Title: Cosmology Lecture

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Collection: Cosmology 2023/24

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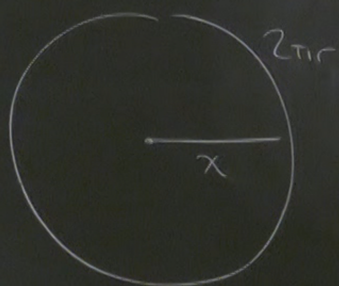
SPATIAL CURVATURE

$$K = [\text{COMOVING LENGTH}]^{-2}$$



$$d\ell^2 = d\chi^2 + S_k(\chi)^2\left[d\theta^2 + \sin^2\theta\,(d\phi)^2\right]$$

$$r = S_k(\chi) = \begin{cases} \frac{1}{\sqrt{k}}\sin(\sqrt{k}\,\chi) & k > 0 \\ \chi & k = 0 \\ \frac{1}{\sqrt{-k}}\sinh(\sqrt{-k}\,\chi) & k < 0 \end{cases}$$

$$d\ell^2 = \frac{dr^2}{1-Kr^2} + r^2\left[d\theta^2 + \sin^2\theta\,(d\phi)^2\right]$$

$$\chi = \begin{cases} \frac{1}{\sqrt{k}}\sin^{-1}(\sqrt{k}\,r) & k > 0 \\ r & k = 0 \\ \frac{1}{\sqrt{-k}}\sinh^{-1}(\sqrt{-k}\,r) & k < 0 \end{cases}$$

> 0

$= 0$

< 0

$$ds^2 = -dt^2 + a(t)^2 \, d\ell^2$$

$$K \in \{0, \pm 1\}$$

$$x \to \lambda x$$

$$a \to \frac{a}{\lambda}$$

$$K \to \frac{K}{\lambda^2}$$

$$\chi = \begin{cases} r \\ \frac{1}{\sqrt{-k}} \sinh^{-1}(\sqrt{-k}\, r \end{cases}$$

DYNAMICS

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p)$$

$$\dot{\rho} = -3H(\rho + p)$$

UNCHANGED

$$H^2 + \frac{k^2}{a^2} =$$

$$\chi = \begin{cases} \ldots & k>0 \\ r & k=0 \\ \frac{1}{\sqrt{-k}}\sinh^{-1}(\sqrt{-k}\,r) & k<0 \end{cases}$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda\right)$$

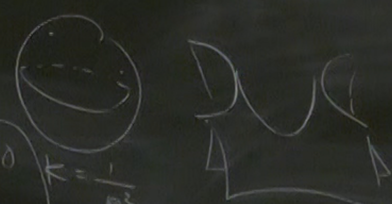
"PHYSICS" PARAMETERIZATION

$$H(a) = H_0\left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right]^{1/2}$$

$$\Omega_k = -\frac{k}{H_0^2}$$

THINGS WE HAVEN'T SHOWN

- METRIC CORRESPONDS TO PICTURES
- $K \neq 0$ HAS AS MUCH SYMMETRY AS $K=0$ $\quad k = \frac{1}{R^2}$
- FRIEDMANN EQS ARE AS CLAIMED



$\frac{1}{\sqrt{-k}} \sinh^{-1}(\sqrt{-k}\, r) \quad k<0$

$$H^2 + \frac{k^2}{a^2} = \frac{8\pi G}{3}\left(\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda\right)$$

"PHYSICS" PARAMETERIZATION

$$\longrightarrow H(a) = H_0\left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda\right]^{1/2}$$

$$\Omega_k = -\frac{k}{H_0^2}$$

THINGS WE HAVEN'T SHOWN

- METRIC CORRESPONDS TO PICTURES
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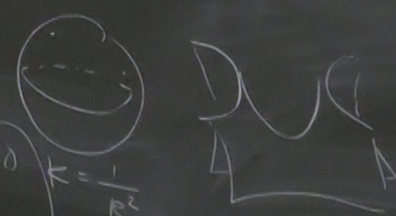


