Title: Cosmology Lecture

Speakers: Kendrick Smith

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LAST TIME

MASSLESS SCALAR IN NONDYNAMICAL FLRW

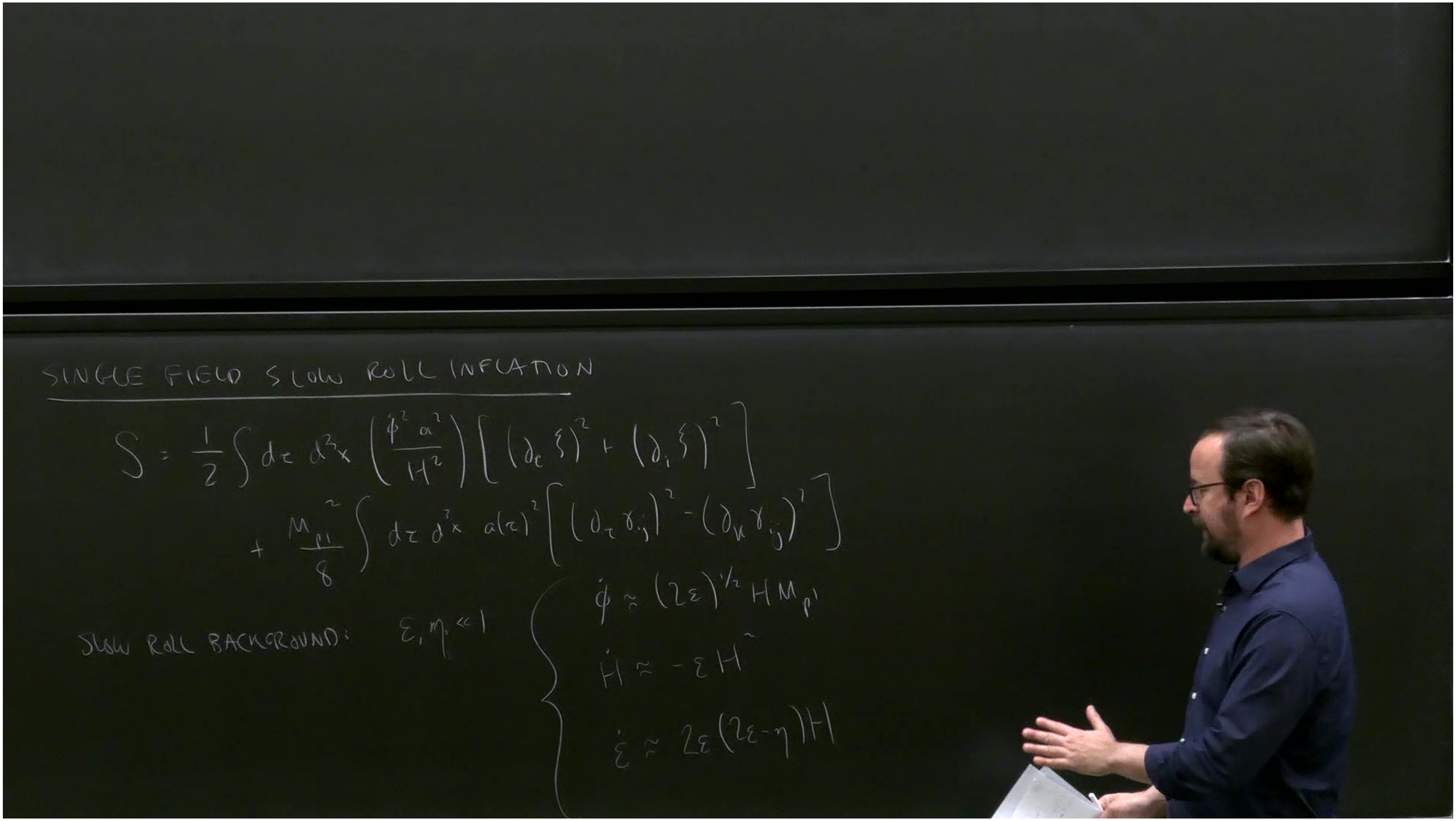
$$S = \frac{1}{2}\int d\tau\, d^3x\; a(\tau)^2\left[(\partial_\tau\phi)^2 - (\partial_i\phi)^2\right]$$

