Title: Cosmology Lecture

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Collection: Cosmology 2023/24

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INFLATION

"HORIZON PROBLEM": CMB IS NEARLY ISOTHERMAL,
EVEN THOUGH IT'S CAUSALLY DISCONNECTED

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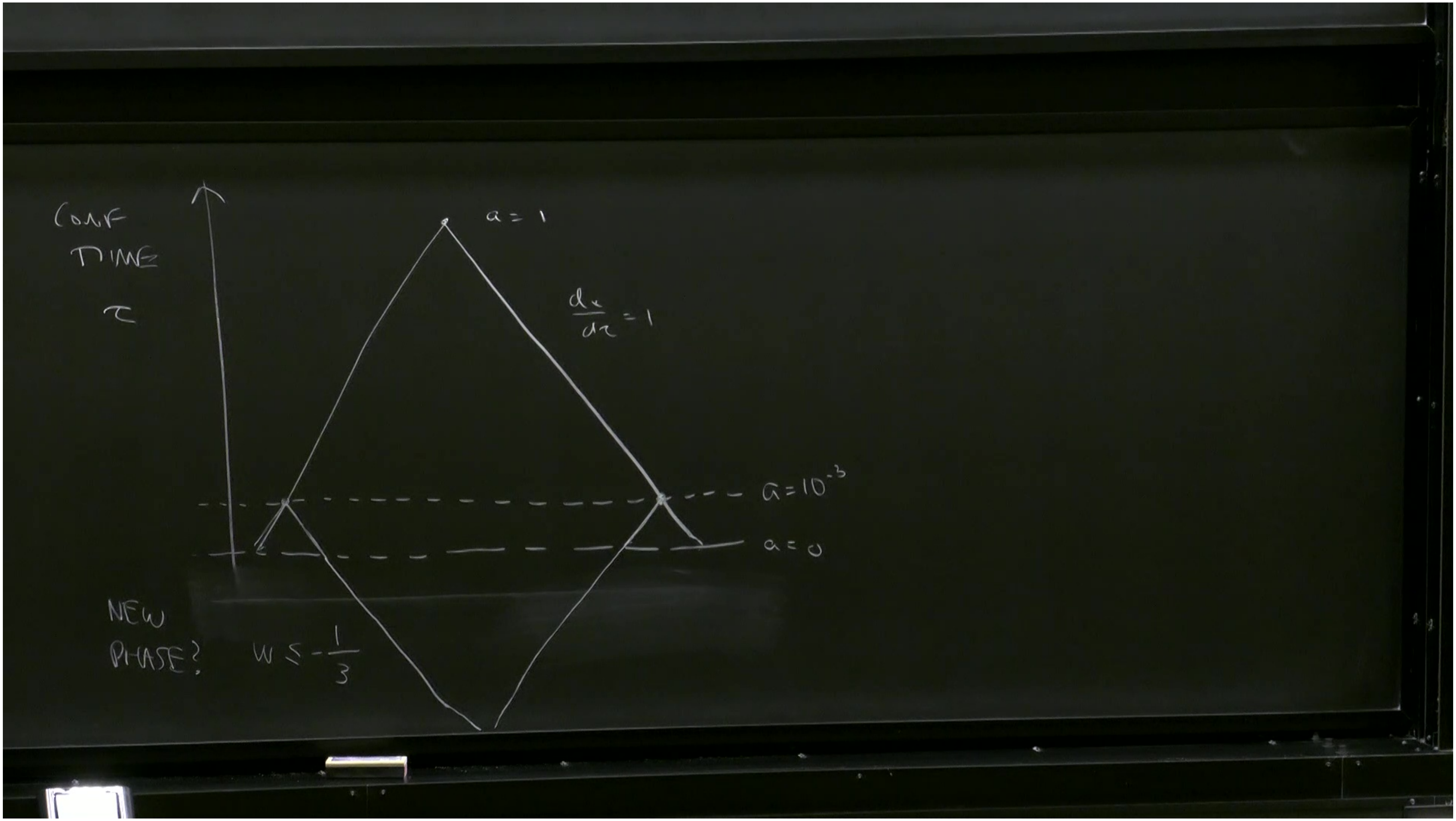
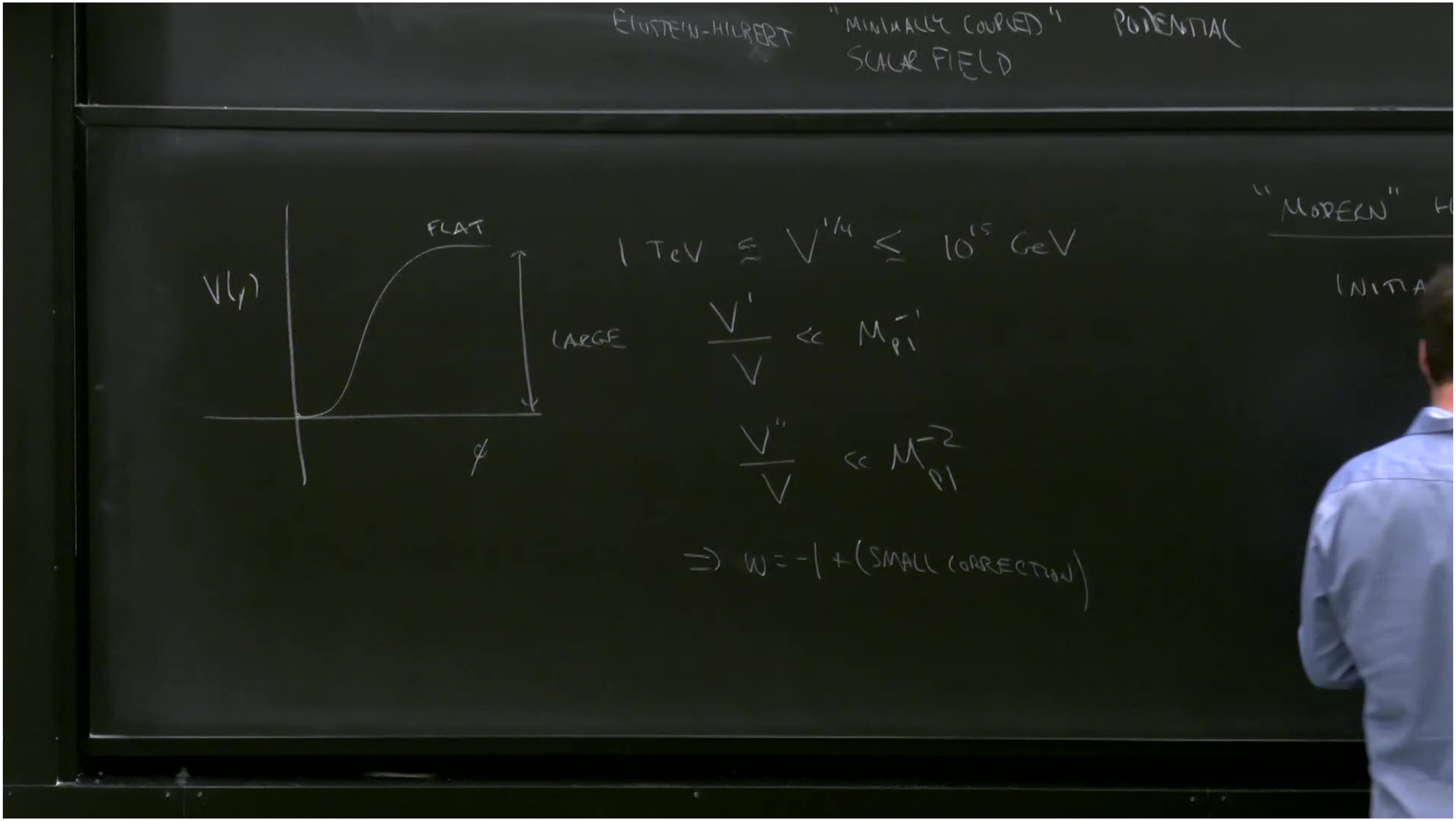
SINGLE FIELD SLOW ROLL INFLATION.

