Title: Cosmology Lecture

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Collection: Cosmology 2023/24

Date: March 26, 2024 - 11:30 AM

URL: https://pirsa.org/24030069

MASSLESS SCALAR IN NONDYNAMICAL DE SITTER

$$S = \int d^4x \sqrt{-g} \left( -\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) \right)$$

$$= \frac{1}{2} \int d\tau \frac{d^3k}{(2\pi)^3} \frac{1}{(H\tau)^2} \left[ \frac{\partial \phi_k^*}{\partial \tau} \frac{\partial \phi_k}{\partial \tau} - k^2 \phi_k^* \phi_k \right]$$

$$g_{\mu\nu} = \begin{pmatrix} -a^2 & \\ & a^2 \delta_{ij} \end{pmatrix}$$

$a(\tau) = -\frac{1}{H\tau}$ WHERE $-\infty < \tau < 0$

CHANGE VARS TO "CANONICALLY NORMALIZED"

FIELD $\varphi = \frac{1}{H\tau} \phi$

$$\phi = H\tau \varphi$$

$$\partial_\tau \phi = \left( H\varphi + H\tau \frac{\partial \varphi}{\partial \tau} \right)$$

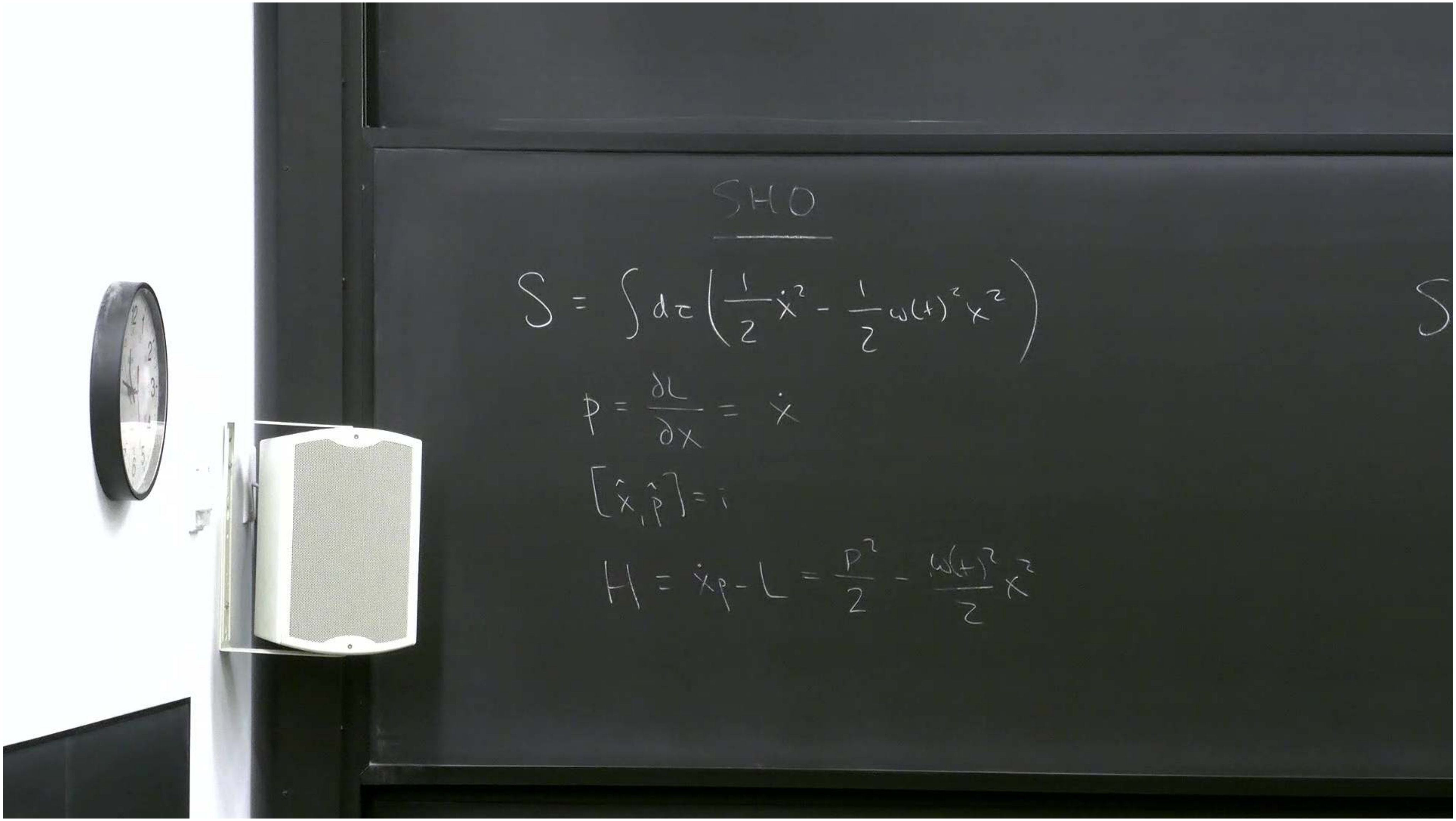
$S = \ldots$

$$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\frac{1}{(H\tau)^2}\left[\left(H\tau\frac{\partial\psi_k}{\partial\tau} + H\psi_k\right)^*\left(H\tau\frac{\partial\psi_k}{\partial\tau} + H\psi_k'\right) - k^2(H\tau)^2\psi_k\psi_k^*\right]$$