IF $a(\tau)$ IS "QUASI DE SITTER" $(w \simeq -1)$

THEN ϕ GETS NEARLY SCALE INVARIANT FLUCTUATIONS

$$\langle \phi(k)\, \phi(k')^* \rangle = \frac{H_*^2}{2k^3}(2\pi)^3\, \delta^3(k-k')$$ WELL AFTER HORIZON CROSSING

$(\cdot)_* =$ "EVALUATED AT HORIZON CROSSING $k = aH$"


SINGLE FIELD SLOW ROLL INFLATION
$S = \frac{1}{2}\int d\tau\, d^3x \left(\frac{\dot\phi^2 a^2}{H^2}\right)\left[(\partial_\tau \zeta)^2 + (\partial_i \zeta)^2\right]$
$+ \frac{M_{pl}^2}{8}\int d\tau\, d^3x\, a(\tau)^2\left[(\partial_\tau \gamma_{ij})^2 - (\partial_k \gamma_{ij})^2\right]$
SLOW ROLL BACKGROUND: $\varepsilon, \eta \ll 1$
$\dot\phi \simeq (2\varepsilon)^{1/2} H M_{pl}$
$\dot H \simeq -\varepsilon H^2$
$\dot\varepsilon \simeq 2\varepsilon(2\varepsilon - \eta)H$

GRAVITATIONAL WAVES FIRST

$\gamma_{ij} = \gamma_{ji} \qquad \gamma_{ii} = 0 \qquad k_i \gamma_{ij}(k) = 0$

E.G. IF $\vec{k} = k\hat{z}$ $\quad \gamma_{ij}(\vec{k}) = h_1 \underbrace{\begin{pmatrix} \frac{1}{2} & & \\ & -\frac{1}{2} & \\ & & 0 \end{pmatrix}}_{\varepsilon^{(1)}_{ij}} + h_2 \underbrace{\begin{pmatrix} 0 & 1/2 & \\ 1/2 & 0 & \\ & & 0 \end{pmatrix}}_{\varepsilon^{(2)}_{ij}}$

GENERALIZE: FOR ANY $\vec{k}$ DEFINE BASIS TENSORS $\varepsilon^{(s)}_{ij}(\vec{k})$

SO THAT $\quad \varepsilon^{(s)}_{ij}\,\varepsilon^{(s')}_{ij} = \delta_{ss'} \qquad s \in \{1,2\}$

$\varepsilon^{(s)}_{ii} = 0 \qquad \varepsilon^{(s)}_{ij} = \varepsilon^{(s)}_{ji} \qquad k_i \varepsilon^{(s)}_{ij} = 0$

THEN

$$\gamma_{ij}(\vec{k}) = \sum_{s=1}^{2} h_s(\vec{k})\, \varepsilon^{(s)}_{ij}(\vec{k})$$

