Title: Physics Meets Geometry: a fuzzy sphere Odyssey in critical phenomena

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Abstract: Historically, the synergy between physics and geometry, from the times of Archimedes and Newton to the era of Einstein, has repeatedly been the catalyst for pivotal breakthroughs in physics and mathematics. In this talk, I will introduce a new narrative demonstrating how physics and geometry intertwine, leading to unexpected and significant results in critical phenomena in physics. Specifically, I will elucidate how non-commutative geometry--a mathematical framework born from the insights of physicists--offers fresh perspectives on conformal field theory, a subject with profound applications across various physics domains, from condensed matter to quantum gravity, and string theory.

Zoom link

Physics Meets Geometry:
A Fuzzy Sphere Odyssey in Critical Phenomena

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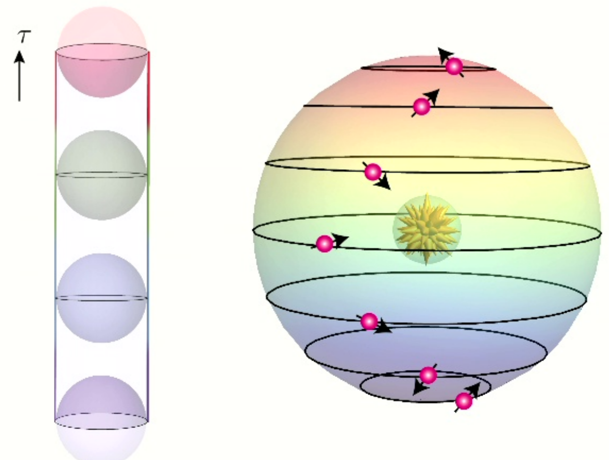
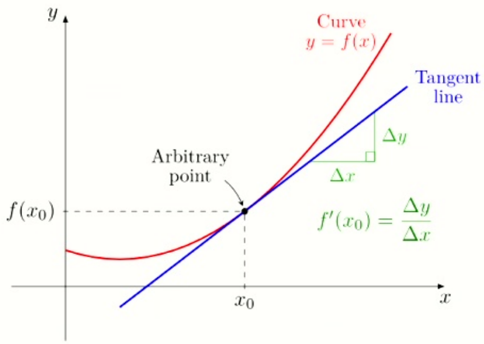
PI, March 2024

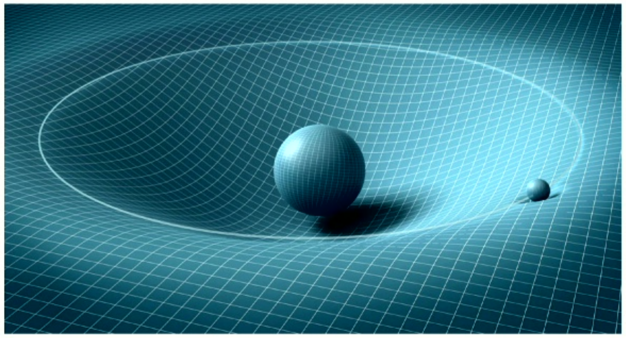
arXiv:2210.13482 (PRX 13, 021009);
arXiv:2303.08844 (PRL 131, 031601);
arXiv:2306.04681 (PRB 108, 235123);
arXiv:2306.16435
arXiv:2308.01903
arXiv:2401.00039
arXiv:2401.17362



Physics meets Geometry

In the history of science, the interplay between physics and geometry has led to lots of profound work in both fields.





This talk:

Physics: Critical phenomena ⟷ Geometry: non-commutative geometry

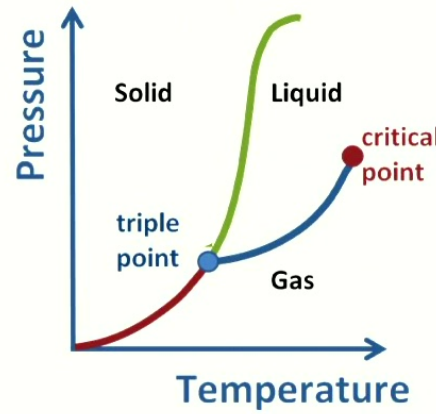
Outline

- Physics: Critical phenomena and conformal field theory
- Geometry: non-commutative geometry and fuzzy sphere
- Physics meets geometry: fuzzy sphere regularization of 3D CFTs

Critical phenomena

Liquid-gas transition

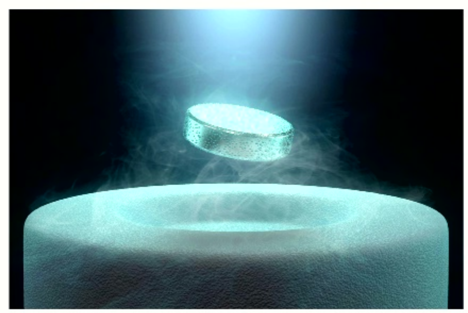
Charles Cagniard de la Tour,
Ann. Chim. Phys., (1822)



Order-disorder transition

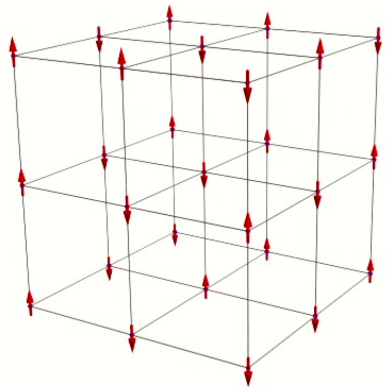
Magnet

Superconductor



Superfluid

Ising model

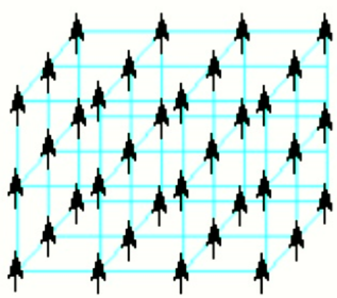


$$H = -\sum_{\langle ij \rangle} s_i \cdot s_j$$

$$s_i = \pm 1$$

Ising transition

A competition between energy and entropy.