$$= \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left\{\frac{\partial\psi_k^*}{\partial\tau}\frac{\partial\psi_k}{\partial\tau} + \frac{1}{\tau}\frac{\partial\psi_k^*}{\partial\tau}\psi_k + \frac{1}{\tau}\psi_k^*\frac{\partial\psi_k}{\partial\tau} - \left(k^2 - \frac{1}{\tau^2}\right)\psi_k^*\psi_k\right\}$$

$< 0$

$$= \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{\partial\psi_k^*}{\partial\tau}\frac{\partial\psi_k}{\partial\tau} - \underbrace{\left(k^2 - \frac{2}{\tau^2}\right)}_{\omega(t)^2 = k^2 - \frac{2}{\tau^2}}\psi_k^*\psi_k\right]$$


SHO
$S = \int d\tau \left( \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega(t)^2 x^2 \right)$
$p = \frac{\partial L}{\partial x} = \dot{x}$
$[\hat{x}, \hat{p}] = i$
$H = \dot{x}p - L = \frac{p^2}{2} - \frac{\omega(t)^2}{2} x^2$
S

QFT

$$S = \int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}\frac{\partial \psi_k^*}{\partial \tau}\frac{\partial \psi_k}{\partial \tau} - \frac{1}{2}\left(k^2 - \frac{2}{\tau^2}\right)\psi_k^* \psi_k\right]$$

$$\pi_k = \frac{\partial \mathcal{L}}{\partial(\partial_\tau \psi)} = \frac{\partial \psi_k}{\partial \tau}$$