METRIC $g_{\mu\nu}$ + SCALAR FIELD $\phi(x)$ "INFLATON"

$$S = \int d^4x \sqrt{-g}\left[\underbrace{\frac{M_{pl}^2}{2}R}_{\text{EINSTEIN-HILBERT}} - \underbrace{\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi)}_{\text{"MINIMALLY COUPLED" SCALAR FIELD}} - \underbrace{V(\phi)}_{\text{POTENTIAL}}\right]$$


CONF
TIME
τ
a = 1
dx/dτ = 1
a = 10^-3
a = 0
NEW
PHASE?
w ≤ -1/3


EINSTEIN-HILBERT
"MINIMALLY COUPLED"
SCALAR FIELD
POTENTIAL
"MODERN"
$V(\phi)$
FLAT
LARGE
ϕ
$1\ \mathrm{TeV} \lesssim V^{1/4} \lesssim 10^{15}\ \mathrm{GeV}$
$\frac{V'}{V} \ll M_{Pl}^{-1}$
$\frac{V''}{V} \ll M_{Pl}^{-2}$
$\Rightarrow w = -1 + (\text{SMALL CORRECTION})$

PHASE? $w \leq -\frac{1}{3}$

"MODERN" HORIZON PROBLEM

- INITIAL CONDITIONS.

$$ds^2 = -dt^2 + e^{2\zeta(x)} a(t)^2 dx^2$$

"ADIABATIC MODE"

$\zeta(x,t)$ = "ADIABATIC CURVATURE"

- ζ IS GAUSSIAN RANDOM FIELD

$\Rightarrow$ EACH FOURIER MODE $\zeta(k) = \int d^3x\, \zeta(x)\, e^{-ik\cdot x}$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE

- TWO-POINT FUNCTION

$$\langle \zeta(k)\zeta(k')^* \rangle = 2\pi^2 \Delta_\zeta^2 k^{-3+\varepsilon} (2\pi)^3 \delta^3(k-k')$$

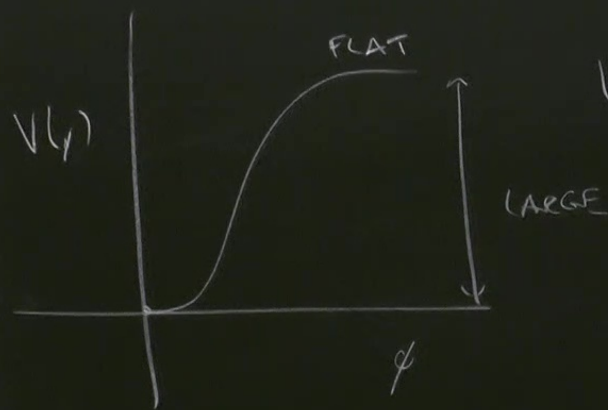
$$\left.\begin{aligned} \Delta_\zeta^2 &= (2.11 \pm 0.03) \times 10^{-9} \\ \varepsilon &= -0.033 \pm 0.004 \end{aligned}\right\}$$

