Title: Quantum Information Lecture

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Quantum Energy Teleportation (Hotta)

Minimal model: Consider two qubits A and B and the following Hamiltoni

where $\hat{H}_A = h\hat{\sigma}_z^A + f(h,k)\mathbb{1}$ $\hat{H}_B = h\hat{\sigma}_z^B + f(h,k)$ $\hat{V} = 2[k\hat{\sigma}_x^A\hat{\sigma}_x^B +$

the ground state of this Hamiltonian is $|g\rangle$ and satisfies $\langle g|\hat{H}_A|g\rangle = \langle g$

$$|g\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1-\frac{f(h,k)}{h}}\,|1\rangle_A|1\rangle_B - \sqrt{1+\frac{f(h,k)}{h}}\,|0\rangle_A|0\rangle_B\right)$$

where $\hat{\sigma}_z^\nu$

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the ground state of this Hamiltonian is $|g\rangle$ and satisfies $\langle g|\hat{H}_A|g\rangle = \langle g|H_B$

$$|g\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1-\frac{f(h,k)}{h}}\,|1\rangle_A|1\rangle_B - \sqrt{1+\frac{f(h,k)}{h}}\,|0\rangle_A|0\rangle_B\right)$$

where $\hat{\sigma}_z^\nu|0\rangle_\nu$

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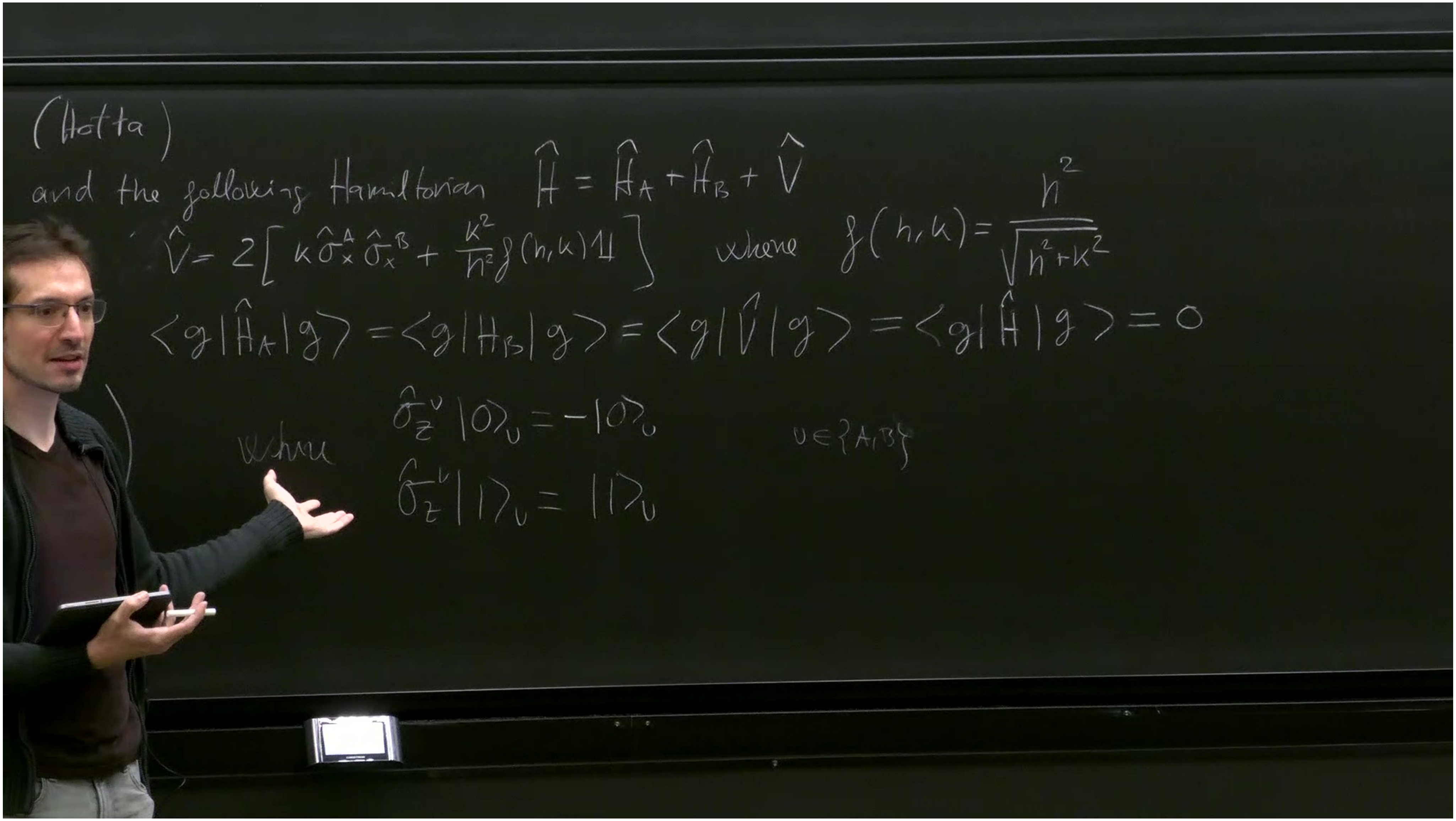
(Hotta)
and the following Hamiltonian $\hat{H} = \hat{H}_A + \hat{H}_B + \hat{V}$
$\hat{V} = 2\left[k\hat{\sigma}_x^A\hat{\sigma}_x^B + \frac{k^2}{h^2} f(h,k)\mathbb{1} \right]$ where $f(h,k) = \frac{h^2}{\sqrt{h^2+k^2}}$
$\langle g|\hat{H}_A|g\rangle = \langle g|H_B|g\rangle = \langle g|\hat{V}|g\rangle = \langle g|\hat{H}|g\rangle = 0$
where $\hat{\sigma}_z^\nu|0\rangle_\nu = -|0\rangle_\nu$
$\nu \in \{A,B\}$
$\hat{\sigma}_z^\nu|1\rangle_\nu = |1\rangle_\nu$

