Title: Cosmology Lecture

Speakers: Kendrick Smith

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FOR ANY $V(\phi)$ DEFINE "SLOW ROLL" PARAMS

$$\varepsilon \equiv \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta \equiv M_{pl}^2 \frac{V''}{V}$$

← WARNING: DIFFERS FROM BAUMANN NOTES

A POTENTIAL IS "SLOW ROLL" IF $\varepsilon, |\eta| \ll 1$

CLAIM. IF $\varepsilon, |\eta| \ll 1$ THEN THE EOMS MAY BE APPROXIMATED

$$(*) \begin{cases} \dot\phi \approx -\dfrac{V'}{3H} & \left[\text{EXACT: } \ddot\phi + 3H\dot\phi + V'(\phi) = 0\right] \\[2ex] H^2 \approx \dfrac{V}{3M_{pl}^2} & \left[\text{EXACT: } H^2 = \dfrac{1}{3M_{pl}^2}\left(\dfrac{1}{2}\dot\phi^2 + V(\phi)\right)\right] \end{cases}$$

$$(\ast)\quad H^2 \approx \frac{V}{3M_{pl}^2} \quad \left[\text{EXACT: } H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)\right]$$

LET (ϕ_{SR}, a_{SR}) = EXACT SOLUTION TO $(\ast)$

DEFINE $[\,\cdot\,]_{SR}$ = "EVALUATED ON SLOW-ROLL SOLUTION (ϕ_{SR}, a_{SR})"

SUFFICES TO SHOW:

$$|\ddot{\phi}|_{SR} \ll |V'|_{SR} \quad (SR1)$$

$$\frac{1}{2}\dot{\phi}^2_{SR} \ll |V|_{SR} \quad (SR2)$$

"SLOW-ROLL-OLOGY": ANY QUANTITY $[\cdot]_{SR}$ CAN BE CONVERTED TO DERIVS OF V, E.G.