(CM

EINSTEIN-HILBERT

"MINIMALLY COUPLED" SCALAR FIELD

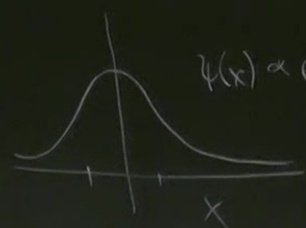
POTENTIAL



$$1\ \text{TeV} \lesssim V^{1/4} \lesssim 10^{15}\ \text{GeV}$$

$$\frac{V'}{V} \ll M_{Pl}^{-1}$$

$$\frac{V''}{V} \ll M_{Pl}^{-2}$$



$\psi(x) \propto e^{-m\omega^2 x/2}$

$\Rightarrow w = -1 + (\text{SMALL CORRECTION})$

"MODERN" HO

- INITIAL

ds^2

- S IS (

- TWO-PO

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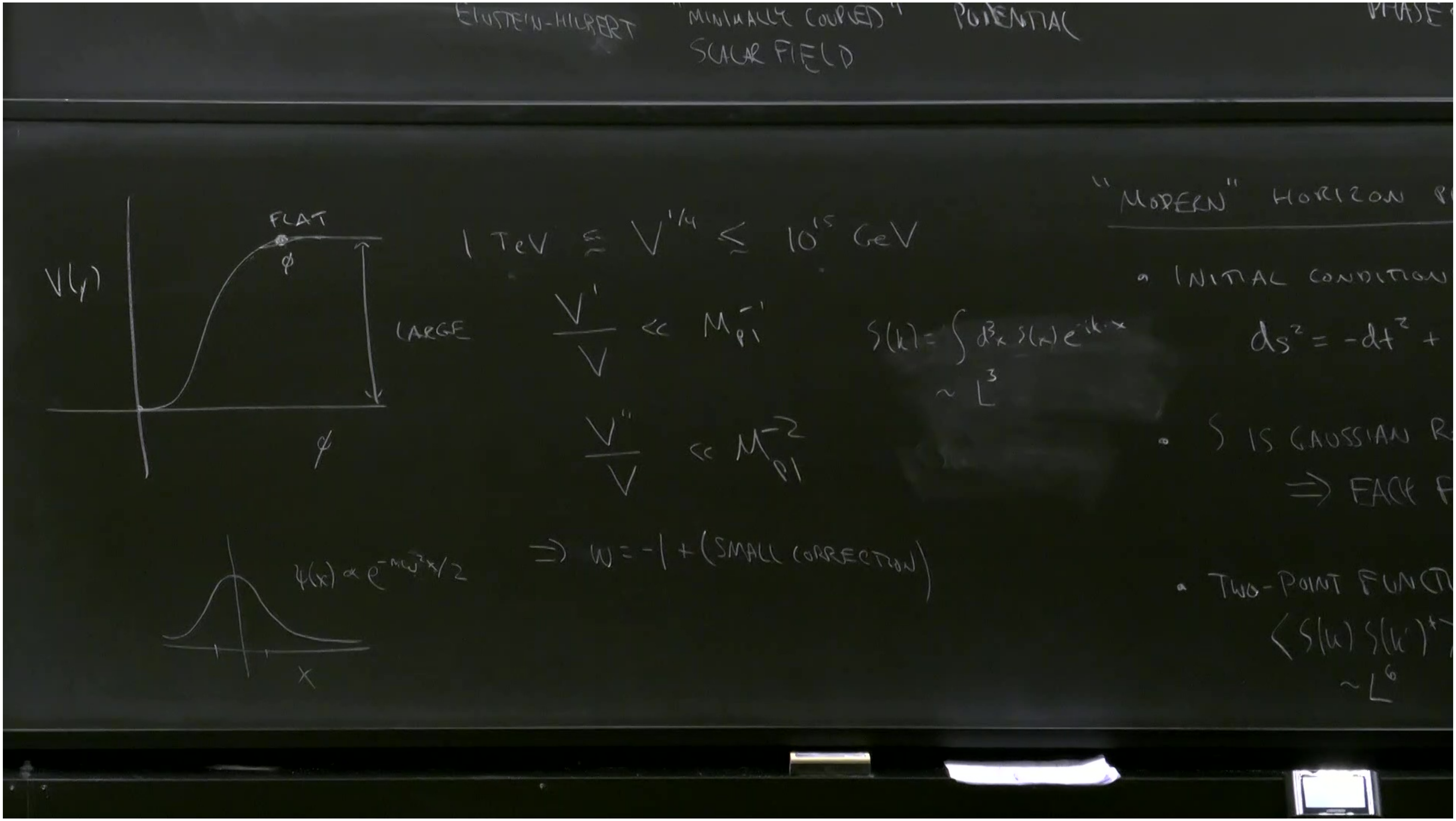

