Title: Cosmology Lecture

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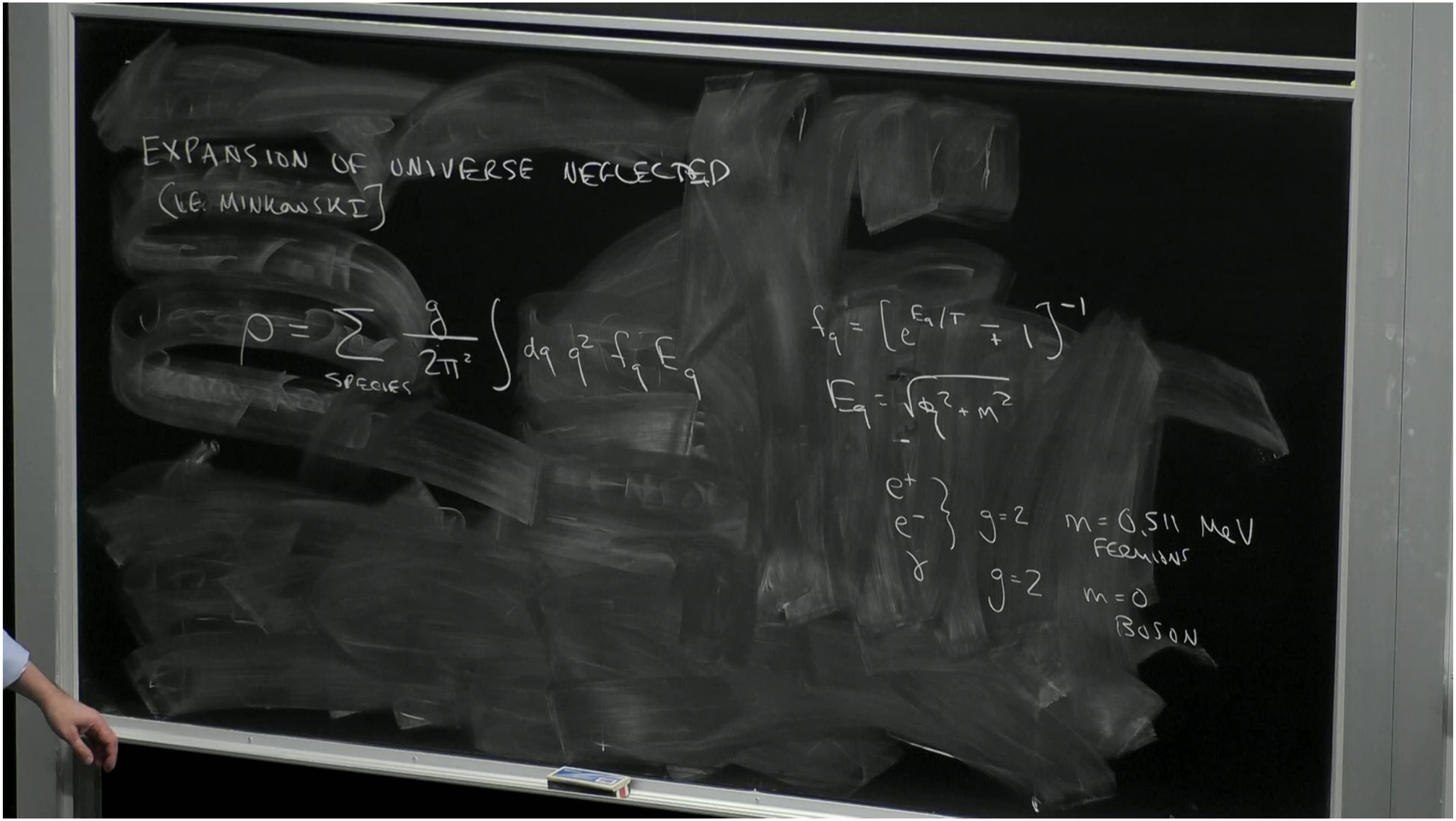

