Title: Quantum Information Lecture

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Collection: Quantum Information 2023/24

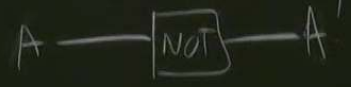
Date: March 20, 2024 - 9:00 AM

URL: https://pirsa.org/24030059

Classical Logic

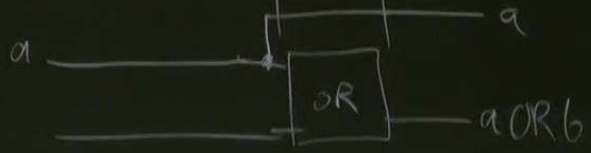
NOT A ≡ A'

Input A	NOT A
0	1
1	0



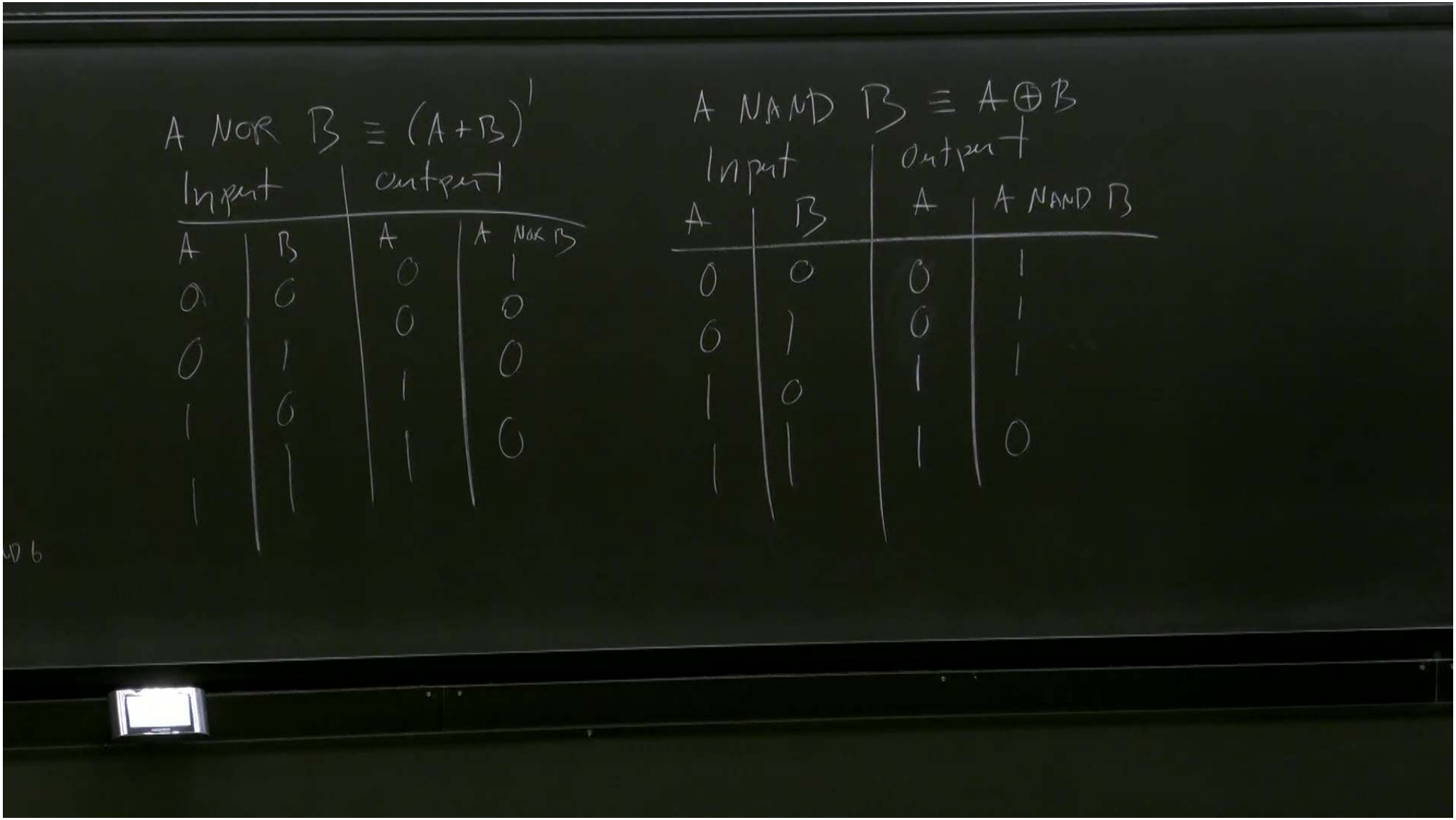
A OR B ≡ A + B

Input		Output	
A	B	A	A OR B
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	1