$$[\dot\phi]_{SR} = -\frac{V'}{3H} \qquad \phi\text{-EOM}$$

$$= -M_{pl}\frac{V'}{(3V)^{1/2}} \qquad a\text{-EOM}$$

$$[\ddot\phi]_{SR} = \frac{d}{dt}\left[\dot\phi\right]$$

$$= \frac{d}{dt}\left[-M_{pl}\frac{V'}{(3V)^{1/2}}\right.$$

$$=$$

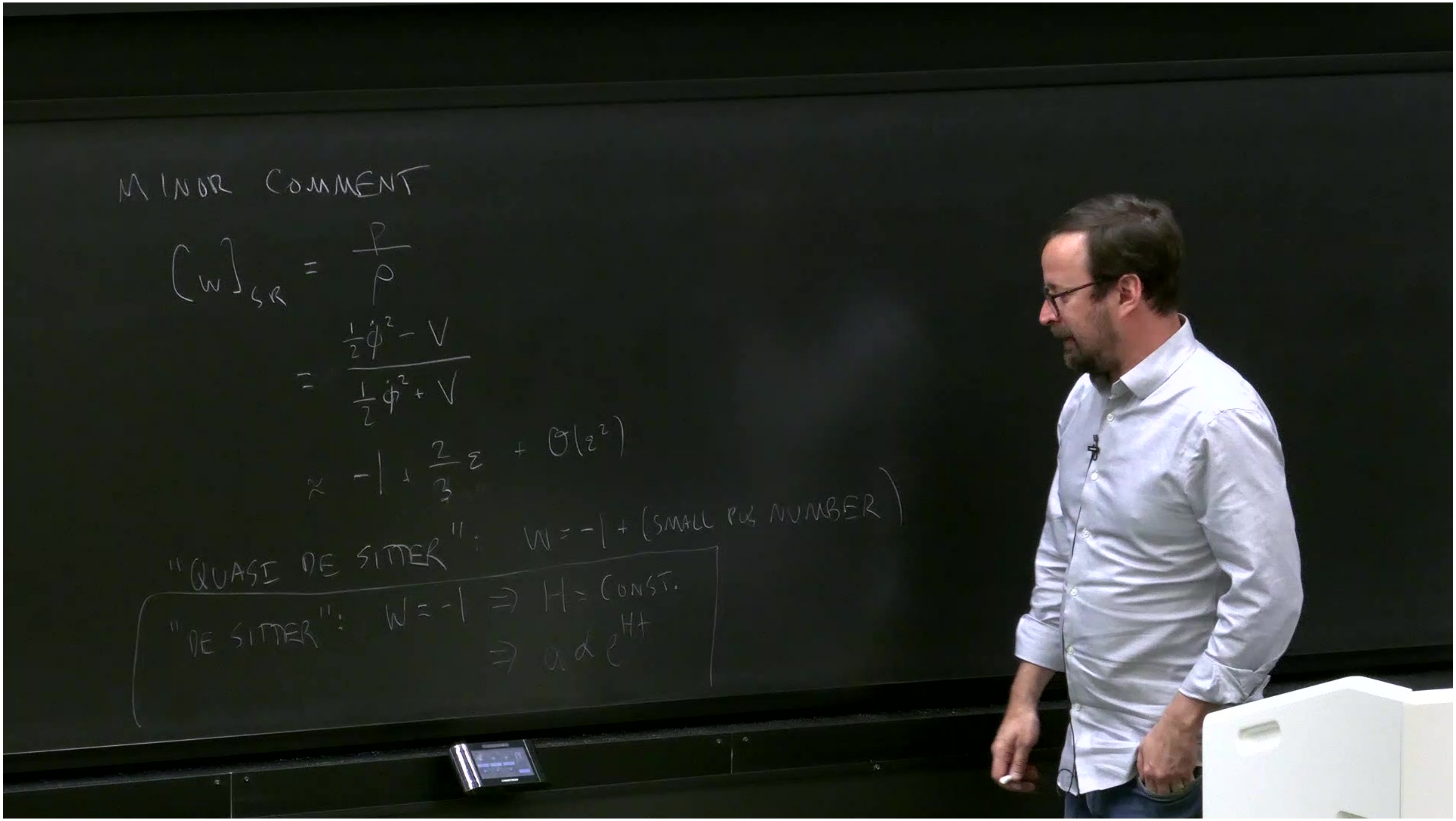
$$[\ddot{\phi}]_{SR} = \frac{d}{dt}[\dot{\phi}]$$

$$= \frac{d}{dt}\left[-M_{pl}\frac{V'}{(3V)^{1/2}}\right]$$

$$= \frac{d}{d\phi}\left(-M_{pl}\frac{V'}{(3V)^{1/2}}\right)\dot{\phi}$$

$$= -M_{pl}\left[\frac{V''}{(3V)^{1/2}} - \frac{1}{2}\frac{(V')}{(3V)^{1/2}V}\right]\left(-M_{pl}\frac{V'}{(3V)^{1/2}}\right)$$

$$= M_{pl}^2\left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\right)$$


MINOR COMMENT
$[w]_{SR} = \frac{P}{\rho}$
$= \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$
$\approx -1 + \frac{2}{3}\varepsilon + \mathcal{O}(\varepsilon^2)$
"QUASI DE SITTER": $w = -1 +$ (SMALL POS NUMBER)
"DE SITTER": $W = -1 \Rightarrow H = $ CONST.
$\Rightarrow a \propto e^{Ht}$

