Title: Quantum Information Lecture

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Multipartite systems

Consider a bipartite system AB composed of two subsystems A,B. The Hilbert space associated to the quantum system AB is $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$

If $\{|a_i\rangle\}$ and $\{|b_i\rangle\}$ are orthonormal bases of $\mathcal{H}_A$ and $\mathcal{H}_B$ respectively then $\{|a_i b_i\rangle\} \equiv \{|a_i\rangle \otimes |b_i\rangle\}$ is an orthonormal basis of $\mathcal{H}_{AB}$

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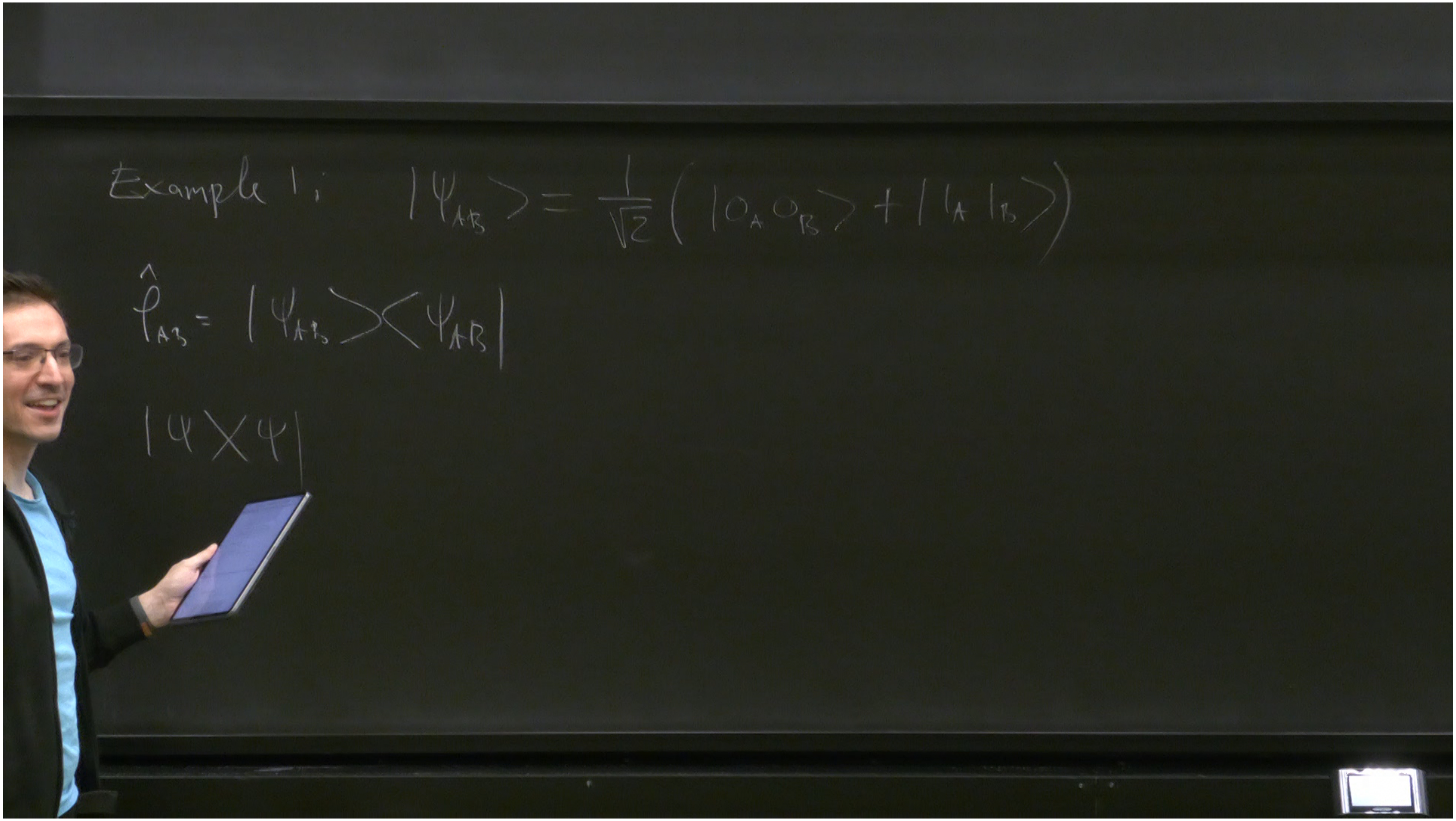