EXPANSION OF UNIVERSE NEGLECTED
(I.E. MINKOWSKI)
$\rho = \sum_{\text{SPECIES}} \frac{g}{2\pi^2} \int dq\, q^2 f_q E_q$
$f_q = \left[e^{E_q/T} \mp 1\right]^{-1}$
$E_q = \sqrt{q^2 + m^2}$
e^+, e^- $g=2$ $m = 0.511$ MeV
FERMIONS
γ $g=2$ $m=0$
BOSON

RELATIVISTIC LIMIT $m \ll T$ $\quad E_q \simeq q$

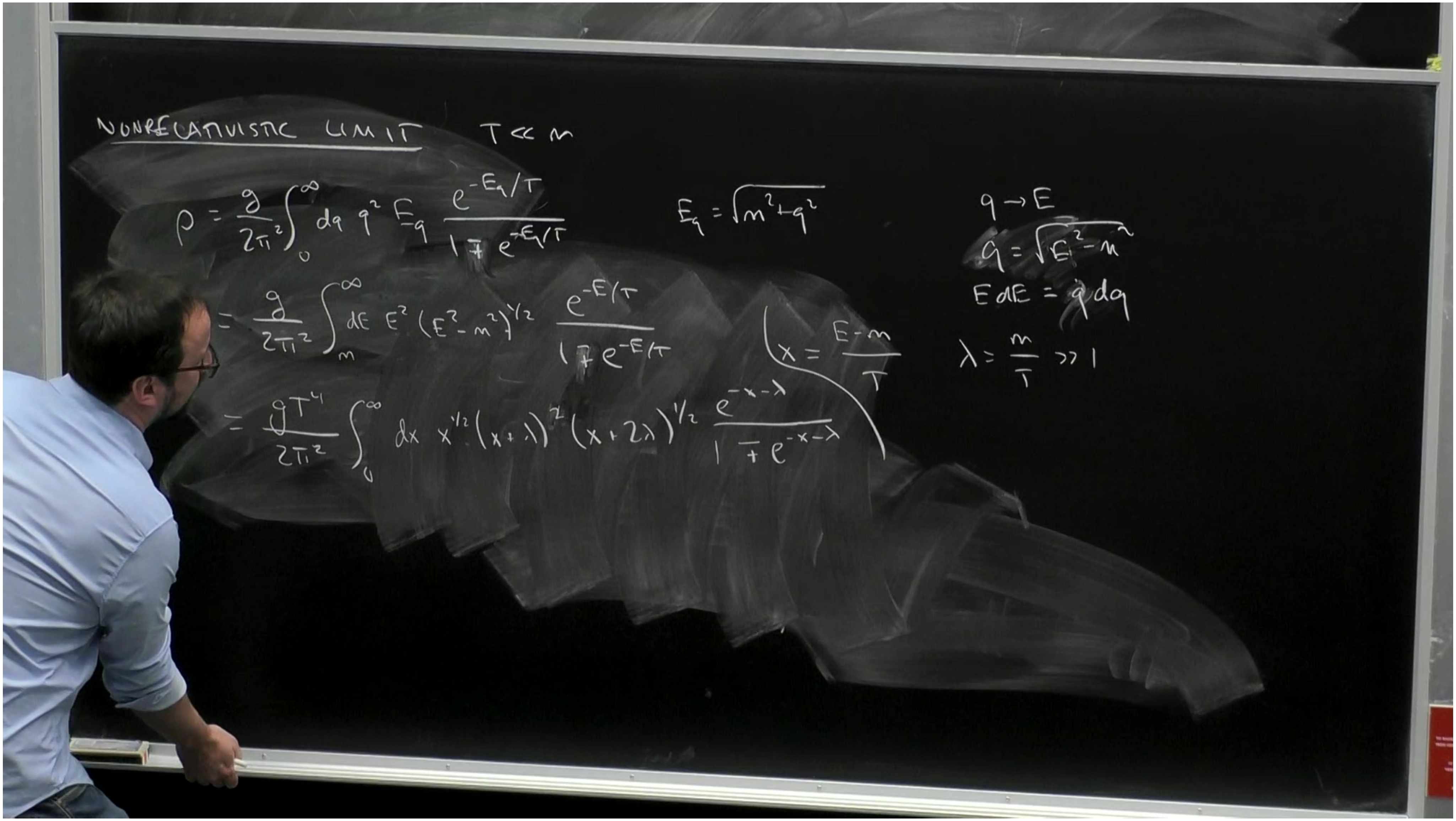
$$\rho \simeq \frac{g}{2\pi^2}\int dq\, \frac{q^3}{e^{q/T} \mp 1}$$

