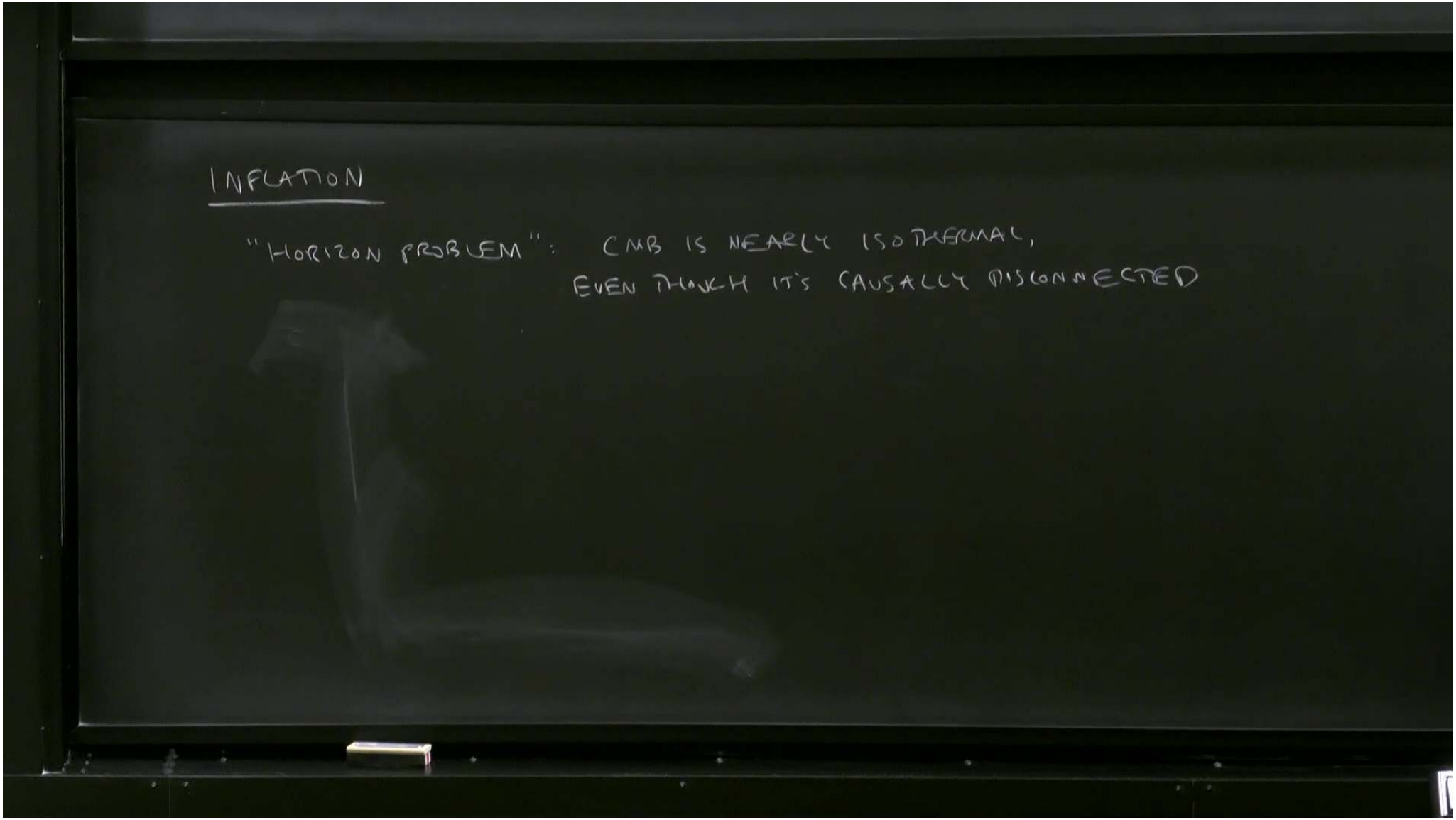
Title: Cosmology Lecture

Speakers: Kendrick Smith

Collection: Cosmology 2023/24

Date: March 19, 2024 - 11:30 AM

URL: https://pirsa.org/24030067


INFLATION
"HORIZON PROBLEM": CMB IS NEARLY ISOTHERMAL,