$= \frac{\varepsilon}{3} \ll 1$

"DE SITTER": $w = -1 \Rightarrow$

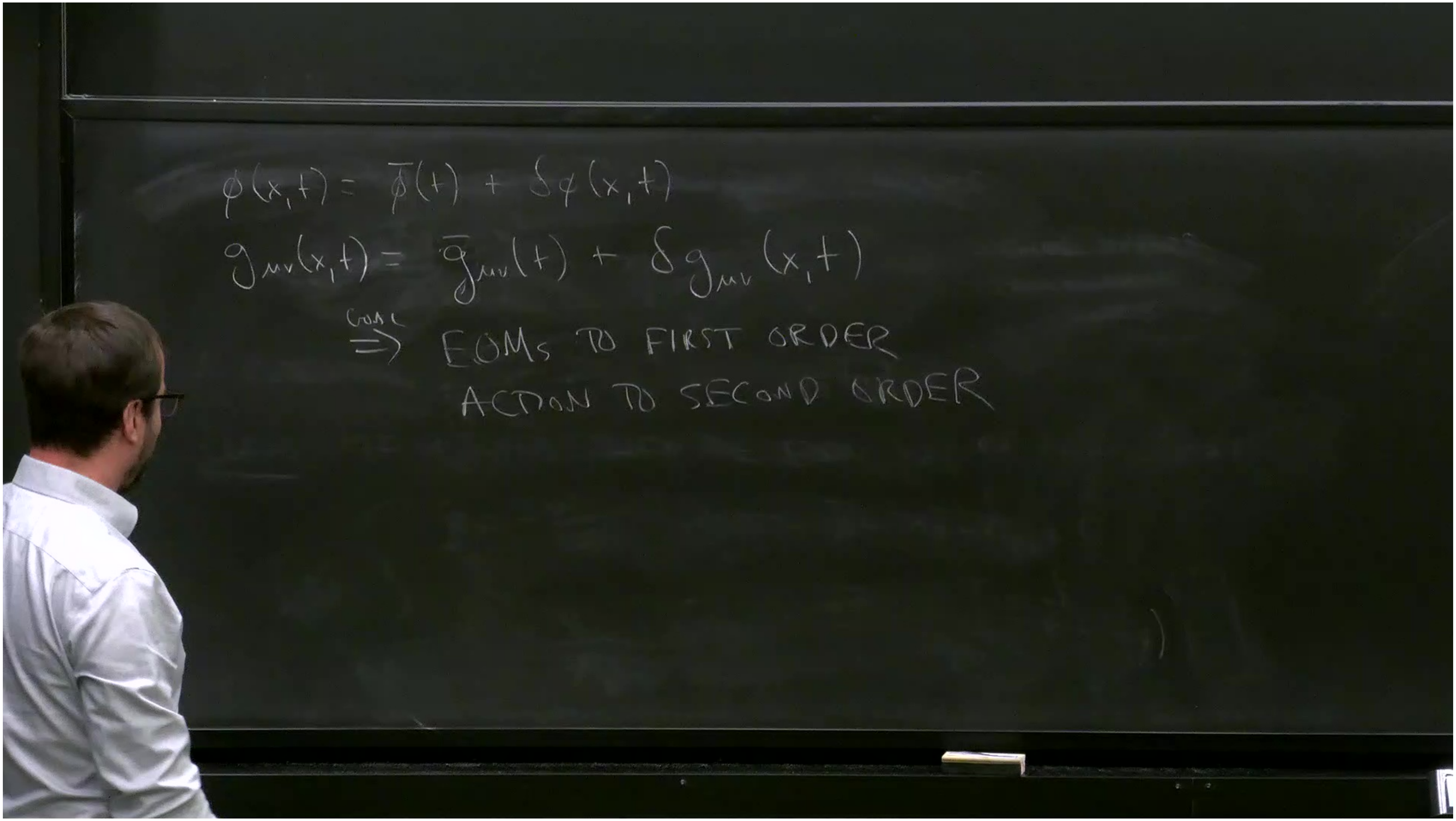
$\Rightarrow$

WARNING. BAUMANN DEFINES

$$\varepsilon_V \equiv \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \kappa = \eta = \frac{\dot{\varepsilon}}{H\varepsilon} \approx 4\varepsilon_V - 2\eta_V$$

$$\eta_V = M_{pl}^2\left(\frac{V''}{V}\right) \qquad \delta = -\frac{\ddot{\phi}}{H\dot{\phi}} \approx -\varepsilon_V + \eta_V$$

$$\varepsilon = -\frac{\dot{H}}{H^2} \approx \varepsilon_V$$


$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$
$g_{\mu\nu}(x,t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(x,t)$
GOAL
$\Rightarrow$ EOMs TO FIRST ORDER
ACTION TO SECOND ORDER

$g_{\mu\nu}(x,t)$

FIRST ORDER

SECOND ORDER

GR

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_p^2}$$

DOFs: $\underbrace{10}_{g_{\mu\nu}} - \underbrace{4}_{\text{GAUGE [COORDS]}} -$

ELECTROMAGNETISM.