Example 1: $|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|0_A 0_B\rangle + |1_A 1_B\rangle\right)$
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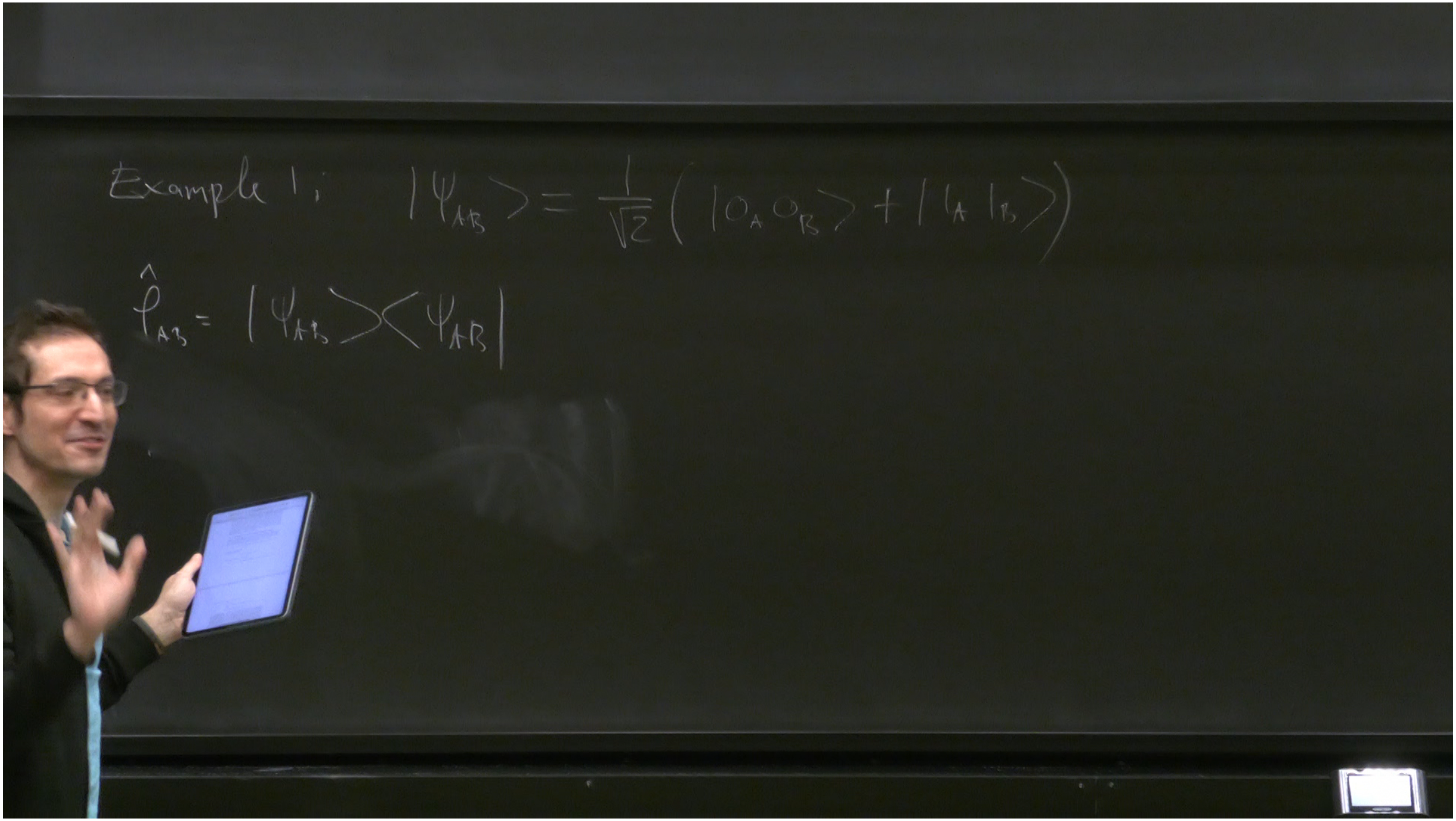

