Title: Quantum Matter Lecture

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Collection: Quantum Matter 2023/24

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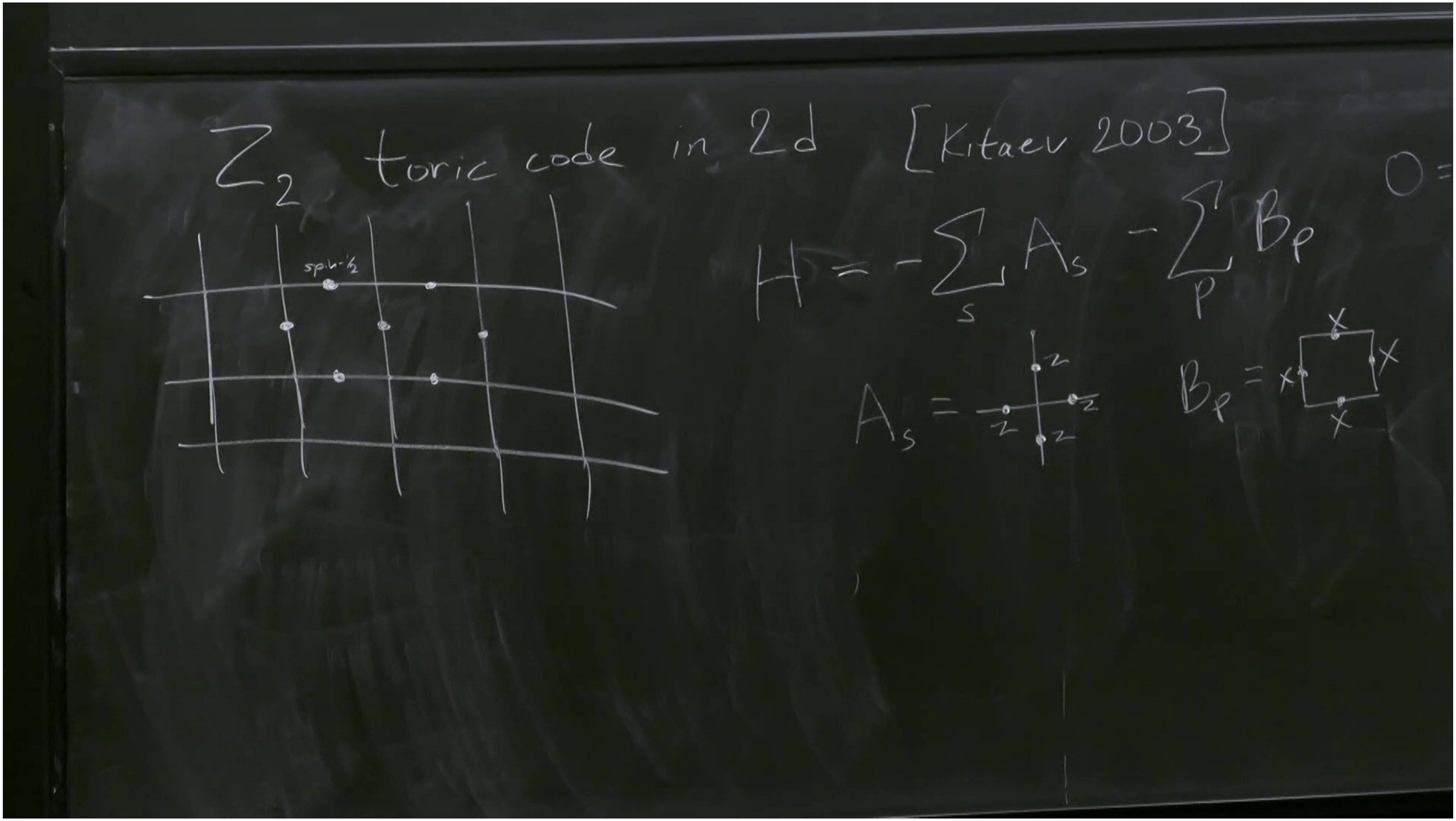
Topological Order (and Quantum Memory)

- distinct phase of matter, regardless of any symmetry
- no local order parameter
- # Ground states depends topology
- "anyon" excitations
- entanglement structure


$\mathbb{Z}_2$ toric code in 2d [Kitaev 2003]
spin-½
$H = -\sum_s A_s - \sum_p B_p$
$A_s =$ z z z z
$B_p =$ x x x x

