Title: Quantum Information Lecture

Speakers: Eduardo Martin-Martinez

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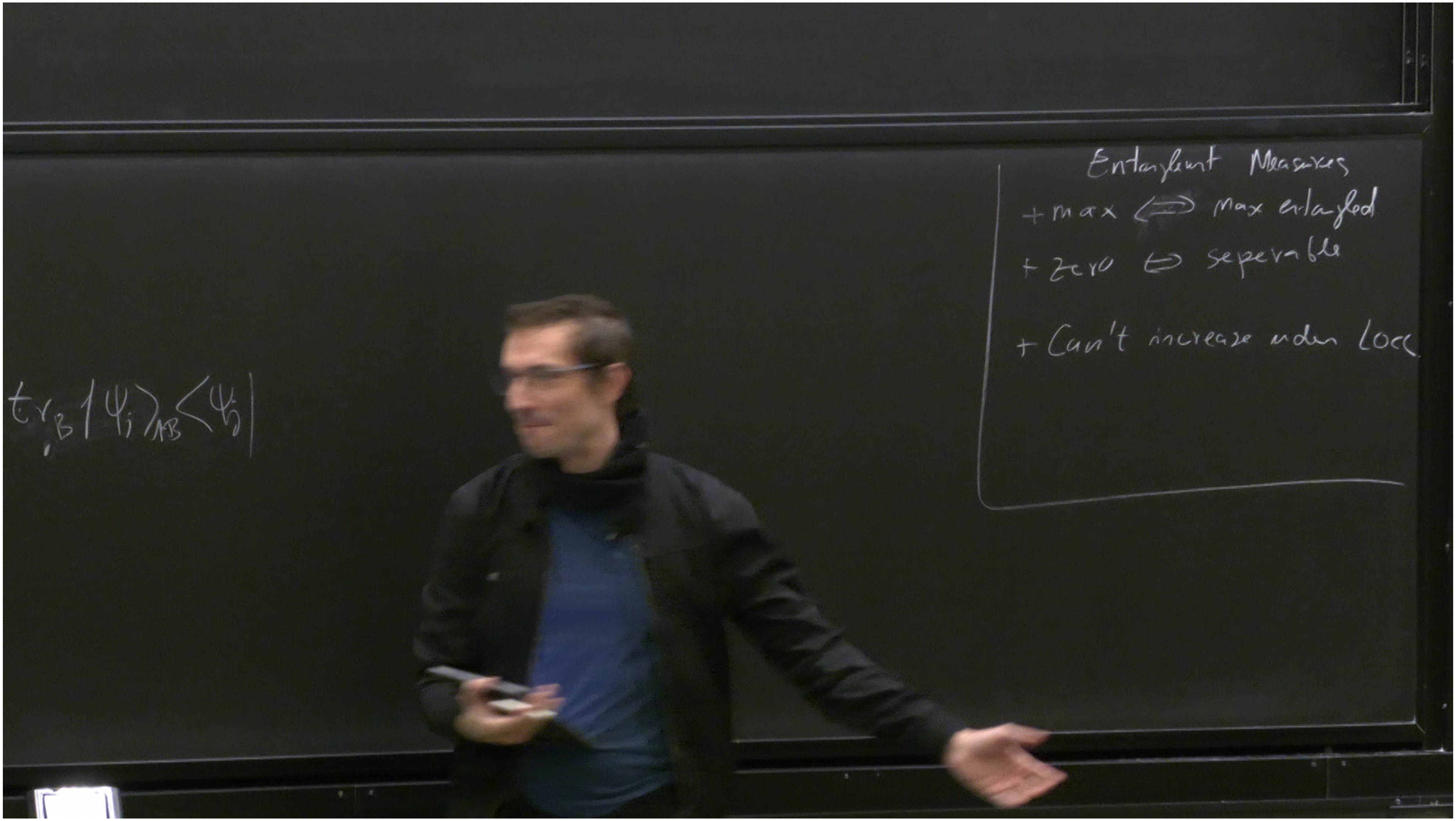
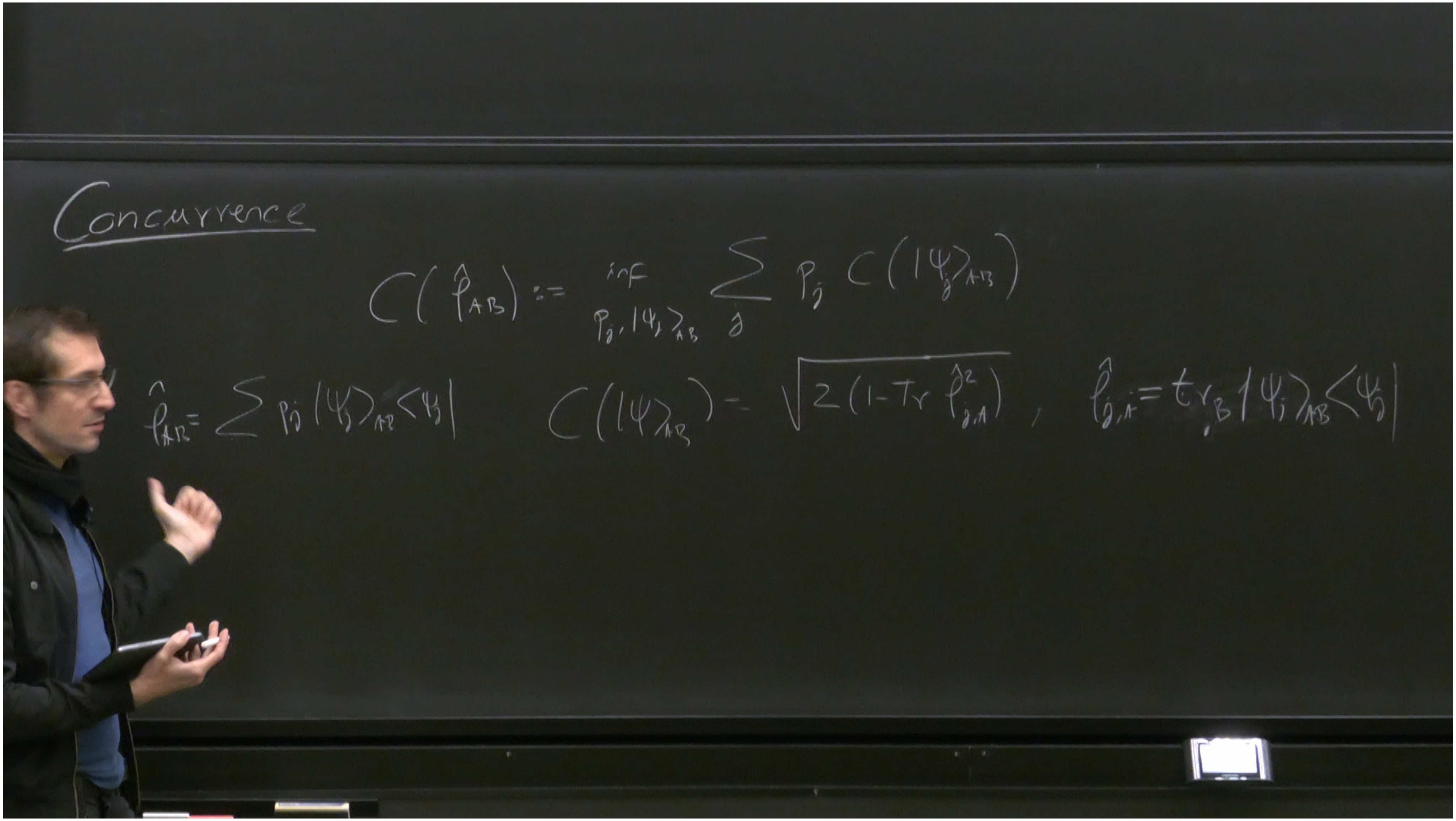
Concurrence

$$C(\hat{\rho}_{AB}) = \inf_{p_j, |\psi_j\rangle_{AB}} \sum_j p_j \, C(|\psi_j\rangle_{AB})$$

$$C(|\psi\rangle_{AB}) = \sqrt{2(1 - \mathrm{Tr}\, \hat{\rho}^2_{j,A})} \, , \quad \hat{\rho}_{j,A} = \mathrm{tr}_B |\psi_j\rangle_{AB}\langle\psi_j|$$


