Title: Quantum Information Lecture

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Collection: Quantum Information 2023/24

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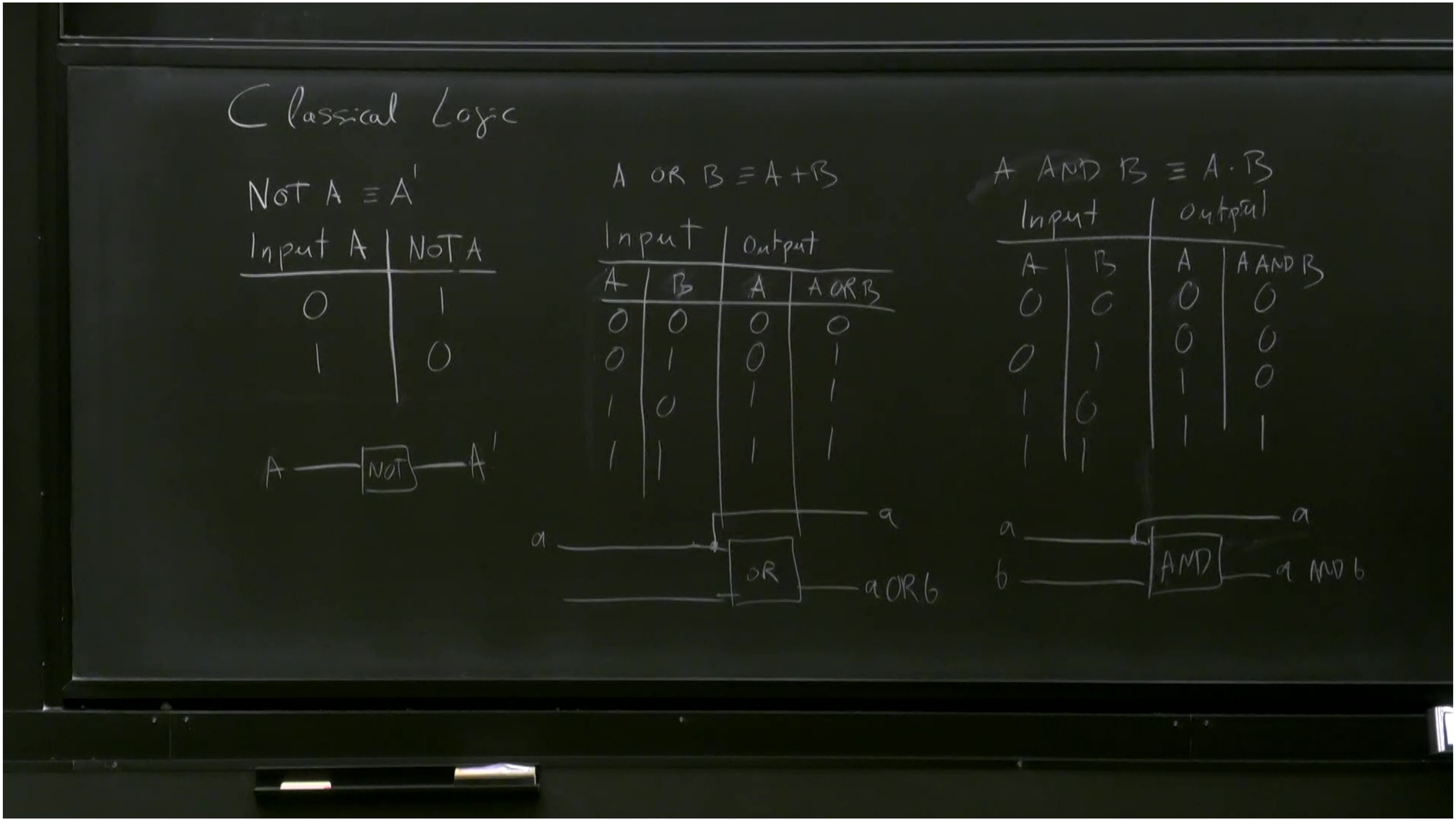