EVEN THOUGH IT'S CAUSALLY DISCONNECTED

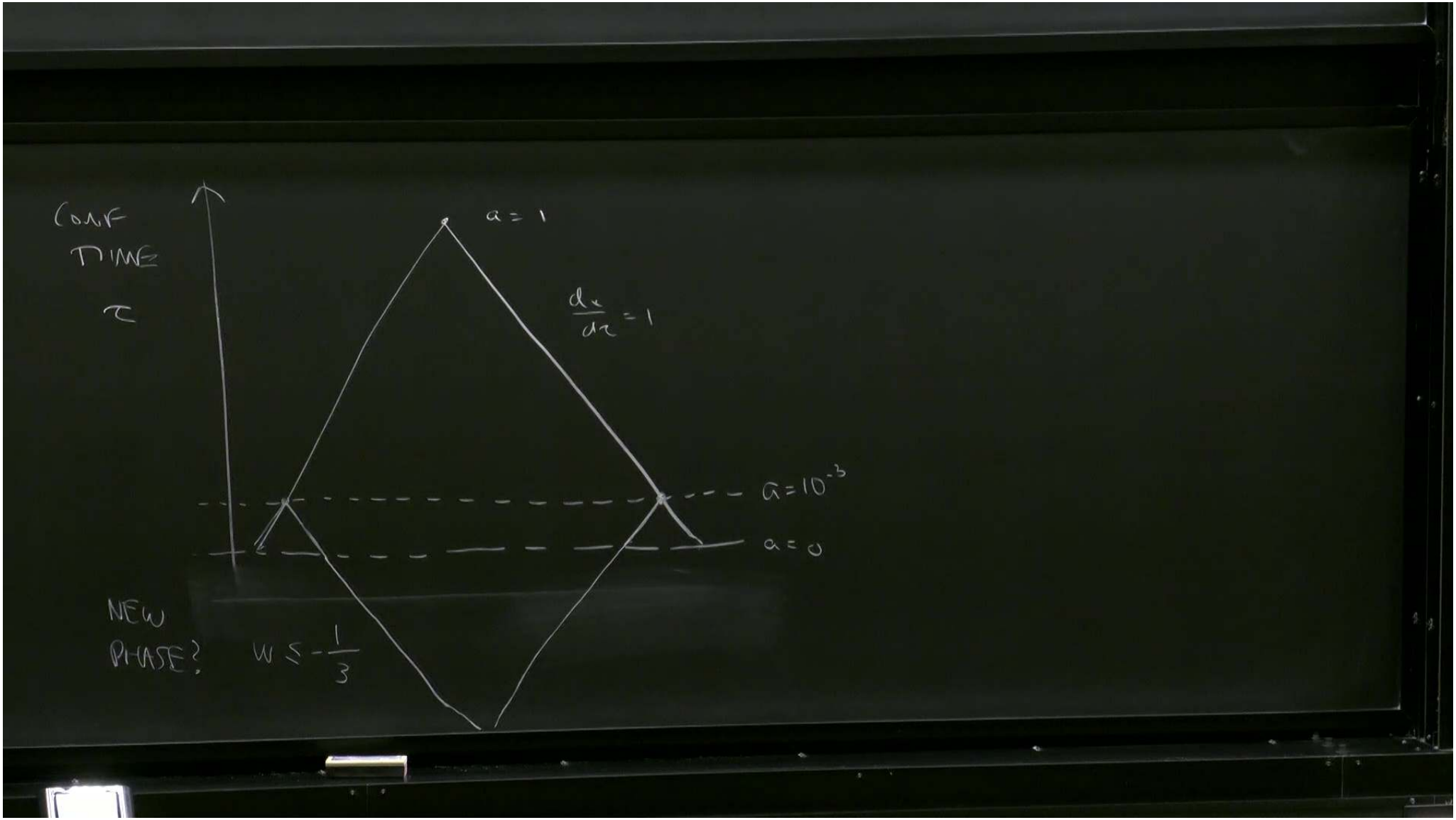
INFLATION

"HORIZON PROBLEM": CMB IS NEARLY ISOTHERMAL,
EVEN THOUGH IT'S CAUSALLY DISCONNECTED

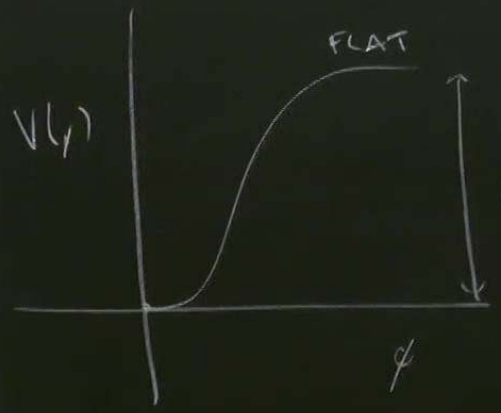
SINGLE FIELD SLOW ROLL INFLATION.

