

Title: Cosmology Lecture

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MASSLESS SCALAR IN NONDYNAMICAL DE

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) \right)$$
$$= \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} \frac{1}{(H\tau)^2} \left[\frac{\partial \phi_k}{\partial \tau} \frac{\partial \phi_k}{\partial \tau} - k^2 \phi_k \phi_k \right]$$

CHANGE VARS TO "CANONICALLY NORMALIZED"

FIELD $\chi = \frac{1}{H\tau} \phi$

$$\phi = H\tau \chi$$
$$\partial_\tau \phi = (H\chi + H\tau \frac{\partial \chi}{\partial \tau})$$

SITTER

$$g_{\mu\nu} = \begin{pmatrix} -a^2 & \\ & a^2 \delta_{ij} \end{pmatrix}$$

$$a(\tau) = -\frac{1}{H\epsilon} \quad \text{WHERE } -\infty < \tau < 0$$

$$S = \dots$$

$$\begin{aligned}
S &= \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} \frac{1}{(\hbar c)^2} \left[\left(\hbar c \frac{\partial \psi_k}{\partial \tau} + \hbar \psi_k \right)^* \left(\hbar c \frac{\partial \psi_k}{\partial \tau} + \hbar \psi_k \right) - k^2 (\hbar c)^2 \psi_k \psi_k^* \right] \\
&= \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} \left[\frac{\partial \psi_k^*}{\partial \tau} \frac{\partial \psi_k}{\partial \tau} + \frac{1}{\tau} \frac{\partial \psi_k^*}{\partial \tau} \psi_k + \frac{1}{\tau} \psi_k^* \frac{\partial \psi_k}{\partial \tau} - \left(k^2 - \frac{1}{c^2} \right) \psi_k^* \psi_k \right] \\
&= \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} \left[\frac{\partial \psi_k^*}{\partial \tau} \frac{\partial \psi_k}{\partial \tau} - \underbrace{\left(k^2 - \frac{2}{c^2} \right)}_{\omega(k)^2 = k^2 - \frac{2}{c^2}} \psi_k^* \psi_k \right]
\end{aligned}$$

SHO

$$S = \int d\tau \left(\frac{1}{2} \dot{x}^2 - \frac{1}{2} \omega(t)^2 x^2 \right)$$

$$p = \frac{\partial L}{\partial \dot{x}} = \dot{x}$$

$$[\hat{x}, \hat{p}] = i$$

$$H = \dot{x}p - L = \frac{p^2}{2} - \frac{\omega(t)^2}{2} x^2$$

QFT

$$S = \int d\tau \frac{d^3k}{(2\pi)^3} \left[\frac{1}{2} \frac{\partial \psi_k^*}{\partial \tau} \frac{\partial \psi_k}{\partial \tau} - \frac{1}{2} \left(k^2 - \frac{2}{\tau^2} \right) \psi_k^* \psi_k \right]$$

$$\pi_k = \frac{\partial \mathcal{L}}{\partial (\partial_\tau \psi)} = \frac{\partial \psi_k}{\partial \tau}$$

$$[\hat{\psi}_k, \hat{\pi}_{k'}] = i (2\pi)^3 \delta^3(k+k')$$

$$\hat{H} = \int d^3x \left[\pi \dot{\psi} - \mathcal{L} \right] = \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{2} \pi_k^* \pi_k - \frac{1}{2} \left(k^2 - \frac{2}{\tau^2} \right) \psi_k^* \psi_k \right]$$

$$\frac{\partial \hat{\psi}_k}{\partial \tau} = \hat{\pi}_k$$

$$\frac{\partial \hat{\pi}_k}{\partial \tau} = -\left(k^2 - \frac{z}{\tau^2}\right) \hat{\psi}_k$$

$$\hat{\psi}_k(\tau) = u_\psi(k, \tau) a_k + u_\psi^*(k, \tau) a_{-k}^\dagger$$

$$\hat{\pi}_k(\tau) = \dot{u}_\psi(k, \tau) a_k + \dot{u}_\psi^*(k, \tau) a_{-k}^\dagger$$

$$\ddot{u}_\psi(k, \tau) = -\left(k^2 - \frac{z}{\tau^2}\right) u_\psi(k, \tau)$$

$$u_\psi(k, \tau) \rightarrow \frac{1}{(2k)^{1/2}} e^{-ik\tau} \quad \text{AS } \tau \rightarrow -\infty$$