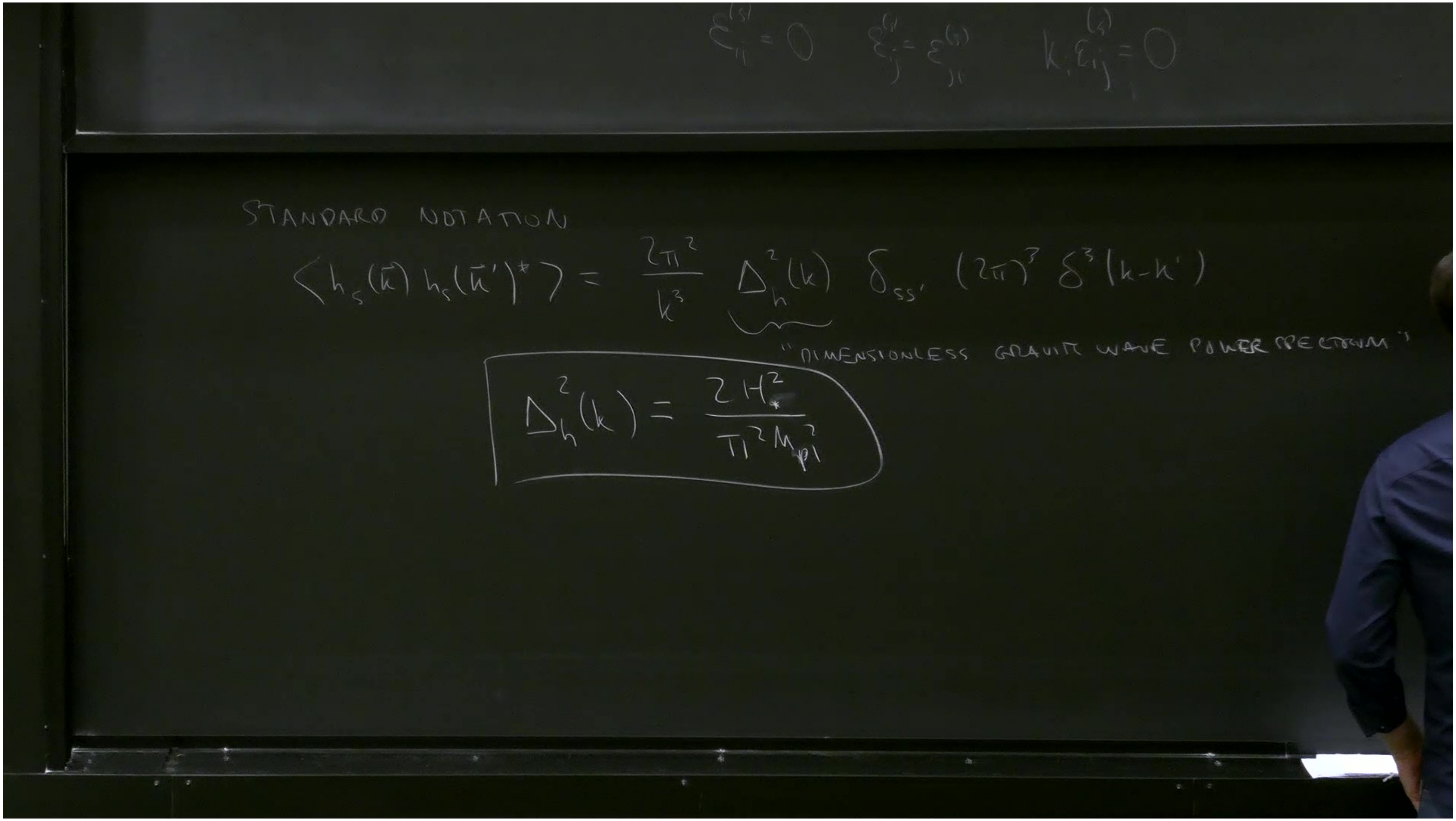
IN TERMS OF $h_s(\vec{k})$ THE GW ACTION IS:

$$S_{gw} = \frac{M_{pl}^2}{16} \sum_s \int d\tau \frac{d^3k}{(2\pi)^3} a(\tau)^2 \left[\frac{\partial h_s(\vec{k})^*}{\partial \tau} \frac{\partial h_s(\vec{k})}{\partial \tau} - k^2 h_s(k)^* h_s(k) \right]$$

SAME AS SCALAR IN NONDYNAMICAL FLRW $\phi = \frac{M_{pl}}{\sqrt{8}} h_s \quad (s \in \{1,2\})$

$\Rightarrow$ LATE TIME TWO-POINT FUNCTION

$$\boxed{\begin{aligned} \langle h_s(\vec{k}) h_{s'}(\vec{k}')^* \rangle &= \frac{8}{M_{pl}^2} \langle \phi(\vec{k}) \phi(\vec{k}')^* \rangle \delta_{ss'} \\ &= \frac{8}{M_{pl}^2} \frac{H_k^2}{2k^3} (2\pi)^3 \delta^3(k-k') \delta_{ss'} \end{aligned}}$$