$$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = -j^\nu$$

DOFs. $\underbrace{4}_{A_\mu} -$

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

CHOOSE GAUGE: COULOMB $\partial_i A_i = 0$

$$\Rightarrow \text{EOMs} \begin{cases} \nabla^2 A^0 = -j^0 \\ (\partial_t^2 - \nabla^2)\vec{A} = \vec{j} - \vec{\nabla} A^0 \end{cases}$$

GR $\quad G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{pl}^2}$

DOFs: $\underbrace{10}_{g_{\mu\nu}} - \underbrace{4}_{\text{GAUGE [COORDS]}} - \underbrace{4}_{\text{NONDYNAMICAL}} = 2$

ELECTROMAGNETISM.

$$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = -j^\nu$$

DOFs. $\underbrace{4}_{A_\mu} - \underbrace{1}_{\text{GAUGE}} - \underbrace{1}_{\text{NONDYNAMICAL}} = 2$

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

CHOOSE GAUGE: COULOMB $\partial_i A_i = 0$ & NO TIME DERIVS

$$\Rightarrow \text{EOMs} \begin{cases} \nabla^2 A^0 = -j^0 \\ (\partial_t^2 - \nabla^2)\vec{A} = \vec{J} - \vec{\nabla} A^0 \end{cases}$$

GR

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{pl}^2}$$

DOFs: $\underbrace{10}_{g_{\mu\nu}} - \underbrace{4}_{\text{GAUGE [COORDS]}} - \underbrace{4}_{\text{NONDYNAMICAL}} = 2$

ELECTROMAGNETISM.

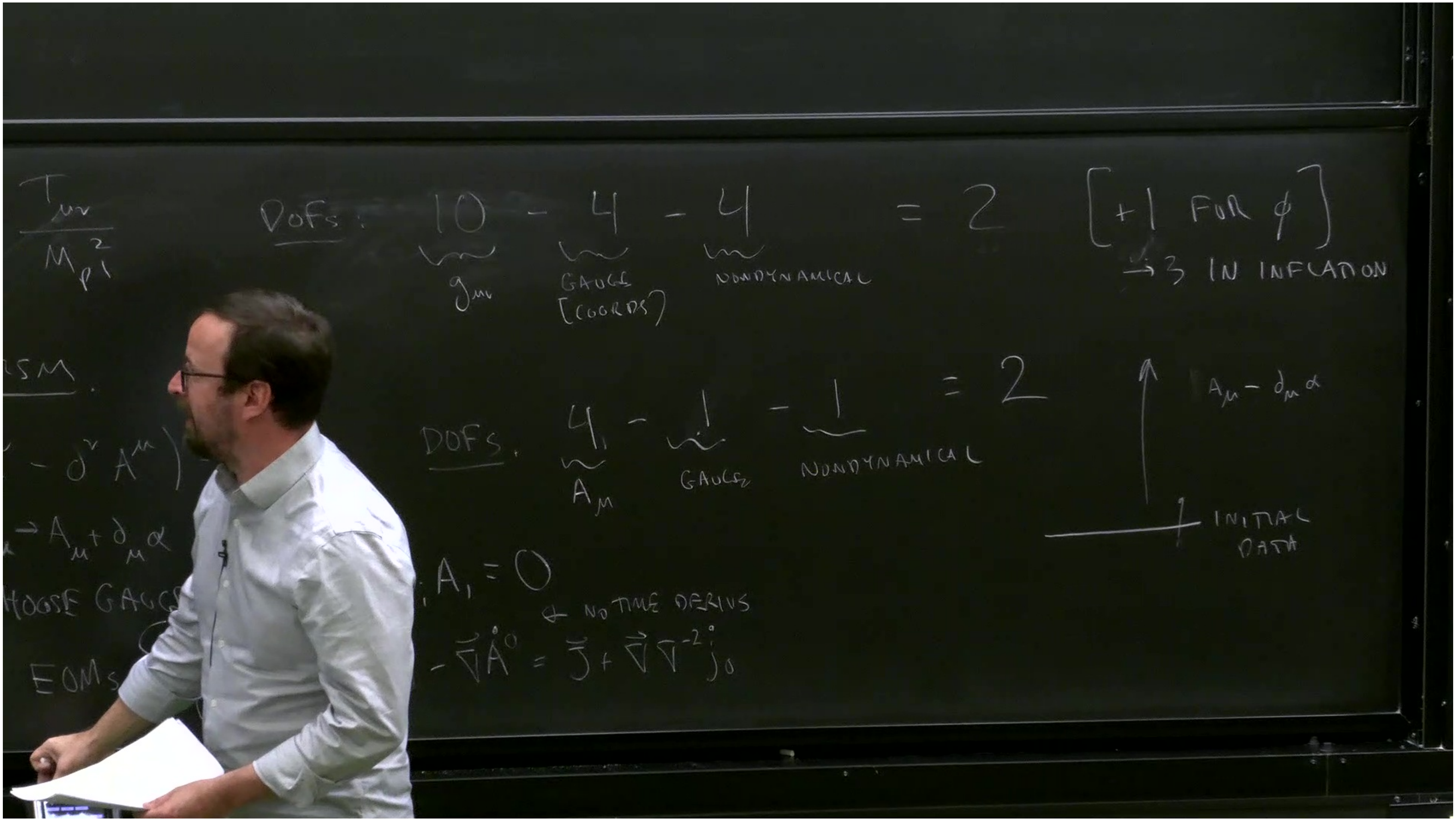
$$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = -j^\nu$$

