Title: Cosmology Lecture

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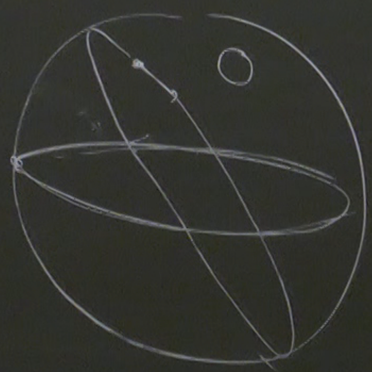
$T_{CNB} = 1.94 \pm 0.03 \quad K$



$2\pi R$

$$\rho_{\text{blackbody}} = \int \frac{d^3q}{(2\pi)^3} \frac{q^3}{e^{q/T}-1}$$

$\hbar = c = k_B = 1$

$$= \frac{\pi^2}{15} T^4$$

THERMAL GAS OF PHOTONS

$P(\text{STATE}) \propto e^{-E/T}$

- MULTIPARTICLE PLASMA
  - $m \neq 0$
  - ARB. SPIN $g = (2s+1)$
  - BOSON OR FERMION

$e^+ e^- \gamma$

$$\Gamma(n+1) \equiv \int_0^\infty dx\; x^n\, e^{-x}$$

$$\Gamma(n+1) = n\,\Gamma(n)$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$$\Gamma\left(\tfrac{5}{2}\right) = \frac{3}{2}\Gamma\left(\tfrac{3}{2}\right)$$

$$= \frac{3}{2}\,\frac{1}{2}\,\Gamma\left(\tfrac{1}{2}\right)$$

$$= \frac{3}{4}\sqrt{\pi}$$

$$\sum_{n=0}^{\infty} x^n = (1-x)^{-1}$$

$$\sum_{n=0}^{\infty} n x^n = x\frac{d}{dx}\left[\sum_{n=0}^{\infty} x^n\right]$$

$$= x\frac{d}{dx}\left[(1-x)^{-1}\right]$$

$$= \frac{x}{(1-x)^2}$$

$\zeta(2)$

$\zeta(4)$

$$\zeta(2) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$$

$$\zeta(4) = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$$

$$\zeta(s) \equiv \sum_{n=0}^{\infty} n^{-s}$$ "RIEMANN ZETA FUNCTION"

$$\zeta(3) = 1.202\cdots$$

$$\zeta(5) = 1.036\cdots$$

$= \frac{3}{4}\sqrt{\pi}$

$$\frac{1}{1^4} - \frac{1}{2^4} + \frac{1}{3^4} - \frac{1}{4^4} + \cdots$$

$$= \left(1 - 2\cdot\frac{1}{2^4}\right)\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\right) = \frac{7}{8}\,\frac{\pi^4}{90}$$

MORE GENERALLY, $\sum_{n=0}^{\infty} (-1)^n n^{-s} = \left(1 - \frac{1}{2^{s-1}}\right) \zeta(s)$

"THERMAL BOSO

$\int_0^\infty dx\, \frac{x}{e^x}$

"THERMAL BOSONIC INTEGRAL"

$$\int_0^\infty dx\, \frac{x^k}{e^x - 1} = \int_0^\infty dx\, x^k \frac{e^{-x}}{1-e^{-x}}$$

$$= \int_0^\infty dx\, x^k\, e^{-x} \sum_{n=0}^{\infty} (e^{-x})^n$$

$$= \int_0^\infty dx\, x^k \left(e^{-x} + e^{-2x} + e^{-3x} + \cdots \right)$$

$$= \Gamma(k+1) + \frac{1}{2^{k+1}} \Gamma(k+1) + \frac{1}{3^{k+1}} \Gamma(k+1) + \cdots$$

$$= \zeta(k+1)\,\Gamma(k+1)$$

$\frac{\pi^4}{90}$

"FERMIONIC THERMAL INTEGRAL"

$$\int_0^\infty dx\, \frac{x^k}{e^x+1} = \int_0^\infty dx\, x^k \frac{e^{-x}}{1+e^{-x}}$$

$$= \int_0^\infty dx\, x^k \left[e^{-x} - e^{-2x} + e^{-3x} - e^{-4x} + \cdots \right]$$

$$= \Gamma(k+1) - \frac{1}{2^{k+1}}\Gamma(k+1) + \frac{1}{3^{k+1}}\Gamma(k+1) - \frac{1}{4^{k+1}}\Gamma(k+1)$$