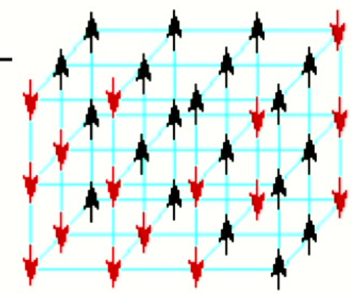
Low Temperature | High Temperature

Ordered phase

$\langle s \rangle \neq 0$

Disordered phase

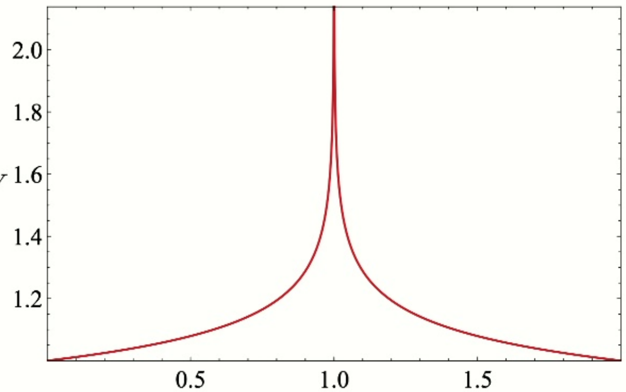
$\langle s \rangle = 0$



Characteristic features of critical phenomena

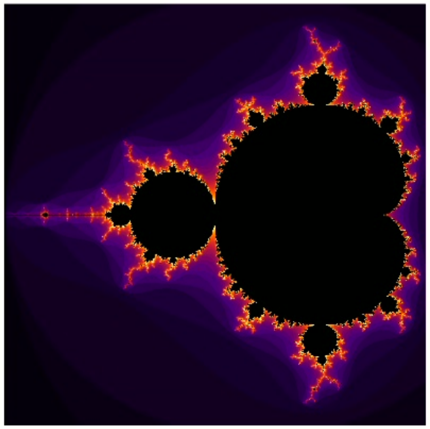
- Diverging physical quantities.
- Power-law correlation functions.
- Infinitely correlated, sensitive to small perturbations.
- Scale invariance.

$$C_V \sim |T - T_c|^{-\alpha}$$



C_V

T/T_c

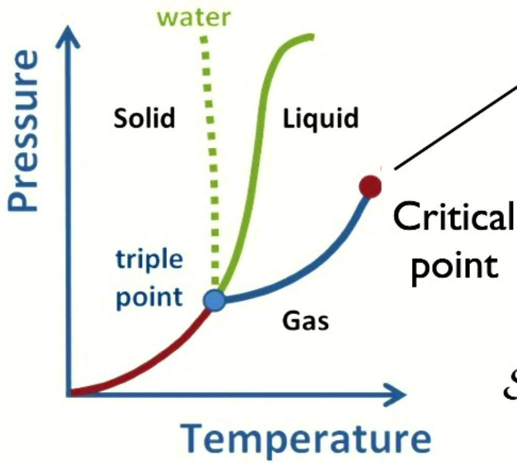


Universality

- Critical points are characterized by a set of universal numbers (critical exponents), e.g.

$$C_V \sim |T - T_c|^{-\alpha} \qquad \xi \sim |T - T_c|^{-\nu}$$

- Critical points in seemingly unrelated systems can belong to the same universality with the exactly same properties.

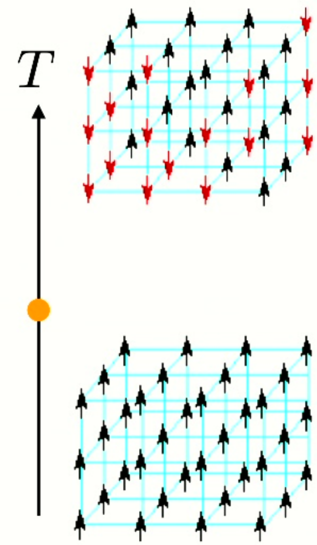


Same critical exponents!

$$\alpha = 0.11008(1)$$

$$\nu = 0.629971(4)$$

$$\mathcal{S} = \int d^3x((\partial_\mu \phi(x))^2 + \phi(x)^4)$$



Critical phenomena are everywhere

Stock market

Flocks of birds

Critical brain hypothesis

Social network

Earthquake

Deep learning

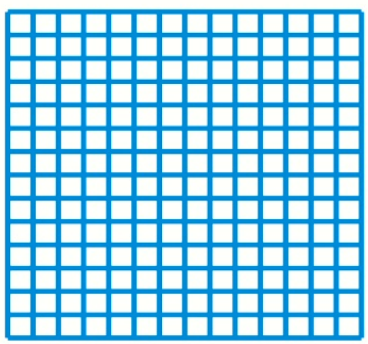
Critical phenomena in modern physics

- The study of critical phenomena gives birth to a number of fundamentally important physics concepts/theories:

  A. Universality.

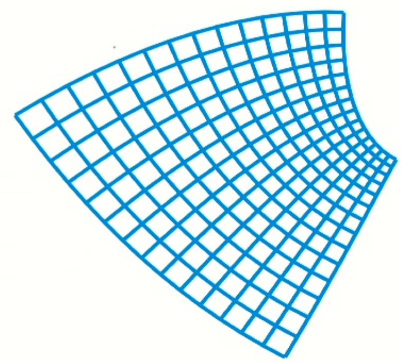
  B. Renormalization group.

  C. Conformal field theory (CFT).



Conformal transformation
(angle preserving transformation)

$$x^\mu \to \frac{x^\mu - x^2 b^\mu}{1 - 2b\cdot x + b^2 x^2}$$

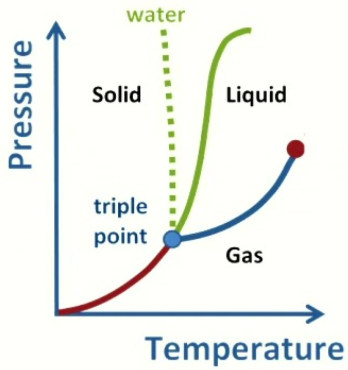


Polyakov 1970: discovered the 2D Ising transition has an emergent conformal symmetry, and conjectured it is also true for the 3D Ising transition.

Conformal field theories (CFTs)

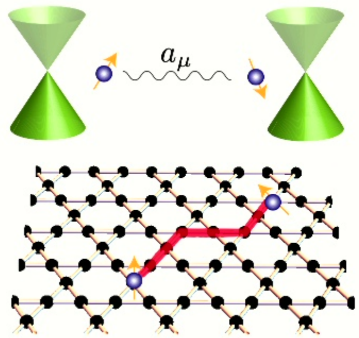
Statistical mechanics

Example:
Ising model,
liquid-gas
transition



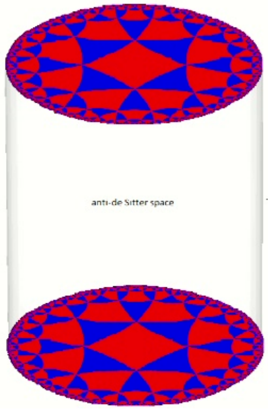
Quantum matter

Example:
Quantum criticality,
gapless spin liquid



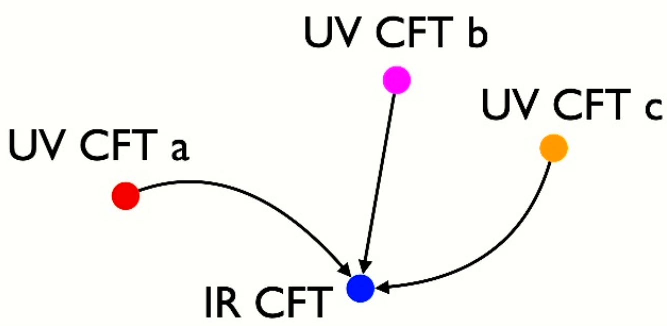
Quantum gravity, String theory

Example:
AdS/CFT



Quantum field theory

RG fixed points of QFTs



CFTs are strongly interacting

- Many 2D CFTs are exactly solvable. Belavin, Polyakov & Zamolodchikov (1984)
- 3D and higher dimensional CFTs are not well understood:
  1. Perturbative RG computation.
     Wilson, Fisher 1972…
  2. Lattice model simulations (mostly Monte Carlo).
  3. Conformal bootstrap.
     Polyakov 1974, Rattazzi, Rychkov, Tonni, Vichi 2008

Outstanding challenges

- Do phase transitions generically have emergent conformal symmetry?
- What is the nature of each CFT?
- What is the landscape of CFTs?
- New type of critical phenomenon and CFT in nature?