DOFs. $\underbrace{4}_{A_\mu} - \underbrace{1}_{\text{GAUGE}} - \underbrace{1}_{\text{NONDYNAMICAL}} = 2$

$$A_\mu \to A_\mu + \partial_\mu \alpha$$

CHOOSE GAUGE: COULOMB $\partial_i A_i = 0$ & NO TIME DERIVS

$$\Rightarrow \text{EOMs} \begin{cases} \nabla^2 A^0 = -j^0 \\ (\partial_t^2 - \nabla^2)\vec{A} = \vec{J} - \vec{\nabla}\dot{A}^0 = \vec{J} + \vec{\nabla}\nabla^{-2}\dot{j}_0 \end{cases}$$


DOFs: 10 - 4 - 4 = 2 [+1 FOR φ]
g_μν
GAUGE [COORDS]
NONDYNAMICAL
→ 3 IN INFLATION
DOFs. 4 - 1 - 1 = 2
A_μ
GAUGE
NONDYNAMICAL
A_μ - ∂_μ α
INITIAL DATA
A_μ + ∂_μ α
NO TIME DERIVS
EOMs

$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$$

$$g_{\mu\nu}(x,t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(x,t)$$

$\overset{\text{CALC}}{\Longrightarrow}$ EOMs TO FIRST ORDER

ACTION TO SECOND ORDER

DEFINE CHANGE OF VARS $(\delta g_{ij}) \to (\zeta, \gamma_{ij})$

$$\delta g_{ij}(x,t) = a(t)^2 \left[(1 + 2\zeta(x,t))\,\delta_{ij} + \gamma_{ij}(x,t) \right]$$

WHERE $\gamma_{ii} = 0$

GR

$G_{\mu\nu} =$

ELECTROMAGNETIS

$\partial_\mu(\partial^\mu A^\nu$

$A_\mu \to$

(CHOO

$\Rightarrow$ E

WHERE $\gamma_{ii} = 0$

$\Rightarrow$ EOMs

CHOOSE "UNITARY GAUGE": $\boxed{\delta\phi = 0 \qquad \partial_j \gamma_{ij} = 0}$ $(\star)$

CLAIM: STARTING FROM ARB PERTURBATIONS $(\delta\phi, \zeta, \gamma_{ij})$

$\exists$ A UNIQUE GAUGE TRANSFORMATION $x^\mu \to x^\mu + \varepsilon^\mu(x)$

$\hookrightarrow$ FIRST OR

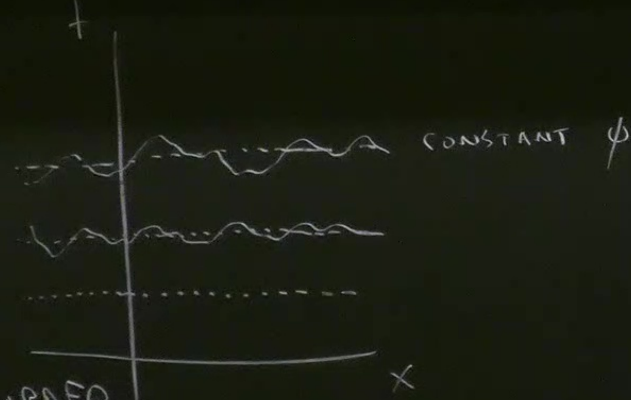
$$\delta\phi \to \delta\phi - \mathcal{L}_\varepsilon \bar{\phi}$$