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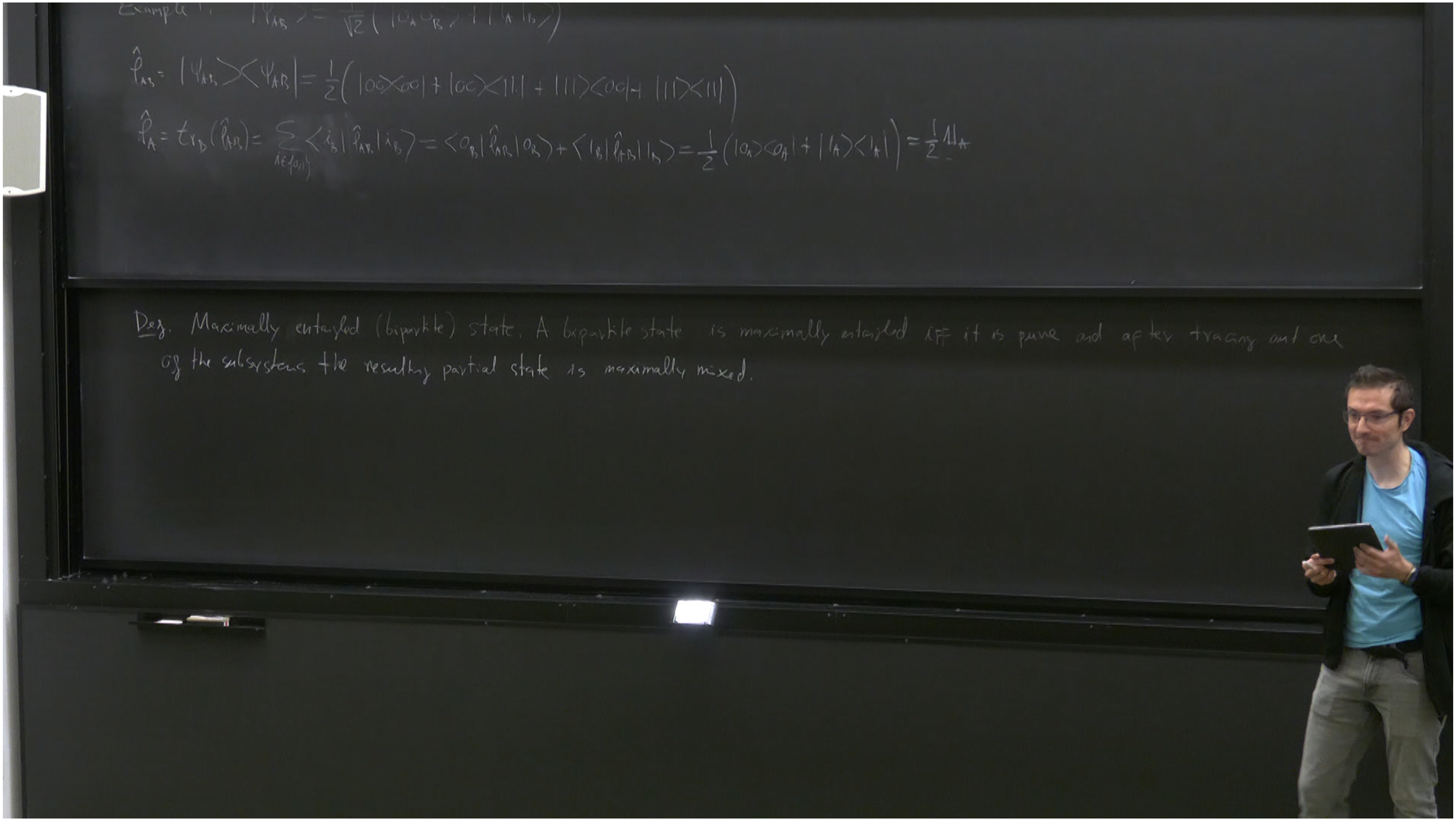

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Def. Maximally entangled (bipartite) state: A bipartite state is maximally entangled iff it is pure and after tracing out one of the subsystems the resulting partial state is maximally mixed.

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Example 3: $\hat{\rho}_{AB} = \frac{1}{4}\left(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11| \right) = \frac{1}{4} \mathbb{1}_{AB} = \frac{1}{4} \mathbb{1}_A \otimes \mathbb{1}_B$

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A bipartite state $\hat{\rho}_{AB}$ is said to be a <u>product state</u> iff $\hat{\rho}_{AB} = \hat{\rho}_A \otimes \hat{\rho}_B$

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A bipartite state is entangled (or non-separable) iff it cannot be written as $\hat{\rho}_{AB} = \sum_i p_i \, \hat{\rho}_A^i \otimes \hat{\rho}_B^i$

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$\hat{\rho}_{AB}$ product $\Longrightarrow \hat{\rho}_{AB}$ separable

$\hat{\rho}_{AB}$ pure and separable $\Longrightarrow \hat{\rho}_{AB}$ product

$\hat{\rho}_{AB}$ separable but not a product $\Longleftrightarrow$

$\hat{\rho}_{AB}$ only has classical correlations iff $\hat{\rho}_{AB} = \hat{\rho}_A \otimes \hat{\rho}_B$

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$\hat{\rho}_{AB}$ only has classical correlations

$\hat{\rho}_{AB}$ product $\Longleftrightarrow$ A and B uncorrelated is maximally entangled iff it is pure and after tracing out one