$$\psi_k^* = \psi_{-k} \quad \hat{\psi}_k^\dagger = \hat{\psi}_{-k}$$

$$\text{WHERE } [a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k - k')$$

$$\frac{d\hat{x}}{dt} = \hat{p}$$

$$\frac{d\hat{p}}{dt} = -\omega(t)^2 \hat{x}$$

$$\frac{\partial \hat{\psi}_k}{\partial \tau} = \frac{\hat{\pi}_k}{\hbar}$$

$$\hat{x}_H(t) = u(t)a + u(t)^* a^\dagger$$

$$\hat{p}_H(t) = \dot{u}(t)a + \dot{u}(t)^* a^\dagger$$

$$\ddot{u}(t) = -\omega(t)^2 u(t)$$

$$u(t) \rightarrow \frac{1}{(2\omega_0)^{1/2}} e^{-i\omega_0 t}$$

$$[a, a^\dagger] = 1$$

$$[\hat{x}_H, \hat{p}_H] = \text{Det} \begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix} = i$$

$$[\hat{x}, \hat{p}] = \text{Det} \begin{pmatrix} u(H) & u(H^*) \\ \dot{u}(H) & \dot{u}(H^*) \end{pmatrix} = i$$

$$[\hat{\phi}_k, \hat{\pi}_k] = i$$

$$u_{\phi}(k, \tau) = \frac{1}{(2k)^{1/2}} \left(1 - \frac{i}{k\tau} \right) e^{-ik\tau}$$

LATE-TIME

HARD TO FIND

LOOKS DIVERGENT AT LATE TIMES $\tau \rightarrow 0$

IN ORIGINAL FIELD VARIABLE $\hat{\phi} = H\tau \hat{\psi}$

$$\hat{\phi}_k(\tau) = u_{\phi}(k, \tau) a_{k+} + u_{\phi}^*(k, \tau) a_{-k+}$$

WHERE $u_{\phi}(k, \tau) \equiv H\tau u_{\psi}(k, \tau) = \frac{H}{(2k^3)^{1/2}} (1 + ik\tau) e^{-ik\tau}$

$$(2\pi)^3 \delta^3(k+k')$$

2-POINT FUNCTION

$$c) \phi_k(\tau) |0\rangle = \langle 0 | \left(u_\phi(k, \tau) a_k + u_\phi^*(k, \tau) a_{-k}^\dagger \right) \left(u_\phi(k', \tau) a_{k'} + u_\phi^\dagger(k', \tau) a_{-k'}^\dagger \right) |0\rangle$$

$$= u_\phi(k, \tau) u_\phi^\dagger(k', \tau) \langle 0 | a_k a_{-k'}^\dagger |0\rangle$$

$$= \frac{H^2}{(2k^3)} (1+ik\tau)(1-ik'\tau) e^{-ik(\tau-\tau')} (2\pi)^3 \delta^3(k+k')$$

$$\tau \rightarrow 0 \quad = \frac{H^2}{2k^3} (2\pi)^3 \delta^3(k+k') \quad \text{AS } \tau \rightarrow 0$$

$$u(t) \rightarrow \frac{1}{(2\omega_0)^{1/2}} e^{-i\omega_0 t}$$

$$[\hat{x}, \hat{p}] = \text{Det} \begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix} = i$$

$$u_\varphi(k, \tau) \rightarrow \frac{1}{(2k)^{1/2}} e^{ik\tau}$$

$$[\varphi_k, \varphi_{k'}] = i(2\pi)^3 \delta^3(k+k')$$

$$u_\varphi(k, \tau) = \frac{1}{(2k)^{1/2}} \left(1 - \frac{i}{k\tau} \right) e^{-ik\tau}$$

HARD TO FIND

LOOKS DIVERGENT AT LATE TIMES $\tau \rightarrow 0$

IN ORIGINAL FIELD VARIABLE $\hat{\phi} = H\tau \hat{\psi}$

$$\hat{\phi}_k(\tau) = u_\varphi(k, \tau) a_{\varphi k} + u_\varphi^*(k, \tau) a_{-\varphi k}$$

WHERE $u_\varphi(k, \tau) \equiv H\tau u_\psi(k, \tau) = \frac{H}{(2k^3)^{1/2}} (1 + ik\tau) e^{-ik\tau}$

