

Title: Cosmology Lecture

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EXPANSION OF UNIVERSE NEGLECTED  
(I.E. MINKOWSKI)

$$\rho = \sum_{\text{SPECIES}} \frac{g}{2\pi^2} \int dq q^2 f_q E_q$$

$$f_q = \left[ e^{E_q/T} \pm 1 \right]^{-1}$$

$$E_q = \sqrt{q^2 + m^2}$$

$\left. \begin{matrix} e^+ \\ e^- \end{matrix} \right\} g=2 \quad m=0.511 \text{ MeV}$   
FERMIONS  
 $\gamma \quad g=2 \quad m=0$   
BOSON

RELATIVISTIC LIMIT  $m \ll T$

$$E_q \approx q$$

$$\rho \approx \frac{g}{2\pi^2} \int dq \frac{q^3}{e^{q/T} \mp 1}$$

$$= \begin{cases} \frac{g}{2\pi^2} \Gamma(4) \zeta(4) T^4 & \text{BOSON} \end{cases}$$

$$\begin{cases} \frac{g}{2\pi^2} \left(1 - \frac{1}{2^4}\right) \Gamma(4) \zeta(4) T^4 & \text{FERMION} \end{cases}$$

$$= \begin{cases} g \frac{\pi^2}{30} T^4 & \text{BOSON} \end{cases}$$

$$\begin{cases} g \left(\frac{7}{8}\right) \frac{\pi^2}{30} T^4 & \text{FERMION} \end{cases}$$

$$\Gamma(4) = 3! = 6$$

$$\zeta(4) = \frac{\pi^4}{90}$$

NONRELATIVISTIC LIMIT  $T \ll m$

$$\rho = \frac{g}{2\pi^2} \int_0^\infty dq q^2 \frac{E_q e^{-E_q/T}}{1 + e^{-E_q/T}}$$

$$E_q = \sqrt{m^2 + q^2}$$

$$q \rightarrow E$$

$$q = \sqrt{E^2 - m^2}$$

$$E dE = q dq$$

$$= \frac{g}{2\pi^2} \int_m^\infty dE E^2 (E^2 - m^2)^{1/2} \frac{e^{-E/T}}{1 + e^{-E/T}}$$

$$x = \frac{E - m}{T}$$

$$\lambda = \frac{m}{T} \gg 1$$

$$= \frac{g T^4}{2\pi^2} \int_0^\infty dx x^{1/2} (x + \lambda)^2 (x + 2\lambda)^{1/2} \frac{e^{-x-\lambda}}{1 + e^{-x-\lambda}}$$

$$\rho = \frac{g}{2\pi^2} \int_0^\infty dq q^2 \frac{e^{-E/T}}{1 + e^{-E/T}}$$

$$= \frac{g}{2\pi^2} \int_m^\infty dE E^2 (E^2 - m^2)^{1/2} \frac{e^{-E/T}}{1 + e^{-E/T}}$$

$$= \frac{gT^4}{2\pi^2} \int_0^\infty dx x^{1/2} (x+\lambda)^2 (x+2\lambda)^{1/2} \frac{e^{-x-\lambda}}{1 + e^{-x-\lambda}}$$

$$\approx \frac{gT^4}{2\pi^2} \int_0^\infty dx x^{1/2} \lambda^2 (2\lambda)^{1/2} e^{-x-\lambda}$$

$$= \frac{gT^4}{2\pi^2} \lambda^2 (2\lambda)^{1/2} e^{-\lambda} \int_0^\infty dx x^{1/2} e^{-x}$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{1}{2} \sqrt{\pi}$$

$$q = \sqrt{E^2 - m^2}$$

$$E dE = q dq$$

$$\lambda = \frac{m}{T} \gg 1$$



to 5 P 25

$$\rho = g_m \left( \frac{mT}{2\pi} \right)^{3/2} \underbrace{e^{-m/T}}_{\text{"BOLTZMANN SUPPRESSION"}}$$

$$\rho = \frac{g}{2\pi^2} \int dq q^2 f_q E_q$$

$$\rightarrow \left\{ \begin{array}{ll} \frac{\pi^2}{30} g T^4 & \text{REL BOSON} \\ \frac{7}{8} \frac{\pi^2}{30} g T^4 & \text{REL FERMION} \\ g_m \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T} & \text{NUMRE } \rho \end{array} \right.$$

NUMBER DENSITY  $n = \frac{g}{2\pi^2} \int dq q^2 f_q$

→  $\left\{ \begin{array}{l} \frac{5(3)}{\pi^2} gT^3 \quad \text{REL BOSON} \\ \frac{3}{4} \frac{5(3)}{\pi^2} gT^3 \quad \text{REL FERMION} \\ g^m \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \quad \text{NONREL} \end{array} \right.$

NONTRIVIAL!