Entanglement Measures
+ max ⟺ max entangled
+ zero ⟺ seperable
+ Can't increase under LOCC
$tr_B |\psi_i\rangle_{AB}\langle\psi_j|$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$$\left(1 - \mathrm{tr}(\rho_j^2)\right)$$

$\mathrm{tr}_B |\psi_i\rangle_{AB}\langle\psi_j|$

Concurrence for 2×2 systems

$$C(\hat{\rho}_{AB}) = \max(0, (\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4))$$

where $\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \lambda_4$ are the eigenvalues of $\hat{R}$

$$\hat{R} := \sqrt{\sqrt{\hat{\rho}_{AB}}\,\hat{\tilde{\rho}}\,\sqrt{\hat{\rho}_{AB}}}$$

$$\hat{\tilde{\rho}} := (\hat{\sigma}_{y_A} \otimes \hat{\sigma}_{y_B})\,\hat{\rho}^*_{AB}\,(\hat{\sigma}_{y_A} \otimes \hat{\sigma}_{y_B})$$

ρ^* is the complex conjugate of the matrix representation of $\hat{\rho}_{AB}$ in the computational basis (eigenbasis of $\hat{\sigma}_z$)

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$\ldots r(\rho^i_1))$

$$EoF(\hat{\rho}_{AB}) = h\left(\frac{1+\sqrt{1+C(\hat{\rho}_{AB})}}{2}\right)$$

where $h(x) = -x\log_2(x) - (1-x)\log_2(1-x)$

Conc...