EINSTEIN-HILBERT
"MINIMALLY COUPLED" SCALAR FIELD
POTENTIAL
"MODERN" HORIZON P
FLAT
V(φ)
φ
LARGE
1 TeV ≲ V^{1/4} ≤ 10^{15} GeV
V'/V << M_{pl}^{-1}
V''/V << M_{pl}^{-2}
δ(k) = ∫ d³x δ(x) e^{ik·x}
~ L³
• INITIAL CONDITION
ds² = -dt² +
• δ IS GAUSSIAN R
⇒ EACH F
ψ(x) ∝ e^{-mωx²/2}
⇒ w = -1 + (SMALL CORRECTION)
• TWO-POINT FUNCT
⟨δ(k) δ(k')*⟩
~ L⁶

PHASE? $w \lesssim -\frac{1}{3}$

"MODERN" HORIZON PROBLEM

$$T \sim (1\ \mathrm{MeV}) \left(\frac{t}{1\ \mathrm{sec}}\right)^{-1/2}$$

• INITIAL CONDITIONS.

$$ds^2 = -dt^2 + e^{2\zeta(x)}\, a(t)^2\, dx^2$$

"ADIABATIC MODE"

$\zeta(x,t)$ = "ADIABATIC CURVATURE"

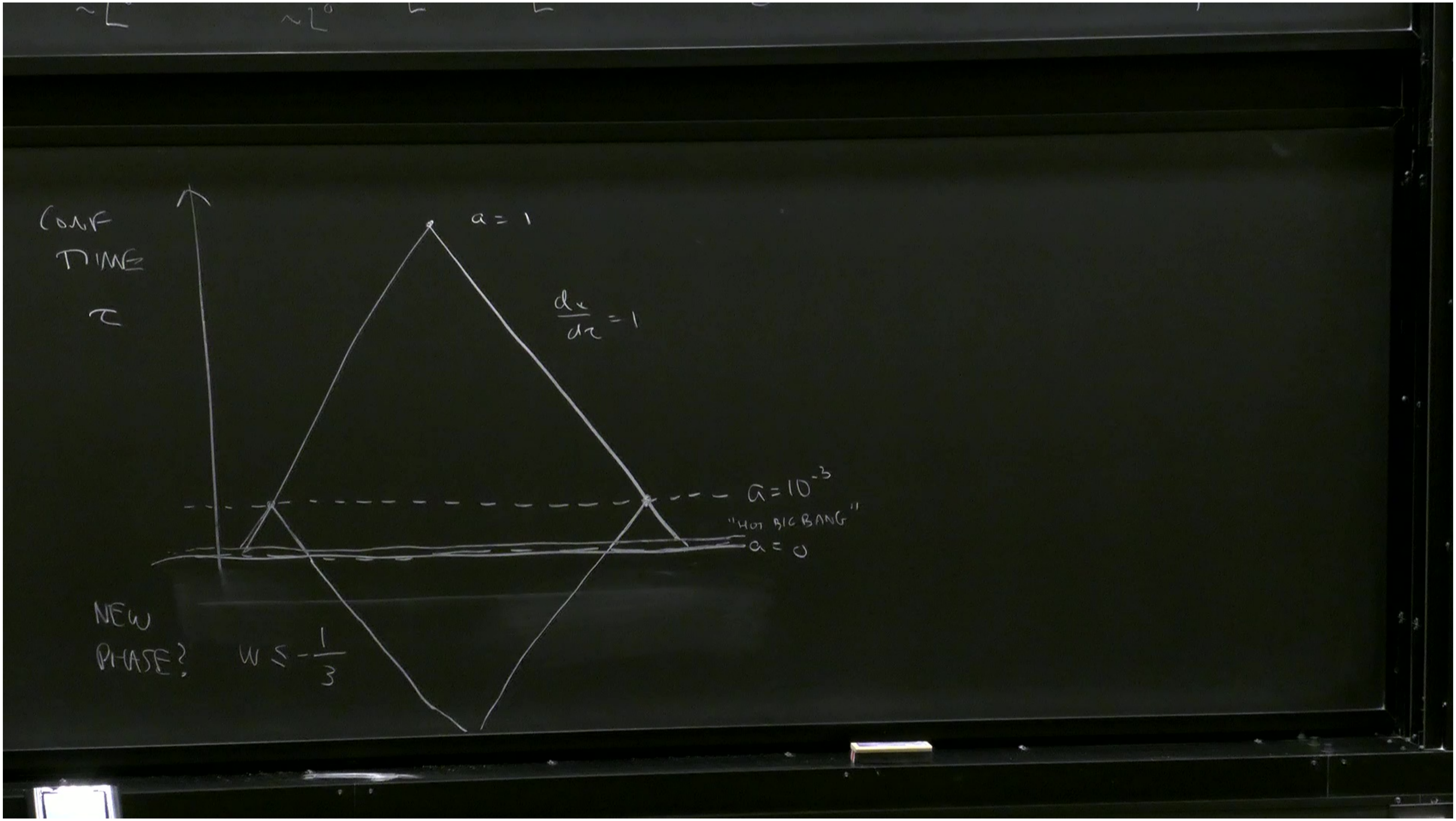
• ζ IS GAUSSIAN RANDOM FIELD

$\Rightarrow$ EACH FOURIER MODE $\zeta(k) = \int d^3x\, \zeta(x)\, e^{-ik\cdot x}$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE

• TWO-POINT FUNCTION

$$\underbrace{\langle \zeta(k)\, \zeta(k')^* \rangle}_{\sim L^6} = \underbrace{2\pi^2 \Delta_\zeta^2}_{\sim L^0}\, \underbrace{k^{-3}}_{\sim L^3}\, \underbrace{(2\pi)^3 \delta^3(k-k')}_{\sim L^3}$$

