Title: Quantum Information Lecture

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Locality $A \neq A(b)$ $B \neq B(a)$

λ is a random hidden va

Realism $A = A(a,\lambda)$ $B = B(b,\lambda)$

Consider a two level system so that $A(a,\lambda) = \pm 1$, $B(b,\lambda) = \pm 1$

"two-point" correlator $C(a,b) = \int d\lambda \, \Lambda(\lambda) \, A(a,\lambda) \, B(b,\lambda)$

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$$C(a,b) - C(a,b') = \int d\lambda\, \Lambda(\lambda) \Big(A(a,\lambda)\, B(b,\lambda) - A(a,\lambda$$

$$C(a,b) - C(a,b') = \int d\lambda\, \Lambda(\lambda) \Big(A(a,\lambda)\, B(b,\lambda) \big(1 \pm A(a',\lambda) B(b',\lambda)\big) - A(a,\lambda) B(b',\lambda) \big(1$$

...n hidden variable with probability distribution $\Lambda(\lambda) \geq 0$ so that $\int d\lambda \, \Lambda(\lambda) = 1$

two possible lab settings for Alice and Bob

a, a' $\qquad\qquad$ b, b'

$$\ldots, \lambda) \, B(b', \lambda) \Big) \Big)$$

$$\ldots) \big(1 \pm A(a', \lambda) B(b, \lambda) \big) \Big) \Big)$$

$$|A(a, \lambda)| \leq 1$$

$$|B(b, \lambda)| \leq 1$$

Locality $A \neq A(b)$ $\quad B \neq B(a)$ $\qquad \lambda$ is a random

Realism $A = A(a,\lambda)$ $\quad B = B(b,\lambda)$ $\qquad \Lambda(\lambda)$ $\qquad \int d\lambda \Lambda(\lambda)$

Consider a two level system so that $\int d\lambda$ $A(a,\lambda) = \pm 1$, $B(b,\lambda) = \pm 1$

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$$|C(a,b) - C(a,b') \leq \int d\lambda\, \Lambda(\lambda) \left[\left(1 \pm A(a',\lambda)\, B(b',\lambda)\right) + \left(1 \pm A(a',\lambda)\, B(b,\lambda)\right) \right] =$$

$$= 2 \underbrace{\int d\lambda\, \Lambda(\lambda)}_{1} \pm \int d\lambda\, \Lambda(\lambda)\, A(a',\lambda)\, B(b',\lambda) \pm \int d\lambda\, \Lambda(\lambda)\, A(a',\lambda)\, B(b,\lambda)$$

$$|C(a,b) - C(a,b')| \leq 2 \pm \left(C(a',b') + C(a',b) \right) \Rightarrow |C(a,b) - C(a,b')| \mp \left(C(a',b') + C(a',b) \right) \leq 2$$

$$\boxed{|C(a,b) - C(a,b')| + |C(a',b') + C(a',b)| \leq 2}$$

$|\Phi^{\pm}\rangle := \frac{1}{\sqrt{2}}\left(|0_A 0_B\rangle \pm |1_A 1_B\rangle\right)$, $|\Psi^{\pm}\rangle := \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle \pm |1_A 0_B\rangle\right)$

Let's pick $|\Psi^{-}\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - |1_A 0_B\rangle\right)$

Alice can do two kind of expernts:

$a \equiv$ Alice measures spin along the z axis

$a' \equiv$ " " " " " " x "

Bob can do two kind of experiments

$b \equiv$ Bob measures $\frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B})\right)$

$b' \equiv$ " " " " $\frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right.$

$$|\Phi^{\pm}\rangle := \frac{1}{\sqrt{2}}\left(|0_A 0_B\rangle \pm |1_A 1_B\rangle\right), \quad |\Psi^{\pm}\rangle := \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle \pm |1_A 0_B\rangle\right)$$

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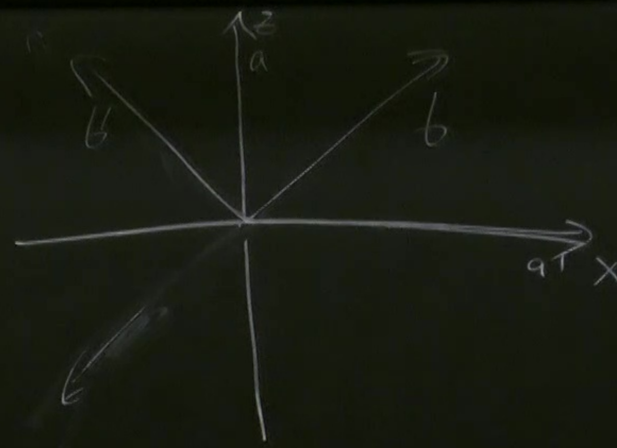
$1_A\, 0_B\rangle)\Big)$

...o kind of experiments

$$\frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}\right)\right)$$

$$\frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right)\right)$$



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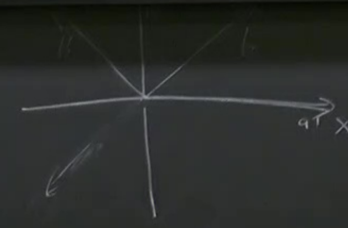

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$b' \equiv$ " " " " $\frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat\sigma_{z_B} - \hat\sigma_{x_B})\right)$

$\hat A(a) = \hat\sigma_{z_A} \otimes \mathbb{1}_B \qquad \hat B(b) = \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat\sigma_{z_B} + \hat\sigma_{x_B})\right)$