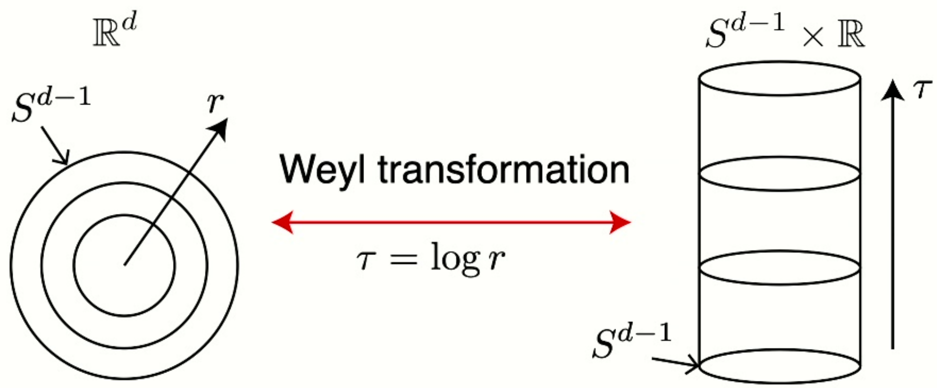
We need powerful non-perturbative tools!

This talk: a condensed matter approach—study strongly interaction models.

Leveraging conformal symmetry

Study CFTs (e.g. Ising) on a conformally flat manifold $\mathbb{R}^d$, $S^{d-1} \times \mathbb{R}$, S^d.

Radial quantization (state-operator correspondence)



Eigenstates of the quantum Hamiltonian defined on S^{d-1} are in one-to-one correspondence with CFT's scaling operators.

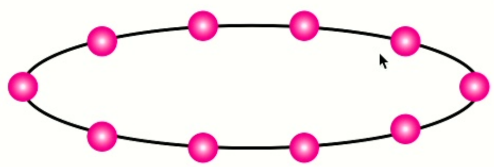
Energy gaps∼scaling dimensions: $\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$

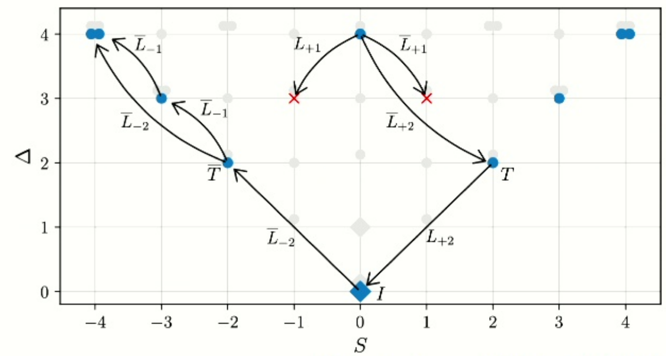
Radial quantization on a lattice

2D CFT: We can just study a quantum Hamiltonian on a circle.

Most conformal data can be extracted.

Cardy

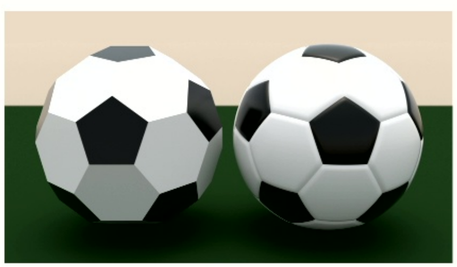


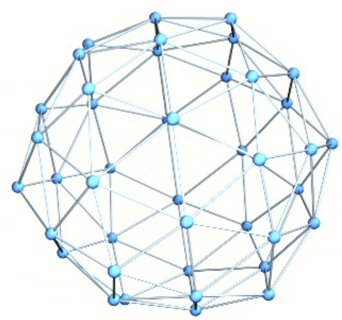


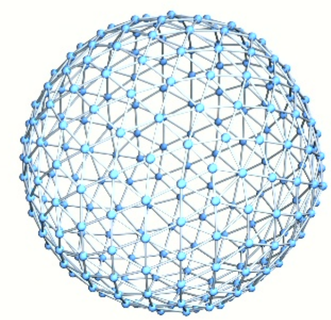
Milsted and Vidal 2017

3D CFT: We need to put a quantum Hamiltonian on a two-sphere.

But a regular lattice won't fit since two-sphere has a curvature…

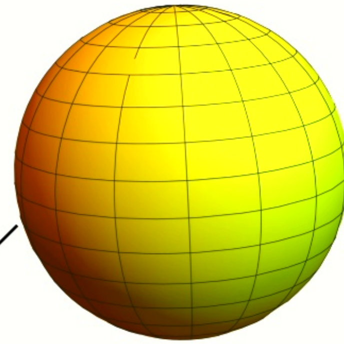






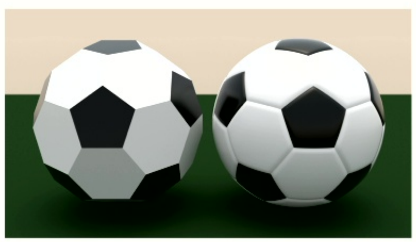
Our recipe: make it fuzzy!

Sphere is a curved space.



Discretize

Spherical tiling



Spherical rotation
is broken badly.

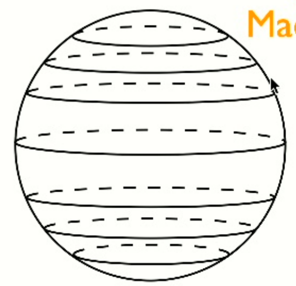
Fuzzify

Lowest Landau
level projection

Haldane 1983

fuzzy (non-commutative) sphere

Madore 1992



Spherical rotation
is kept exactly.

Even 4 spins work!!!

Gaps of ALL the excited states of the system with N=4 spins.