$C(\hat{\rho}\ldots$

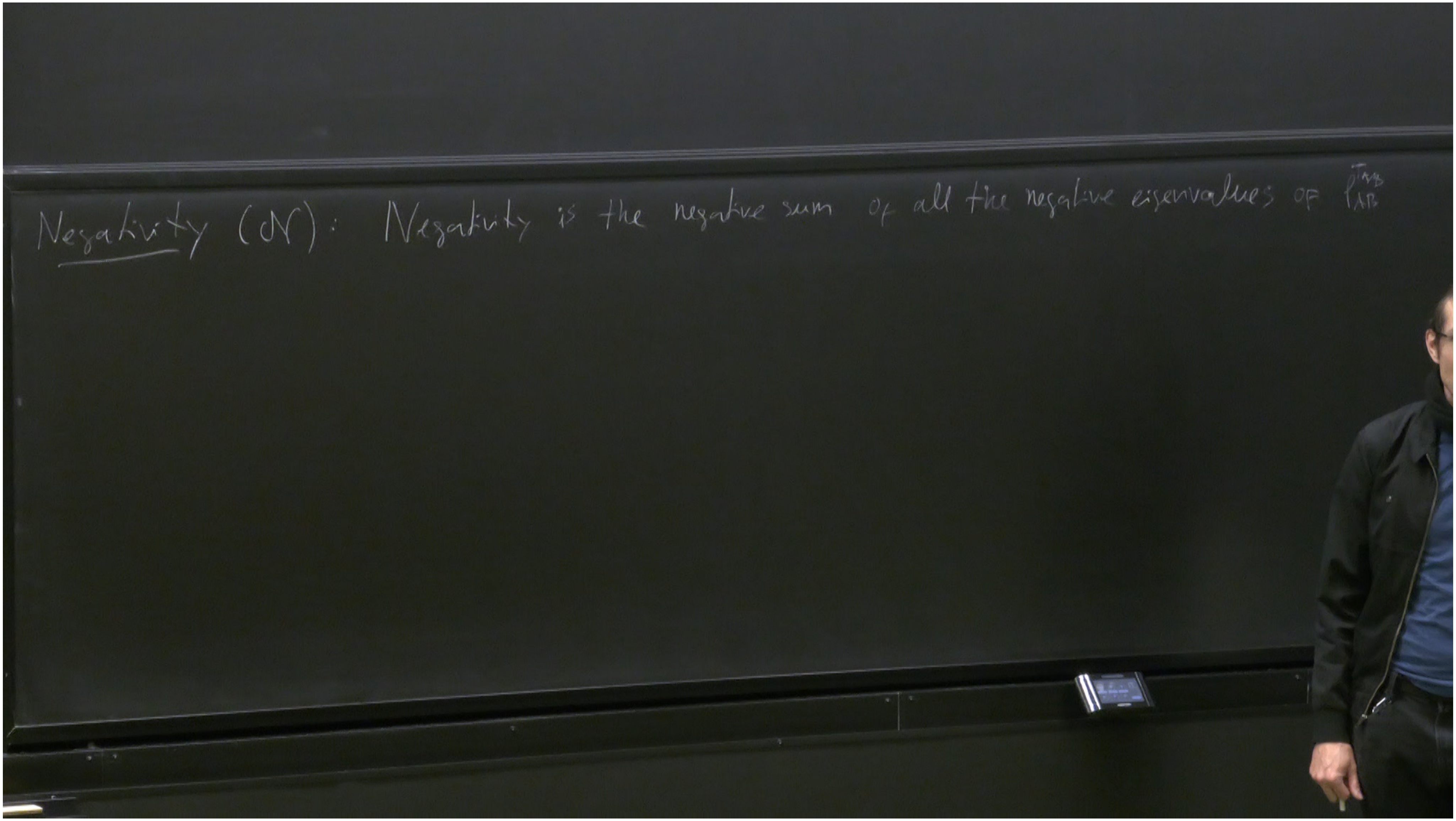
whe...