$\epsilon^{(s)}_{ii} = 0 \quad \epsilon^{(s)}_{ij} = \epsilon^{(s)}_{ji} \quad k_i \epsilon^{(s)}_{ij} = 0$
STANDARD NOTATION
$\langle h_s(\vec{k})\, h_s(\vec{k}')^* \rangle = \frac{2\pi^2}{k^3} \underbrace{\Delta_h^2(k)}_{\text{"DIMENSIONLESS GRAVITY WAVE POWER SPECTRUM"}} \delta_{ss'} (2\pi)^3 \delta^3(k-k')$
$\boxed{\Delta_h^2(k) = \frac{2H_*^2}{\pi^2 M_{pl}^2}}$

$\Rightarrow$ LATE TIME TWO-POINT FUNCTION

$$\boxed{\langle h_s(\vec{k}) h_{s'}(\vec{k}')^* \rangle = \frac{8}{M_{pl}^2} \langle \phi(k)\phi(k)^* \rangle \delta_{ss'} = \frac{8}{M_{pl}^2} \frac{H_k^2}{2k^3} (2\pi)^3 \delta^3(k-k') \delta_{ss'}} \lesssim 10^{-10}$$

SPECTRAL INDEX OR "TILT"

$$n_t = \frac{d \log \Delta_h^2(k)}{d \log k}$$

$$= 2 \frac{d \log H_k}{d \log k}$$

GENERAL RULE (WHEN X_* DEPENDS ON k VIA $k = aH$)

$$\frac{dX_*}{d\log k} = \frac{dX}{d\log aH}$$

$$= \left(\frac{d\log aH}{dt}\right)^{-1}\frac{dX}{dt}$$

$$\simeq \Big(\underbrace{H}_{\frac{d\log a}{dt}} - \underbrace{\varepsilon H}_{\frac{d\log H}{dt}}\Big)^{-1}\frac{dX}{dt}$$

$$= \frac{1}{H}\frac{dX}{dt}$$

$\Rightarrow$ LATE TIME TWO-POINT FUNCTION

$$\boxed{\langle h_s(k) h_{s'}(k') \rangle = \frac{8}{M_{pl}^2} \langle \varphi(k)\varphi(k') \rangle \delta_{ss'} = \frac{8}{M_{pl}^2} \frac{H_k^2}{2k^3} (2\pi)^3 \delta^3(k-k')\, \delta_{ss'}} \lesssim 10^{-10}$$

SPECTRAL INDEX OR "TILT"

$$n_t = \frac{d \log \Delta_h^2(k)}{d \log k}$$

$$= 2 \frac{d \log H_k}{d \log k}$$

$$\approx 2 \frac{1}{H} \frac{d \log H}{dt}$$

$$\approx -2\varepsilon$$

$$\varepsilon \equiv \frac{M_{pl}^2}{2} \left(\frac{V'}{V} \right)^2$$

$aH)$

MOVING ON TO SCALARS (ζ)

$$S = \frac{1}{2}\int d\tau\, d^3x\, \underbrace{\frac{\dot\phi^2 a^2}{H^2}}\left[(\partial_\tau \zeta)^2 - (\partial_i \zeta)^2\right]$$

SAME AS MASSLESS SCALAR IN FAKE EXPANSION HISTORY $z(\tau) = \frac{\dot\phi a}{H}$

TWO HUBBLES

$$H_a = \frac{\partial_\tau a}{a^2} \qquad H_z = \frac{\partial_\tau z}{z^2}$$

$\Rightarrow$ LATE TIME TWO POINT FUNCTION $\langle \zeta(k)\, \zeta(k')^* \rangle = \left(\frac{H_z^2}{2k^3}\right)_* (2\pi)^3 \delta^3(k-k')$

