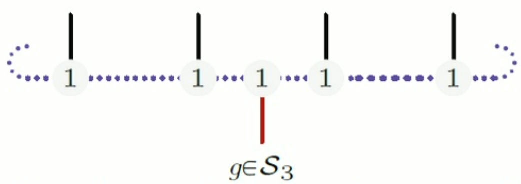
$$\mathcal{Z}(\mathsf{Vec}_{\mathbb{Z}/2\mathbb{Z}}) \xrightarrow{\sim} \mathcal{Z}(\mathsf{Rep}(\mathbb{Z}/2\mathbb{Z})) : (\mathsf{P}/\mathsf{AP}, \mathsf{even}) \mapsto (\mathsf{P}, \mathsf{even}/\mathsf{odd}),\ (\mathsf{P}/\mathsf{AP}, \mathsf{odd}) \mapsto (\mathsf{AP}, \mathsf{even}/\mathsf{odd})$$

$\mathsf{Rep}(\mathcal{S}_3)$ symmetry

[Lootens, CD, Verstraete '22]

$\mathsf{Rep}(\mathcal{S}_3) \circlearrowleft \mathsf{Vec}$

- Hilbert space



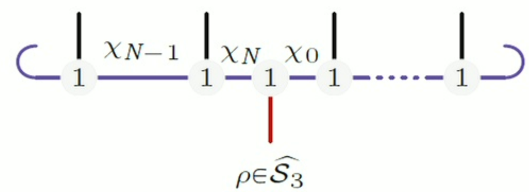
- Hamiltonian (periodic boundary cond.)

$$\mathbb{H} = -\sum_{\mathsf{i}=1}^{N-1} \left(X_{\mathsf{i}-\frac{1}{2}} X_{\mathsf{i}+\frac{1}{2}} + Y_{\mathsf{i}-\frac{1}{2}} Y_{\mathsf{i}+\frac{1}{2}} + Z_{\mathsf{i}-\frac{1}{2}} Z_{\mathsf{i}+\frac{1}{2}} \right)$$

- $\mathsf{Vec}_{\mathcal{S}_3} \simeq (\mathsf{Rep}(\mathcal{S}_3))^\star_{\mathsf{Vec}}$ symmetry

$\mathsf{Rep}(\mathcal{S}_3) \circlearrowleft \mathsf{Rep}(\mathbb{Z}/2\mathbb{Z})$

- Hilbert space



- Hamiltonian (periodic boundary cond.)

$$\mathbb{H} = -\sum_{\mathsf{i}=1}^{N-1} \left(Z_{\mathsf{i}-1} \mathbb{1}_{\mathsf{i}} Z_{\mathsf{i}+1} + Z_{\mathsf{i}-1} X_{\mathsf{i}} Z_{\mathsf{i}+1} - X_{\mathsf{i}} \right)$$

- $\mathsf{Rep}(\mathcal{S}_3) \simeq (\mathsf{Rep}(\mathcal{S}_3))^\star_{\mathsf{Rep}(\mathbb{Z}/2\mathbb{Z})}$ symmetry

$$\mathcal{Z}(\mathsf{Vec}_{\mathcal{S}_3}) \xrightarrow{\sim} \mathcal{Z}(\mathsf{Rep}(\mathcal{S}_3)) : ([r], \underline{0}_{\mathbb{Z}/3\mathbb{Z}}) \mapsto ([\mathbb{1}], \underline{2})$$

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Topological Sectors

[Wang, Wen '15] [CD '17] [Bullivant, CD '19]

Torus

- Two closed boundary conditions
  $\rightsquigarrow (z_1, z_2) \in G^2$ s.t. $z_1 z_2 = z_2 z_1$
- One charge sector
  $\rightsquigarrow$ Irrep of stabilizer of orbit of (z_1, z_2)
- $\mathsf{Mod}(\mathbb{C}[\Lambda^2 \overline{G}]^{\mathrm{t}^2(\pi)}) \simeq \mathcal{Z}(\mathsf{Vec}_{\Lambda \overline{G}}^{\mathrm{t}(\pi)})$
  $\simeq \mathsf{DW}_G^{\pi}(\mathbb{T}^2)$
  $\rightsquigarrow$ Loop-like excitations

[Bullivant, CD '20]

Infinite strip

- Two (local) open boundary conditions
  $\rightsquigarrow$ $2\mathsf{Vec}_G^{\pi}$-module 2-categories $\mathcal{M}(A, \lambda)$
  $A \subset G$ and $\pi^{-1}_{A \times A \times A \times A} = \mathrm{d}\lambda$
- Twisted sector in $A \backslash G / B \ni [c]$
- $\mathsf{Vec}^{\varpi_c}_{A \cap cBc^{-1}}$-symmetry for the quasi 1d model

[Kong, Tian, Zhou '19] [Bullivant, CD '20] [Bartsch, Bullimore, Grigoletto '23]

Cylinder

- One closed boundary condition
- Two open boundary conditions
- $\mathcal{Z}(2\mathsf{Vec}_G^{\pi}) \simeq \boxplus_{[c] \in \mathsf{Cl}(G)} \mathsf{Mod}(\mathsf{Vec}_{Z_c}^{\mathrm{t}_c(\pi)}) \simeq \mathsf{DW}_G^{\pi}(\mathbb{S}^1)$

Outlook

[Lootens, CD, Ortiz, Verstraete '21 '22][CD, Tiwari '23]

Fusion category $\mathcal{C} \rightsquigarrow$ Hilbert space: $\mathcal{M} \in \mathsf{Mod}(\mathcal{C})$

Symmetry operators: $\mathcal{C}^{\star}_{\mathcal{M}}$

Topological sectors: $\mathcal{Z}(\mathcal{C}^{\star}_{\mathcal{M}})$ (closed) and $\mathsf{Mod}(\mathcal{C}^{\star}_{\mathcal{M}})$ (open)

Gauging map: $\mathsf{Fun}_{\mathcal{C}}(\mathcal{M}, \mathcal{N}) \rightsquigarrow$ Mapping sectors: $\mathcal{Z}(\mathcal{C}^{\star}_{\mathcal{M}}) \simeq \mathcal{Z}(\mathcal{C}^{\star}_{\mathcal{N}}) \rightsquigarrow$ Isometries relating spectra
$\rightsquigarrow$ Mapping sectors in open case?

Higher-dimensional generalisation