1- We start from $|g\rangle$

2- Alice performs a projective measurement of $\hat{\sigma}_x^A$ on many copies of $|g\rangle$. on avera

3- The result of the measurement is announced to Bob through a classical channel (

4- Bob performs a classically controlled local unitary on B that depends on es of $|g\rangle$. on average this will cost $E_{P_A} > 0$

classical channel (fast)

B that depends on Alice's outcome. Bob will be able to extract energy from the system.

Step 1: Alice measures $\hat{\sigma}_x^A$ obtains an outcome $\alpha \in \{-1, 1\}$. Th

$$|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}} \hat{P}_A(\alpha)|g\rangle \qquad p_A(\alpha) = \langle g|\hat{P}_A(\alpha)|g\rangle$$

$$\hat{\rho}_1 = \sum_{\alpha=\pm 1} p_A(\alpha)\, |\psi_{PM}(\alpha)\rangle\langle\psi_{PM}(\alpha)| = \sum_{\alpha=\pm 1} \hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)$$

Average Energy cost $\quad E_{P_A} = tr(\hat{\rho}_1 \hat{H}) - \underbrace{tr(|g\rangle\langle g|\hat{H})}_{0}$

The state update is $\hat{P}_A(\alpha) = \frac{1}{2}\left(\mathbb{1} + \alpha\,\hat{\sigma}_x^A\right)$

$$E_{P_A} = \sum_{\alpha=\pm1} \langle g | \hat{P}_A(\alpha)\, \hat{H}\, \hat{P}_A(\alpha) | g\rangle =$$

$$= \sum_{\alpha=\pm1} \Big(\langle g | \hat{P}_A(\alpha)\, \hat{H}_A\, \hat{P}_A(\alpha) | g\rangle + \langle g | \hat{P}_A(\alpha)\, \hat{H}_B\, \hat{P}_A(\alpha) | g\rangle + \langle g | P_A(\alpha)\, \hat{V}\, \hat{P}_A(\alpha) | g\rangle \Big)$$