COMPUTE H_z IN SLOW ROLL APPROX

$$k = aH_a$$

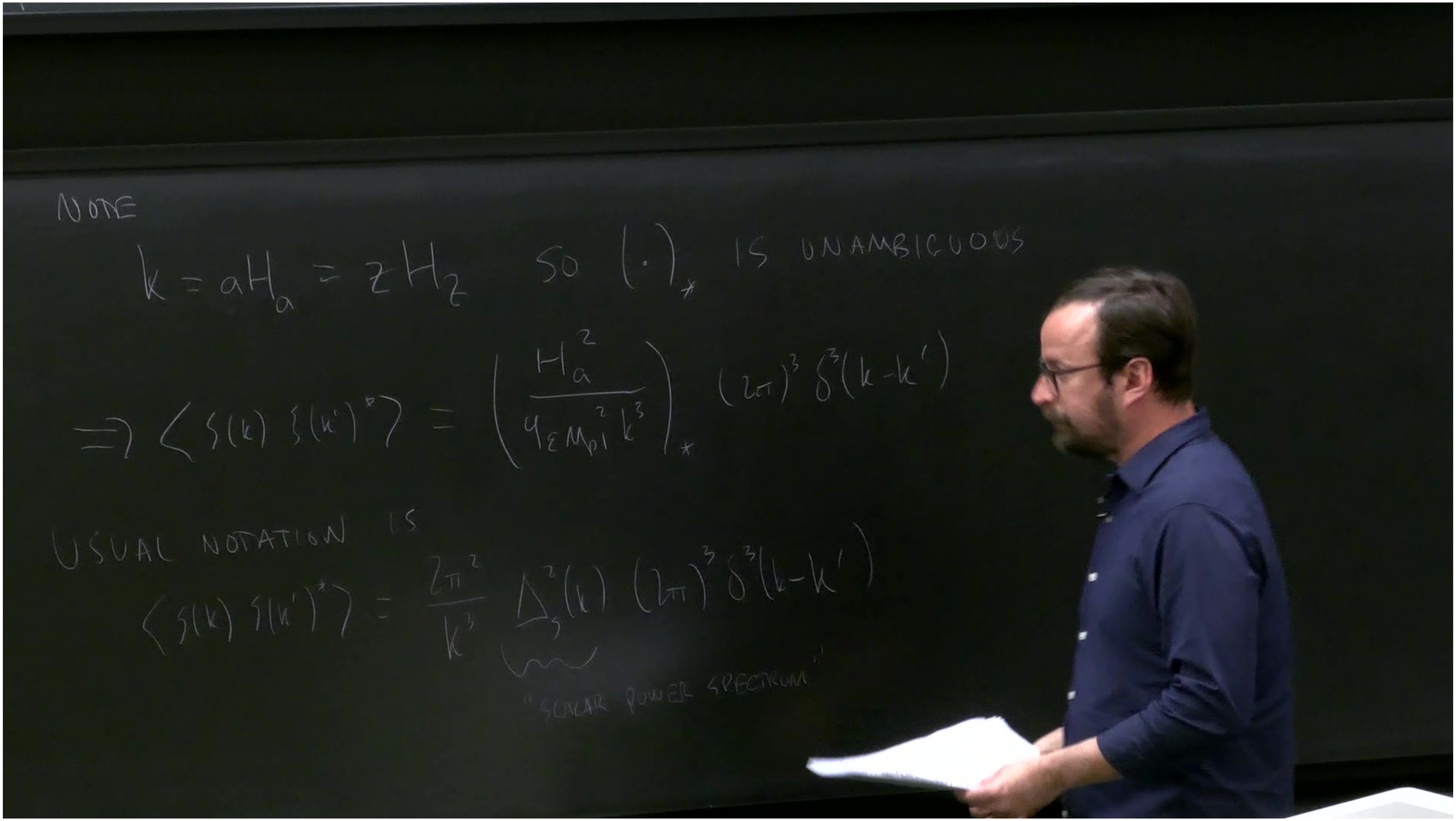
$$z(\tau) \equiv \frac{\dot{\phi}\, a}{H} \approx (2\varepsilon)^{1/2} M_{pl}\, a(\tau)$$