$$\left[\hat{\psi}_k, \hat{\pi}_{k'}\right] = i(2\pi)^3 \delta^3(k+k')$$

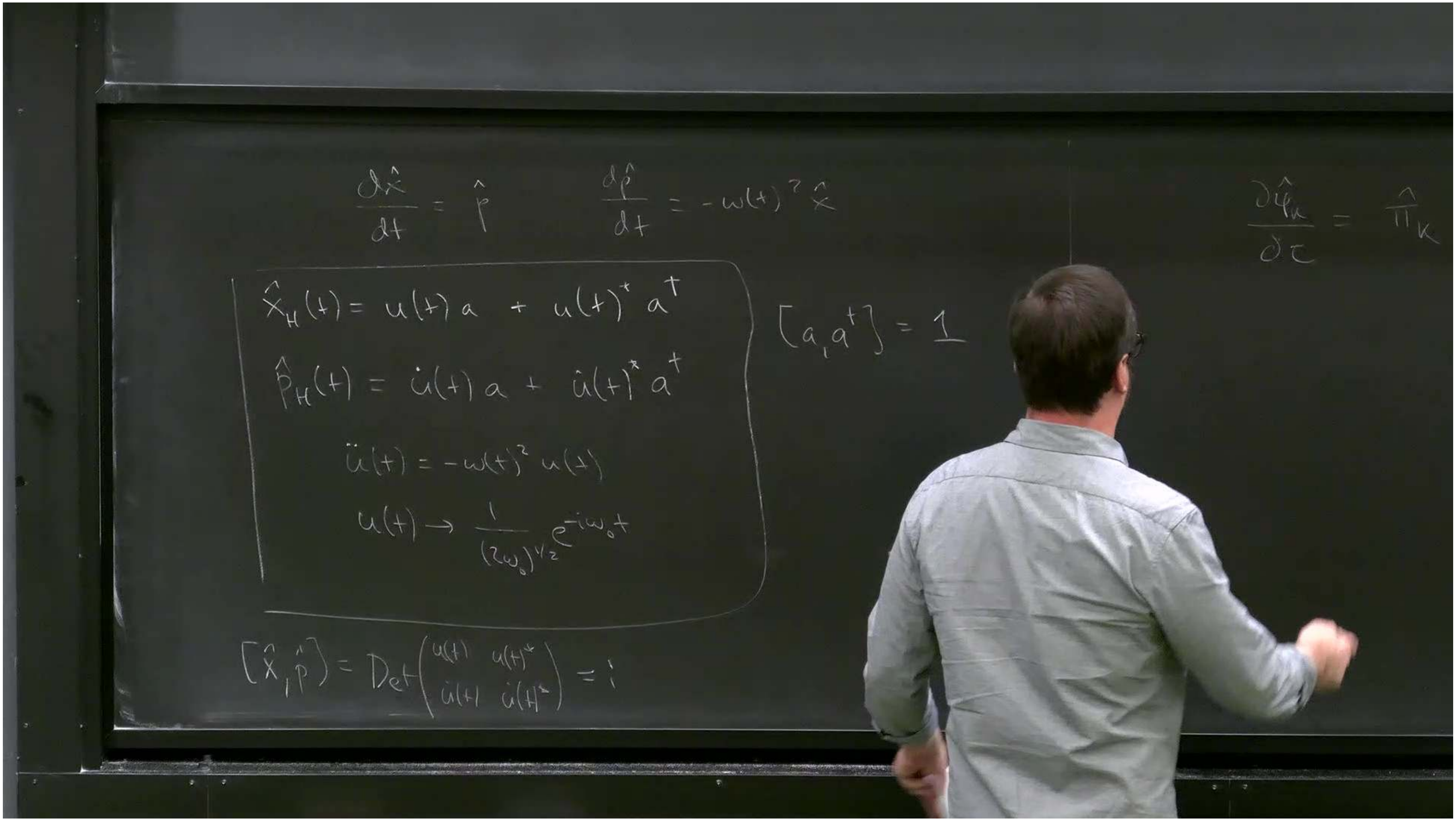
$$H = \int d^3x \left[\pi \dot{\psi} - \mathcal{L}\right] = \int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}\pi_k^* \pi_k - \frac{1}{2}\left(k^2 - \frac{2}{\tau^2}\right)\psi_k^* \psi_k\right]$$

$$\frac{\partial \hat{\psi}_k}{\partial \tau} = \hat{\pi}_k \qquad \frac{\partial \hat{\pi}_k}{\partial \tau} = -\left(k^2 - \frac{2}{\tau^2}\right) \hat{\psi}_k$$