We know $[\hat{P}_A(\alpha), \hat{H}_B] = [\hat{P}_A(\alpha), \hat{V}] = 0$

$$\sum_{\alpha=\pm1} \langle g | \hat{P}_A(\alpha)\, \hat{H}_B\, \hat{P}_A(\alpha) | g\rangle = 0$$

$$\sum_{\alpha=\pm1} \langle g | \hat{P}_A(\alpha)\, \hat{V}\, \hat{P}_A(\alpha) | g\rangle = \sum_{\alpha=\pm1} \langle g | \hat{V}\, \hat{P}_A(\alpha) | g\rangle = 0$$

$$E_{P_A} = \mathrm{tr}(\hat{\rho}_1 \hat{H}) =$$

$$= \sum_{\alpha=\pm1} \langle g | \hat{P}_A(\alpha)\, \hat{H}_A\, \hat{P}_A(\alpha) | g\rangle = f(h,k) > 0$$

$tr(\hat O|\psi\rangle\langle\psi|) = \langle\psi|\hat O|\psi\rangle$ $\quad \hat O$

Bob applies $\hat U_B(\alpha) = \cos\theta\, \mathbb{1} - i\alpha \sin\theta\, \hat\sigma_y^B$ $\quad$ θ is give

$$\hat U_B(\alpha)|\psi_{PM}\rangle = \frac{1}{\sqrt{p_A(\alpha)}}\hat U_B(\alpha)\hat P_A(\alpha)|g\rangle$$

$$\hat\rho_2 = \sum_{\alpha=\pm1} \hat U_B \hat P_A(\alpha)|g\rangle\langle g|\hat P_A(\alpha)\hat U_B^\dagger(\alpha)$$

$$E_{0_B} = Tr(\hat\rho_2\hat H) - tr(\hat\rho_1\hat H) = Tr(\hat\rho_2\hat H) - E_{PA}$$

$$tr(\hat\rho_2\hat{\mathbb{1}}) = \sum_{\alpha=\pm1} \langle g$$

$$\sum_{\alpha=\pm1} \langle g|\hat{P}_A(\alpha)\, V\, \hat{P}_A(\alpha)|g\rangle = \sum_{\alpha=\pm1}$$

$\sin\theta\, \hat{\sigma}_y^B$ θ is given $\cos 2\theta = \dfrac{h^2+2k^2}{\sqrt{(h^2+2k^2)^2+h^2k^2}}$, $\sin 2\theta = \dfrac{hk}{\sqrt{\;}}$

$$tr(\hat{\rho}_2 \hat{H}) = \sum_{\alpha=\pm1} \langle g|P_A(\alpha)\, \hat{U}_B^\dagger(\alpha)\, \underbrace{\hat{H}}_{\hat{H}_A+\hat{H}_B+\hat{V}}\, \hat{U}_B(\alpha)\, P_A(\alpha)|g\rangle$$

$$\sum_{\alpha=\pm1} \langle g|P_A(\alpha)\, \hat{U}_B^\dagger(\alpha)\, \hat{H}_A\, \hat{U}_B(\alpha)\, \hat{P}_A(\alpha)|g\rangle = \sum_{\alpha=\pm1} \langle g|\hat{P}_A(\alpha)\, \hat{H}_A\, P_A(\alpha)|g\rangle = E_{P_A}$$

$$E_{U_B} = tr\left[\hat{\rho}_2(\hat{H}_B+\hat{V})\right] = \sum_{\alpha} \langle g|\hat{P}_A(\alpha)\, \hat{U}_B^\dagger(\alpha)\left[\hat{H}_B+\hat{V}\right] U_B(\alpha)|g\rangle = \frac{-1}{h^2+k^2}\left[hk\sin(2\alpha) + (h^2+2k^2)(1-\cos 2\alpha)\right]$$

for $0<\theta\ll 1 \Rightarrow E_{U_B} \simeq \dfrac{-2kh\theta}{h^2+k^2} < 0$

$$\sum_{\alpha=\pm1} \langle g | P_A(\alpha) V P_A(\alpha) | g \rangle = \sum_{\alpha=\pm1}$$