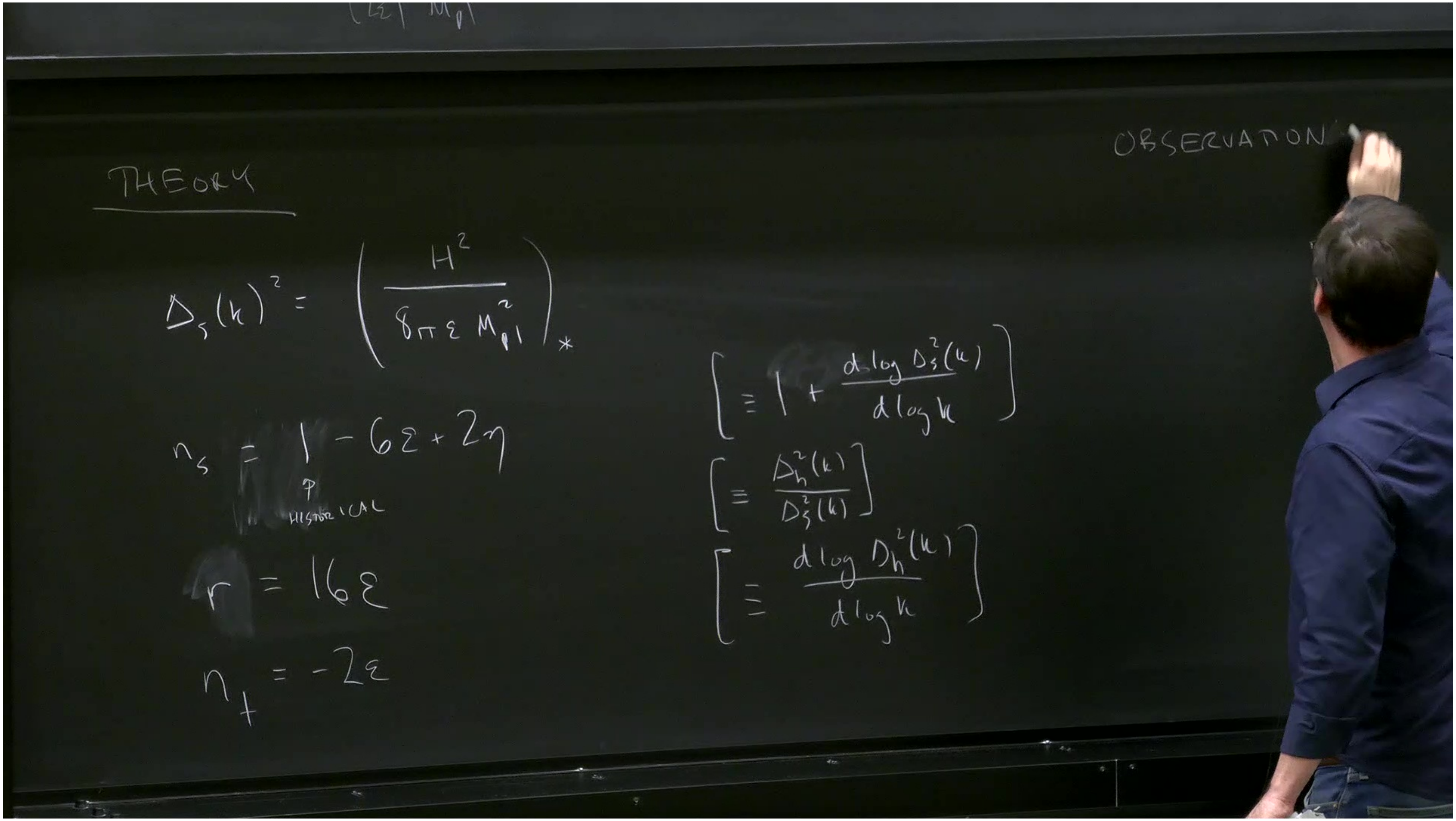
$$H_z \equiv \frac{\partial_\tau z}{z^2} = \frac{1}{z}\, \partial_\tau \log z$$