– FRIEDMANN EQS ARE AS

SUPPOSE WE CAN WRITE $\rho = \rho(a)$

ANALOGY W/ PARTICLE IN 1-D POTENTIAL $V(x)$

$$\frac{\dot{x}^2}{2} + V(x) = \text{CONST} \qquad \ddot{x} = -V'(x)$$

IF $\rho = \rho(a)$ THEN FRIEDMANN EQS CAN BE WRITTEN

$$\frac{\dot{a}^2}{2} - \frac{4\pi G}{3} a^2 \rho(a) + \frac{K}{2} = 0 \qquad \ddot{a} = \frac{d}{da}\left[\frac{4\pi G}{3} a^2 \rho(a)\right]$$

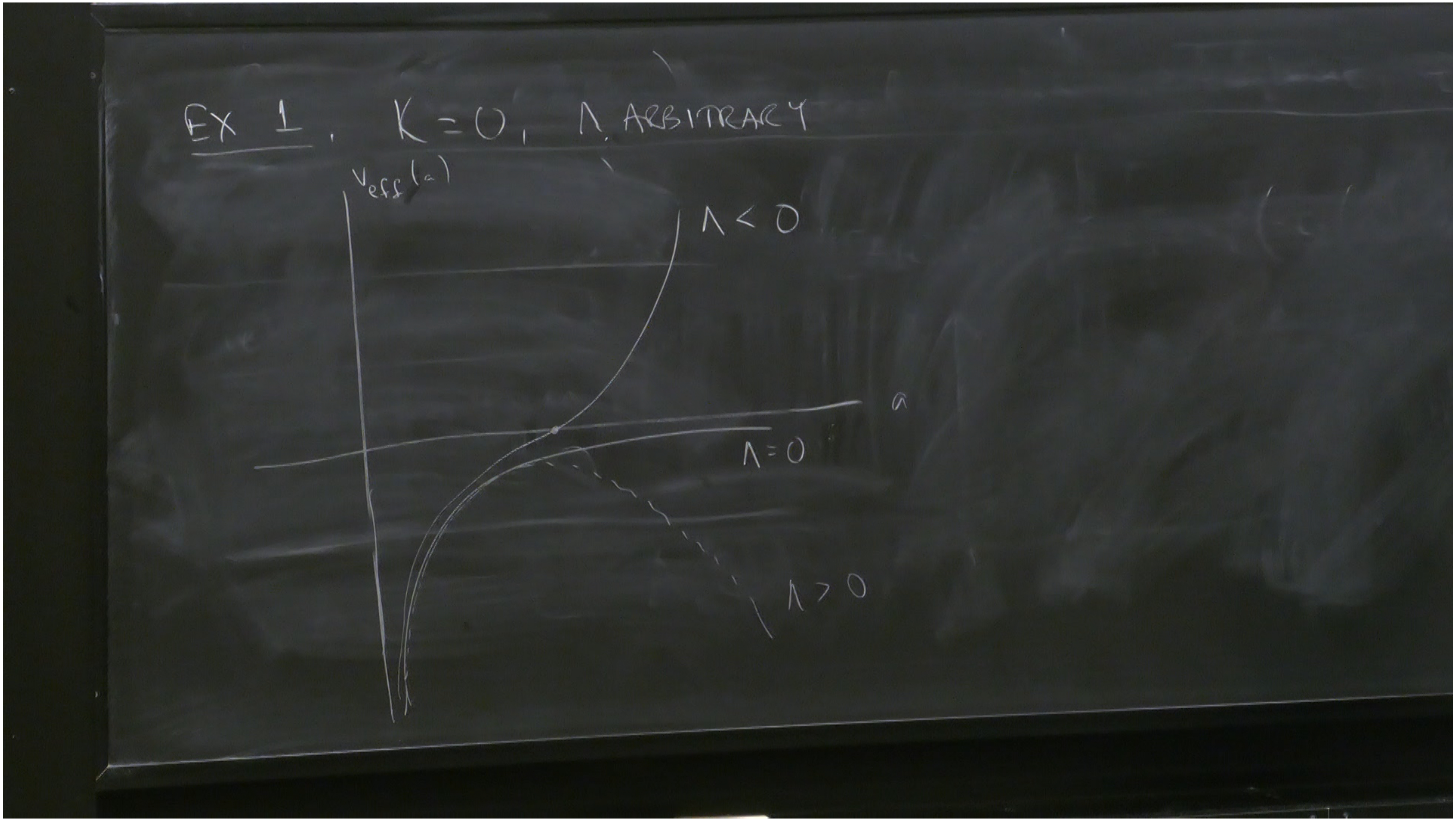
...QS ARE AS CLAIMED

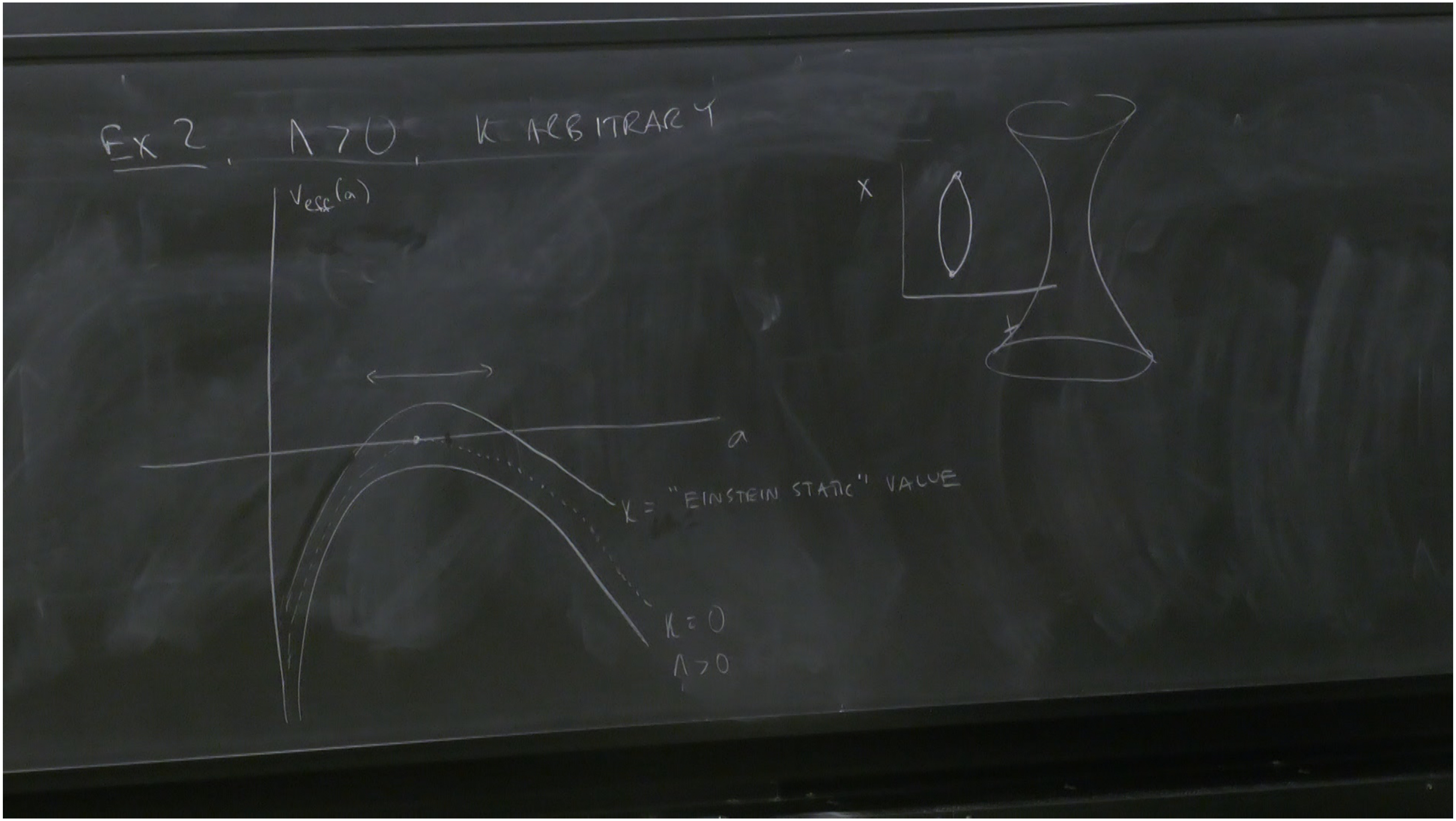
SAME EQS WITH

$$V_{eff}(a) = -\frac{4\pi G}{3} a^2 \rho(a) + \frac{K}{2}$$

$$= -\frac{4\pi G}{3}\left[\frac{\rho_{r0}}{a^2} + \frac{\rho_{m0}}{a} + \Lambda a^2\right] + \frac{K}{2}$$