Bootstrap data from Simmons-Duffin, 2017

	CB	4 spins	Errors
σ	0.518	0.530	2.3%
$\partial_{\mu_1}\sigma$	1.518	1.522	0.3%
$\Box\sigma$	2.518	2.427	3.6%
$\partial_{\mu_1}\partial_{\mu_2}\sigma$	2.518	2.428	3.6%
$\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\sigma$	3.518	2.847	20%
$\partial_{\mu_1}\Box\sigma$	3.518	3.291	6.5%
$\sigma_{\mu_1\mu_2}$	4.180	4.241	1.5%
$\sigma_{\mu_1\mu_2\mu_3}$	4.638	4.618	0.4%

	CB	4 spins	Errors
ϵ	1.413	1.382	2.2%
$\partial_{\mu_1}\epsilon$	2.413	2.337	3.1%
$T_{\mu_1\mu_2}$	3	3	NA
$\partial_{\mu_1}\partial_{\mu_2}\epsilon$	3.413	3.126	8.4%
$\Box\epsilon$	3.413	3.577	4.8%
$\partial_{\mu_3}T_{\mu_1\mu_2}$	4	3.663	8.4%
$\varepsilon_{\mu_2\rho\tau}\partial_\rho T_{\mu_1\mu_2}$	4	4.054	1.4%
ϵ'	3.830	4.019	4.9%
$\partial_{\mu_3}\partial_{\mu_4}T_{\mu_1\mu_2}$	5	4.856	2.9%

- 6 primaries and 11 descendants in the fuzzy sphere model with 4 spins!!
- In the lattice Ising model, only 3 primaries were identified even if millions of spins are simulated!

Welcome to the era of fuzzy sphere!!

Simulating 3D (2+1D) CFT? Easier than ever!

Lattice model simulation	Fuzzy sphere
1000~100,000 spins	4~20 spins
Millions of CPU hours	30 mins on a laptop
Very limited information	Almost everything
No access to conformal symmetry	Fingerprint of conformal symmetry

Zhu, Han, Huffman, Hofmann, YCH, arXiv:2210.13482 (PRX); Hu, YCH, Zhu, arXiv:2303.08844 (PRL); Han, Hu, Zhu, YCH, arXiv:2306.04681 (PRB); Zhou, Hu, Zhu, YCH, arXiv:2306.16435; Hu, YCH, Zhu, arXiv:2308.01903; Zhou, Gaiotto, YCH, Zou, arxiv:2401.00039; Hu, Zhu, YCH, arXiv:2401.17362

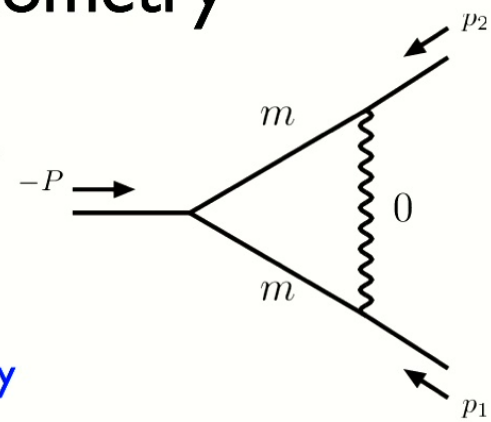
Outline

- Physics: Critical phenomena and conformal field theory
- Geometry: non-commutative geometry and fuzzy sphere
- Physics meets geometry: fuzzy sphere regularization of 3D CFTs

Non-commutative geometry

$$[x_i, p_j] = i\pi\delta_{ij} \Longrightarrow [x_i, x_j] = i\theta_{ij}$$

non-commutativity in phase space → non-commutativity in real space



Heisenberg's original idea in 1930s: to cure the infamous UV divergence in quantum field theory

Heisenberg

A letter to → 1930

Peierls

→

Pauli

→

Oppenheimer

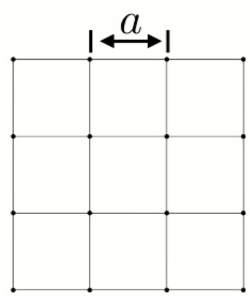
A PhD project → 1947

Snyder

Mathematical foundation was developed by Connes during 1970s-1980s.

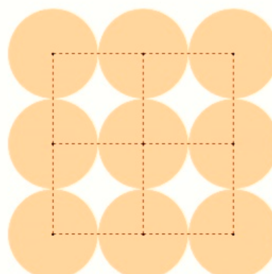
Why non-commutative geometry helps?

Lattice regularization
Wilson 1974
Continuum: $a \to 0$



UV finite
Discrete space
(no continuous space symmetry)
Hard to fit curved space

Fuzzy regularization
Snyder 1947
$$[x, y] = ia^2$$
$$\Delta x \cdot \Delta y \geq a^2/2$$



Uncertainty principle $\Longrightarrow$ UV finite
No rigid lattice $\Longrightarrow$ Continuous space
Fits curved space: e.g. sphere

Fuzzy two-sphere: $[x_i, x_j] = i\varepsilon_{ijk}x_k, \quad \sum_{i=1}^{3} x_i x_i = \text{const} \cdot \mathbf{1}$
Madore 1992

Non-locality and UV-IR mixing

Non-commutative field theory: $S = \int \mathrm{Tr}\,\mathcal{L}(\phi(\hat{x}))$

Review article Douglas & Nekrasov 2001

The making and breaking of non-commutative geometry:

$$[\hat{x}, \hat{y}] = ia^2$$

UV finite	No sharp sense of position
	$\Downarrow$
Continuous space	Non-locality
	$\Downarrow$
Amenable to curved space	UV-IR mixing

The idea of fuzzy regularization was not successful as far as QFT is concerned.

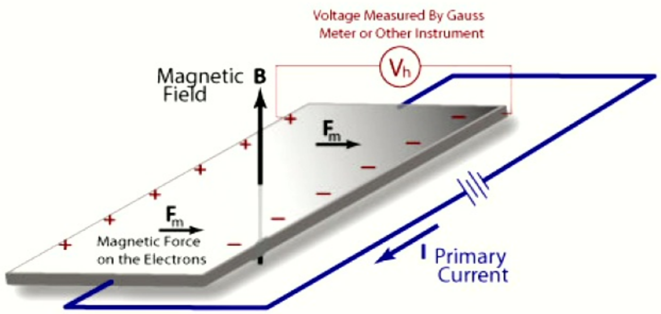
Non-commutative field theory in physics

Review article Douglas & Nekrasov 2001

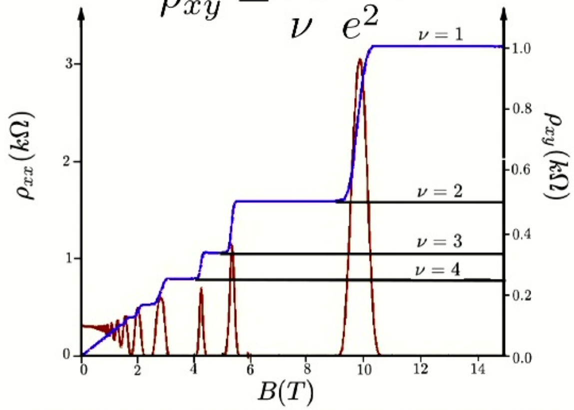
- Non-commutative Yang-Mills theory, Standard model. Connes, Lott 1991;…
- String theory, D-branes, M-theory. Kabat, Taylor, 1997; Seiberg & Witten 1999; Myers 1999;…
- Quantum gravity. Doplicher, Fredenhagen & Roberts 1995; Ahluwalia 1993;…
- Solitons and instantons. Nekrasov & Schwarz 1998; Gopakumar, Minwalla & Strominger 2000;…

A Renaissance for the fuzzy regularization

An inspiration from the condensed matter physics—Quantum Hall effect.



$$\rho_{xy} = \frac{1}{\nu}\frac{2\pi\hbar}{e^2}$$



Klitzing 1980; Stormer, Tsui 1982; Laughlin 1983…

Quantum Hall physics is related to the non-commutative geometry.