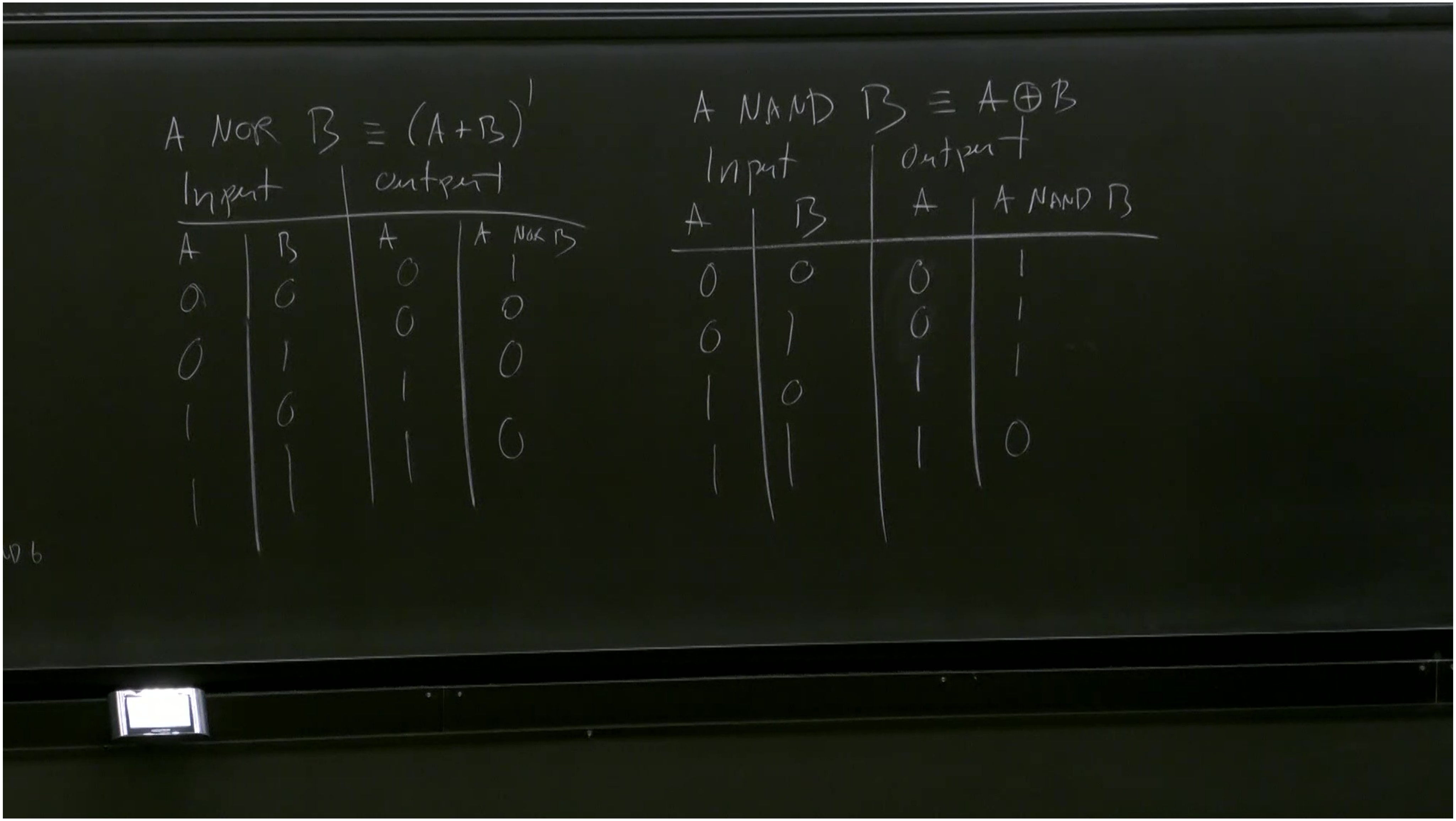
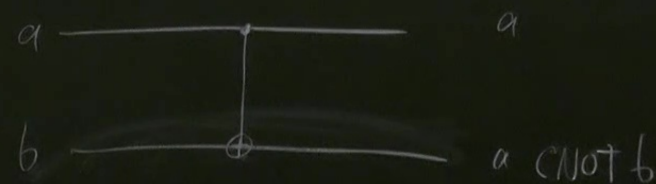
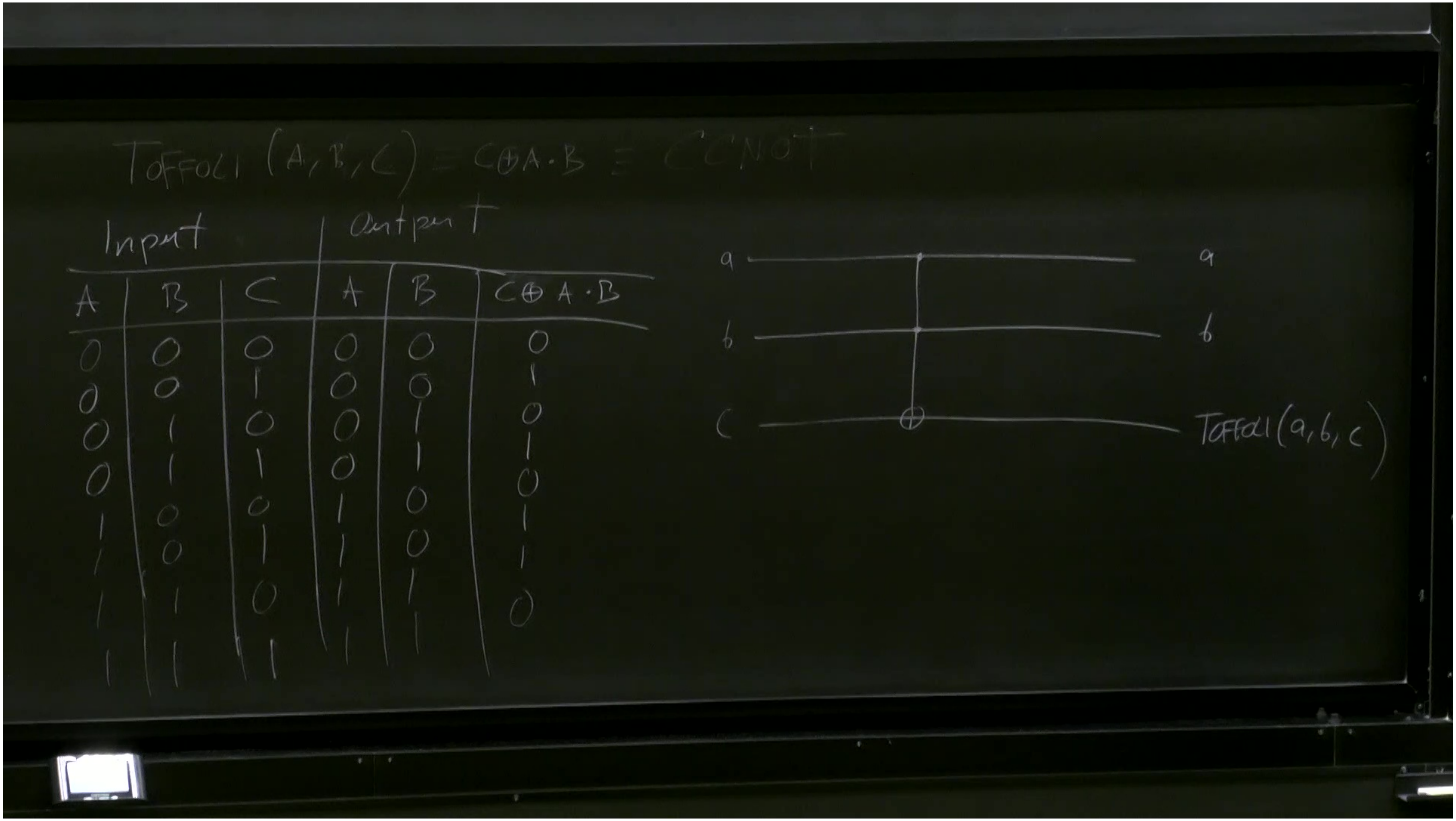