METRIC $g_{\mu\nu}$ + SCALAR FIELD $\phi(x)$ "INFLATON"

$$S = \int d^4x \sqrt{-g} \left[\underbrace{\frac{M_{Pl}^2}{2} R}_{\text{EINSTEIN-HILBERT}} - \underbrace{\frac{1}{2}(\nabla^\mu \phi)(\nabla_\mu \phi)}_{\text{"MINIMALLY COUPLED" SCALAR FIELD}} - \underbrace{V(\phi)}_{\text{POTENTIAL}} \right]$$


CONF
TIME
τ
a = 1
$\frac{da}{d\tau} = 1$
$a = 10^{-3}$
a = 0
NEW
PHASE?
$w \leq -\frac{1}{3}$

EINSTEIN-HILBERT "MINIMALLY COUPLED" POTENTIAL
SCALAR FIELD



$1\ \text{TeV} \lesssim V^{1/4} \lesssim 10^{15}\ \text{GeV}$

$\frac{V'}{V} \ll M_{Pl}^{-1}$

$\frac{V''}{V} \ll M_{Pl}^{-2}$

$\Rightarrow w = -1 + (\text{SMALL CORRECTION})$

"MODERN" H

INITIA

PHASE? $w \leq -\frac{1}{3}$

"MODERN" HORIZON PROBLEM

- INITIAL CONDITIONS.

$$ds^2 = -dt^2 + e^{2\zeta(x)} a(t)^2 dx^2$$

"ADIABATIC MODE"

$\zeta(x,t)$ = "ADIABATIC CURVATURE"

- ζ IS GAUSSIAN RANDOM FIELD

$\Rightarrow$ EACH FOURIER MODE $\zeta(k) = \int d^3x\, \zeta(x) e^{-ik\cdot x}$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE

- TWO-POINT FUNCTION

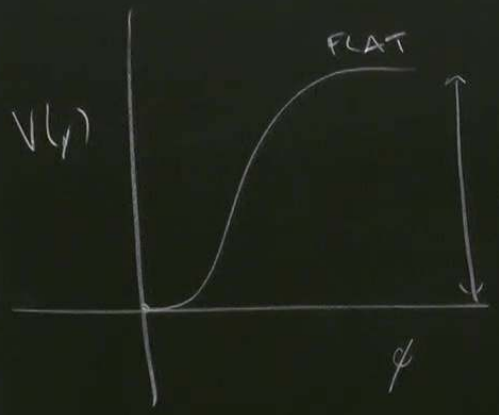
$$\langle \zeta(k)\zeta(k')^* \rangle = 2\pi^2 \Delta_\zeta^2 k^{-3+\varepsilon} (2\pi)^3 \delta^3(k-k')$$

$$\Delta_\zeta^2 = (2.11 \pm 0.03) \times 10^{-9}$$

$$\varepsilon = -0.033 \pm 0.004$$

CM

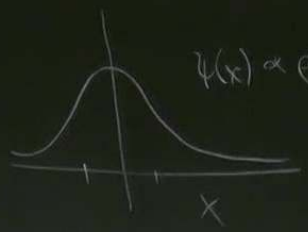
EINSTEIN-HILBERT MINIMALLY COUPLED SCALAR FIELD POTENTIAL



$1\ \text{TeV} \lesssim V^{1/4} \lesssim 10^{15}\ \text{GeV}$

$\frac{V'}{V} \ll M_{Pl}^{-1}$

$\frac{V''}{V} \ll M_{Pl}^{-2}$

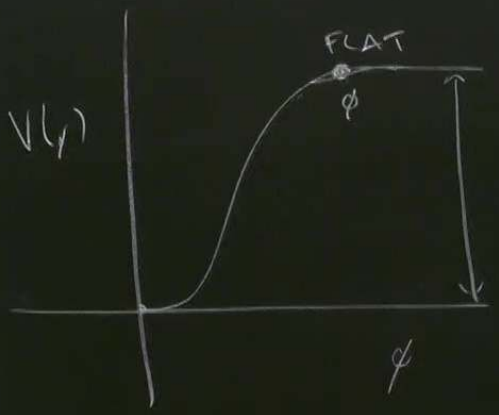


$\psi(x) \propto e^{-m\omega^2 x/2} \Rightarrow w = -1 + (\text{SMALL CORRECTION})$