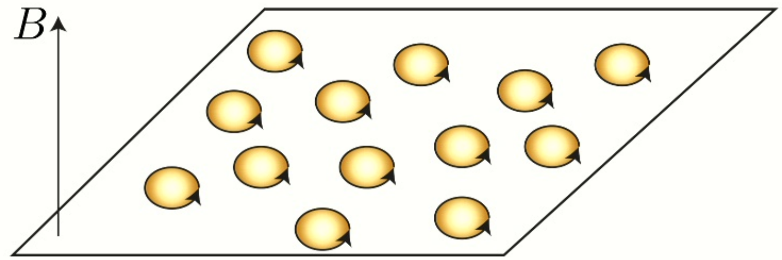
Read 1998; Susskind 2001; Polychronakos 2001; Haldane 2011; Dong & Senthil 2020; Du, Mehta & Son 2021…

Landau level and non-commutative geometry

Particles moving in a strong magnetic field leads to non-commutative geometry!

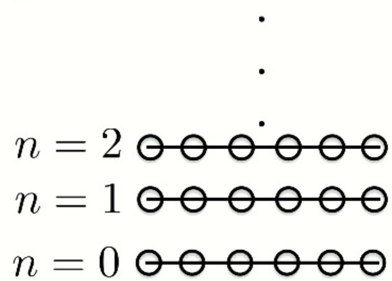
$$\mathcal{L} = \frac{M}{2}\dot{\vec{x}}^2 - \dot{\vec{x}}\cdot\vec{A}$$

$$A_i = -\frac{B}{2}\epsilon_{ij}x^j$$



Landau level: single particle states in the presence of magnetic field.

- Quantized energy: $E_n = \frac{B}{M}(n + 1/2)$
- Complete flat.
- Massive degeneracy at each level: $\frac{B\mathcal{A}}{2\pi}$



Restrict/Project to the lowest Landau level:

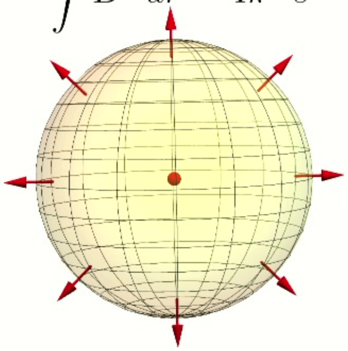
$$\mathcal{L}_0 = -\dot{\vec{x}}\cdot\vec{A} = \frac{B}{2}\epsilon_{ij}\dot{x}^i x^j \Rightarrow [x^i, x^j] = \frac{i}{B}\epsilon^{ij}$$

Fuzzy sphere and spherical Landau levels

Haldane 1983

Electrons move under a monopole.

$$\int \vec{B}\cdot d\vec{r} = 4\pi \cdot s$$



- Quantized Landau level (LL) n=0, 1, 2,…
- The states (orbitals) in each LL form a spin-(n+s) SO(3) rep.
- The wavefunctions of each LL are monopole Harmonics.

Lowest LL $\quad m = -s, -s+1, \cdots, s$

$$Y^{(s)}_{s,m}(\theta,\varphi) = \mathcal{N}_{s,m} e^{im\varphi} \cos^{s+m}\left(\frac{\theta}{2}\right) \sin^{s-m}\left(\frac{\theta}{2}\right)$$

On the lowest LL the coordinates become:

$$(X_i)_{m_1,m_2} = \int d\Omega\, x_i(\Omega) \bar{Y}^{(s)}_{s,m_1}(\Omega) Y^{(s)}_{s,m_2}(\Omega)$$

$$[X_i, X_j] = \frac{1}{s+1} i\varepsilon_{ijk} X_k \qquad \sum_{i=1}^{3} X_i X_i = \frac{s}{s+1} \mathbf{1}_{2s+1}$$

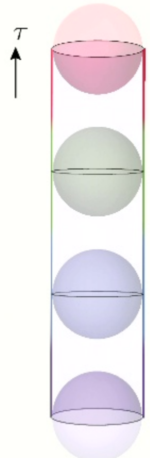
Fuzzy two-sphere: $[x_i, x_j] = i\varepsilon_{ijk} x_k, \quad \sum_{i=1}^{3} x_i x_i = \mathrm{const} \cdot \mathbf{1}$

Madore 1992

Outline

- Physics: Critical phenomena and conformal field theory
- Geometry: non-commutative geometry and fuzzy sphere
- Physics meets geometry: fuzzy sphere regularization of 3D CFTs

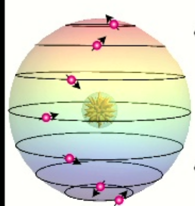
Recap



Physical motivation:

- A UV finite Realization of 3D (2+1D) CFTs on the $S^2 \times \mathbb{R}$ geometry.
- Curved sphere motivates fuzzy sphere, i.e. non-commutative geometry.

Geometric perspective:

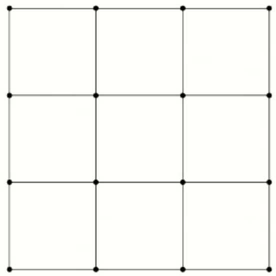


- Non-commutative field theory—QFT on non-commutative geometry has been pursued since 1930s-1940s.
- Non-commutative field theory has novel non-locality, UV-IR mixing.
- Lowest Landau level physics relates to non-commutative geometry.

Fuzzy sphere regularization of 3D CFTs

Quantum mechanical model realizations of 2+1D CFTs.

Lattice model



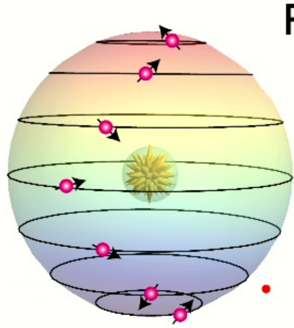
Spin-1/2 on each site $\begin{pmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{pmatrix}$ $\sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$H = -\sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Spins point to +z or -z — Spins point to +x — h

2+1D Ising CFT

Fuzzy sphere model



Particles moving on sphere in the presence of a monopole.