Classical Logic
NOT A ≡ A'
Input A | NOT A
0 | 1
1 | 0
A — NOT — A'
A OR B ≡ A + B
Input | Output
A | B | A | A OR B
0 | 0 | 0 | 0
0 | 1 | 0 | 1
1 | 0 | 1 | 1
1 | 1 | 1 | 1
a
OR
a OR b
A AND B ≡ A · B
Input | Output
A | B | A | A AND B
0 | 0 | 0 | 0
0 | 1 | 0 | 0
1 | 0 | 1 | 0
1 | 1 | 1 | 1
a
b
AND
a AND b


A NOR B ≡ (A+B)'
Input | Output
A | B | A | A NOR B
0 | 0 | 0 | 1
0 | 1 | 0 | 0
1 | 0 | 1 | 0
1 | 1 | 1 | 0
A NAND B ≡ A ⊕ B
Input | Output
A | B | A | A NAND B
0 | 0 | 0 | 1
0 | 1 | 0 | 1
1 | 0 | 1 | 1
1 | 1 | 1 | 0

$A \text{ XOR } B \equiv A \oplus B$ / CNOT

Input		Output	
A	B	A	A XOR B
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0




TOFFOLI (A, B, C) = C ⊕ A · B ≡ CCNOT
Input
Output
A B C
A B C ⊕ A · B
0 0 0 0 0 0
0 0 1 0 0 1
0 1 0 0 1 0
0 1 1 0 1 1
1 0 0 1 0 0
1 0 1 1 0 1
1 1 0 1 1 1
1 1 1 1 1 0
a
b
c
a
b
Toffoli(a, b, c)

Universal set of gates

A set of gates is universal if it can be used to build any function $f: \mathbb{Z}_2^n$

$$f(x_1, \dots x_n) = (x_1', \dots, x_n')$$

Examples of Universal sets: $\{$AND, OR, NOT$\}$, $\{$AND, NOT$\}$, $\{$XO

Examples of equivalence XOR from $\{$AND, OR, NOT$\}$ $\Rightarrow$ a XOR b = (a AND (NOT b))

OR from $\{$AND, NOT$\}$ $\Rightarrow$ a OR b = NOT ((NOT a) AND (NOT b))

Universal set of gates

A set of gates is universal if it can be used to build any function $f: \mathbb{Z}_2^n \to$

$$f(x_1, \dots x_n) = (x_1', \dots, x_m')$$

Examples of Universal sets: {AND, OR, NOT}, {AND, NOT}, {XOR

Examples of equivalence XOR from {AND, OR, NOT} $\Rightarrow$ a XOR b = (a AND (NOT b))

OR from {AND, NOT} $\Rightarrow$ a OR b = NOT ((NOT a) AND (NOT b))