$$= \left(1 - \frac{1}{2^{k+1}} + \frac{1}{3^{k+1}} - \frac{1}{4^{k+1}} + \cdots\right)\Gamma(k+1)$$

$$= \boxed{\left(1 - \frac{1}{2^k}\right)\zeta(k+1)\,\Gamma(k+1)}$$

- MULTIPARTICLE PLASMA

"MODE" OF PLASMA = CHOICE OF SPECIES, SPIN STATE, AND MOMENTUM q

N = OCCUPATION NUMBER

$$P(N) \propto e^{-N E_q / T} \qquad E_q = \sqrt{m^2 + q^2}$$

FERMION $\Rightarrow N \in \{0, 1\}$

BOSON $\Rightarrow N \in \{0, 1, 2, \ldots\}$

f_q = MEAN OCCUPATION NUMBER $\langle N \rangle$

$$= \sum_N N\, P(N) \quad \text{WHERE} \quad \sum_N P(N) = 1$$

- FERMION

$$p(N) = \frac{e^{-NE_q/T}}{\sum_{N=0}^{1} e^{-NE_q/T}} = \frac{e^{-NE_q/T}}{1 + e^{-E_q/T}} \qquad N \in \{0, 1\}$$

$$f_q = \sum_{N=0}^{1} N\, p(N) = \frac{e^{-E_q/T}}{1 + e^{-E_q/T}}$$

$$= \boxed{\frac{1}{e^{E_q/T} + 1}}$$

• BOSON $\quad p(N) = \dfrac{e^{-NE_q/T}}{\sum_{N=0}^{\infty} e^{-NE_q/T}}$

$$= \frac{e^{-NE_q/T}}{(1-e^{-E_q/T})^{-1}}$$

MEAN OCCUPATION $\quad f_q = \sum_{N=0}^{\infty} N\, p(N)$

$$= \frac{1}{(1-e^{-E_q/T})^{-1}} \sum_{N=0}^{\infty} N\, e^{-NE_q/T}$$

$$= \frac{1}{(1-e^{-E_q/T})^{-1}} \frac{e^{-E_q/T}}{(1-e^{-E_q/T})^{2}}$$

$$= \frac{e^{-E_q/T}}{1-e^{-E_q/T}}$$

$$= \frac{1}{e^{E_q/T}-1}$$

$$= \frac{\quad}{e^{E_q/T} + 1}$$

NOTATION

$$\boxed{f_q = \frac{1}{e^{E_q/T} \mp 1}}$$

UPPER SIGN = BOSON

LOWER SIGN = FERMION

$$= \frac{3}{4}\sqrt{\pi}$$

$$\rho = \frac{1}{V} \sum_{\text{SPECIES}} \sum_{\text{SPINS}} \sum_{q} f_q E_q \qquad \sum_q \to V \int \frac{d^3q}{(2\pi)^3}$$

$$= \frac{3}{4}\sqrt{\pi}$$

$$\rho = \frac{1}{V} \sum_{\text{SPECIES}} \sum_{\text{SPINS}} \sum_{q} f_q E_q$$

$$\sum_q \rightarrow V \int \frac{d^3q}{(2\pi)^3}$$

$$= \sum_{\text{SPECIES}} \sum_{\text{SPINS}} \int \frac{d^3q}{(2\pi)^3} f_q E_q$$

$$= \sum_{\text{SPECIES}} g \int \frac{d^3q}{(2\pi)^3} \frac{E_q}{e^{E_q/T} \mp 1}$$