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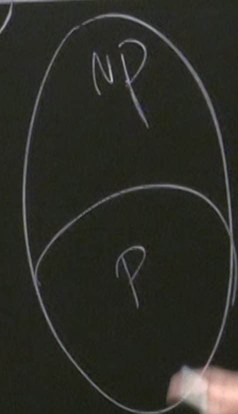
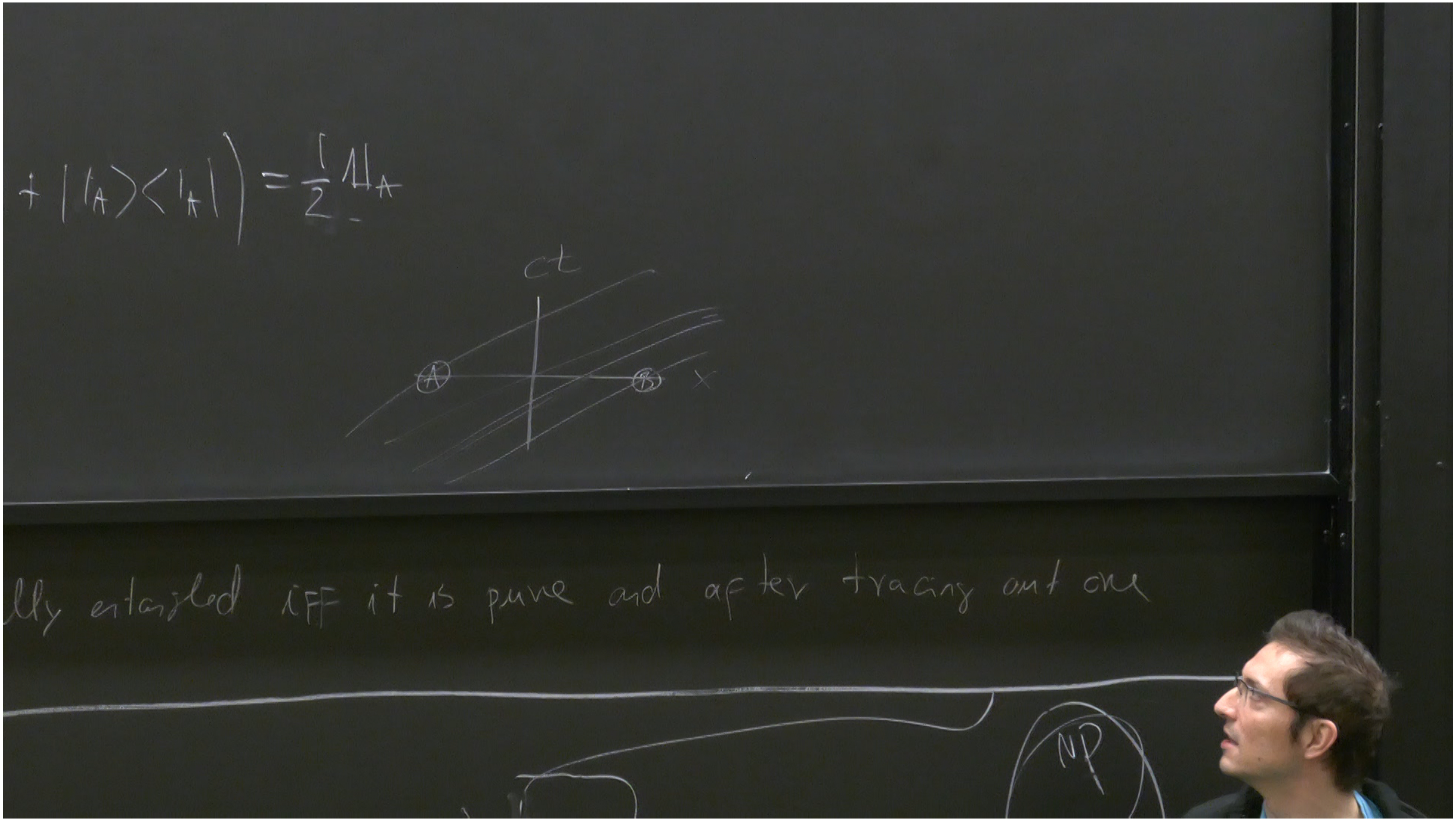


$$\frac{1}{4}\mathbb{1}_A \otimes \mathbb{1}_B \qquad \frac{1}{4} tr_B(\mathbb{1}_A \otimes \mathbb{1}_B) = \frac{1}{4}\mathbb{1}_A\, tr(\mathbb{1}_B) = \frac{1}{2}\mathbb{1}_A$$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$\hat{\rho}_{AB}$ product $\Longleftrightarrow$ A and B uncorrelated

Locality, Realism and Bell inequalities

We have two labs Alice and Bob, Lab Alice measures quantities A about a bipartite system with lab parameters a and Bob measures quantities B with lab parameters b of the same bipartite system

- Locality: $A \neq A(b)$, $B \neq B(a)$

in general $A = A(a, \lambda)$, $B = B(b, \lambda)$ $\quad \lambda \equiv$ Hidden variables

- Realism: Objects have definite properties independent of observation.