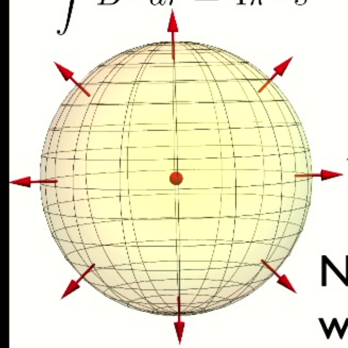
$$H = \frac{1}{2m} \sum_{i=1}^{N_e} (\vec{p}_i + \vec{A}(\vec{x}_i))^2 + \sum_{i,j=1}^{N_e} U(\vec{x}_i - \vec{x}_j)$$

Kinetic term — Interaction term

- The model is local if interactions are local.
- 2+1D CFTs can be realized by tuning the interaction form.

Fuzzy sphere model for the 2+1D Ising CFT

$\int \vec{B} \cdot d\vec{r} = 4\pi \cdot s$



$$H = \frac{1}{2Mr^2} \int d\Omega \psi^\dagger(\Omega)(\partial_\mu + iA_\mu)^2 \psi(\Omega) + H_{int}$$

$$H_{int} = -\int d\Omega_a d\Omega_b \, U(\Omega_a, \Omega_b) n^z(\Omega_a) n^z(\Omega_b) - h \int d\Omega \, n^x(\Omega)$$

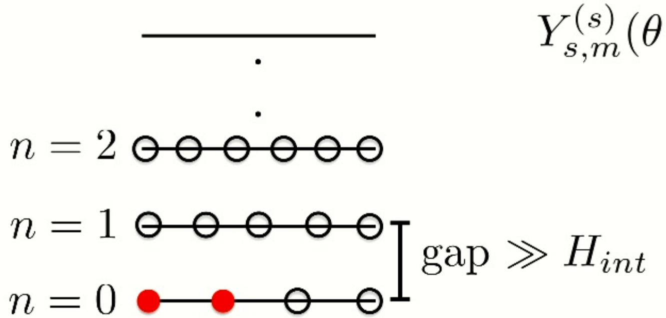
Non-relativistic fermions with an isospin.

$$n^\alpha(\Omega) = (\hat{\psi}_\uparrow^\dagger(\Omega), \, \hat{\psi}_\downarrow^\dagger(\Omega)) \, \sigma^\alpha \begin{pmatrix} \hat{\psi}_\uparrow(\Omega) \\ \hat{\psi}_\downarrow(\Omega) \end{pmatrix}$$

$$U(\Omega_a, \Omega_b) = g_0 \, \delta(\Omega_{ab}) + g_1 \, \nabla^2 \delta(\Omega_{ab})$$

Lowest Landau level projection Haldane 1983

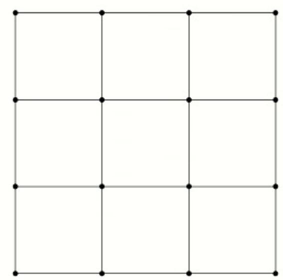
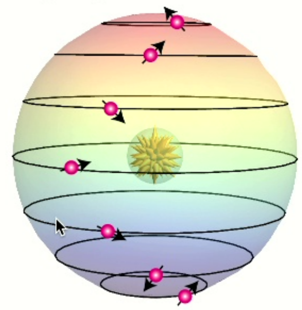
Landau levels



$$Y^{(s)}_{s,m}(\theta, \varphi) = \mathcal{N}_{s,m} e^{im\varphi} \cos^{s+m}\left(\frac{\theta}{2}\right) \sin^{s-m}\left(\frac{\theta}{2}\right)$$

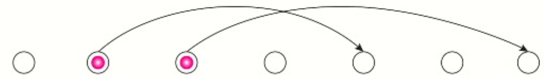
$$\psi_\alpha(\vec{\Omega})^\dagger = \sum_{m=-s}^{s} c^\dagger_{\alpha,m} \, Y^{(s)}_{s,m}(\vec{\Omega})$$

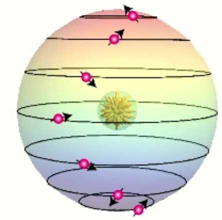
Lattice model versus fuzzy sphere model

	Lattice model	Fuzzy sphere model
		
Hilbert space	Each site two states $\vert\uparrow\rangle, \vert\downarrow\rangle$	Each orbital four states $\vert 0\rangle, c^\dagger_\uparrow\vert 0\rangle, c^\dagger_\downarrow\vert 0\rangle, c^\dagger_\uparrow c^\dagger_\downarrow\vert 0\rangle$
Order phase	$\prod \vert\uparrow\rangle_i$ and $\prod \vert\downarrow\rangle_i$	$\prod_{m=-s}^{s} c^\dagger_{m\uparrow}\vert 0\rangle$ and $\prod_{m=-s}^{s} c^\dagger_{m\downarrow}\vert 0\rangle$
Disorder phase	$\prod \frac{\vert\uparrow\rangle_i + \vert\downarrow\rangle_i}{\sqrt{2}}$	$\prod_{m=-s}^{s} \frac{c^\dagger_{m\uparrow} + c^\dagger_{m\downarrow}}{\sqrt{2}}\vert 0\rangle$
Hamiltonian	$H = -\sum_{\langle ij\rangle} \sigma^z_i \sigma^z_j - h\sum_i \sigma^x_i$	$H = -\sum_{m_{1,2},m=-s}^{s} V_{m_1,m_2,m_2-m,m_1+m} (\mathbf{c}^\dagger_{m_1}\sigma^z \mathbf{c}_{m_1+m})(\mathbf{c}^\dagger_{m_2}\sigma^z \mathbf{c}_{m_2-m}) - h\sum_{m=-s}^{s} \mathbf{c}^\dagger_m \sigma^x \mathbf{c}_m$ $V_{m_1,m_2,m_3,m_4} = \sum_l V_l (4s-2l+1) \begin{pmatrix} s & s & 2s-l \\ m_1 & m_2 & -m_1-m_2 \end{pmatrix} \begin{pmatrix} s & s & 2s-l \\ m_3 & m_4 & -m_3-m_4 \end{pmatrix}$

A closer look at the fuzzy sphere model

$2s+1$-site fermionic model





Many-body Hilbert space $\prod_{i=1}^{2s+1} c^\dagger_{m_i,\alpha_i}|0\rangle$ $\quad m_i = -s, -s+1, \cdots, s$ spin-s rep of $SO(3)$ $\quad \alpha_i = \uparrow, \downarrow$

Continuum limit: $s \to \infty$

Hamiltonian for the 2+1D Ising model $\mathbf{c}^\dagger_m = (c^\dagger_{m,\uparrow}, c^\dagger_{m,\downarrow})$