$\sin\theta\, \hat{\sigma}_y^B$ $\qquad \theta$ is given $\qquad \cos 2\theta = \dfrac{h^2+2k^2}{\sqrt{(h^2+2k^2)^2+h^2k^2}}$, $\sin 2\theta = \dfrac{hk}{\sqrt{\;\;}}$

$$\mathrm{tr}\left(\hat{\rho}_2 \hat{H}\right) = \sum_{\alpha=\pm1} \langle g | P_A(\alpha)\, U_B^\dagger(\alpha)\, \underbrace{H}_{\hat{H}_A+\hat{H}_B+\hat{V}}\, \hat{U}_B(\alpha)\, P_A(\alpha) | g \rangle$$

$$\sum_{\alpha=\pm1} \langle g | P_A(\alpha)\, U_B^\dagger(\alpha)\, \hat{H}_A\, U_B(\alpha)\, \hat{P}_A(\alpha) | g \rangle = \sum_{\alpha=\pm1} \langle g | \hat{P}_A(\alpha)\, \hat{H}_A\, P_A(\alpha) | g \rangle = E_{P_A}$$

$$E_{U_B} = \mathrm{tr}\left[\hat{\rho}_2 \left(\hat{H}_B + \hat{V}\right)\right] = \sum_{\alpha} \langle g | \hat{P}_A(\alpha)\, U_B^\dagger(\alpha) \left[\hat{H}_B + \hat{V}\right] U_B(\alpha) | g \rangle = \frac{-1}{h^2+k^2}\left[hk \sin(2\alpha) + (h^2+2k^2)(1-\cos 2\alpha)\right]$$

$$\text{for } 0 < \theta \ll 1 \Rightarrow E_{U_B} \simeq \frac{-2k\, h\theta}{h^2+k^2} < 0 \qquad E_{U_B} \leq E_{P_A}$$

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Natural Energy flow from A to B $\langle \hat{H}_B \rangle(t)$ $\langle \hat{V} \rangle(t)$

$$\langle H_B \rangle(t) = \sum_{a=\pm 1} \langle g| \hat{P}_A(a)\, e^{i\hat{H}t} \hat{H}_B\, e^{-i\hat{H}t} P_A(a)|g\rangle = \frac{1}{2} f(h,k)\left[1 - \cos(4ut)\right]$$

$$\langle \hat{V} \rangle(t) = 0$$

Imagine Bob performs an optimal unitary $\hat{W}_B$ on $|\Psi_{PM}\rangle$

$$\hat{W}_B|\Psi_{PM}\rangle = \frac{1}{\sqrt{p_A(\alpha)}}\,\hat{W}_B\,\hat{P}_A(\alpha)\,|g\rangle$$

$$\hat{\rho}_2' = \hat{W}_B\,\hat{\rho}_1\,\hat{W}_B^\dagger = \hat{W}_B\left(\sum_{\alpha=\pm1}\hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)\right)\hat{W}_B^\dagger$$

$$E_{W_B} = \mathrm{Tr}(\hat{\rho}_2'\hat{H}) - \mathrm{Tr}(\hat{\rho}_1\hat{H}) = \mathrm{tr}(\hat{\rho}_2'\hat{H}) - E_{PA} = \sum_\alpha \langle g|\hat{P}_A(\alpha)\,\hat{W}_B^\dagger\,[\hat{H}_B+\hat{V}]\,\hat{W}_B|g\rangle$$

$$[\hat{H}_A, W_B] = 0 \Rightarrow \langle g|\hat{W}_B^\dagger \hat{H}_A \hat{W}_B|g\rangle =$$

$$= \langle g|\hat{H}_A|g\rangle = 0$$

$$\langle P_A(\alpha)| = \mathbb{1}$$

$$g\rangle = \langle g|W_B^\dagger [\overset{\overset{H_A}{\downarrow}}{\hat{H}_B} + \hat{V}] W_B|g\rangle = \langle g|W_B^\dagger \hat{H} \hat{W}_B|g\rangle \geq 0$$