AND TOTAL ENERGY "TUNED" TO ZERO


EX 1. K = 0, Λ ARBITRARY
$V_{eff}(a)$
Λ < 0
a
Λ = 0
Λ > 0


Ex 2 , Λ > 0 , K ARBITRARY
Veff(a)
a
x
K = "EINSTEIN STATIC" VALUE
K = 0
Λ > 0

CONFORMAL TIME

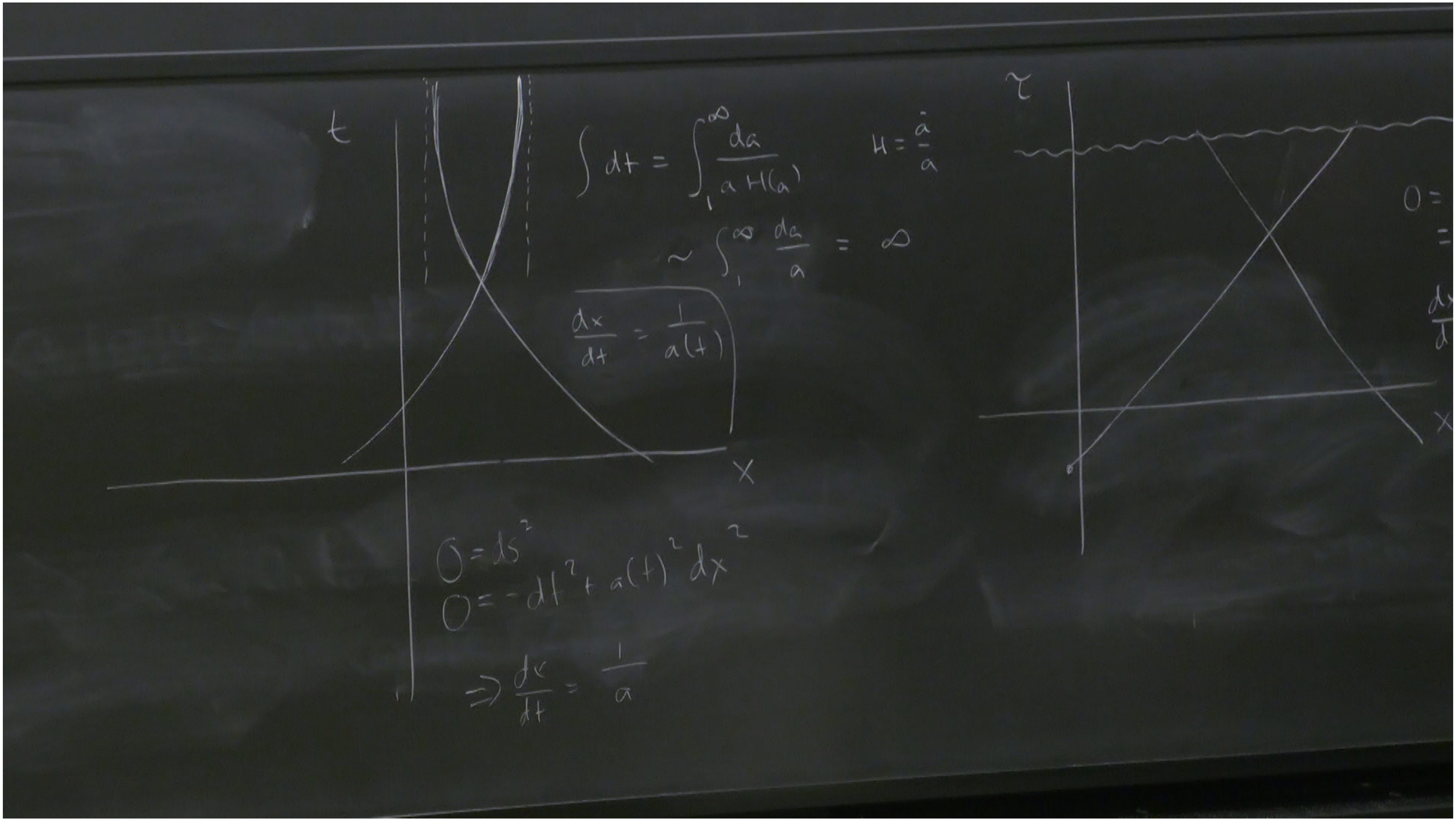
$$ds^2 = -dt^2 + a(t)^2 dx^2$$

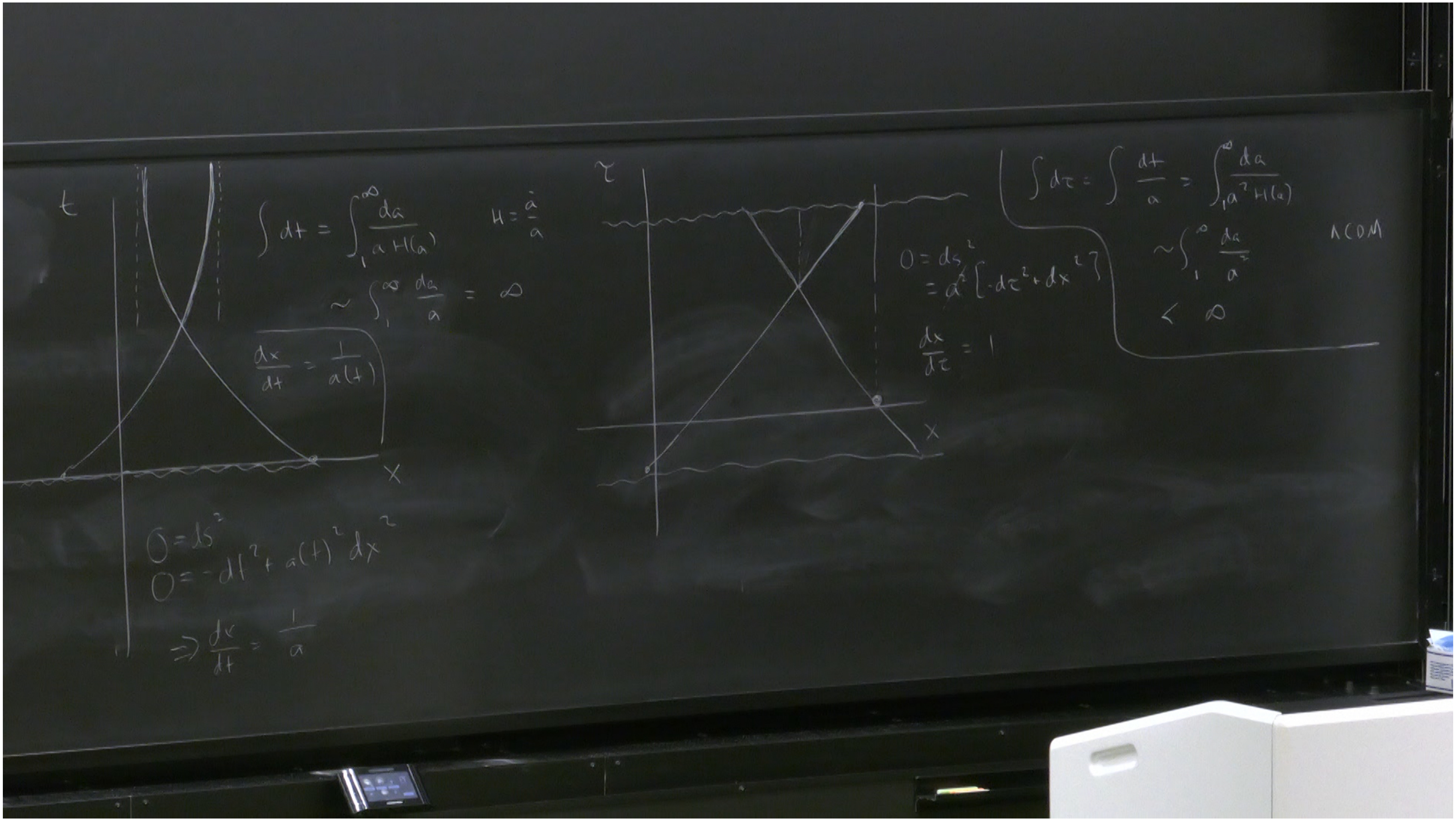
DEFINE CONFORMAL TIME τ BY:

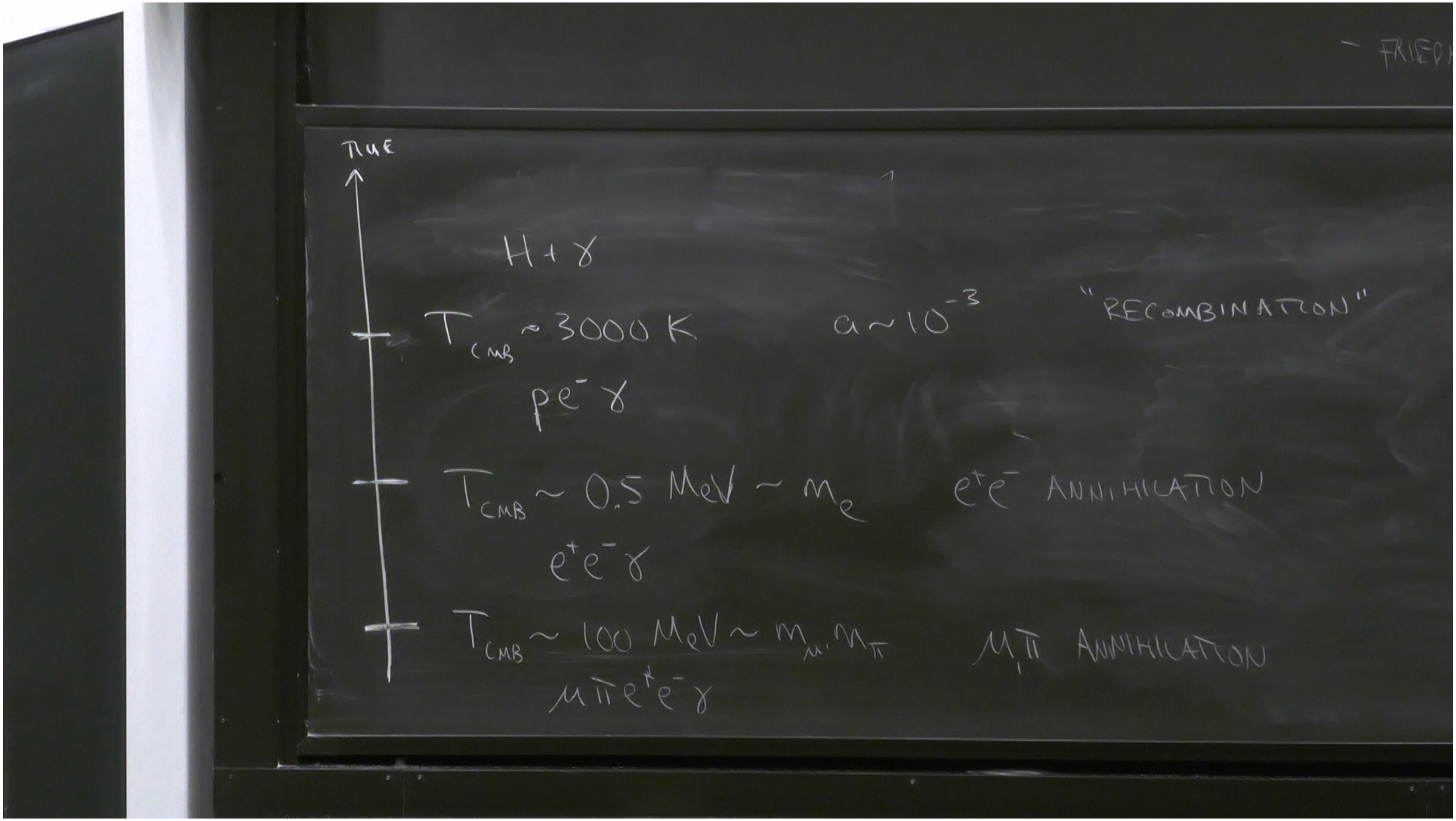
$$dt = a\, d\tau \qquad \tau = \int_0^t \frac{dt'}{a(t')}$$