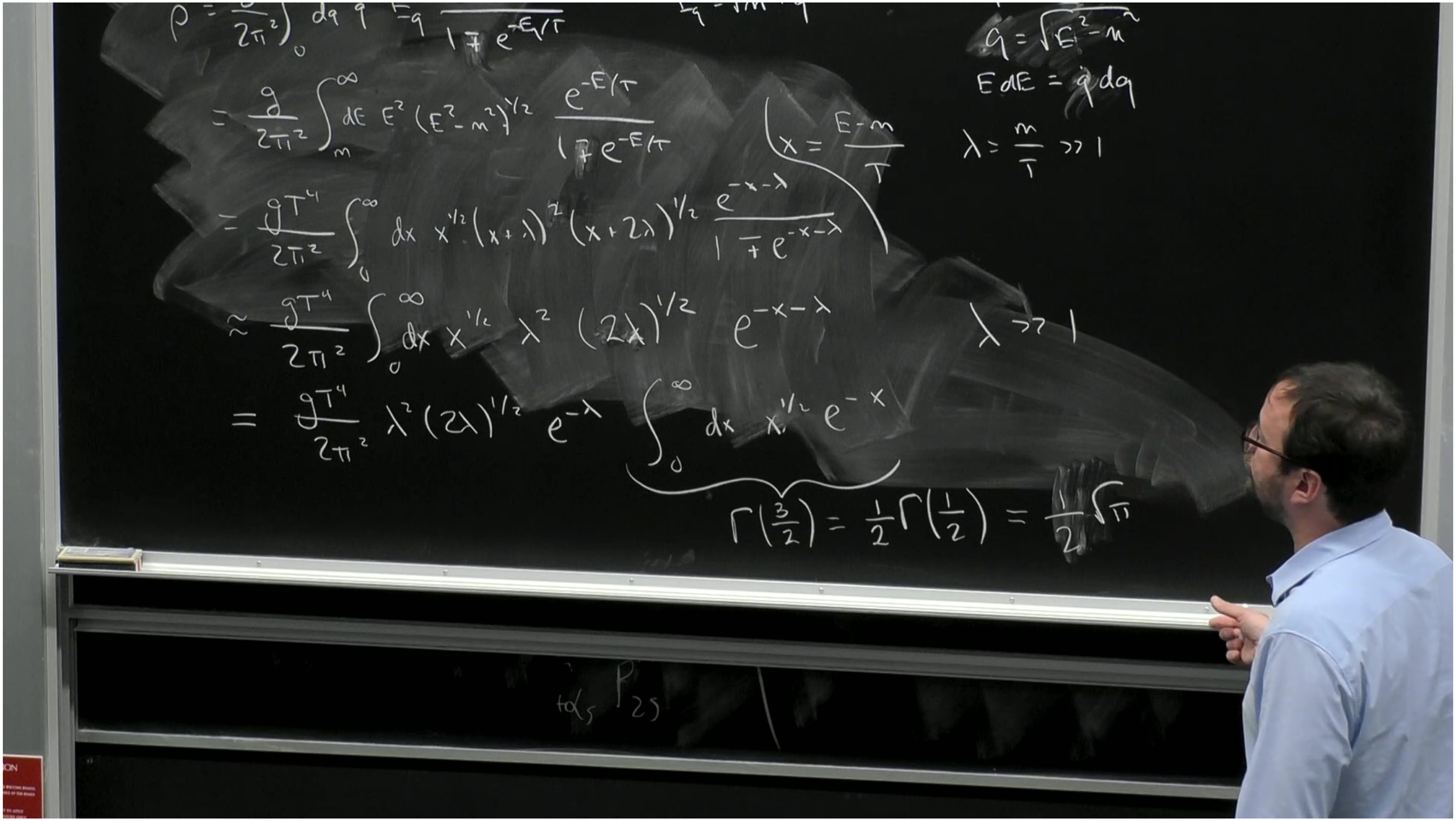
$$= \begin{cases} \frac{g}{2\pi^2}\Gamma(4)\,\zeta(4)\,T^4 & \text{BOSON} \\ \frac{g}{2\pi^2}\left(1-\frac{2}{2^4}\right)\Gamma(4)\,\zeta(4)\,T^4 & \text{FERMION} \end{cases}$$