$\hat A(a') = \hat\sigma_{x_A} \otimes \mathbb{1}_B \qquad \hat B(b') = \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat\sigma_{z_B} - \hat\sigma_{x_B})\right)$

$\hat\sigma_z|0\rangle = -|0\rangle$

$\sigma_z|1\rangle = |1\rangle$

$$\langle A(a)B(b)\rangle = \langle\Psi^-|\,\hat\sigma_{z_A}\otimes\frac{-1}{\sqrt{2}}(\hat\sigma_{z_B}+\hat\sigma_{x_B})\,|\Psi^-\rangle = \frac{1}{2\sqrt{2}}\Big(\overbrace{\langle 0_A 1_B|\,\hat\sigma_{z_A}\otimes(\hat\sigma_{z_B}+\cancel{\hat\sigma_{x_B}})\,|0_A 1_B\rangle}^{1} - \overbrace{\langle 0_A 1_B|\,\cancel{\hat\sigma_{z_A}}\otimes(\hat\sigma_{z_B}+\hat\sigma_{x_B})\,|1_A 0_B\rangle}^{0} -$$

$$- \underbrace{\langle 1_A 0_B|\,\cancel{\hat\sigma_{z_A}}\otimes(\hat\sigma_{z_B}+\hat\sigma_{x_B})\,|0_A 1_B\rangle}_{0} + \underbrace{\langle 1_A 0_B|\,\hat\sigma_{z_A}\otimes(\hat\sigma_{z_B}+\cancel{\hat\sigma_{x_B}})\,|1_A 0_B\rangle}_{1}\Big) = \frac{1}{\sqrt{2}}$$

$$\hat{A}(a) = \hat{\sigma}_{z_A} \otimes \mathbb{1}_B$$

$$\hat{A}(a') = \hat{\sigma}_{x_A} \otimes \mathbb{1}_B$$

$$\hat{B}(b) = \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}\right)\right)$$

$$\hat{B}(b') = \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right)\right)$$

$$\langle A(a') B(b') \rangle = \langle \psi^- | \hat{\sigma}_{x_A} \otimes \frac{1}{\sqrt{2}}\left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right) | \psi^- \rangle = \frac{+1}{2\sqrt{2}} \Big(\langle O_A 1_B$$

$$- \langle 1_A O_B$$

$$\hat{\sigma}_z |0\rangle = -|0\rangle$$

$$\sigma_z |1\rangle = |1\rangle$$

$$= \Big(\overbrace{\langle 0_A 1_B | \hat{\sigma}_{x_A} \otimes (\hat{\sigma}_{z_B} - \cancel{\sigma_{x_B}}) | 0_A 1_B \rangle}^{0} - \overbrace{\langle 0_A 1_B | \hat{\sigma}_{x_A} \otimes (\cancel{\sigma_{z_B}} - \hat{\sigma}_{x_B}) | 1_A 0_B \rangle}^{-(-1) = 1} - \underbrace{\langle 1_A 0_B | \hat{\sigma}_{x_A} \otimes (\cancel{\hat{\sigma}_{z_B}} - \sigma_{x_B}) | 0_A 1_B \rangle}_{-(-1) = 1} + \underbrace{\langle 1_A 0_B | \cancel{\hat{\sigma}_{x_A}} \otimes (\hat{\sigma}_{z_B} - \cancel{\hat{\sigma}_{x_B}}) | 1_A 0_B \rangle}_{1} \Big) = \frac{\pm 1}{\sqrt{2}}$$

$$\hat{\sigma}_z|0\rangle = -|0\rangle$$

$$\sigma_z|1\rangle = |1\rangle$$

$$\overbrace{1_B|\,\hat{\sigma}_{x_A}\otimes\left(\hat{\sigma}_{z_B}+\cancel{\hat{\sigma}_{x_B}}\right)|0_A 1_B\rangle}^{0} - \overbrace{\langle 0_A 1_B|\,\hat{\sigma}_{x_A}\otimes\left(\cancel{\hat{\sigma}_{z_B}}+\hat{\sigma}_{x_B}\right)|1_A 0_B\rangle}^{-1} -$$

$$\underbrace{0_B|\,\hat{\sigma}_{x_A}\otimes\left(\cancel{\hat{\sigma}_{z_B}}+\hat{\sigma}_{x_B}\right)|0_A 1_B\rangle}_{-1} + \underbrace{\langle 1_A 0_B|\,\cancel{\hat{\sigma}_{x_A}}\otimes\left(\hat{\sigma}_{z_B}+\cancel{\hat{\sigma}_{x_B}}\right)|1_A 0_B\rangle}_{0} \Bigg) = \frac{1}{\sqrt{2}}$$

$$\langle \hat{A}(a)\,\hat{B}(b)\rangle = \frac{1}{\sqrt{2}} \qquad \langle \hat{A}(a)\,\hat{B}(b')\rangle = \frac{-1}{\sqrt{2}} \qquad \langle \hat{A}(a')\,\hat{B}(b')\rangle = \frac{1}{\sqrt{2}}$$

$$\underbrace{\left|\underbrace{\langle A(a) B(b)\rangle}_{1/\sqrt{2}} - \underbrace{\langle \hat{A}(a)\,\hat{B}(b')\rangle}_{-1/\sqrt{2}}\right|}_{2/\sqrt{2}} + \underbrace{\left|\underbrace{\langle \hat{A}(a')\,\hat{B}(b')\rangle}_{1/\sqrt{2}} + \underbrace{\hat{A}(a')\,\hat{B}(b)}_{1/\sqrt{2}}\right|}_{2/\sqrt{2}} = 2\sqrt{2} > 2$$