Title: Quantum Information Lecture

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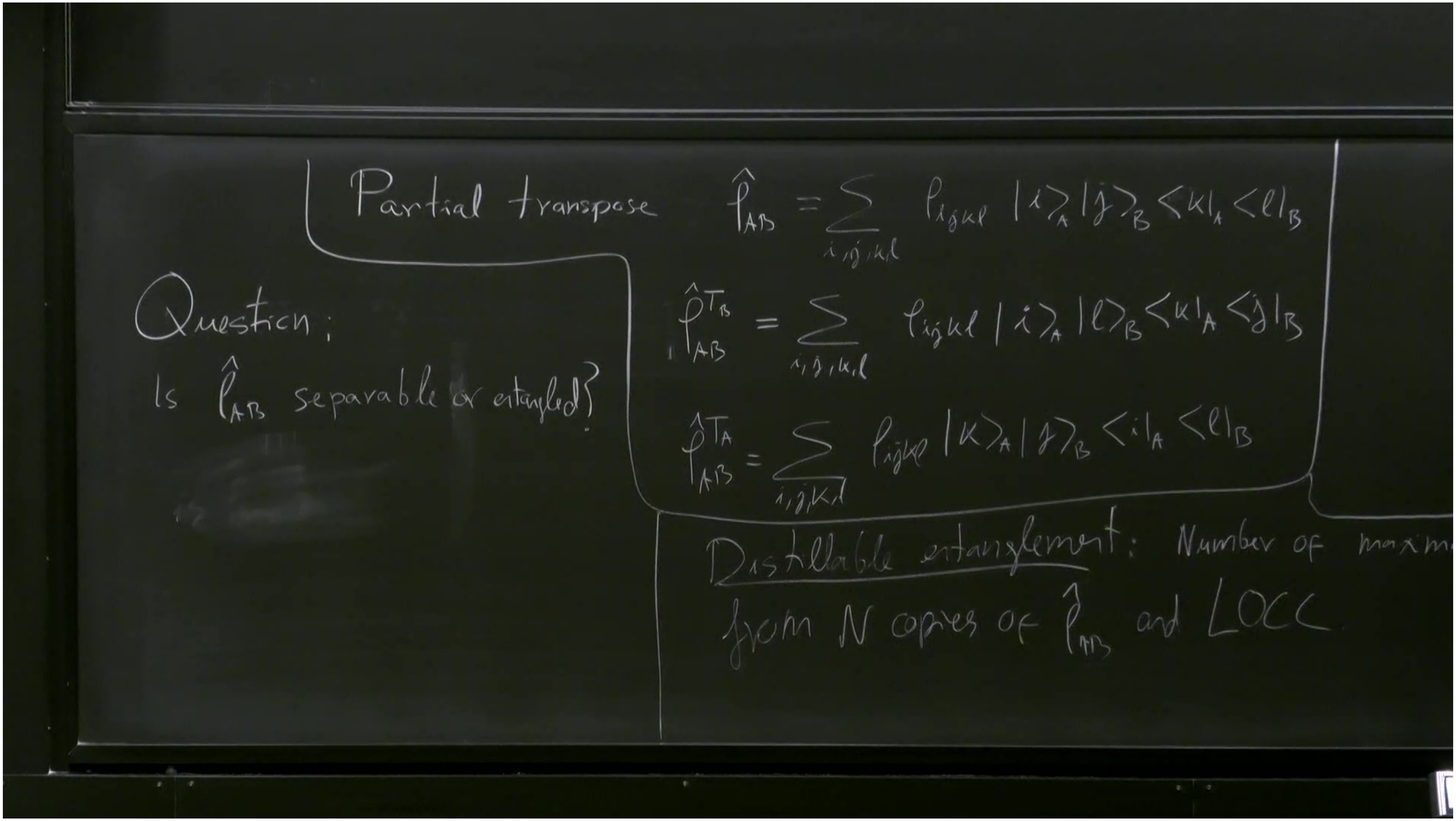
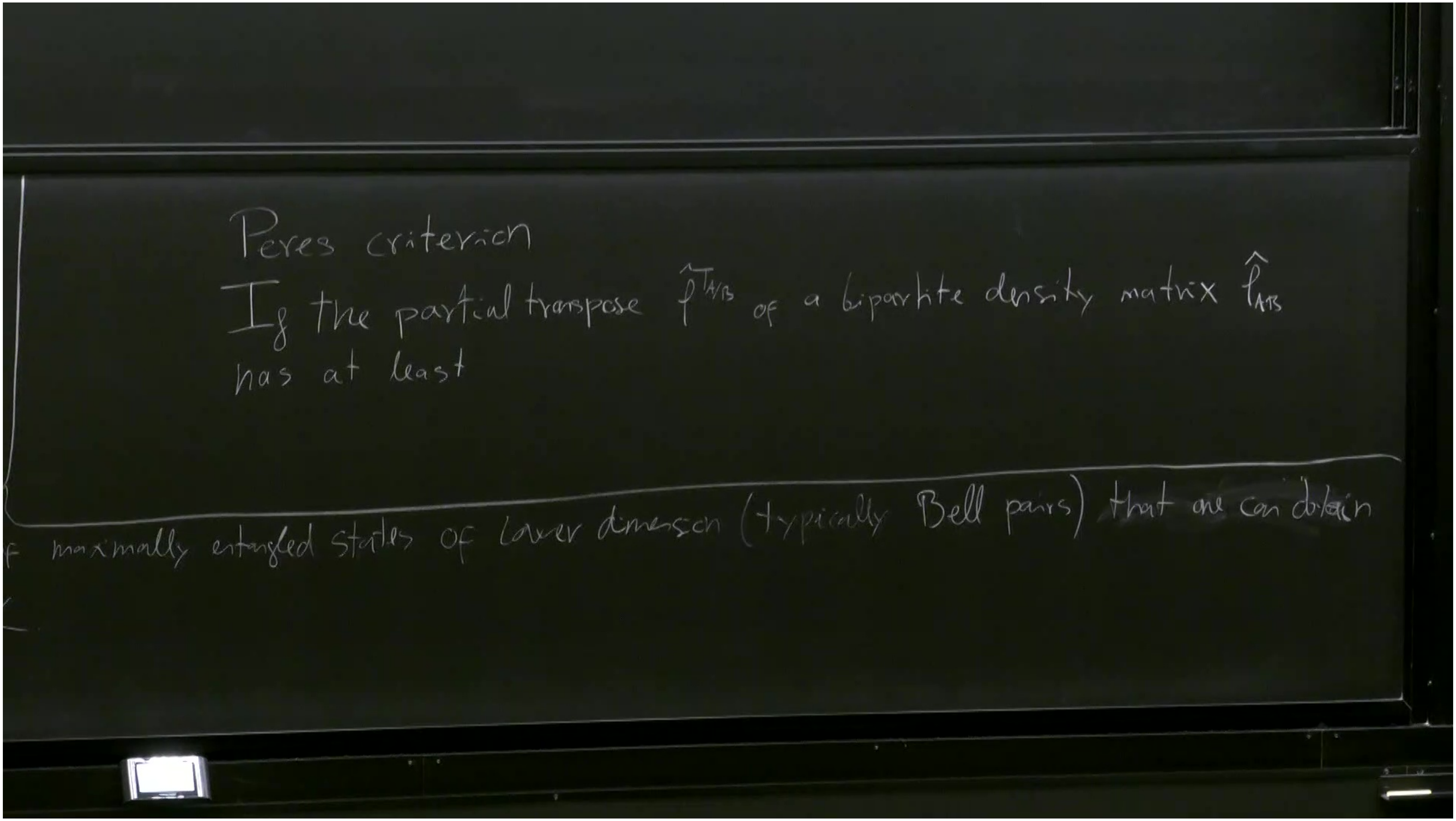