$$= \begin{cases} g\frac{\pi^2}{30}T^4 & \text{BOSON} \\ g\left(\frac{7}{8}\right)\frac{\pi^2}{30}T^4 & \text{FERMION} \end{cases}$$

$\Gamma(4) = 3! = 6$

$\zeta(4) = \frac{\pi^4}{90}$


NONRELATIVISTIC LIMIT
$T \ll m$
$\rho = \frac{g}{2\pi^2}\int_0^\infty dq\, q^2 E_q \frac{e^{-E_q/T}}{1 \mp e^{-E_q/T}}$
$E_q = \sqrt{m^2+q^2}$
$q \to E$
$q = \sqrt{E^2-m^2}$
$E\,dE = q\,dq$
$= \frac{g}{2\pi^2}\int_m^\infty dE\, E^2 (E^2-m^2)^{1/2} \frac{e^{-E/T}}{1 \mp e^{-E/T}}$
$x = \frac{E-m}{T}$
$\lambda = \frac{m}{T} \gg 1$
$= \frac{gT^4}{2\pi^2}\int_0^\infty dx\, x^{1/2}(x+\lambda)^2(x+2\lambda)^{1/2} \frac{e^{-x-\lambda}}{1 \mp e^{-x-\lambda}}$


$q = \sqrt{E^2 - m^2}$
$E\,dE = q\,dq$
$= \frac{g}{2\pi^2}\int_m^\infty dE\, E^2 (E^2-m^2)^{1/2} \frac{e^{-E/T}}{1 \mp e^{-E/T}}$
$x = \frac{E-m}{T}$
$\lambda = \frac{m}{T} \gg 1$
$= \frac{gT^4}{2\pi^2}\int_0^\infty dx\, x^{1/2}(x+\lambda)^2(x+2\lambda)^{1/2}\frac{e^{-x-\lambda}}{1\mp e^{-x-\lambda}}$
$\approx \frac{gT^4}{2\pi^2}\int_0^\infty dx\, x^{1/2}\lambda^2(2\lambda)^{1/2} e^{-x-\lambda}$
$\lambda \gg 1$
$= \frac{gT^4}{2\pi^2}\lambda^2(2\lambda)^{1/2}e^{-\lambda}\underbrace{\int_0^\infty dx\, x^{1/2}e^{-x}}$
$\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi}$

$$\rho \simeq g m \left(\frac{mT}{2\pi}\right)^{3/2} \underbrace{e^{-m/T}}_{\text{“BOLTZMANN SUPPRESSION”}}$$

$$\rho = \frac{g}{2\pi^2}\int dq\, q^2\, f_q E_q \rightarrow \begin{cases} \frac{\pi^2}{30} g T^4 & \text{REL BOSON} \\ \frac{7}{8}\frac{\pi^2}{30} g T^4 & \text{REL FERMION} \\ g m \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} & \text{NONREL} \end{cases}$$

NUMBER DENSITY $n = \frac{g}{2\pi^2}\int dq\, q^2 f_q$

$$\to \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{REL BOSON} \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3 & \text{REL FERMION} \\ g\, m\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} & \text{NONREL} \end{cases}$$

$\longrightarrow$ NONTRIVIAL!