"MODERN" H

- INITAL
- ds^2
- S IS G
- TWO-P

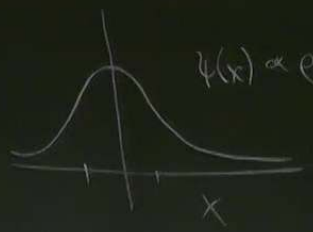
EINSTEIN-HILBERT "MINIMALLY COUPLED" SCALAR FIELD POTENTIAL PHASE



$1\ \text{TeV} \lesssim V^{1/4} \lesssim 10^{15}\ \text{GeV}$

$\frac{V'}{V} \ll M_{Pl}^{-1}$

$\frac{V''}{V} \ll M_{Pl}^{-2}$



$\Rightarrow w = -1 + (\text{SMALL CORRECTION})$

"MODERN" HORIZON P

- INITIAL CONDITION

$ds^2 = -dt^2 +$

$S(k) = \int d^3x\, S(x) e^{-ik\cdot x}$
$\sim L^3$

- S IS GAUSSIAN R

$\Rightarrow$ FACT F

- TWO-POINT FUNCTI

$\langle S(k) S(k')^* \rangle$
$\sim L^6$

PHASE? $w \leq -\frac{1}{3}$

"MODERN" HORIZON PROBLEM

$T \sim (1\ \mathrm{MeV}) \left(\frac{t}{1\ \mathrm{sec}}\right)^{-1/2}$

- INITIAL CONDITIONS.

$$ds^2 = -dt^2 + e^{2\zeta(x)}\, a(t)^2\, dx^2$$

"ADIABATIC MODE"

$\zeta(x,t)$ = "ADIABATIC CURVATURE"

- ζ IS GAUSSIAN RANDOM FIELD

$\Rightarrow$ EACH FOURIER MODE $\zeta(k) = \int d^3x\, \zeta(x)\, e^{ik\cdot x}$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE