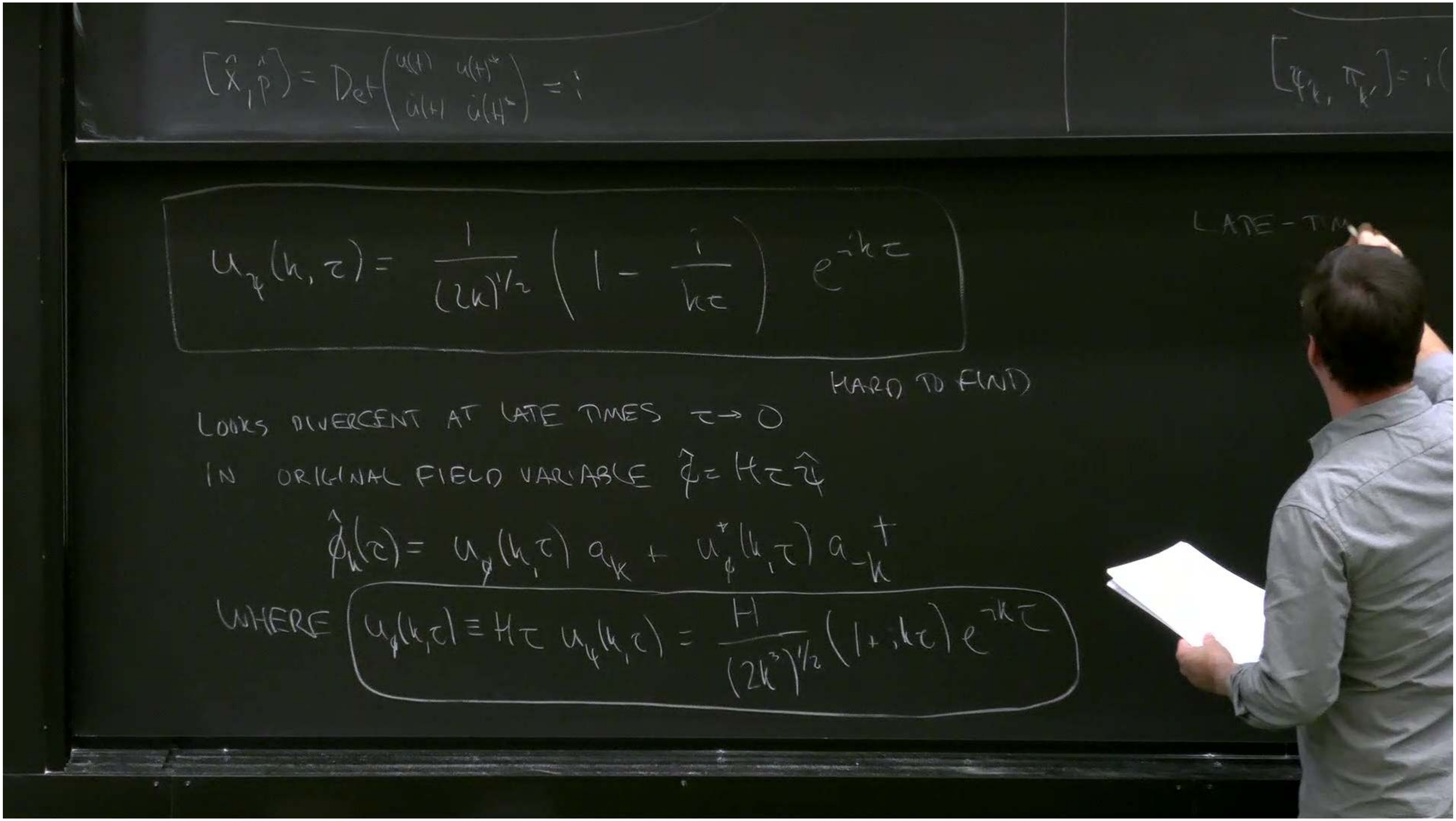
$$\boxed{\begin{aligned} \hat{\psi}_k(t) &= u_\psi(k,\tau)\, a_k + u_\psi^*(k,\tau)\, a_{-k}^\dagger \\ \hat{\pi}_k(t) &= \dot{u}_\psi(k,\tau)\, a_k + \dot{u}_\psi^*(k,\tau)\, a_{-k}^\dagger \\ \ddot{u}_\psi(k,\tau) &= -\left(k^2 - \frac{2}{\tau^2}\right) u_\psi(k,\tau) \\ u_\psi(k,\tau) &\to \frac{1}{(2k)^{1/2}} e^{-ik\tau} \quad \text{AS } \tau \to -\infty \end{aligned}}$$

$$\psi_k^* = \psi_{-k} \qquad \hat{\psi}_k^\dagger = \hat{\psi}_{-k}$$

WHERE $[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k - k')$



$\frac{d\hat{x}}{dt} = \hat{p}$
$\frac{d\hat{p}}{dt} = -\omega(t)^2\hat{x}$
$\frac{\partial\hat{\varphi}_k}{\partial\tau} = \hat{\pi}_k$
$\hat{x}_H(t) = u(t)\,a + u(t)^* a^\dagger$
$\hat{p}_H(t) = \dot{u}(t)\,a + \dot{u}(t)^* a^\dagger$
$\ddot{u}(t) = -\omega(t)^2 u(t)$
$u(t) \to \frac{1}{(2\omega_0)^{1/2}} e^{-i\omega_0 t}$
$[a, a^\dagger] = 1$
$[\hat{x}, \hat{p}] = \mathrm{Det}\begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix} = i$