LATE-TIME TWO POINT FUNCTION

$$\langle 0 | \phi_k(\tau) \phi_{k'}(\tau) | 0 \rangle = \langle 0 |$$

"BUNCH-DAVIES
VACUUM"

$$(2\pi)^3 \delta^3(k+k')$$

2-POINT FUNCTION

$$c) \phi_k(\tau) |0\rangle = \langle 0 | \left(u_\phi(k, \tau) a_k + u_\phi^*(k, \tau) a_{-k}^\dagger \right) \left(u_\phi(k', \tau) a_{k'} + u_\phi^\dagger(k', \tau) a_{-k'}^\dagger \right) |0\rangle$$

$$= u_\phi(k, \tau) u_\phi^\dagger(k', \tau) \langle 0 | a_k a_{-k'}^\dagger |0\rangle$$

$$= \frac{H^2}{(2k^3)} (1+ik\tau)(1-ik'\tau) e^{-ik(\tau-\tau')} (2\pi)^3 \delta^3(k+k')$$

$$\tau \rightarrow 0 \quad = \frac{H^2}{2k^3} (2\pi)^3 \delta^3(k+k') \quad \text{AS } \tau \rightarrow 0$$

$$(2\pi)^3 \delta^3(k+k')$$

2-POINT FUNCTION

$$c) \phi_k(\tau) |0\rangle = \langle 0 | \left(u_\phi(k, \tau) a_k + u_\phi^*(k, \tau) a_{-k}^\dagger \right) \left(u_\phi(k', \tau) a_{k'} + u_\phi^*(k', \tau) a_{-k'}^\dagger \right) |0\rangle$$

$$= u_\phi(k, \tau) u_\phi^*(k', \tau) \langle 0 | a_k a_{-k'}^\dagger |0\rangle$$

$$= \frac{H^2}{(2k^3)} (1+ik\tau)(1-ik'\tau) e^{-i(k-k')\tau} (2\pi)^3 \delta^3(k+k')$$

$$\tau \rightarrow 0 \quad = \frac{H^2}{2k^3} (2\pi)^3 \delta^3(k-k') \quad \text{AS } \tau \rightarrow 0$$

$$a(\tau) = -\frac{1}{H\tau}$$

$$-\infty < \tau < 0$$

MASSLESS FIELD IN NONDYNAMICAL FLRW

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$g_{\mu\nu} = \begin{pmatrix} -a(z)^2 & \\ & a(z)^2 \delta_{ij} \end{pmatrix}$$

$$= \frac{1}{2} \int dt d^3x a(t)^2 \left[(\partial_t \phi)^2 - (\partial_i \phi)^2 \right]$$

CAN QUANTIZE EITHER USING ϕ OR $\psi = a\phi$ (CANONICALLY NORMALIZED)

$$\hat{\phi}_k(\tau) = u_\phi(k, \tau) a_k + u_\phi^*(k, \tau) a_{-k}^\dagger$$

$$\hat{\psi}_k(\tau) = u_\psi(k, \tau) a_k + u_\psi^*(k, \tau) a_{-k}^\dagger$$

MASSLESS FIELD IN NONDYNAMICAL FLRW

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{mn} \partial_m \phi \partial_n \phi \right]$$

$$g_{mn} = \begin{pmatrix} -a(\tau)^2 & \\ & a(\tau)^2 \delta_{ij} \end{pmatrix}$$

$$= \frac{1}{2} \int d\tau d^3x a(\tau)^2 \left[(\partial_\tau \phi)^2 - (\partial_i \phi)^2 \right]$$

(CANONICALLY NORMALIZED)

CAN QUANTIZE EITHER USING ϕ OR $\psi = a\phi$

$$\hat{\phi}_k(\tau) = u_\phi(k, \tau) a_k + u_\phi^*(k, \tau) a_{-k}^\dagger$$

CONVENIENT AT LATE TIMES

$$\hat{\psi}_k(\tau) = u_\psi(k, \tau) a_k + u_\psi^*(k, \tau) a_{-k}^\dagger$$

CONVENIENT AT EARLY TIMES

QUAS

$$\frac{\partial^2 u_\psi(k, \tau)}{\partial \tau^2} + \left(k^2 - \frac{\partial^2 a}{a} \right) u_\psi(k, \tau) = 0$$

$$\frac{\partial}{\partial t} \left(\frac{1}{a} \frac{\partial u_\psi(k, \tau)}{\partial \tau} \right) + k^2 a u_\psi(k, \tau) = 0$$