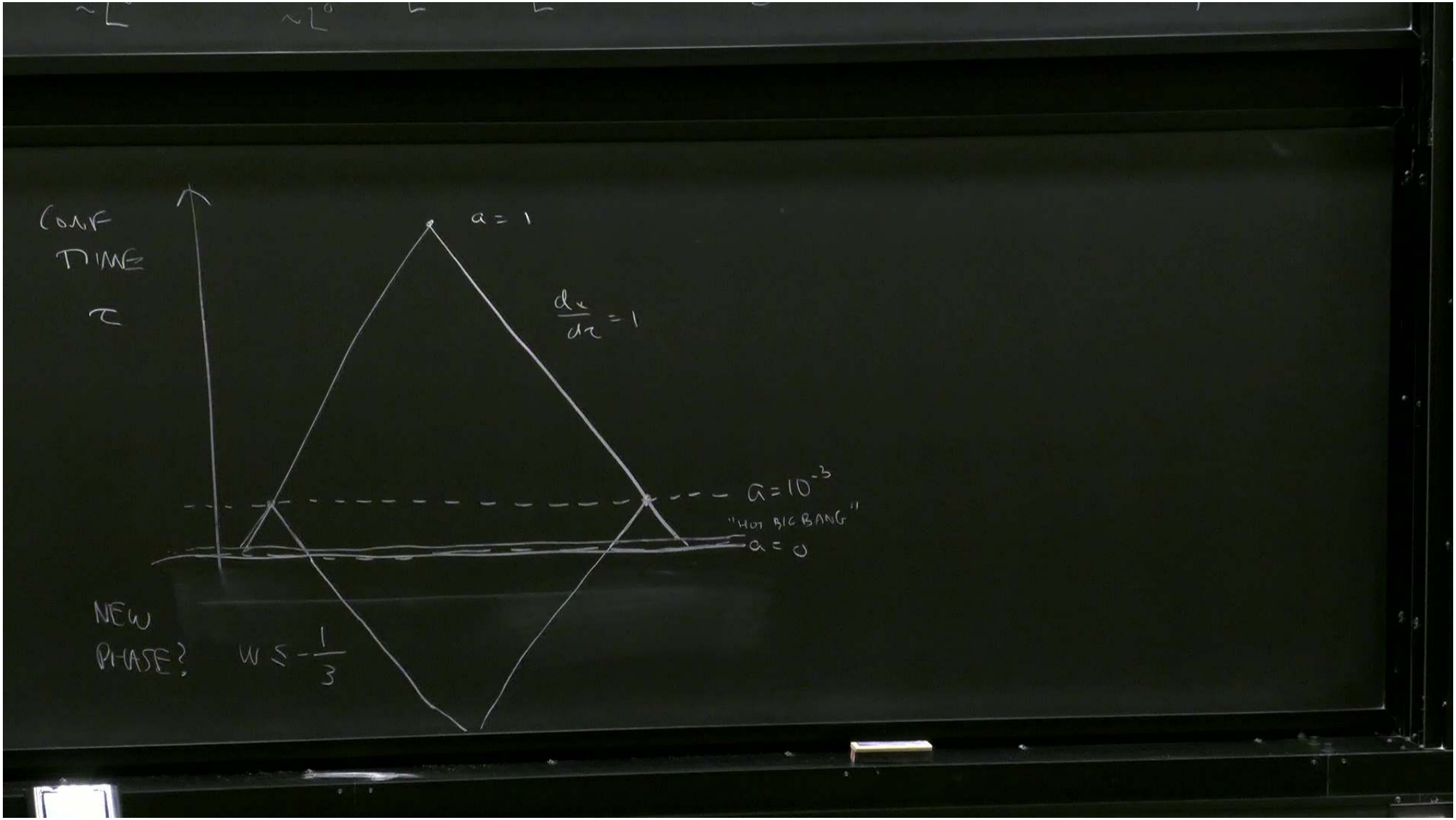
- TWO-POINT FUNCTION

$$\underbrace{\langle \zeta(k)\,\zeta(k')^* \rangle}_{\sim L^6} = \underbrace{2\pi^2 \Delta_\zeta^2}_{\sim L^0} \underbrace{k^{-3}}_{\sim L^3} \underbrace{(2\pi)^3 \delta^3(k-k')}_{\sim L^3}$$

$$\Delta_\zeta^2 = (2.11 \pm 0.03) \times 10^{-9}$$

$$\varepsilon = -0.033 \pm 0.004$$

} CMB


CONF TIME
τ
a = 1
$\frac{dx}{d\tau} = 1$
$a = 10^{-3}$
"HOT BIG BANG"
a = 0
NEW PHASE?
$w \leq -\frac{1}{3}$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} (\nabla^\mu \phi)(\nabla_\mu \phi) - V(\phi) \right]$$

EQUATIONS OF MOTION, ARBITRARY $g_{\mu\nu}$ (NOT FRW) AND $V(\phi)$

$$\begin{cases} \phi\text{-EOM} & \nabla^\mu \nabla_\mu \phi = V'(\phi) \\ g_{\mu\nu}\text{-EOM} & G_{\mu\nu} = \dfrac{T_{\mu\nu}}{M_{Pl}^2} \\ \text{STRESS ENERGY} & T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) - \frac{1}{2}(\nabla_\rho \phi)(\nabla^\rho \phi) g_{\mu\nu} - V(\phi) g_{\mu\nu} \end{cases}$$

$$\ddot{\phi} = -\partial_i^2 \phi - V'(\phi)$$

$$M_{Pl} = (8\pi G)^{-1/2} \sim 10^{18}\ \text{GeV}$$

DERIVATION OMITTED!

DERIVATION OMITTED!

SPECIALIZE TO CASE OF ROTATION + TRANSLATION SYMMETRY

$$g_{\mu\nu} = \begin{pmatrix} -1 & & \\ & a(t)^2 \delta_{ij} & \end{pmatrix} \qquad T_{\mu\nu} = \begin{pmatrix} \rho(t) & & \\ & a(t)^2 p(t) \delta_{ij} & \end{pmatrix}$$