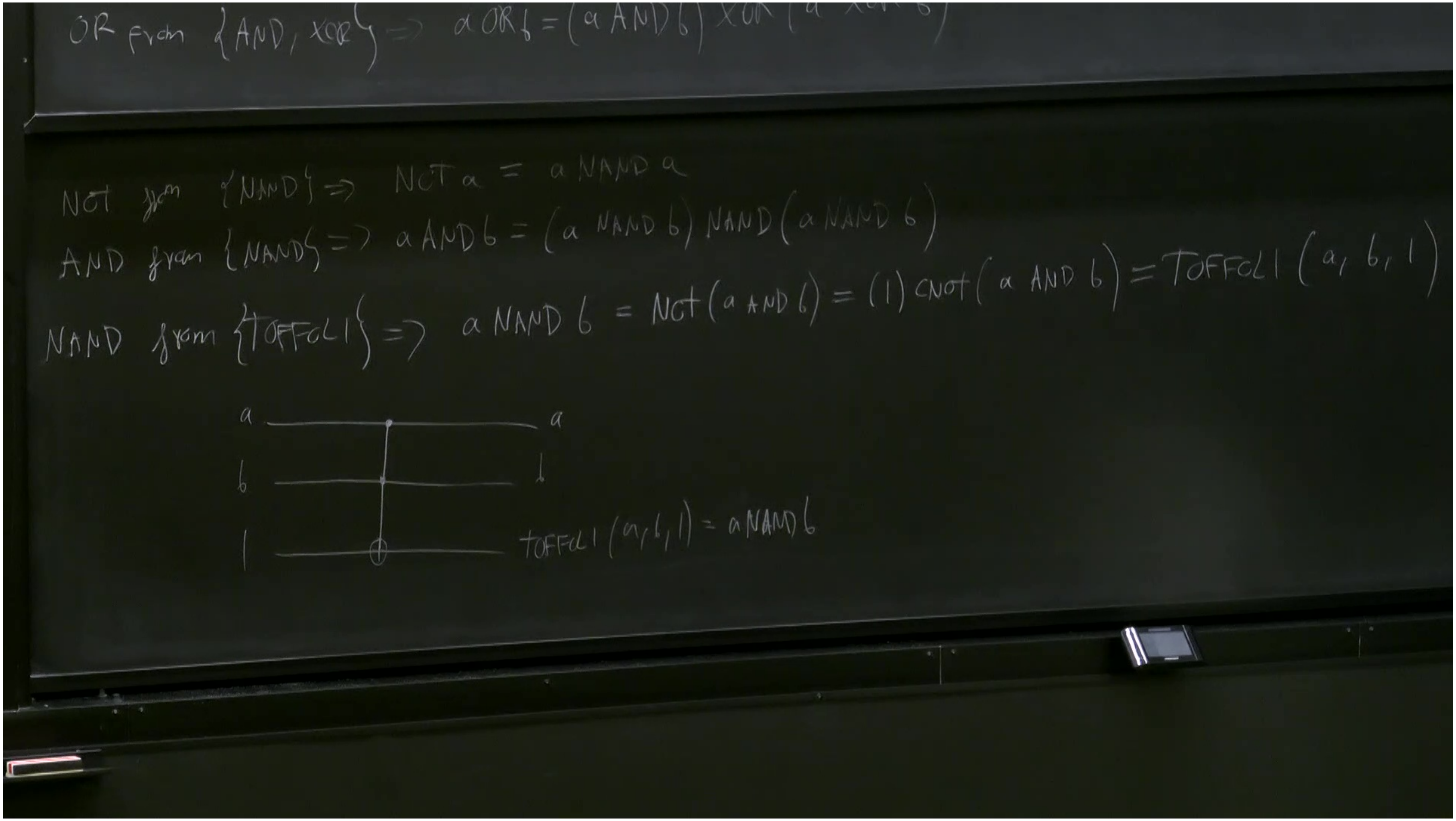
OR from {AND, XOR} $\Rightarrow$ a OR b = (a AND b) XOR (a XOR b) be used to build any function $f: \mathbb{Z}_2^n \to \mathbb{Z}_2^m$ $\quad n, m \in \mathbb{Z}^+ \quad m \le n$

D, OR, NOT}, {AND, NOT}, {XOR, AND}, {NAND}, {NOR}, {TOFFOLI}

R, NOT} ⇒ a XOR b = (a AND (NOT b)) OR ((NOT a) AND b)

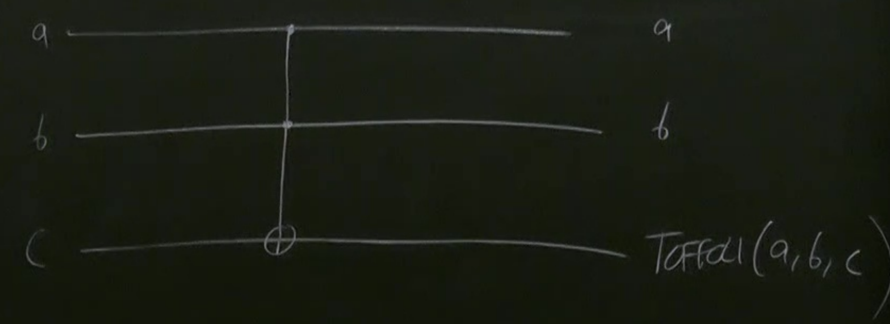
t a) AND (NOT b))