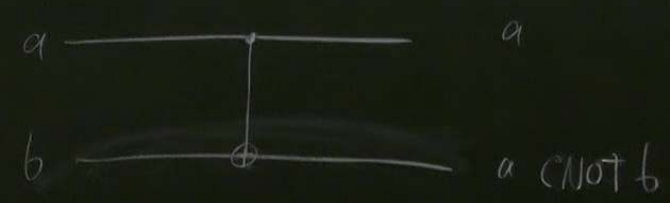
A AND B ≡ A · B

Input		Output	
A	B	A	A AND B
0	0	0	0
0	1	0	0
1	0	1	0
1	1	1	1


A NOR B ≡ (A+B)'
Input
Output
A
B
A NOR B
A NAND B ≡ A⊕B
Input
Output
A
B
A NAND B

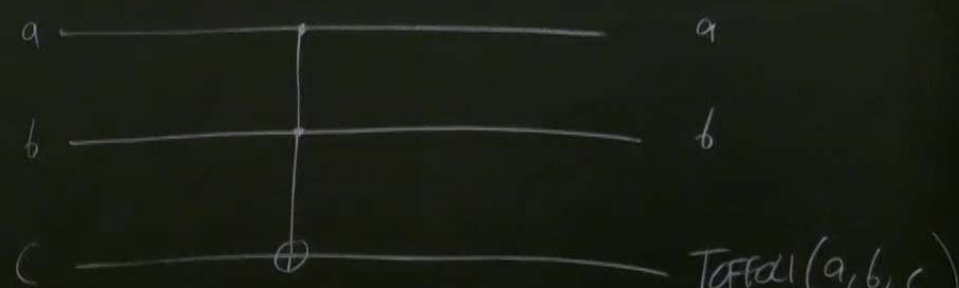
A XOR B $\equiv$ A $\oplus$ B /CNOT

Input		Output	
A	B	A	A XOR B
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0



Toffoli $(A, B, C) = C \oplus A \cdot B$ = CCNOT

Input			Output		
A	B	C	A	B	$C \oplus A \cdot B$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	1	1	1



Universal set of gates

A set of gates is universal if it can be used to build any function $f: Z_2^n \to Z_2^n$

$$f(x_1, \ldots x_n) = (x_1', \ldots, x_n')$$

Examples of Universal sets: {AND, OR, NOT}, {AND, NOT}, {XO

Examples of equivalence XOR from {AND, OR, NOT} $\Rightarrow$ a XOR b = (a AND (NOT b))

OR from {AND, NOT} $\Rightarrow$ a OR b = NOT((NOT a) AND (NOT b))

Universal set of gates

A set of gates is universal if it can be used to build any function $f: Z_2^n \to$

$$f(x_1, \ldots x_n) = (x_1', \ldots, x_m')$$

Examples of Universal sets: {AND, OR, NOT}, {AND, NOT}, {XOR

Examples of equivalence XOR from {AND, OR, NOT} $\Rightarrow$ a XOR b = (a AND (NOT b))

OR from {AND, NOT} $\Rightarrow$ a OR b = NOT((NOT a) AND (NOT b))

OR from {AND, XOR} $\Rightarrow$ a OR b = (a AND b) XOR (a XOR b)

be used to build any function $f: \mathbb{Z}_i^n \to \mathbb{Z}_i^m$ $n, m \in \mathbb{Z}^+$ $m \leq n$

D, OR, NOT }, { AND, NOT }, { XOR, AND }, { NAND }, { NOR }, { TOFFOLI }

R, NOT } ⇒ a XOR b = (a AND (NOT b)) OR ((Not a) AND b)

t, a) AND (NOT b))

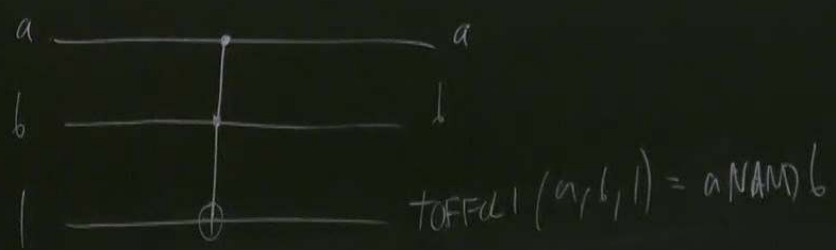
b) XOR (a XOR b)

OR from {AND, XOR} => a OR b = (a AND b) XOR (a XOR b)