Partial transpose $\hat{\rho}_{AB} = \sum_{i,j,k,l} \rho_{ijkl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$
$\hat{\rho}_{AB}^{T_B} = \sum_{i,j,k,l} \rho_{ijkl} |i\rangle_A |l\rangle_B \langle k|_A \langle j|_B$
$\hat{\rho}_{AB}^{T_A} = \sum_{i,j,k,l} \rho_{ijkl} |k\rangle_A |j\rangle_B \langle i|_A \langle l|_B$
Question:
Is $\hat{\rho}_{AB}$ separable or entangled?
Distillable entanglement: Number of maxim
from N copies of $\hat{\rho}_{AB}$ and LOCC


Peres criterion
If the partial transpose $\hat{\rho}^{T_{A/B}}$ of a bipartite density matrix $\hat{\rho}_{AB}$
has at least
maximally entangled states of lower dimension (typically Bell pairs) that one can obtain

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If the partial transpose $\hat{\rho}_{AB}^{T_{A/B}}$ of a bipartite density matrix $\hat{\rho}_{AB}$ has at least one negative eigenvalue $\hat{\rho}_{AB}$ is entangled.

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Peres criterion (sufficient condition for entanglement)

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$\hat{\rho}_{AB}^{T_{A/B}}$ has negative eigenvalues $\Leftarrow$ $\hat{\rho}_{AB}$ has distillable entanglement

If $\hat{\rho}_{AB} = \sum_i c_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i \Rightarrow \hat{\rho}_{AB}^{T_B} = \sum c_i \hat{\rho}_A \otimes \hat{\rho}_B^T$

($\hat{\rho}_{AB}$ is separable) $\Rightarrow$ ($\hat{\rho}^{T_{AB}}$ is still a positive operator)

LOCC: Local operations and Classical Communication

States with bound entanglement but PPT (positive partial transpose) cannot be

Entanglement measures

Def. An entanglement measure is a map $\mathcal{E}: \hat{\rho}_{AB} \longrightarrow \mathbb{R}$ th

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1. Must be maximum iff the state is maximally entangled
2. Must be zero iff the state is separable.
3. Must not increase under LOCC.