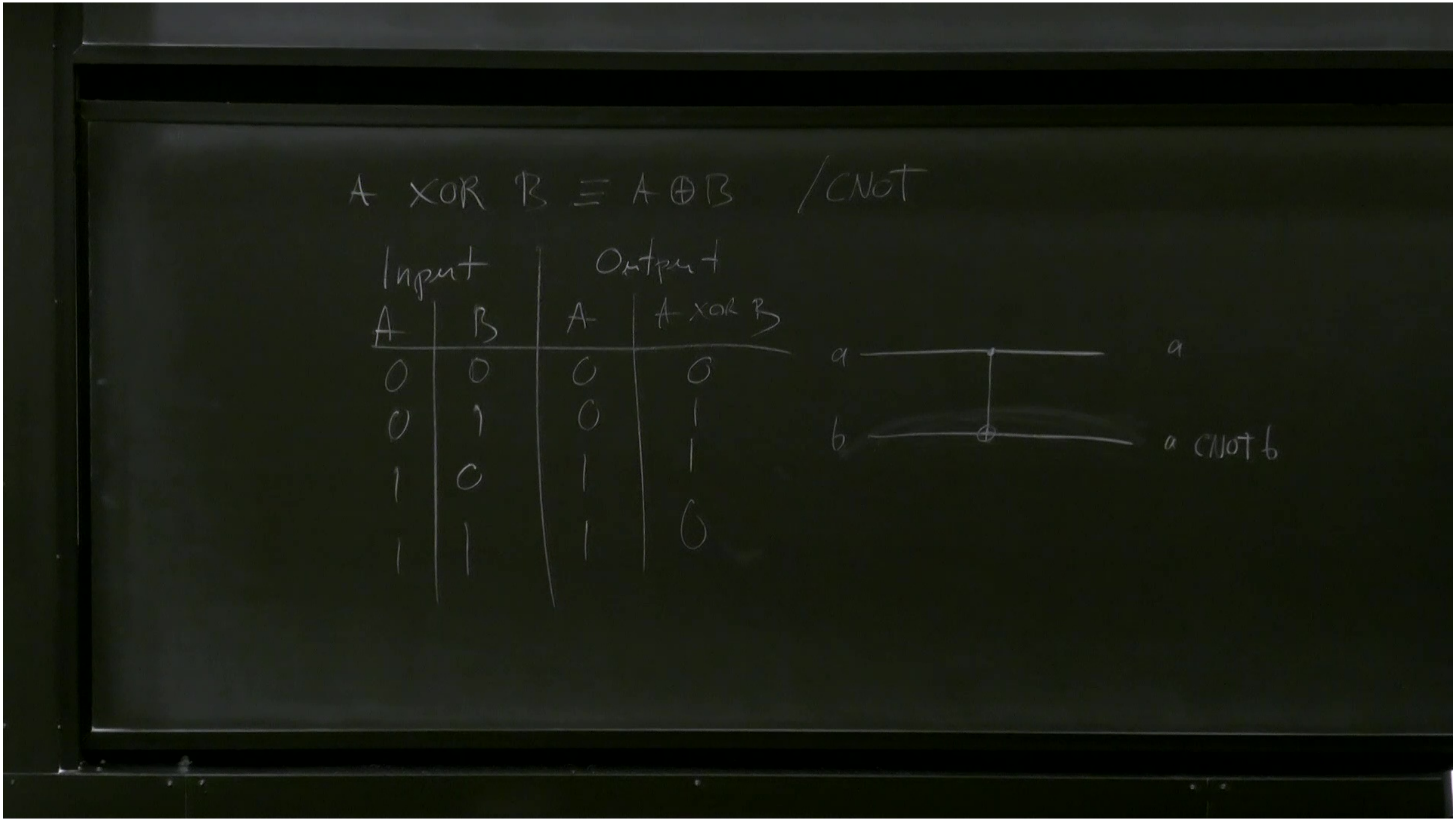
b) XOR (a XOR b)