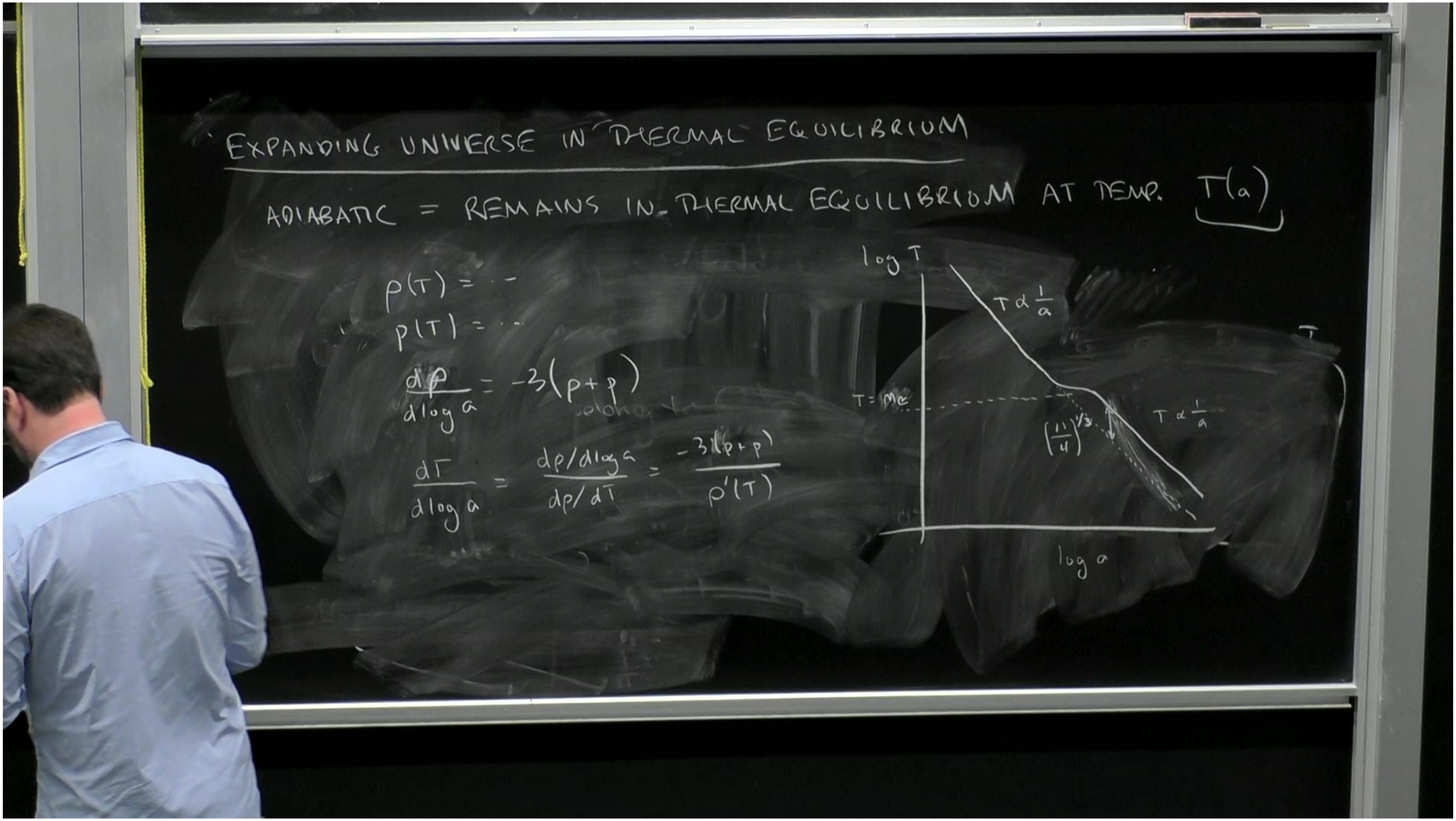
PRESSURE $p = \frac{g}{2\pi^2}\int dq\, q^2 f_q \left(\frac{q^2}{3E_q}\right)$