QUASI DE SITTER $\Rightarrow \frac{\partial^2 u_\psi}{\partial \tau^2} = -k^2 u_\psi \Rightarrow u_\psi(k, \tau) \rightarrow \frac{1}{(2k)^{1/2}} e^{-ik\tau}$

$$\frac{\partial}{\partial t} \left(a^2 \frac{\partial u_\psi(k, \tau)}{\partial \tau} \right) + k^2 a^2 u_\psi(k, \tau) = 0$$

QUASI DE SITTER \Rightarrow

$$\frac{\partial^2 u_\psi}{\partial \tau^2} = -k^2 u_\psi$$

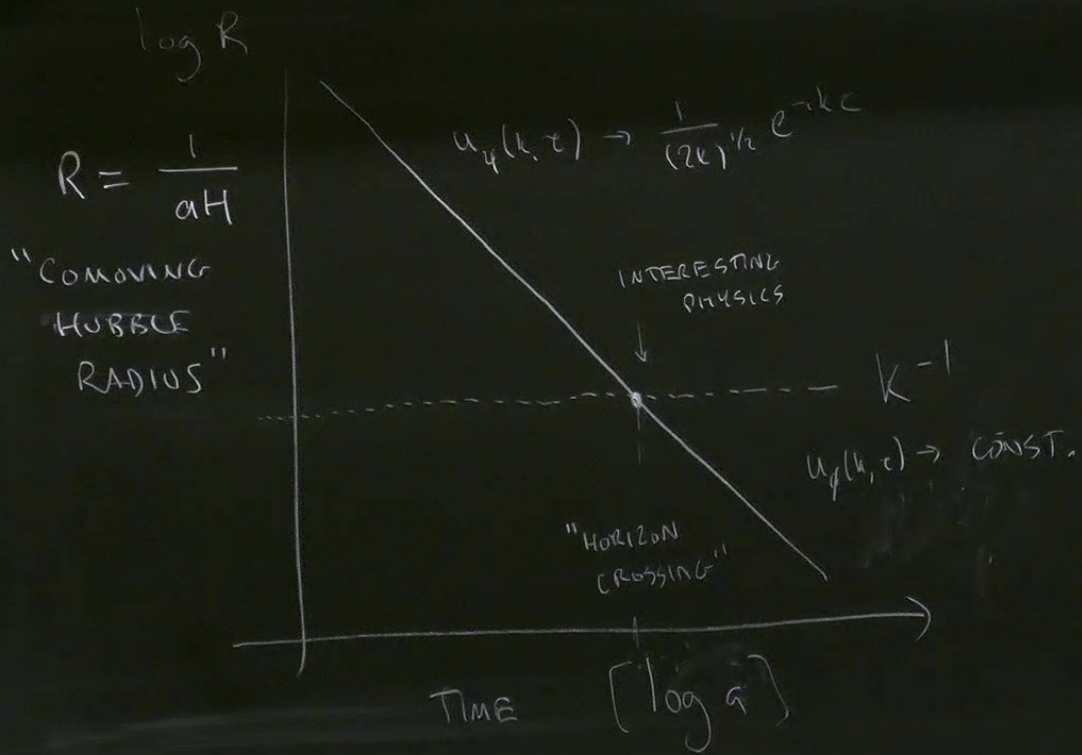
\Rightarrow

$$u_\psi(k, \tau) \rightarrow \frac{1}{(2k)^{1/2}} e^{-k\tau}$$

AT EARLY TIMES

AT LATE TIMES

$$\frac{\partial u_\psi}{\partial \tau} \rightarrow 0$$



ANSATZ

QUASI DE SITTER

$$\Rightarrow u_{\phi}(k, \tau) \rightarrow \frac{H_*}{(2k^3)^{1/2}}$$

WHERE H_* = "VALUE OF $H(\tau)$ AT HORIZON CROSSING $k=aH$ "

ANSATZ

QUASI DE SITTER

$$\Rightarrow u_{\phi}(k, \tau) \rightarrow \frac{H_*}{(2k^3)^{1/2}}$$

$$\langle 0 | \phi_k \phi_{k'} | 0 \rangle \rightarrow \left(\frac{H^2}{2k^3} \right)_* (2\pi)^3 \delta^3(k+k')$$

WHERE H_* = "VALUE OF $H(\tau)$ AT HORIZON CROSSING $k=aH$ "

$$Q_k(t) = u_\psi(k, t) a_k + u_\psi^*(k, t) a_{-k}$$

CONVENIENT AT EARLY TIMES