$[\hat{x}, \hat{p}] = \mathrm{Det}\begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix} = i$
$[\varphi_k, \pi_{k'}] = i($
$u_\varphi(k, \tau) = \frac{1}{(2k)^{1/2}}\left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}$
LATE-TIM
HARD TO FIND
LOOKS DIVERGENT AT LATE TIMES $\tau \to 0$
IN ORIGINAL FIELD VARIABLE $\hat{\phi} = H\tau\hat{\varphi}$
$\hat{\phi}_k(\tau) = u_\phi(k, \tau)\, a_k + u_\phi^*(k, \tau)\, a_{-k}^\dagger$
WHERE $u_\phi(k, \tau) \equiv H\tau\, u_\varphi(k, \tau) = \frac{H}{(2k^3)^{1/2}}(1 + ik\tau)\, e^{-ik\tau}$

$$(2\pi)^3\delta^3(k+k')$$

<u>...O POINT FUNCTION</u>

$$\ldots\tau)\ \phi_{k'}(\tau)|0\rangle = \langle 0|\left(u_\phi(k,\tau)\,a_k + u_\phi^*(k,\tau)\,a^\dagger_{-k}\right)$$
$$\left(u_\phi(k',\tau)\,a_{k'} + u_\phi^*(k',\tau)\,a^\dagger_{-k'}\right)|0\rangle$$

$$= u_\phi(k,\tau)\,u_\phi^*(k',\tau)\,\langle 0|a_k\,a^\dagger_{-k'}|0\rangle$$

$$= \frac{H^2}{(2k^3)}(1+ik\tau)(1-ik'\tau)\,e^{-ik(\tau-\tau')}\quad (2\pi)^3\delta^3(k+k')$$

$$\tau\to 0 \qquad = \frac{H^2}{2k^3}(2\pi)^3\delta^3(k+k') \quad \text{AS } \tau\to 0$$

$$u(t) \to \frac{1}{(2\omega_0)^{1/2}} e^{-i\omega_0 t}$$

$$u_\varphi(k,\tau) \to \frac{1}{(2k)^{1/2}} e^{-ik\tau}$$

$$[\hat{x}, \hat{p}] = \mathrm{Det}\begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix} = i$$

$$[\varphi_k, \pi_{k'}] = i(2\pi)^3 \delta^3(k+k')$$

$$\boxed{u_\varphi(k,\tau) = \frac{1}{(2k)^{1/2}}\left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}}$$

HARD TO FIND

LOOKS DIVERGENT AT LATE TIMES $\tau \to 0$

IN ORIGINAL FIELD VARIABLE $\hat{\phi} = H\tau\hat{\varphi}$

$$\hat{\phi}_k(\tau) = u_\phi(k,\tau)\, a_k + u_\phi^*(k,\tau)\, a_{-k}^\dagger$$

WHERE $\boxed{u_\phi(k,\tau) \equiv H\tau\, u_\varphi(k,\tau) = \frac{H}{(2k^3)^{1/2}}(1 + ik\tau)\, e^{-ik\tau}}$

LATE-TIME TWO POINT FUNCTION

$$\langle 0 | \phi_k(\tau)\, \phi_k(\tau) | 0 \rangle = \langle 0 |$$

"BUNCH-DAVIES VACUUM"

$(2\pi)^3\delta^3(k+k')$

WO POINT FUNCTION

$$\ldots\tau)\ \phi_{k'}(\tau)\,|0\rangle = \langle 0|\left(u_\phi(k,\tau)\,a_k + u^*_\phi(k,\tau)\,a^\dagger_{-k}\right)\left(u_\phi(k',\tau)\,a_{k'} + u^*_\phi(k',\tau)\,a^\dagger_{-k'}\right)|0\rangle$$

$$= u_\phi(k,\tau)\,u^*_\phi(k',\tau)\,\langle 0|\,a_k\,a^\dagger_{-k'}\,|0\rangle$$

$$= \frac{H^2}{(2k^3)}(1+ik\tau)(1-ik'\tau)\,e^{-ik(\tau-\tau')}\quad(2\pi)^3\,\delta^3(k+k')$$

$$\tau\to 0 \qquad = \frac{H^2}{2k^3}(2\pi)^3\delta^3(k+k')\quad \text{AS } \tau\to 0$$