$$ds^2 = -(a\, d\tau)^2 + a^2 dx^2$$

$$= a(\tau)^2 \underbrace{\left[-d\tau^2 + dx^2\right]}_{\text{MINKOWSKI}}$$


t
x
$\int dt = \int_1^\infty \frac{da}{a H(a)}$
$H = \frac{\dot{a}}{a}$
$\sim \int_1^\infty \frac{da}{a} = \infty$
$\frac{dx}{dt} = \frac{1}{a(t)}$
$0 = ds^2$
$0 = -dt^2 + a(t)^2 dx^2$
$\Rightarrow \frac{dx}{dt} = \frac{1}{a}$
τ
x


t
∫dt = ∫_1^∞ da / (a H(a))
H = ȧ/a
~ ∫_1^∞ da/a = ∞
dx/dt = 1/a(t)
x
0 = ds²
0 = -dt² + a(t)² dx²
⇒ dx/dt = 1/a
τ
0 = ds² = a² [-dτ² + dx²]
dx/dτ = 1
x
∫dτ = ∫ dt/a = ∫_1^∞ da / (a² H(a))
~ ∫_1^∞ da/a²
< ∞
ΛCDM


TIME
H + γ
T_CMB ~ 3000 K
a ~ 10^-3
"RECOMBINATION"
p e^- γ
T_CMB ~ 0.5 MeV ~ m_e
e^+ e^- ANNIHILATION
e^+ e^- γ
T_CMB ~ 100 MeV ~ m_μ, m_π
μ, π ANNIHILATION
μ π e^+ e^- γ

- FRIEDMANN EQS ARE AS CLAIMED

RON"

COSMIC NEUTRINO BACKGROUND

$$T_{CNB} = \left(\frac{4}{11}\right)^{1/3} T_{CMB}$$

$$= 1.95\ K$$

OBSERVED VALUE $(1.94 \pm 0.03)\ K$