$$\to \begin{cases} \frac{\pi^2}{90} g T^4 & \text{REL BOSON} \\ \frac{7}{8}\frac{\pi^2}{90} g T^4 & \text{REL FERMION} \\ g T\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} & \text{NONREL} \end{cases}$$

$w = \frac{p}{\rho}$

$$\to \begin{cases} \frac{1}{3} & \text{REL} \\ \frac{T}{m} & \text{NONREL} \end{cases}$$

$\frac{T}{m} \to 0$


EXPANDING UNIVERSE IN THERMAL EQUILIBRIUM
ADIABATIC = REMAINS IN THERMAL EQUILIBRIUM AT TEMP. T(a)
ρ(T) = ...
p(T) = ...
dρ/dlog a = -3(ρ+p)
dT/dlog a = (dρ/dlog a)/(dρ/dT) = -3(ρ+p)/ρ'(T)
log T
T ∝ 1/a
T = me
(11/4)^1/3
T ∝ 1/a
log a

CONSERVATION LAW. $s(T)\,a^3 = \text{CONST.}$

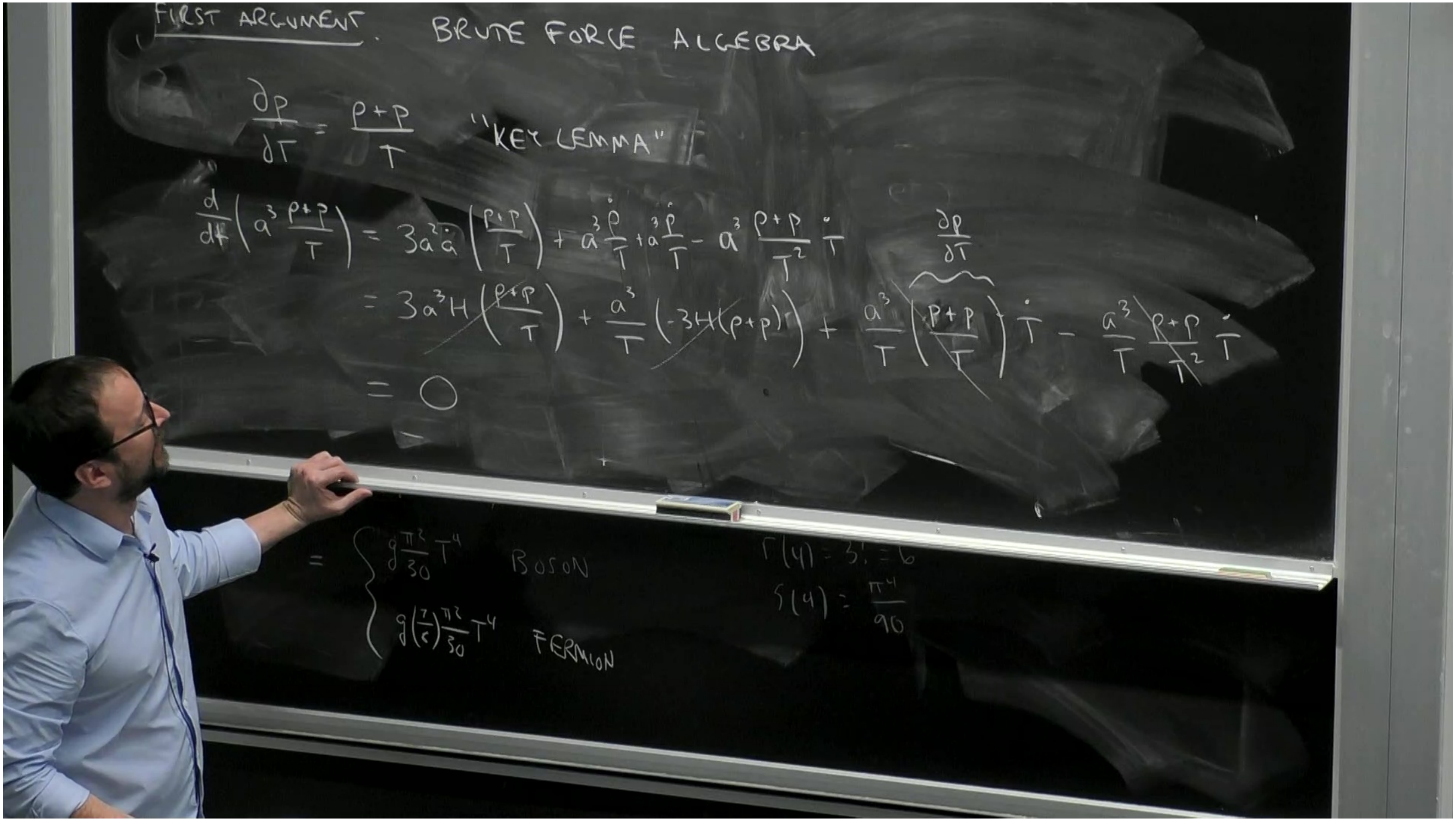
$$s(T) = \frac{\rho(T) + p(T)}{T}$$

ENTROPY DENSITY

FIRST ARGUMENT. BRUTE FORCE ALGEBRA

$$\frac{\partial p}{\partial T} = \frac{\rho + p}{T}$$

"KEY LEMMA"


FIRST ARGUMENT. BRUTE FORCE ALGEBRA
$\frac{\partial p}{\partial T} = \frac{\rho + p}{T}$ "KEY LEMMA"