Entanglement entropy

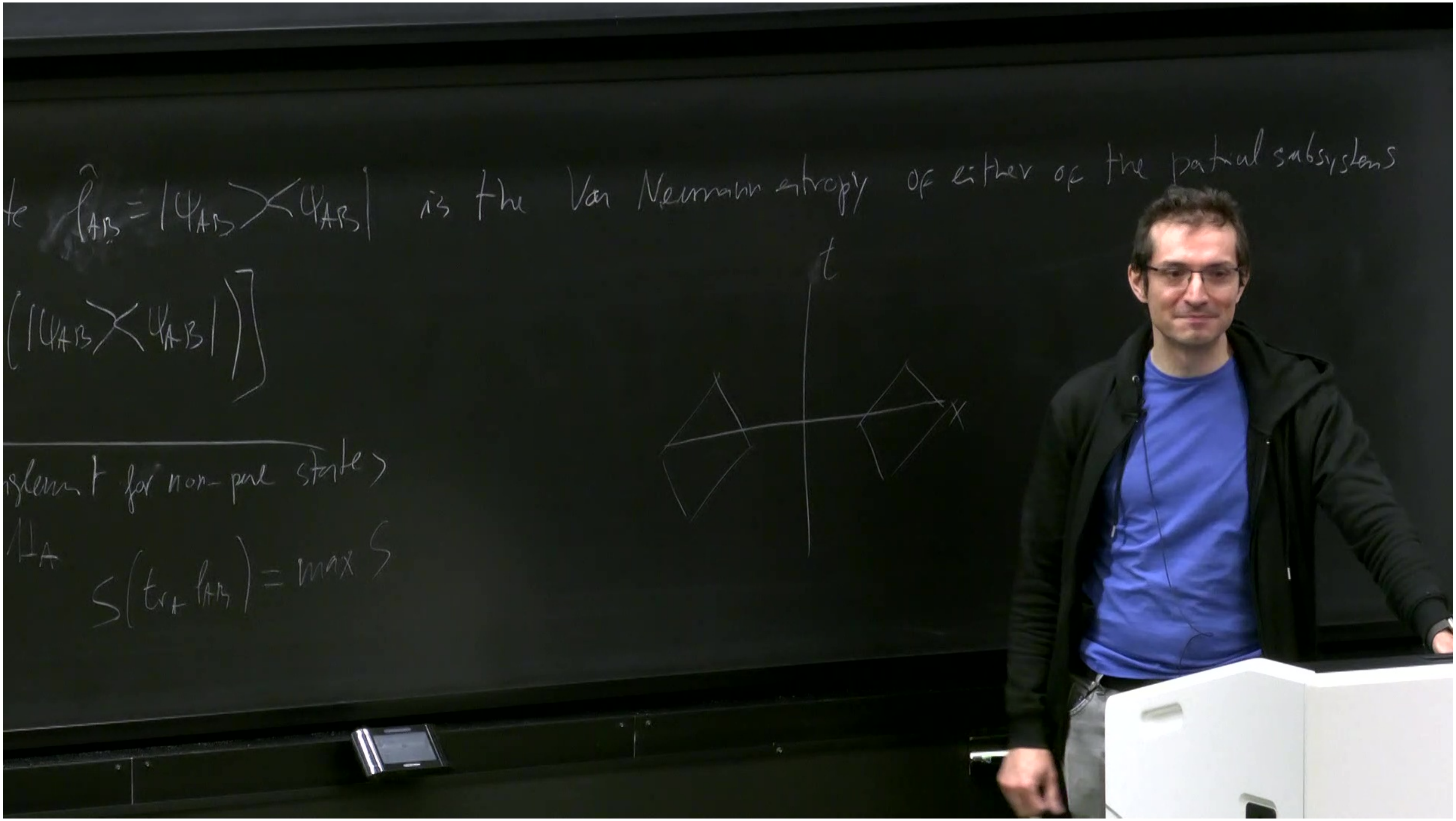
The Entanglement entropy of a pure bipartite state $\hat{\rho}_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}|$ is