$$\left.\begin{aligned} \Delta_\zeta^2 &= (2.11 \pm 0.03) \times 10^{-9} \\ \epsilon &= -0.033 \pm 0.004 \end{aligned}\right\} \text{CMB}$$


CONF
TIME
τ
a = 1
dx/dτ = 1
a = 10^-3
"HOT BIG BANG"
a = 0
NEW
PHASE?
w ≤ -1/3

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{pl}^2}{2} R - \frac{1}{2}(\nabla^\mu \phi)(\nabla_\mu \phi) - V(\phi)\right]$$

EQUATIONS OF MOTION, ARBITRARY $g_{\mu\nu}$ (NOT FRW) AND $V(\phi)$

ϕ-EOM $\quad \nabla^\mu \nabla_\mu \phi = V'(\phi)$ $\qquad \ddot{\phi} = -\partial_i^2 \phi - V'(\phi)$

$M_{pl} = (8\pi G)^{-1/2} \sim 10^{18}$ GeV

$g_{\mu\nu}$-EOM $\quad G_{\mu\nu} = \dfrac{T_{\mu\nu}}{M_{pl}^2}$

STRESS ENERGY $\quad T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) - \frac{1}{2}(\nabla_\rho \phi)(\nabla^\rho \phi) g_{\mu\nu} - V(\phi) g_{\mu\nu}$

DERIVATION OMITTED!

DERIVATION OMITTED!