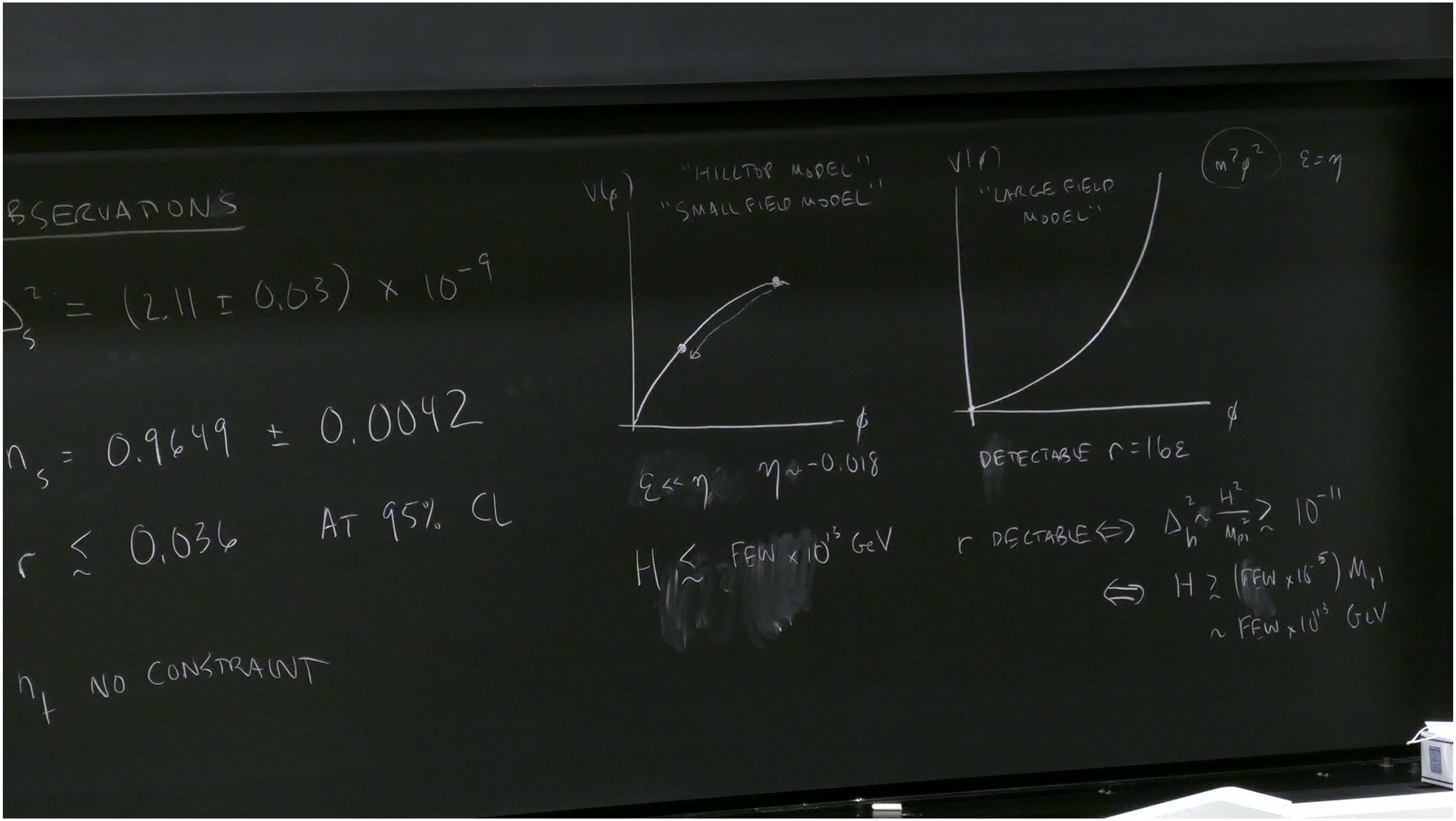
$$\approx \left[(2\varepsilon)^{1/2} M_{pl}\, a \right]^{-1} \left[\frac{1}{2} \frac{\partial_\tau \varepsilon}{\varepsilon} + \frac{\partial_\tau a}{a} \right]$$