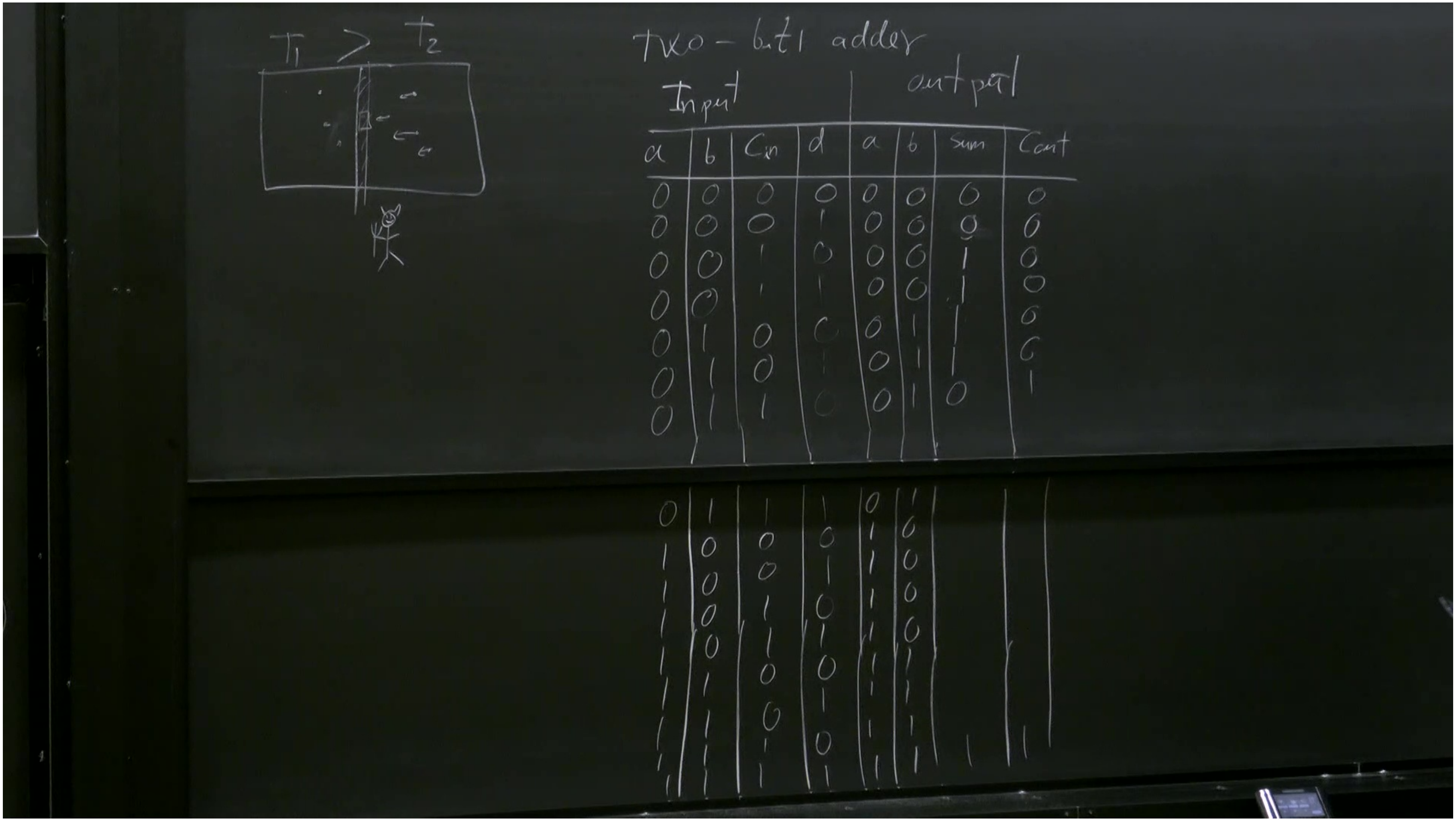

OR from {AND, XOR} => a OR b = (a AND b) XOR (a XOR b)
NOT from {NAND} => NOT a = a NAND a
AND from {NAND} => a AND b = (a NAND b) NAND (a NAND b)
NAND from {TOFFOLI} => a NAND b = NOT (a AND b) = (1) CNOT (a AND b) = TOFFOLI (a, b, 1)
a
b
1
a
b
TOFFOLI (a, b, 1) = a NAND b