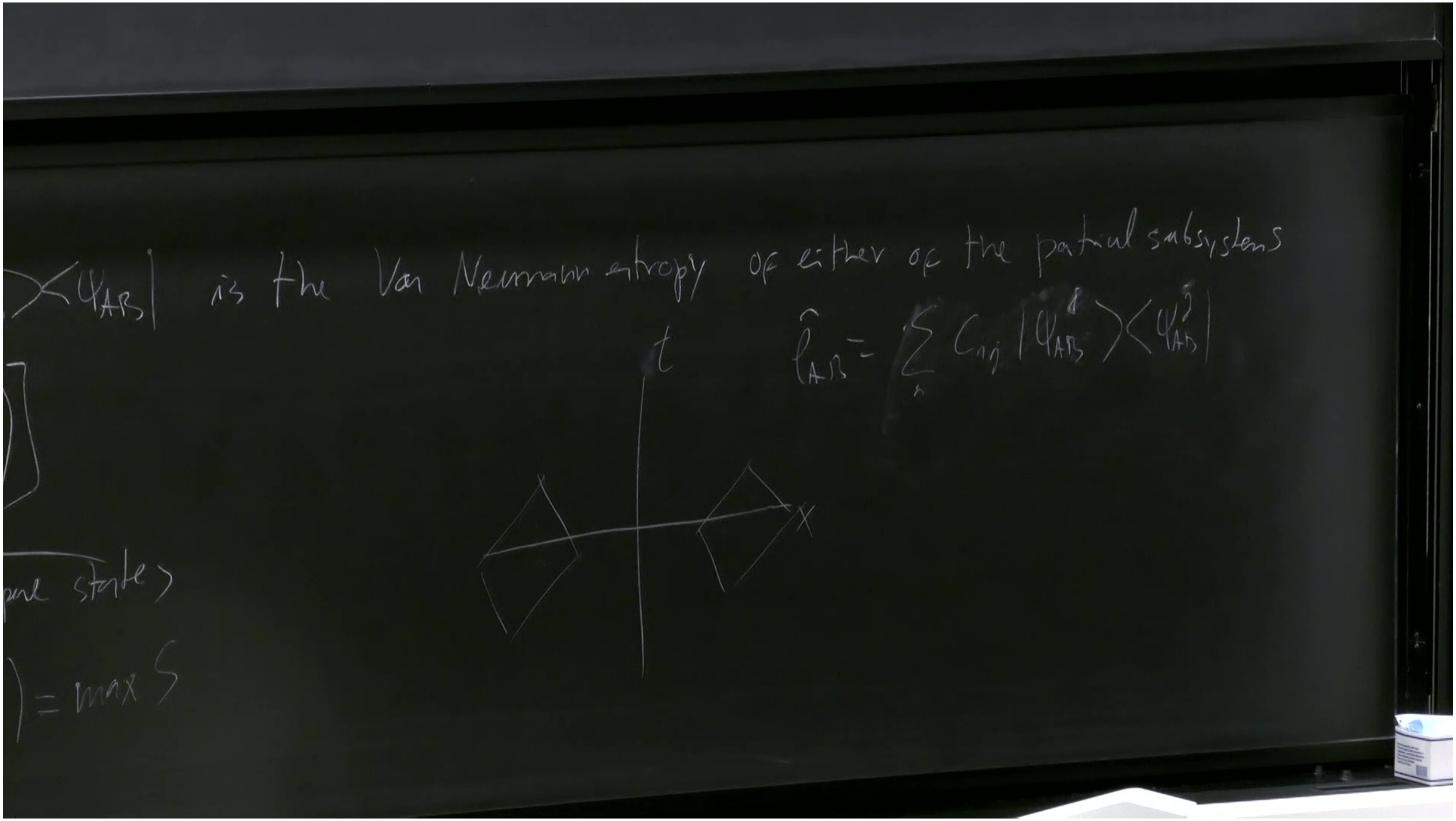
$$S_E(|\psi_{AB}\rangle) := S\left[\mathrm{tr}_A\left(|\psi_{AB}\rangle\langle\psi_{AB}|\right)\right] = S\left[\mathrm{tr}_B\left(|\psi_{AB}\rangle\langle\psi_{AB}|\right)\right]$$

Problem: It dramatically fails to measure entanglement for non-pure states

$$\text{Ex} \quad \hat{\rho}_{AB} = \mathbb{1}_A \otimes \mathbb{1}_B \Rightarrow \hat{\rho}_A = \mathrm{tr}_B(\hat{\rho}_{AB}) = \mathbb{1}_A$$

$$S(\mathrm{tr}_A \, \rho_{AB}) = \max S$$


is the Van Neumann entropy of either of the partial subsystems
for non-pure states
t
x


$\rangle\langle\psi_{AB}|$ is the Von Neumann entropy of either of the partial subsystems
$\hat{\rho}_{AB} = \sum_i C_{ij} |\psi^i_{AB}\rangle\langle\psi^j_{AB}|$
t
x
pure state>
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Problem: It dramatically fails to measure entanglement for non-pure states

Ex $\hat{\rho}_{AB} = \mathbb{1}_A \otimes \mathbb{1}_B \Rightarrow \hat{\rho}_A = \mathrm{tr}_B(\hat{\rho}_{AB}) = \mathbb{1}_A$ $\quad S(\mathrm{tr}_A \rho_{AB}) = \max S$

Entanglement of Formation (EoF)

$$\hat{\rho}_{AB} = \sum_i p_i |\psi^i_{AB}\rangle\langle\psi^i_{AB}| \qquad \langle S_E\rangle(\rho_{AB}) := \sum_i p_i S_E(|\psi_i\rangle_{AB})$$

Def: $$EoF(\hat{\rho}_{AB}) := \inf_{p_i, |\psi_i\rangle_{AB}} \sum_i p_i EoF(|\psi_i\rangle_{AB})$$

$$EoF(|\psi_i\rangle_{AB}) = S_E(|\psi_i\rangle_{AB})$$