$(2\pi)^3 \delta^3(k+k')$

WO POINT FUNCTION

$a(\tau) = -\frac{1}{H\tau}$

$-\infty < \tau < 0$

$$\tau) \ \phi_k(\tau) | 0 \rangle = \langle 0 | \left( u_\phi(k,\tau) a_k + u_\phi^*(k,\tau) a^\dagger_{-k} \right) \left( u_\phi(k',\tau) a_{k'} + u_\phi^*(k',\tau) a^\dagger_{-k'} \right) | 0 \rangle$$

$$= u_\phi(k,\tau) u_\phi^*(k',\tau) \langle 0 | a_k a^\dagger_{-k'} | 0 \rangle$$

$$= \frac{H^2}{(2k^3)} (1+ik\tau)(1-ik'\tau) e^{-i(k-k')\tau} (2\pi)^3 \delta^3(k+k')$$

$$\tau \to 0 \qquad = \frac{H^2}{2k^3} (2\pi)^3 \delta^3(k+k') \quad \text{AS } \tau \to 0$$

MASSLESS FIELD IN NONDYNAMICAL FLRW

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right] \qquad g_{\mu\nu} = \begin{pmatrix} -a(\tau)^2 & \\ & a(\tau)^2\delta_{ij} \end{pmatrix}$$

$$= \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2\left[(\partial_\tau\phi)^2 - (\partial_i\phi)^2\right]$$

CAN QUANTIZE EITHER USING $\phi$ OR $\psi = a\phi$ (CANONICALLY NORMALIZED)

$$\hat{\phi}_k(\tau) = u_\phi(k,\tau)\,a_k + u_\phi^*(k,\tau)\,a^\dagger_{-k}$$

$$\hat{\psi}_k(\tau) = u_\psi(k,\tau)\,a_k + u_\psi^*(k,\tau)\,a^\dagger_{-k}$$

MASSLESS FIELD IN NONDYNAMICAL FLRW

$$S = \frac{1}{2}\int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right]$$

$$g_{\mu\nu} = \begin{pmatrix} -a(\tau)^2 & \\ & a(\tau)^2\delta_{ij} \end{pmatrix}$$

$$= \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2\left[(\partial_\tau\phi)^2 - (\partial_i\phi)^2\right]$$

CAN QUANTIZE EITHER USING $\phi$ OR $\psi = a\phi$ (CANONICALLY NORMALIZED)

$$\hat\phi_k(\tau) = u_\phi(k,\tau)a_k + u_\phi^*(k,\tau)a_{-k}^\dagger$$ CONVENIENT AT LATE TIMES

$$\hat\psi_k(\tau) = u_\psi(k,\tau)a_k + u_\psi^*(k,\tau)a_{-k}^\dagger$$ CONVENIENT AT EARLY TIMES

QUAS

$$\frac{\partial^2 u_\varphi(k\tau)}{\partial \tau^2} + \left(k^2 - \frac{\partial_\tau^2 a}{a}\right) u_\varphi(k,\tau) = 0$$

$$\frac{\partial}{\partial t}\left( a^2 \frac{\partial u_\varphi(k,\tau)}{\partial \tau}\right) + k^2 a^2 u_\varphi(k,\tau) = 0$$

QUASI DE SITTER $\Rightarrow$ $\frac{\partial^2 u_\varphi}{\partial \tau^2} = -k^2 u_\varphi \Rightarrow \boxed{u_\varphi(k,\tau) \to \frac{1}{(2k)^{1/2}} e^{-ik\tau}}$

$$\frac{\partial}{\partial \tau}\left(a^2 \frac{\partial u_\varphi(k,\tau)}{\partial \tau}\right) + k^2 a^2 u_\varphi(k,\tau) = 0$$