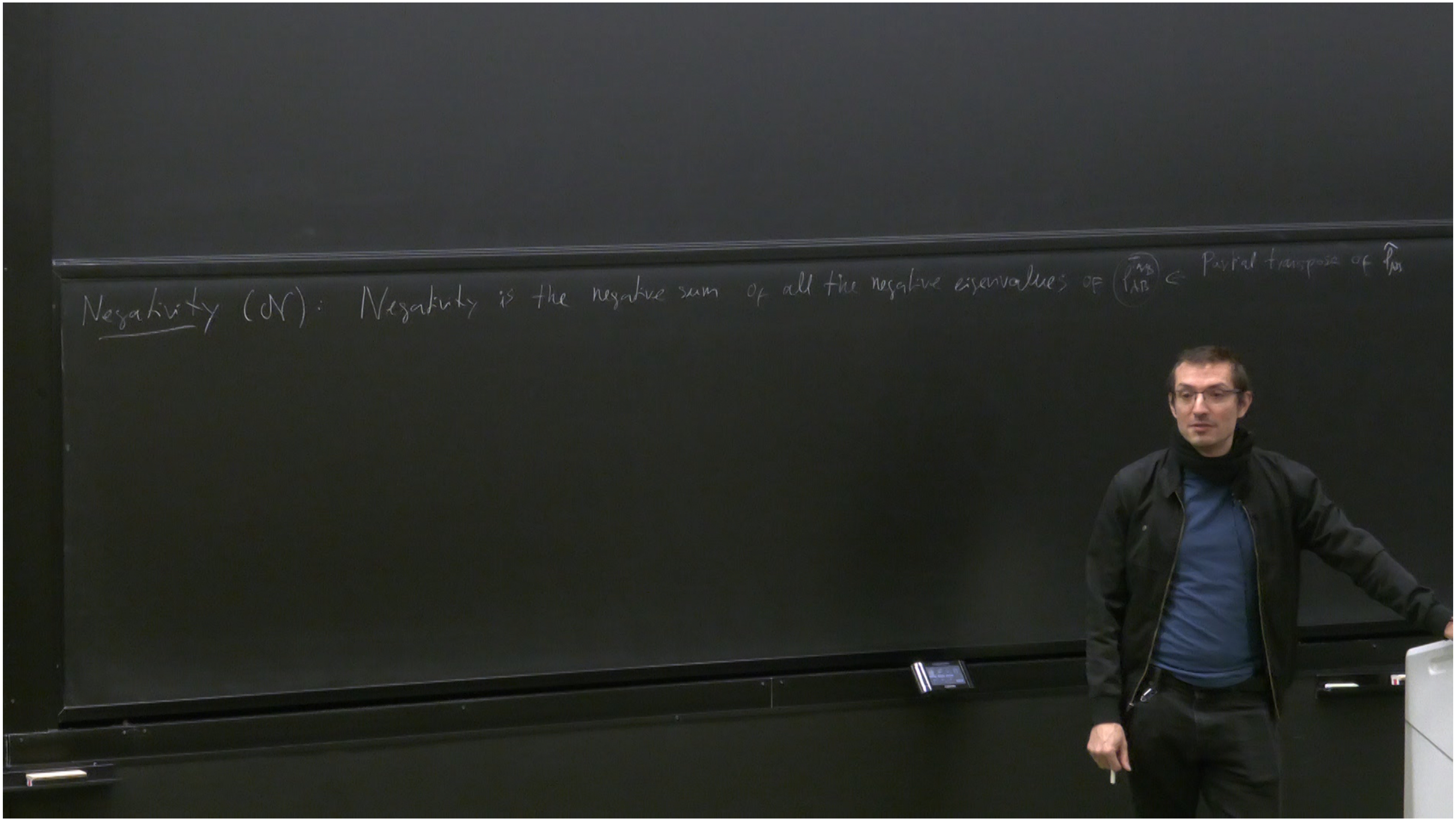
eigen...

$\hat{R} := \ldots$

$\hat{\tilde{\rho}} := (\ldots$

ρ^{*} i...


Negativity ($\mathcal{N}$): Negativity is the negative sum of all the negative eigenvalues of $\rho_{AB}^{T_B}$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for higher dimension it is a measure of distillable entanglement.