$$\delta g_{\mu\nu} \to \delta g_{\mu\nu} - \mathcal{L}_\varepsilon \bar{g}_{\mu\nu}$$

...HOOSE GAUGE: COULOMB $\partial_i A_i = 0$

& NO TIME DERIVS

$$\text{EOMs}\begin{cases}\nabla^2 A^0 = -j^0 \\ (\partial_t^2 - \nabla^2)\vec{A} = \vec{j} - \vec{\nabla}\dot{A}^0 = \vec{j} + \vec{\nabla}\nabla^{-2}\dot{j}_0\end{cases}$$



(x)

→ FIRST ORDER

$$\delta\varphi(\vec{k}) \to \delta\varphi(\vec{k}) + \dot{\bar{\varphi}}\,\varepsilon^0(\vec{k})$$

$$k_i\gamma_{ij}(\vec{k}) \to k_i\gamma_{ij}(\vec{k}) + i\,C_j k^2 \varepsilon_j(\vec{k})$$

WHERE $C_j = \begin{cases} 1 & \text{IF } j \in \{1,2\} \\ \frac{4}{3} & \text{IF } j = 3 \end{cases}$

x = COMOVING 3-VECTOR

k = COMOVING 3-WAVENUMBER

$$= \frac{\epsilon}{3} \ll 1$$

"DE SITTER": $w = -1 \Rightarrow H = $

$$g_{00} = -(1+2A)$$
$$g_{0i} = -aB_i$$
$$g_{ij} = a^2\left[(1+2\zeta)\delta_{ij} + \gamma_{ij}\right]$$

EOMs.

$$A = \frac{\dot{\zeta}}{H}$$
$$B_i = \partial_i\left[\frac{\zeta}{aH} - \frac{\dot{\phi}^2}{2M_{pl}^2 H^2}\partial^{-2}\dot{\zeta}\right]$$

$$\frac{\partial}{\partial t}\left(\frac{\dot{\phi}^2 a^3}{H^2}\frac{\partial \zeta}{\partial t}\right) = \frac{\dot{\phi}^2 a}{H}\partial^2 \zeta$$
$$\frac{\partial}{\partial t}\left(a^3 \frac{\partial \gamma_{ij}}{\partial t}\right) = a\,\partial^2 \gamma_{ij}$$

SITTER : $W = -1 + ($SMALL ...

$W = -1 \Rightarrow H = $ CONST.

$\Rightarrow \ldots e^{Ht}$

DOFs. $\underbrace{1}_{S} + \underbrace{2}_{\gamma_{ij}} = 3$

$$\frac{\dot\phi^2 a}{H}\partial^2 S$$

$$a\,\partial^2 \gamma_{ij}$$

"THE" EQS OF MOTION

$$S = \frac{1}{2}\int dt\, d^3x\, \frac{\dot\phi^2 a^3}{H^2}\left(\dot S^2 - \frac{1}{a^2}(\partial_i S)(\partial_i S)\right)$$

$$+ \frac{M_{pl}^2}{8}\int dt\, d^3x\, a^3\left(\dot\gamma_{ij}\dot\gamma_{ij} - \frac{1}{a^2}(\partial_k \gamma_{ij})(\partial_k \gamma_{ij})\right)$$

$w = -1 \Rightarrow H = \text{const.}$

$\Rightarrow a \propto e^{Ht}$

UNITARY GAUGE: $\delta\phi = 0$ $\quad \partial_i \gamma_{ij} = 0$

SPATIALLY FLAT: $\zeta = 0$ $\quad \partial_i \gamma_{ij} = 0$