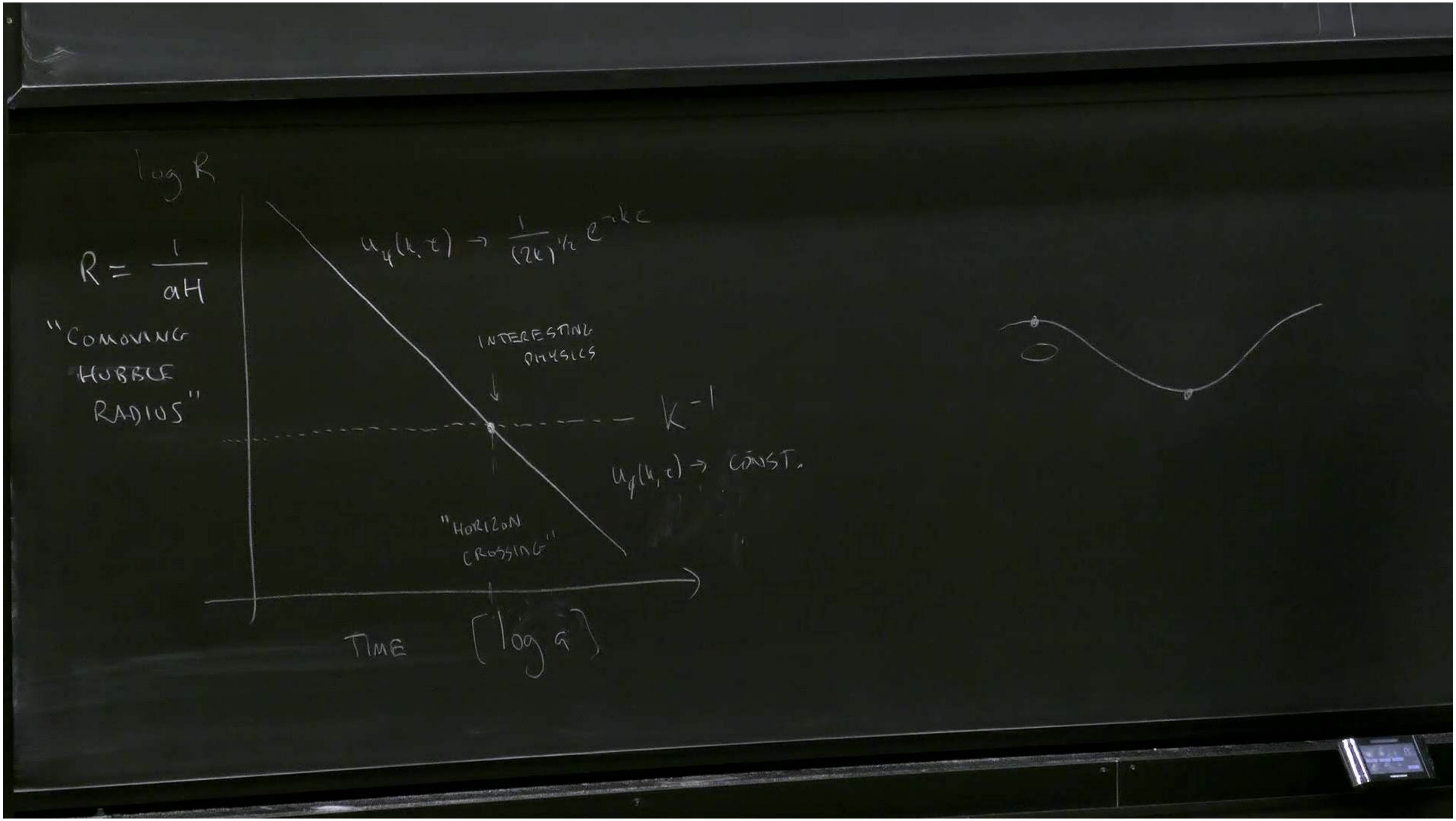
VEO )

QUASI DE SITTER $\Rightarrow \dfrac{\partial^2 u_\varphi}{\partial \tau^2} = -k^2 u_\varphi \Rightarrow \boxed{u_\varphi(k,\tau) \to \dfrac{1}{(2k)^{1/2}} e^{-ik\tau}}$ AT EARLY TIMES

S

AT LATE TIMES $\dfrac{\partial u_\varphi}{\partial \tau} \to 0$

ES


log R
R = 1/aH
"COMOVING HUBBLE RADIUS"
u_φ(k, τ) → 1/(2k)^{1/2} e^{-ikτ}
INTERESTING PHYSICS
k^{-1}
u_φ(k, τ) → CONST.
"HORIZON CROSSING"
TIME
[log a]

ANSATZ

QUASI DE SITTER

$$\Rightarrow \boxed{u_\phi(k,\tau) \rightarrow \frac{H_*}{(2k^3)^{1/2}}}$$

WHERE $H_*$ = "VALUE OF $H(\tau)$ AT HORIZON CROSSING $k = aH$"