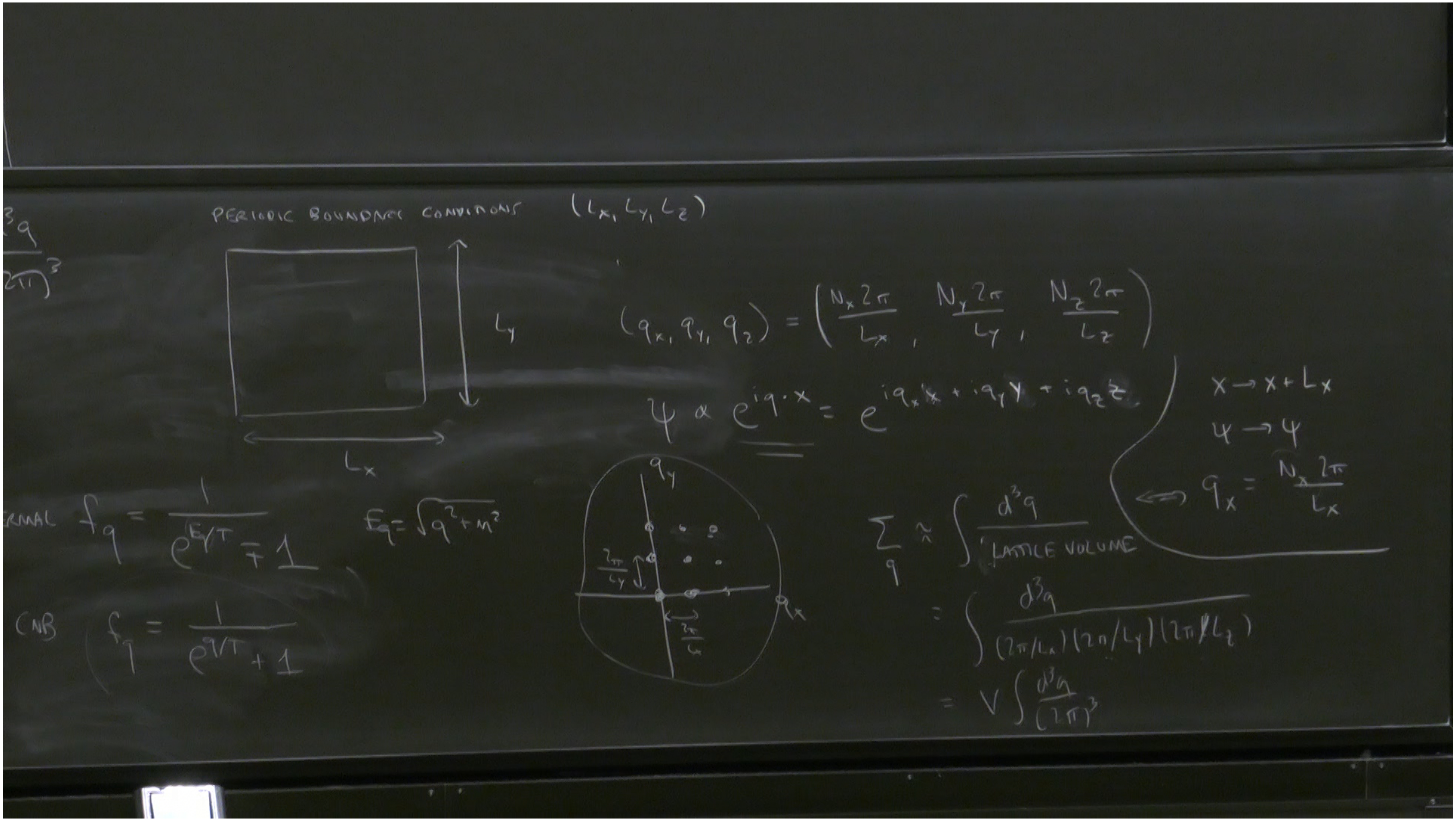
$$E_q = \sqrt{q^2 + m^2}$$

THERMAL $f_q = \frac{1}{e^{E_q/\ldots}}$

$$= \boxed{\sum_{\text{SPECIES}} \frac{g}{2\pi^2} \int dq \frac{q^2 E_q}{e^{E_q/T} \mp 1}}$$

$$d^3q = 4\pi q^2 dq$$

CNB $f_q = \frac{1}{e^{q\ldots}}$


PERIODIC BOUNDARY CONDITIONS (L_x, L_y, L_z)
L_y
L_x
$(q_x, q_y, q_z) = \left(\frac{N_x 2\pi}{L_x}, \frac{N_y 2\pi}{L_y}, \frac{N_z 2\pi}{L_z}\right)$
$\psi \propto e^{iq\cdot x} = e^{iq_x x + iq_y y + iq_z z}$
$x \to x + L_x$
$\psi \to \psi$
$\Longleftrightarrow q_x = \frac{N_x 2\pi}{L_x}$
q_y
q_x
$\frac{2\pi}{L_y}$
$\frac{2\pi}{L_x}$
$\sum_q \approx \int \frac{d^3q}{\text{LATTICE VOLUME}}$
$= \int \frac{d^3q}{(2\pi/L_x)(2\pi/L_y)(2\pi/L_z)}$
$= V \int \frac{d^3q}{(2\pi)^3}$
$f_q = \frac{1}{e^{E/T} \mp 1}$
$E_q = \sqrt{q^2 + m^2}$
CNB $f_q = \frac{1}{e^{q/T} + 1}$