$\frac{d}{dt}\left(a^3 \frac{\rho+p}{T}\right) = 3a^2\dot{a}\left(\frac{\rho+p}{T}\right) + a^3\frac{\dot\rho}{T} + a^3\frac{\dot p}{T} - a^3\frac{\rho+p}{T^2}\dot T$
$= \cancel{3a^3H\left(\frac{\rho+p}{T}\right)} + \frac{a^3}{T}\left(\cancel{-3H(\rho+p)}\right) + \frac{a^3}{T}\underbrace{\left(\frac{\rho+p}{T}\right)}_{\frac{\partial p}{\partial T}}\dot T - \frac{a^3}{T}\frac{\rho+p}{T^2}\dot T$
$= 0$
$= g\frac{\pi^2}{30}T^4$ BOSON
$g\left(\frac{7}{8}\right)\frac{\pi^2}{30}T^4$ FERMION
$\Gamma(4) = 3! = 6$
$\zeta(4) = \frac{\pi^4}{90}$

SECOND ARGUMENT: THERMODYNAMIC

$$dU = T\,dS - p\,dV \qquad \text{FIRST LAW}$$

$$\Leftrightarrow \quad dS = \frac{1}{T}\,dU + \frac{p}{T}\,dV$$

$$= \frac{1}{T}\left[d(\rho V) + p\,dV \right]$$

$$= \frac{1}{T}\left[d((\rho+p)V) - V\,dp \right]$$

$$= \frac{1}{T}\left[d((\rho+p)V) - V\,\frac{\rho+p}{T}\,dT \right] \qquad \text{BY ``WET LEMMA''}$$

$$= d\left(\frac{(\rho+p)V}{T} \right)$$

THIRD ARGUMENT: "MICROPHYSICAL" ENTROPY

$$S = -\sum_x p(x)\log p(x)$$

$$= \dots \frac{(\rho+p)V}{T}\dots$$