NOT from {NAND} => NOT a = a NAND a

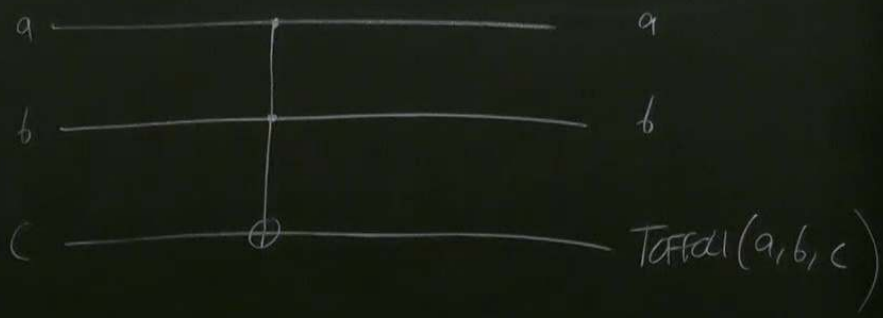
AND from {NAND} => a AND b = (a NAND b) NAND (a NAND b)

NAND from {TOFFOLI} => a NAND b = NOT (a AND b) = (1) CNOT (a AND b) = TOFFOLI (a, b, 1)



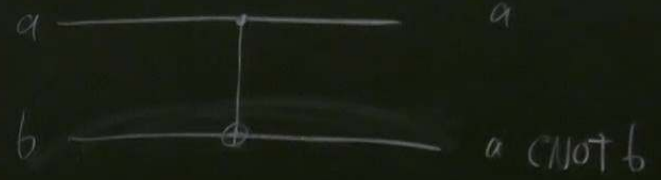
Toffoli $(A, B, C) = C \oplus A \cdot B$ = CCNOT

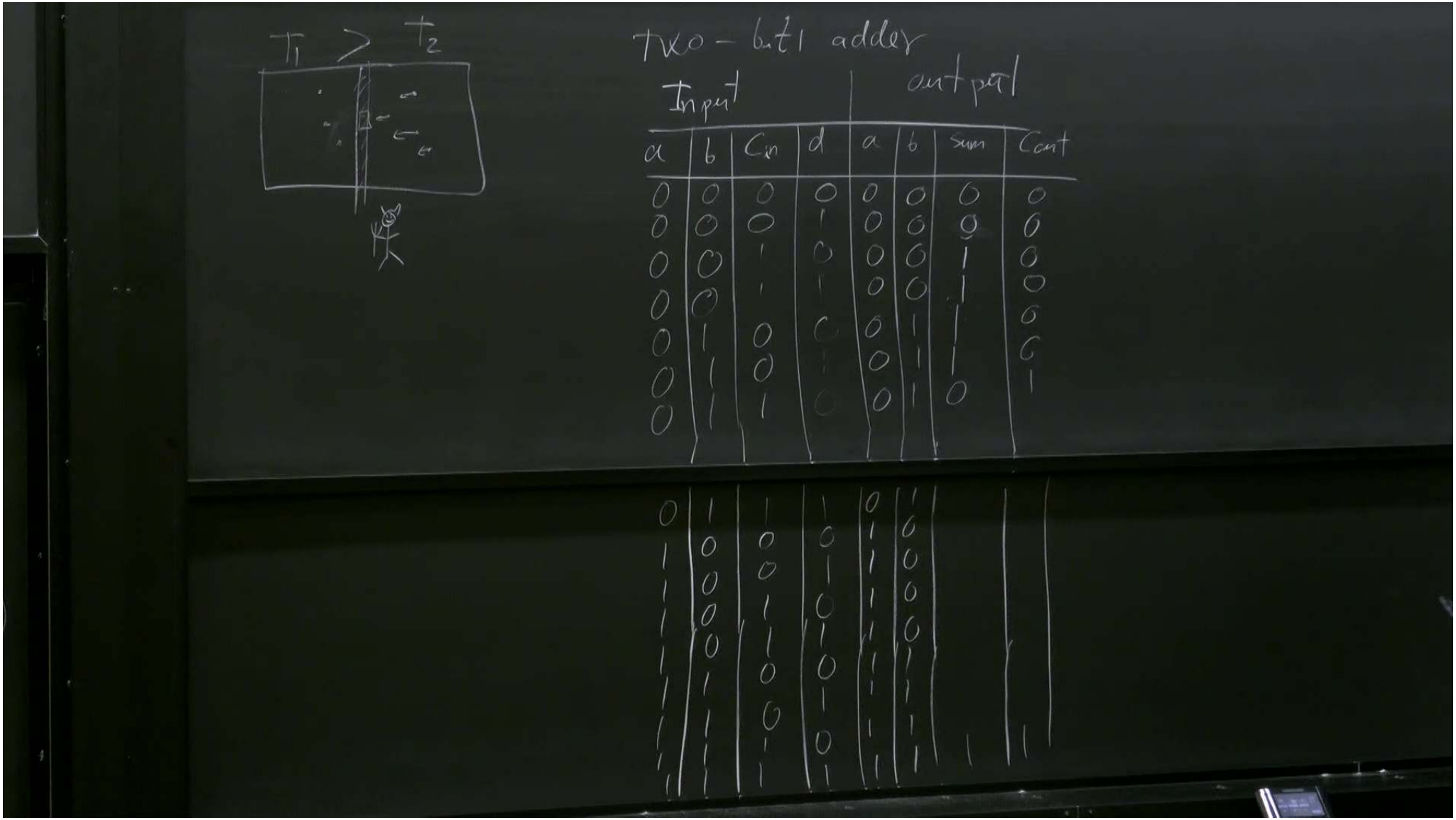
Input			Output		
A	B	C	A	B	$C \oplus A \cdot B$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