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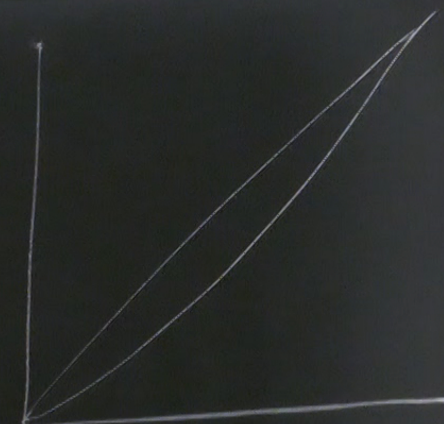
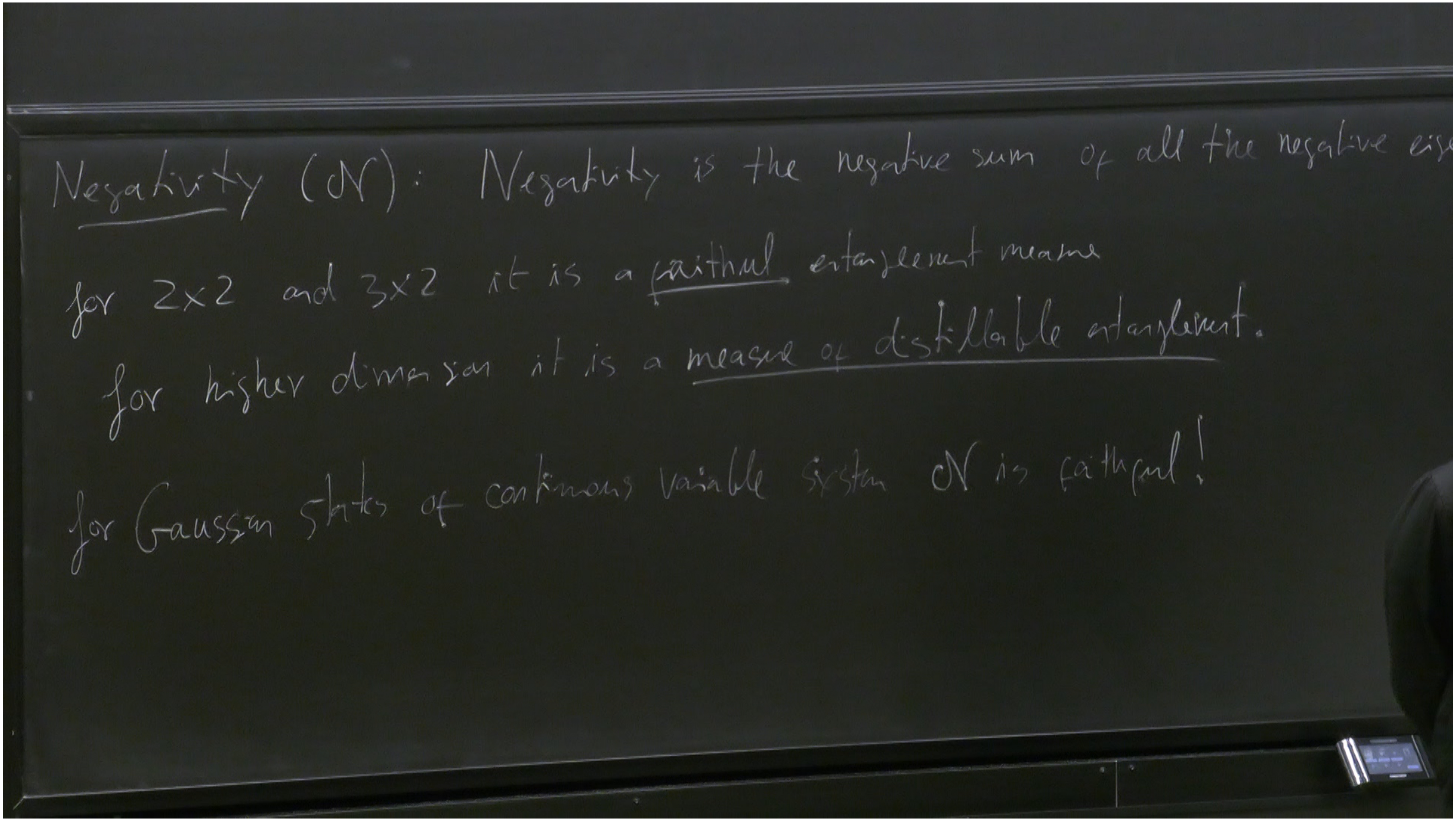
$$C \geqslant 2\mathcal{N} \geq \sqrt{(1-C)^2+C^2} - (1-C)$$

...nt.

