Title: Quantum Information Lecture

Speakers: Eduardo Martin-Martinez

Collection: Quantum Information 2023/24

Date: March 15, 2024 - 9:00 AM

URL: https://pirsa.org/24030057

Def. (Purification of a state) A state $\hat{\rho}_A$ is said to be pu

Stinespring dilation theorem: Given a CPTP map mapping

there exists a Hilbert space $\mathcal{H}_{AB}$ and a state $|\Theta_{AB}\rangle$ for every

If $\dim(\mathcal{H})$ is finite then $\mathcal{H}_{AB} \geq (\dim \mathcal{H})^2$

Def. (Purification of a state) A state $\hat{\rho}_A$ is said to be

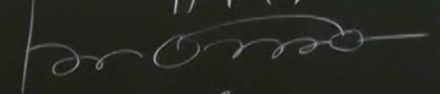
<u>Stinespring dilation theorem</u>: Given a CPTP map mapping

there exists a Hilbert space $\mathcal{H}_{AB}$ and a state $|0_{AB}\rangle$ for every

If $\dim(\mathcal{H})$ is finite then $\mathcal{H}_{AB} \geq (\dim \mathcal{H})^2$

...d to be purified by $|\psi_{AB}\rangle$ if $\hat{\rho}_A = tr_B(|\psi_{AB}\rangle\langle\psi_{AB}|)$

$\hat{H}|\psi\rangle = 0$



...mapping density operators to density operators $T: \hat{\rho} \in \mathcal{O}(\mathcal{H}) \to \hat{\rho}' \in \mathcal{O}(\mathcal{H})$

...or every $\hat{\rho}_A \in \mathcal{O}(\mathcal{H})$, for every T, $T(\hat{\rho}_A) = Tr_B\left[\hat{U}_{AB}|0_{AB}\rangle\langle 0_{AB}|\hat{U}^\dagger_{AB}\right]$.

$$|0_{AB}\rangle \overset{?}{\neq} |0_A\rangle \otimes |0_B\rangle$$

$$\sum_i \frac{\hat{p}_i^2}{2m} + m\omega^2 \hat{x}_i^2 + \lambda(\hat{x}_i - \hat{x}_j)^2$$

Def: Quantum measurement: A measurement of a quantum syst

accompanied by an update rule for the post-measurement st

Postulate 4: Measurements of a quantum system are represented

one per each possible outcome of the measurement that sati

If the system is in a state $|\psi\rangle$, the probability of getting out

is updated as $$|\psi'\rangle = \frac{\hat{M}_n|\psi\rangle}{\sqrt{\langle\psi|\hat{M}_n^\dagger \hat{M}_n|\psi\rangle}}$$

...nt of a quantum system as a map from a quantum state $\hat{\rho}$ to a set of real numbers

post-measurement state.

...ystem are represented by a set of operators $\{\hat{M}_n\}$ acting on the Hilbert space of the system

...asurement that satisfy: $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$

...obability of getting outcome n is $P(n) = \langle \psi | \hat{M}_n^\dagger \hat{M}_n | \psi \rangle$ and the state after the measurement

We call the operators $\hat{E}_n := \hat{M}_n^\dagger \hat{M}_n \Rightarrow \sum_n \hat{E}_n = \mathbb{1}$ POVM elements

(Positive Operator-Valued measures)

nted by a set of operators $\{\hat{M}_n\}$ acting on the Hilbert space of the system

satisfy: $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$

g outcome n is $P(n) = \langle \psi | \hat{M}_n^\dagger \hat{M}_n | \psi \rangle$ and the state after the measurement

operators $\hat{E}_n := \hat{M}_n^\dagger \hat{M}_n \Rightarrow \sum_n \hat{E}_n = \mathbb{1}$ POVM elements

erator-valued measures)

Scenario 2: POVM:

$$\hat{E}_1 = \frac{\sqrt{2}}{1+\sqrt{2}} |1\rangle\langle 1|$$

$$\hat{E}_2 = \frac{\sqrt{2}}{1+\sqrt{2}} \frac{(|0\rangle - |1\rangle)(\langle 0| - \langle 1|)}{2}$$

$$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$$

$$P_1(|\psi_1\rangle) = \langle \psi_1 | \hat{E}_1 | \psi_1 \rangle = 0$$

$$P_1(|\psi_2\rangle) = \langle \psi_2 | \hat{E}_1 | \psi_2 \rangle = \frac{1}{\sqrt{2}+1}$$

$$P_2(|\psi_1\rangle) = \langle \psi_1 | \hat{E}_2 | \psi_1 \rangle = 1 - \frac{1}{\sqrt{2}}$$

$$P_2(|\psi_2\rangle) = \langle \psi_2 | \hat{E}_2 | \psi_2 \rangle = 0$$

$$P_3(|\psi_1\rangle) = \langle \psi_1 | \hat{E}_3 | \psi_1 \rangle = 1/\sqrt{2}$$

$$P_3(|\psi_2\rangle) = \langle \psi_2 | \hat{E}_3 | \psi_2 \rangle = 1 - \frac{1}{\sqrt{2}+1}$$

$$P_1(|\psi_1\rangle) = \langle \psi_1 | \hat{E}_1 | \psi_1 \rangle = 0$$

$$P_1(|\psi_2\rangle) = \langle \psi_2 | \hat{E}_1 | \psi_2 \rangle = \frac{1}{\sqrt{2}+1}$$

Outcome 1

$$P_2(|\psi_1\rangle) = \langle \psi_1 | \hat{E}_2 | \psi_1 \rangle = 1 - \frac{1}{\sqrt{2}}$$

$$P_2(|\psi_2\rangle) = \langle \psi_2 | \hat{E}_2 | \psi_2 \rangle = 0$$

Outcome 2

$$P_3(|\psi_1\rangle) = \langle \psi_1 | \hat{E}_3 | \psi_1 \rangle = 1/\sqrt{2}$$

$$P_3(|\psi_2\rangle) = \langle \psi_2 | \hat{E}_3 | \psi_2 \rangle = 1 - \frac{1}{\sqrt{2}+1}$$

Outcome 3

$\sqrt{\langle\psi|\hat{M}_n^\dagger \hat{M}_n|\psi\rangle}$ | (Positive Operator-Valu

(Coralary of) Naimark's dilation theorem: Consider the

a detector, the environment and a target system. Then the act

the target system

Operator-valued measures)

onsider the unitary evolution of a coupled system consisting of

en the action of a PVM on the detector always yields a POVM on