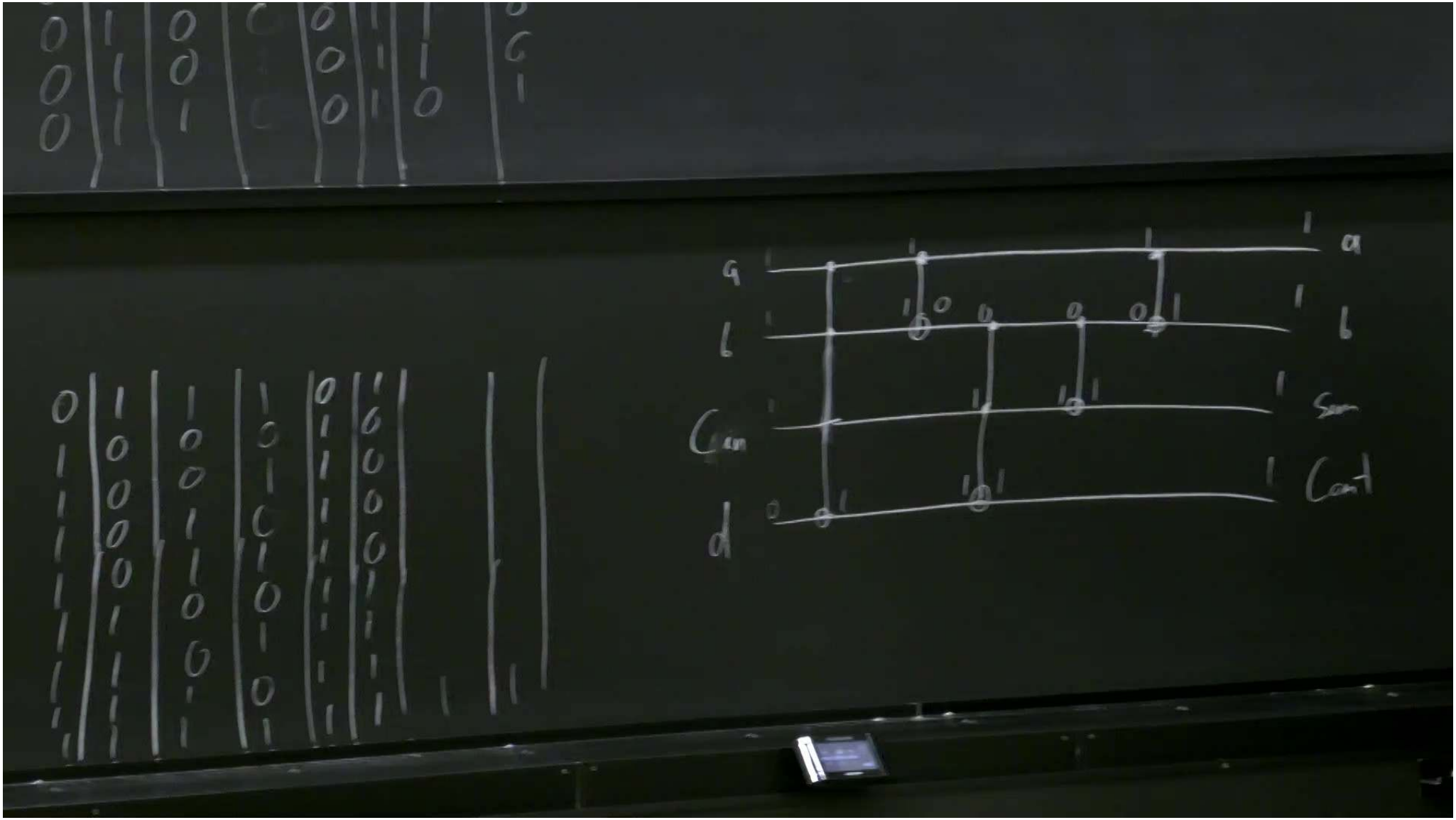


A XOR B $\equiv$ A $\oplus$ B /CNOT

Input		Output	
A	B	A	A XOR B
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0




$T_1 > T_2$
Two-bit adder
Input
output
a | b | Cin | d | a | b | sum | Cout


a
b
Cin
d
a
b
Sum
Cout