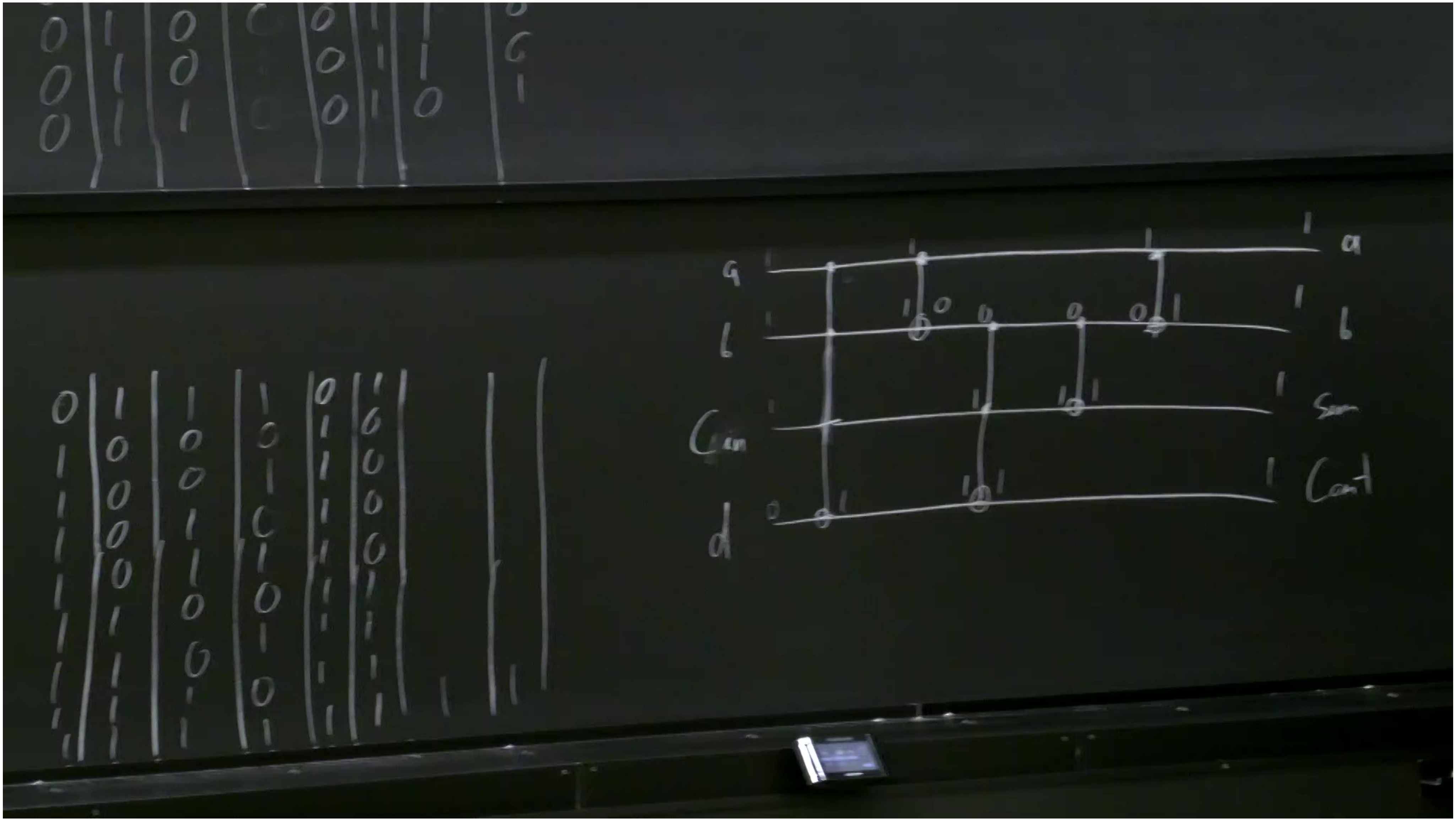
TOFFOLI $(A, B, C) \equiv C \oplus A \cdot B \equiv$ CCNOT

Input			Output		
A	B	C	A	B	$C \oplus A \cdot B$
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0




A XOR B ≡ A ⊕ B /CNOT
Input
Output
A B A A XOR B
0 0 0 0
0 1 0 1
1 0 1 1
1 1 1 0
a
b
a
a CNOT b


T1 > T2
Two-bit adder
Input
output
a b Cin d a b sum Cout


a
b
d
a
b
Cout