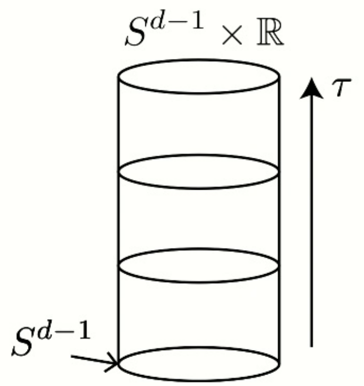
$$H = -\sum_{m_{1,2},m=-s}^{s} V_{m_1,m_2,m_2-m,m_1+m}\left(\mathbf{c}^\dagger_{m_1}\sigma^z\mathbf{c}_{m_1+m}\right)\left(\mathbf{c}^\dagger_{m_2}\sigma^z\mathbf{c}_{m_2-m}\right) - h\sum_{m=-s}^{s}\mathbf{c}^\dagger_m\sigma^x\mathbf{c}_m$$

$$V_{m_1,m_2,m_3,m_3} = \sum_l V_l\,(4s-2l+1)\begin{pmatrix} s & s & 2s-l \\ m_1 & m_2 & -m_1-m_2\end{pmatrix}\begin{pmatrix} s & s & 2s-l \\ m_3 & m_4 & -m_3-m_4\end{pmatrix}$$

$$U(x_i - x_j) = g_0\delta(x_i - x_j) + g_1\nabla^2\delta(x_i - x_j) \Leftrightarrow V_0 = \frac{1}{2}g_0 - \frac{1}{4}g_1, V_1 = \frac{1}{4}g_1$$

State-operator correspondence

Radial quantization
of d-dimensional CFT



$\text{Quantum Hamiltonian on } S^{d-1}$

Eigenstates of the quantum Hamiltonian.

↕ One-to-one correspondence

$$\delta E_n = E_n - E_0 = \frac{v}{R}\Delta_n$$

Scaling operators in the CFT.

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{\delta_{ij}}{|x_1 - x_2|^{2\Delta}}$$

Primaries and descendants

Conformal multiplet

$$\mathcal{O} \longrightarrow \partial_{\mu_1}\mathcal{O} \longrightarrow \partial_{\mu_1}\partial_{\mu_2}\mathcal{O} \quad \cdots$$
$$\Delta \qquad \Delta + 1 \qquad \Delta + 2 \quad \cdots$$

There are infinite number of primary operators in any 3D CFT!

3D Ising			
	$\Delta_\sigma \approx 0.5184189(10)$	$\Delta_\epsilon \approx 1.412625(10)$	$\Delta_{\epsilon'} \approx 3.82968(23)$
	$\eta = 2\Delta_\sigma - 1$	$\nu = 1/(3 - \Delta_\epsilon)$	$\omega = \Delta_{\epsilon'} - 3$

State-operator correspondence

- We identified 15 primary operators, the numerical errors of all primaries are within 1.6%.
- We looked at 70 lowest lying states with L<5, all of them match theoretical expectations with small errors~3%.

	CB	16 spins	Error
σ	0.518	0.524	1.2%
σ'	5.291	5.303	0.2%
$\sigma_{\mu_1\mu_2}$	4.180	4.214	0.8%
$\sigma'_{\mu_1\mu_2}$	6.987	7.048	0.9%
$\sigma_{\mu_1\mu_2\mu_3}$	4.638	4.609	0.6%
$\sigma_{\mu_1\mu_2\mu_3\mu_4}$	6.113	6.069	0.7%
σ^{P-}	NA	11.19	–

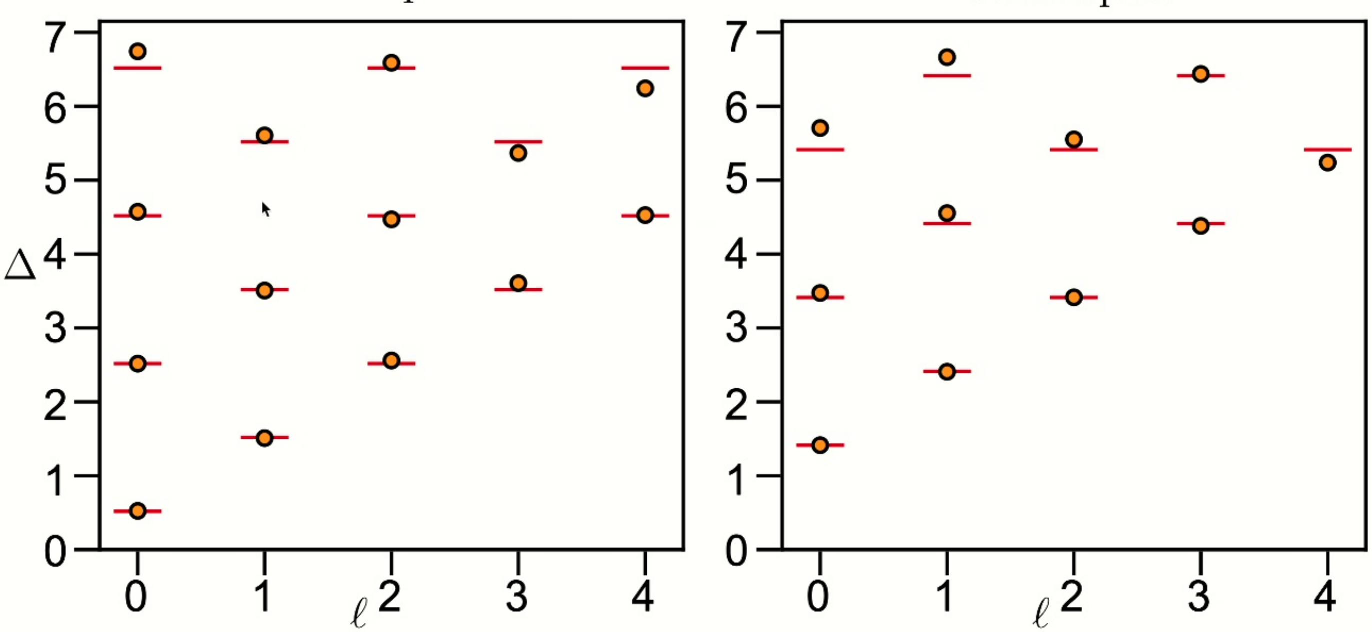
	CB	16 spins	Error
ϵ	1.413	1.414	0.07%
ϵ'	3.830	3.838	0.2%
ϵ''	6.896	6.908	0.2%
$T_{\mu\nu}$	3	3	–
$T'_{\mu\nu}$	5.509	5.583	1.3%
$\epsilon_{\mu_1\mu_2\mu_3\mu_4}$	5.023	5.103	1.6%
$\epsilon'_{\mu_1\mu_2\mu_3\mu_4}$	6.421	6.347	1.2%
ϵ^{P-}	≤ 11.2	10.01	–

Bootstrap data from Simmons-Duffin, 2017

State-operator correspondence

descendents: $\partial_{\mu_1} \cdots \partial_{\mu_j} \Box^n O, \quad n, j \geq 0 \qquad (\Delta + 2n + j, j)$