SPECIALIZE TO CASE OF ROTATION + TRANSLATION SYMMETRY

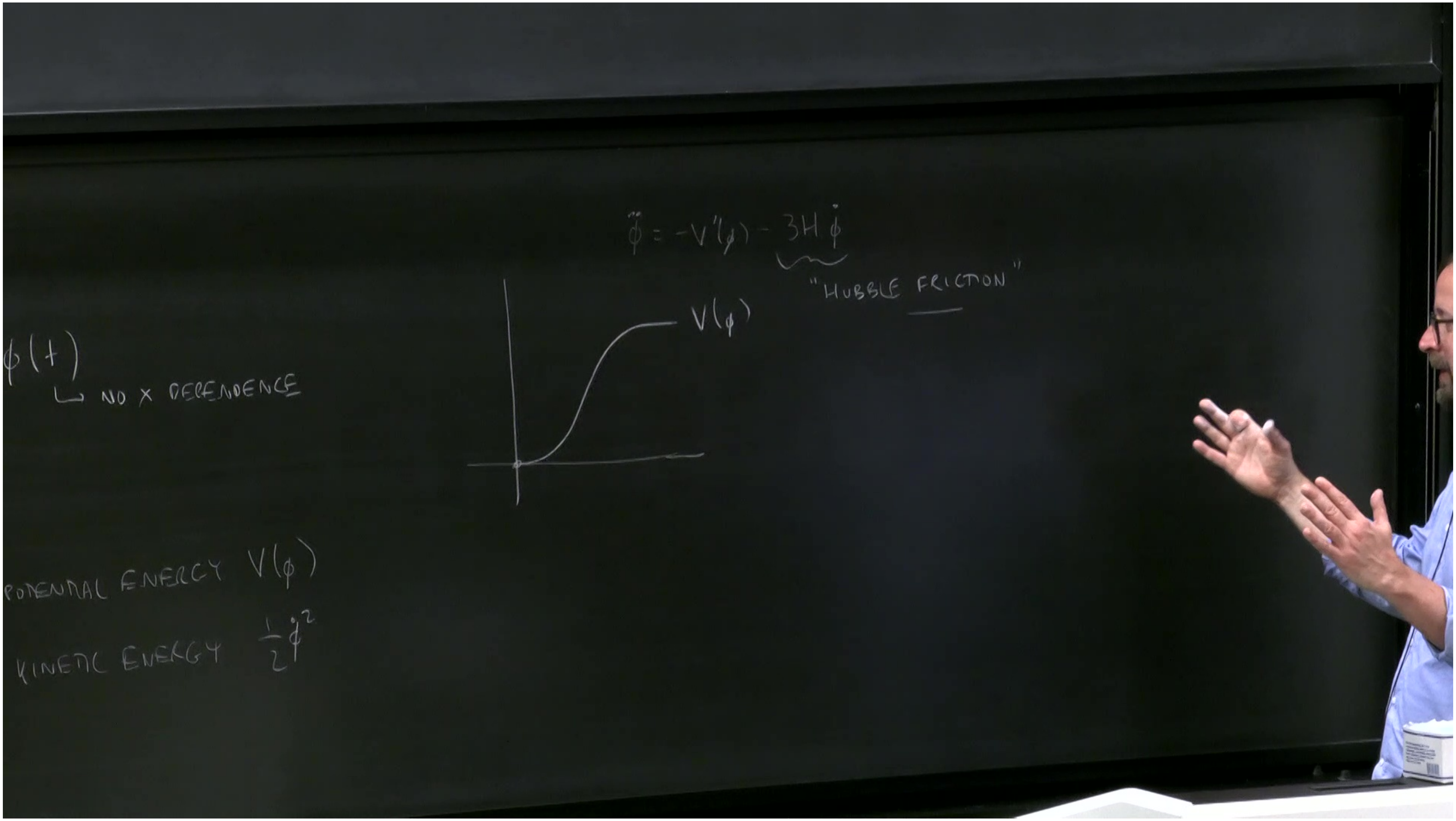
$$g_{\mu\nu} = \begin{pmatrix} -1 & \\ & a(t)^2 \delta_{ij} \end{pmatrix} \qquad T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a(t)^2 p(t) \delta_{ij} \end{pmatrix} \qquad \phi = \phi(t)$$

↳ NO X DEPENDENCE

$$\begin{cases} \phi\text{-EOM} & \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \\ a\text{-EOM} & H^2 = \dfrac{\rho}{3M_{pl}^2} \\ \text{STRESS ENERGY} & \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ & p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases}$$

TODAY'S TUTORIAL

$$w = \frac{p}{\rho} = \begin{cases} -1 & \text{FOR POTENTIAL ENERGY } V(\phi) \\ +1 & \text{FOR KINETIC ENERGY } \frac{1}{2}\dot{\phi}^2 \end{cases}$$


$\ddot{\phi} = -V'(\phi) - 3H\dot{\phi}$
"HUBBLE FRICTION"
$V(\phi)$
$\phi(t)$
NO X DEPENDENCE
POTENTIAL ENERGY $V(\phi)$
KINETIC ENERGY $\frac{1}{2}\dot{\phi}^2$