PRESSURE  $P = \frac{g}{2\pi^2} \int dq q^2 f_q \left( \frac{q^2}{3E_q} \right)$

→  $\left\{ \begin{array}{l} \frac{\pi^2}{90} gT^4 \quad \text{REL BOSON} \\ \frac{7}{8} \frac{\pi^2}{90} gT^4 \quad \text{REL FERMION} \\ gT \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \quad \text{NONREL} \end{array} \right.$

$W = \frac{P}{\rho}$

→  $\left\{ \begin{array}{l} \frac{1}{3} \quad \text{REL} \\ \frac{T}{m} \quad \text{NONREL} \end{array} \right.$

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CAUTION

DO NOT TOUCH THE BOARD SURFACE.  
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# EXPANDING UNIVERSE IN THERMAL EQUILIBRIUM

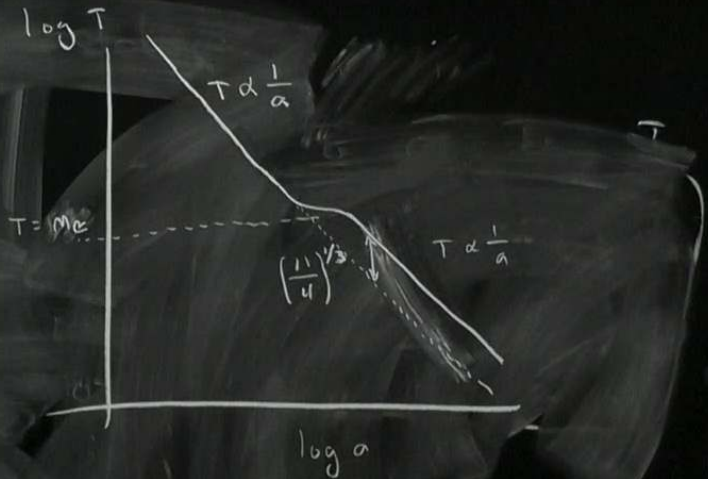
ADIABATIC = REMAINS IN THERMAL EQUILIBRIUM AT TEMP.  $T(a)$

$$p(T) = -$$

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$$\frac{dp}{d \log a} = -3(p + \rho)$$

$$\frac{dT}{d \log a} = \frac{dp/d \log a}{dp/dT} = \frac{-3(p + \rho)}{p'(T)}$$



CONSERVATION LAW

$$S(T) \alpha^3 = \text{CONST.}$$

$$S(T) = \frac{p(T) + P(T)}{T}$$

ENTROPY DENSITY

FIRST ARGUMENT

BRUTE FORCE ALGEBRA

$$\frac{\partial p}{\partial T} = \frac{p + P}{T}$$

"KEY LEMMA"

FIRST ARGUMENT BRUTE FORCE ALGEBRA

$$\frac{\partial p}{\partial T} = \frac{p+p}{T} \quad \text{"KEY LEMMA"}$$

$$\begin{aligned} \frac{d}{dt} \left( a^3 \frac{p+p}{T} \right) &= 3a^2 \dot{a} \left( \frac{p+p}{T} \right) + a^3 \frac{\dot{p}}{T} + a^3 \frac{\dot{p}}{T} - a^3 \frac{p+p}{T^2} \dot{T} \\ &= 3a^2 H \left( \frac{p+p}{T} \right) + \frac{a^3}{T} (-3H(p+p)) + \frac{a^3}{T} \left( \frac{p+p}{T} \right) \dot{T} - \frac{a^3}{T} \frac{p+p}{T^2} \dot{T} \\ &= 0 \end{aligned}$$

$$= \begin{cases} \frac{g \pi^2}{30} T^4 & \text{BOSON} \\ \frac{g(\frac{7}{8}) \pi^2}{30} T^4 & \text{FERMION} \end{cases}$$

$$\begin{aligned} \Gamma(4) &= 3! = 6 \\ \zeta(4) &= \frac{\pi^4}{90} \end{aligned}$$

SECOND ARGUMENT

THERMODYNAMIC

$$dU = T dS - p dV \quad \text{FIRST LAW}$$

$$\Leftrightarrow dS = \frac{1}{T} dU + \frac{p}{T} dV$$

$$= \frac{1}{T} [d(pV) + p dV]$$

$$= \frac{1}{T} [d((p+p)V) - V dp]$$

$$= \frac{1}{T} [d((p+p)V) - V \frac{p+p}{T} dT]$$

$$= d\left(\frac{(p+p)V}{T}\right)$$

BY "WEY LEMMA"

THIRD ARGUMENT "MICROPHYSICAL" ENTROPY

$$S = - \sum_x p(x) \log p(x)$$
$$= \dots \frac{(p+p)V}{T}$$