for 2×2 systems $C = 2\mathcal{N}$

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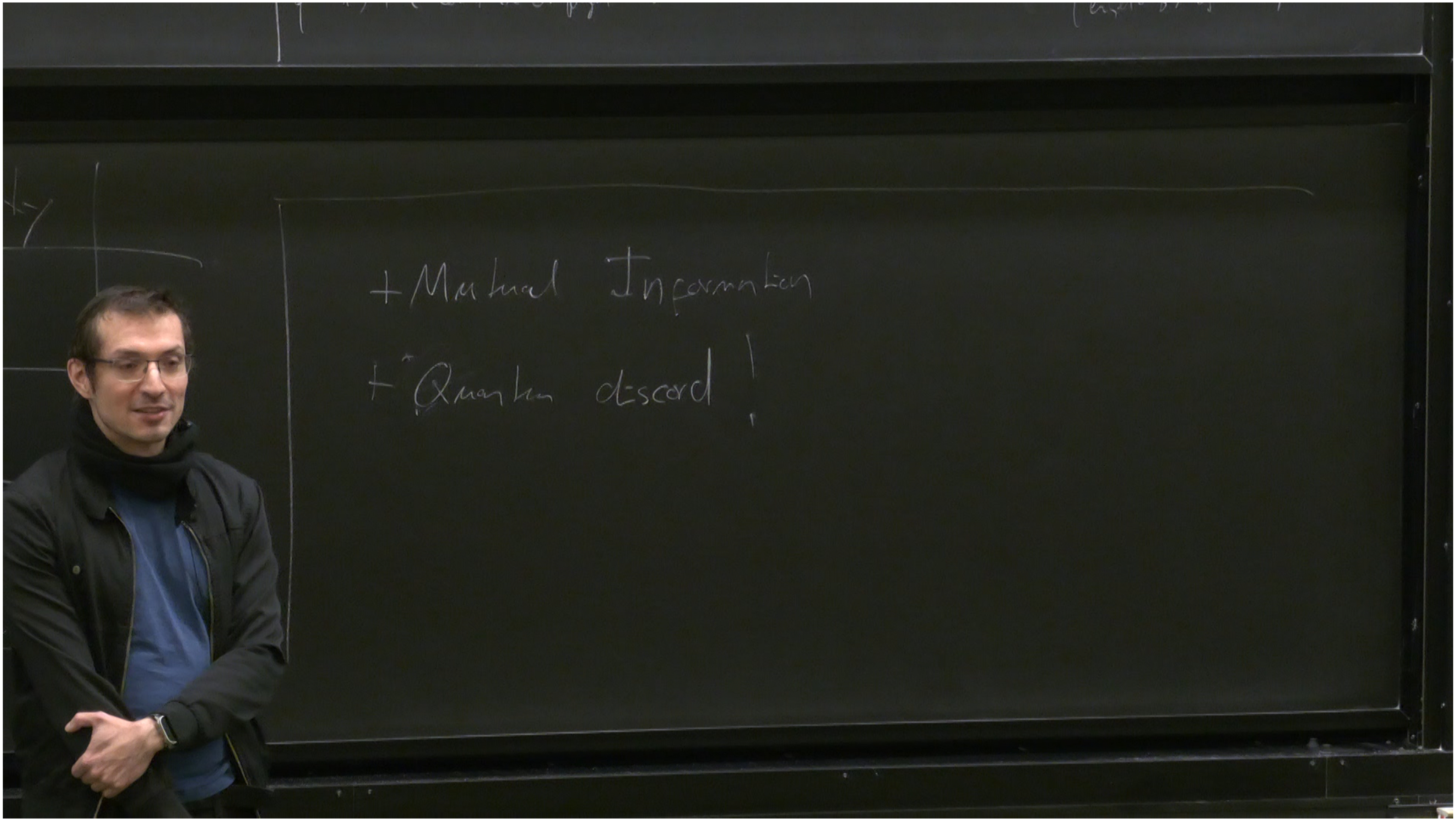
for higher dimension it is a measure of distillable entanglement.
for Gaussian states of continuous variable system $\mathcal{N}$ is faithful!

	Entanglement entropy	EoF	Concurrence	Negativity
Is it limited to small dim?	No	No	No!	No
Is it easy to compute?	Yes	No!	Not in general Yes for 2x2	Yes
Does it work for non-pure states	No!	Yes	Yes.	Yes
Is it an entanglement measure? is it faithful?	No! (Yes for pure states)	Yes	Yes	Yes in 2x2, 2x3, Gaussian Otherwise, means distillable entanglement
Does it have a nice physical interpretation	Yes!	Yes	Yes(?)	Maybe.


+ Mutual Information
+ Quantum discord !