$$\approx \left[(2\varepsilon)^{1/2} M_{pl}\, a \right]^{-1} \left[(2\varepsilon - \eta)\, aH_a + aH_a \right]$$

$$\approx \frac{H_a}{(2\varepsilon)^{1/2} M_{pl}}$$


NOTE
$k = aH_a = zH_z$ SO $(\cdot)_*$ IS UNAMBIGUOUS
$\Rightarrow \langle \zeta(k)\, \zeta(k')^* \rangle = \left(\frac{H_a^2}{4\varepsilon M_{pl}^2 k^3} \right)_* (2\pi)^3 \delta^3(k-k')$
USUAL NOTATION IS
$\langle \zeta(k)\, \zeta(k')^* \rangle = \frac{2\pi^2}{k^3} \underbrace{\Delta_s^2(k)}_{\text{"SCALAR POWER SPECTRUM"}} (2\pi)^3 \delta^3(k-k')$


THEORY
OBSERVATIONS
$\Delta_s(k)^2 = \left(\frac{H^2}{8\pi\varepsilon M_{pl}^2}\right)_*$
$n_s = 1 - 6\varepsilon + 2\eta$
HISTORICAL
$\left[\equiv 1 + \frac{d\log \Delta_s^2(k)}{d\log k}\right]$
$r = 16\varepsilon$
$\left[\equiv \frac{\Delta_h^2(k)}{\Delta_s^2(k)}\right]$
$n_t = -2\varepsilon$
$\left[\equiv \frac{d\log \Delta_h^2(k)}{d\log k}\right]$


BSERVATIONS
Δs² = (2.11 ± 0.03) × 10⁻⁹
ns = 0.9649 ± 0.0042
r ≲ 0.036 AT 95% CL
nt NO CONSTRAINT
V(φ)
"HILLTOP MODEL"
"SMALL FIELD MODEL"
φ
ε << η η ~ -0.018
H ≲ FEW × 10¹³ GeV
V(φ)
"LARGE FIELD MODEL"
m²φ² ε = η
φ
DETECTABLE r = 16ε
r DETECTABLE ⟺ Δh² ~ H²/Mpl² ≳ 10⁻¹¹
⟺ H ≳ (FEW × 10⁻⁵) Mpl
~ FEW × 10¹³ GeV

$\Rightarrow$ LATE TIME TWO-POINT FUNCTION

$$\boxed{\langle h_s(k)\, h_{s'}(k')\rangle = \frac{8}{M_{pl}^2}\langle \varphi(k)\varphi(k')\rangle\,\delta_{ss'} = \frac{8}{M_{pl}^2}\frac{H_k^2}{2k^3}(2\pi)^3\delta^3(k-k')\,\delta_{ss'}} \lesssim 10^{-10}$$

$$E_{infl} \sim \rho^{1/4}$$

$$= \left(3M_{pl}^2\, H^2\right)^{1/4}$$

$$= \left(\frac{3\pi^2\,\Delta_s^2\, M_{pl}^4}{2}\right)^{1/4} r^{1/4}$$

$$= \left(3.23\times 10^{16}\ \text{GeV}\right)\times r^{1/4}$$

DECTABLE $r \Leftrightarrow E_{infl} \gtrsim 10^{16}$ GeV [GUT SCALE]

$\epsilon^{(s)}_{ii} = 0 \qquad \epsilon^{(s)}_{ij} = \epsilon^{(s)}_{ji} \qquad k_i \epsilon^{(s)}_{ij} = 0$

INFLATON "FIELD EXCURSION"

$$\frac{d\phi}{d\log a} = \left(\frac{d\log a}{dt}\right)^{-1} \frac{d\phi}{dt}$$

$$\approx H^{-1}\left[(2\epsilon)^{1/2} H M_{pl}\right]$$

$$= (2\epsilon)^{1/2} M_{pl}$$

LYTH "BOUND"