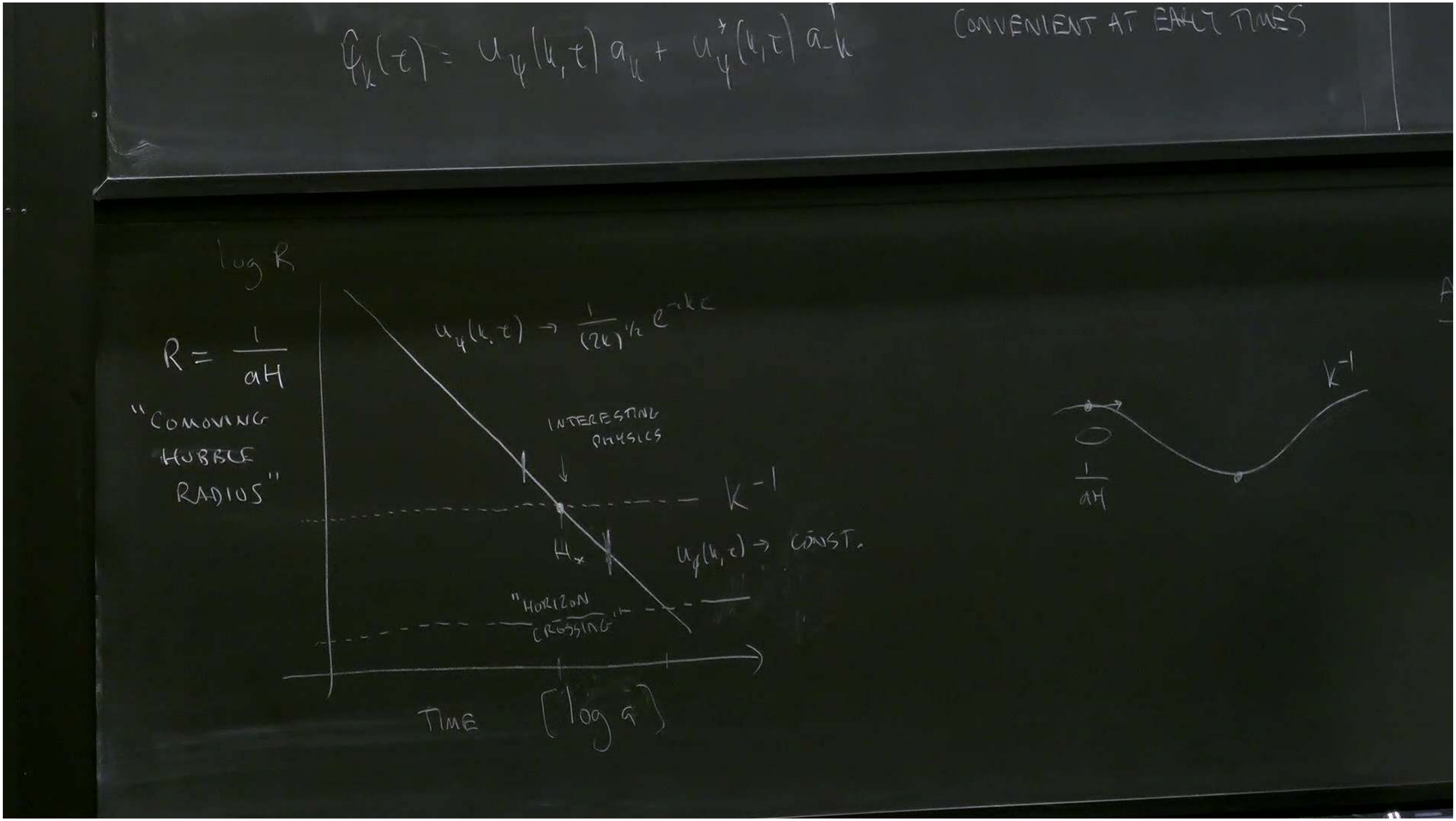
## ANSATZ

QUASI DE SITTER

$$\Rightarrow \boxed{u_\phi(k,\tau) \rightarrow \frac{H_*}{(2k^3)^{1/2}}}$$

$$\langle 0|\phi_k \phi_{k'}|0\rangle \rightarrow \left(\frac{H^2}{2k^3}\right)_* (2\pi)^3 \delta^3(k+k')$$

WHERE $H_*$ = "VALUE OF $H(t)$ AT HORIZON CROSSING $k = aH$"


CONVENIENT AT EARLY TIMES
log R
R = 1/aH
"COMOVING HUBBLE RADIUS"
INTERESTING PHYSICS
k^-1
H_*
u_ψ(k,τ) → CONST.
"HORIZON CROSSING"
TIME
[log a]
1/aH