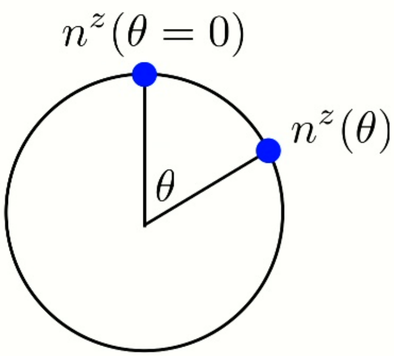
16 electrons



Four-point correlator

Han, Hu, Zhu, YCH, arXiv: 2306.04681

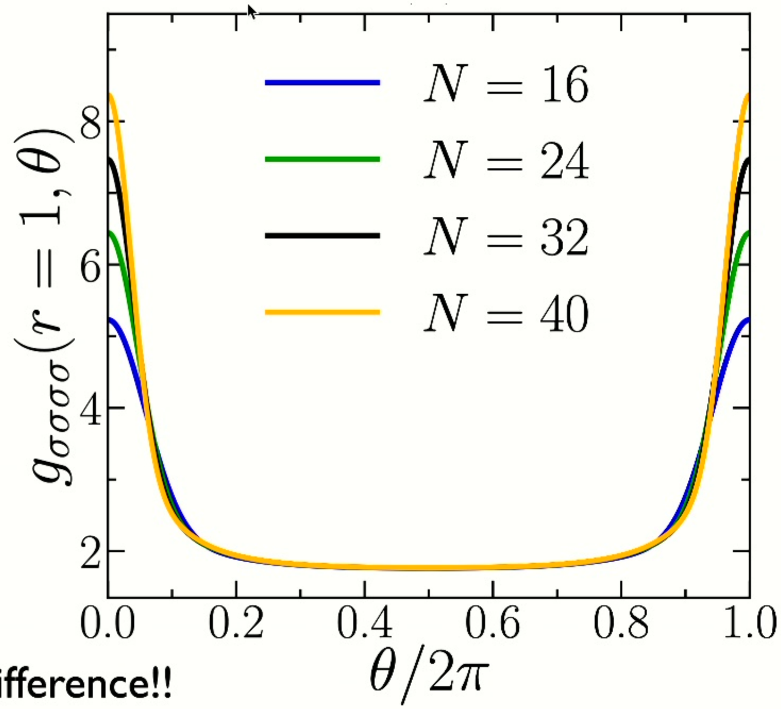
We get a continuous function!



$$g(z = e^{i\theta}, \bar{z} = e^{-i\theta})$$

$$\frac{\langle \sigma | n^z(\theta = 0) n^z(\theta) | \sigma \rangle}{\langle \sigma | n^z(\theta = 0) | 0 \rangle^2}$$

$$1.846 + 0.171 \cos\theta + 0.152 \cos^2\theta + 0.109 \cos^3\theta + 0.109 \cos^4\theta + \cdots$$



0.06% difference!!

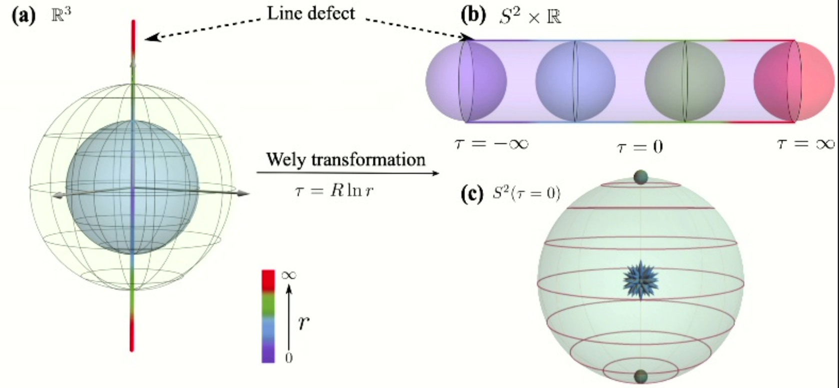
	Bootstrap	$N = 40$	$N = 32$	$N = 24$	$N = 16$
$\theta = \pi$	1.76855	1.76742	1.76671	1.76549	1.76244
$\theta = \pi/3$	2.049	2.03921	2.03495	2.02470	2.01212

Conformal defect

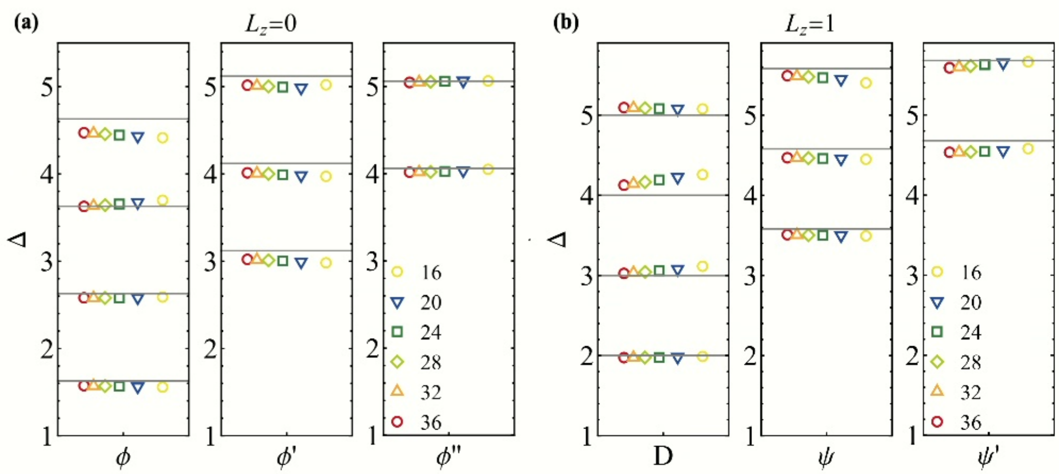
Hu, YCH, Zhu, arXiv:2308.01903
Zhou, Gaiotto, YCH, Zou, arXiv:2401.00039

$$S = S_{CFT} + h \int d^p r\, \mathcal{O}(r)$$

p=1: Line defect; p=2: Plane defect



Results of magnetic line defect of 3D Ising



A lot to explore in this fuzzy world

Direction I

A numerical tool to solve open problems of CFTs/QFTs:

- Critical gauge theories: QED3, QCD3, Chern-Simons matter theories, etc.
- 2+1D CFT at finite temperature, Cardy formula
- Conformal defect
- Non-equilibrium dynamics, quantum chaos
- Complex fixed point, complex CFT
- Landscape of CFTs, new CFTs
- …

Direction II

Unreasonable effectiveness of mathematics (fuzzy geometry):

- Regulating QFTs using non-commutative geometry?!
- Exact solution or hidden structure of 3D CFTs?!

Summary

Thank you!

- Critical phenomenon, particularly CFT, is a cornerstone in physics and it also presents outstanding challenges and opportunities in modern physics.
- Leveraging non-commutative geometry and quantum Hall physics, we propose a non-perturbative scheme called fuzzy sphere regularization for studying 3D CFTs.
- Fuzzy sphere regularization demonstrates unreasonable effectiveness for studying 3D CFTs, implying a deep connection between CFT, QFT and non-commutative geometry.

Let's explore the fuzzy world!