$\phi = \phi(t)$ ↳ NO $\vec{x}$ DEPENDENCE

ϕ-EOM $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

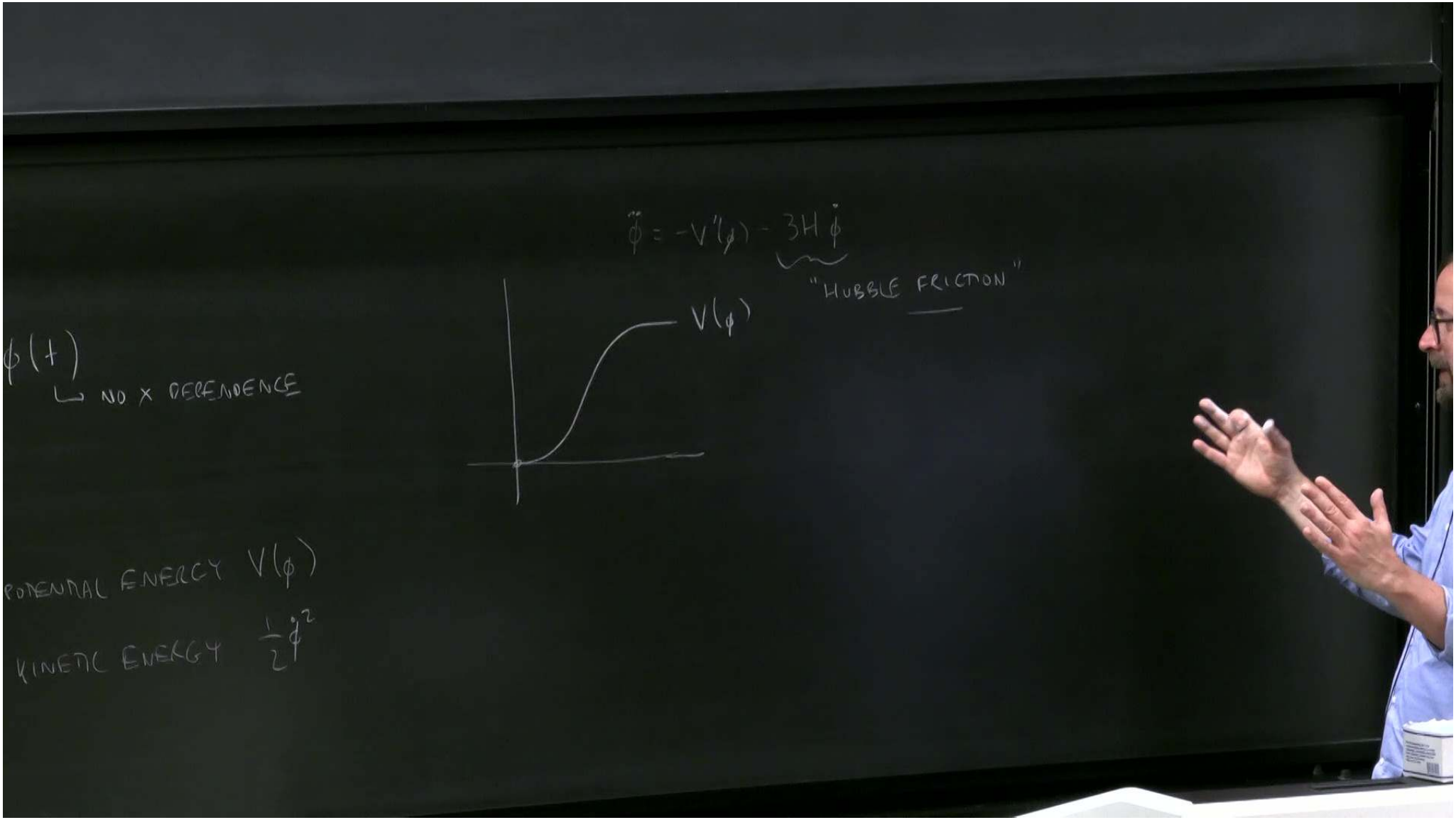
a-EOM $H^2 = \frac{\rho}{3M_{Pl}^2}$

STRESS ENERGY $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

TODAY'S TUTORIAL

$w = \frac{p}{\rho} = \begin{cases} -1 & \text{FOR POTENTIAL ENERGY } V(\phi) \\ +1 & \text{FOR KINETIC ENERGY } \frac{1}{2}\dot{\phi}^2 \end{cases}$


$\ddot{\phi} = -V'(\phi) - 3H\dot{\phi}$
"HUBBLE FRICTION"
$V(\phi)$
$\phi(t)$
↳ NO x DEPENDENCE
POTENTIAL ENERGY $V(\phi)$
KINETIC ENERGY $\frac{1}{2}\dot{\phi}^2$