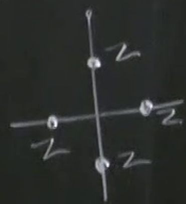
2d [Kitaev 2003]

$$0 = [A_s, A_{s'}] = [B_p, B_{p'}] = [A_s, B_p]$$

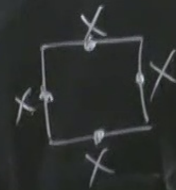
$$H = -\sum_s A_s - \sum_p B_p$$

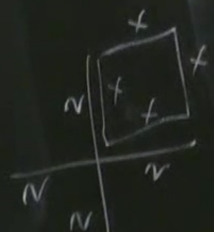
Ground state: $A_s = B_p = 1$

$A_s =$



$B_p =$

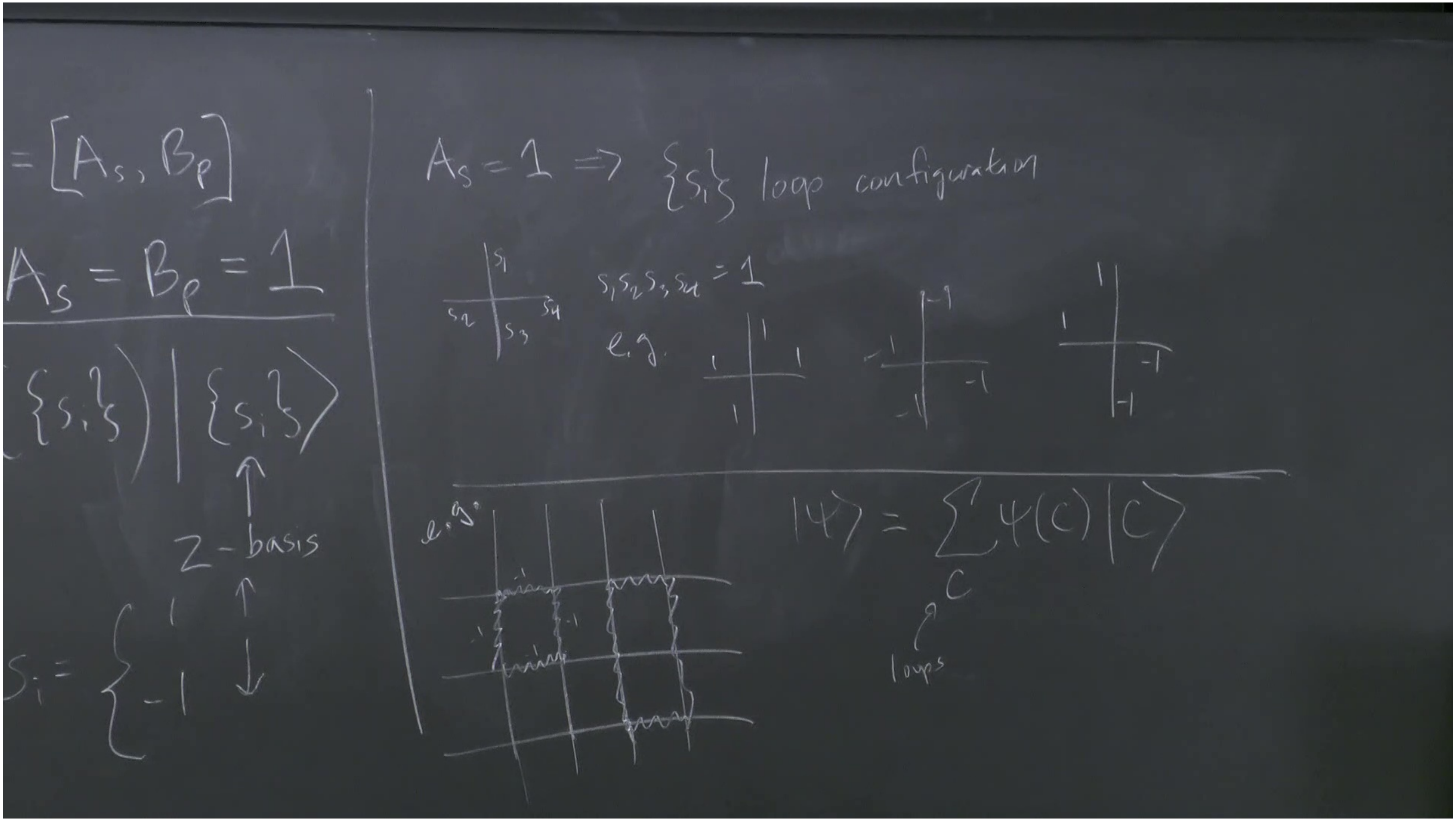


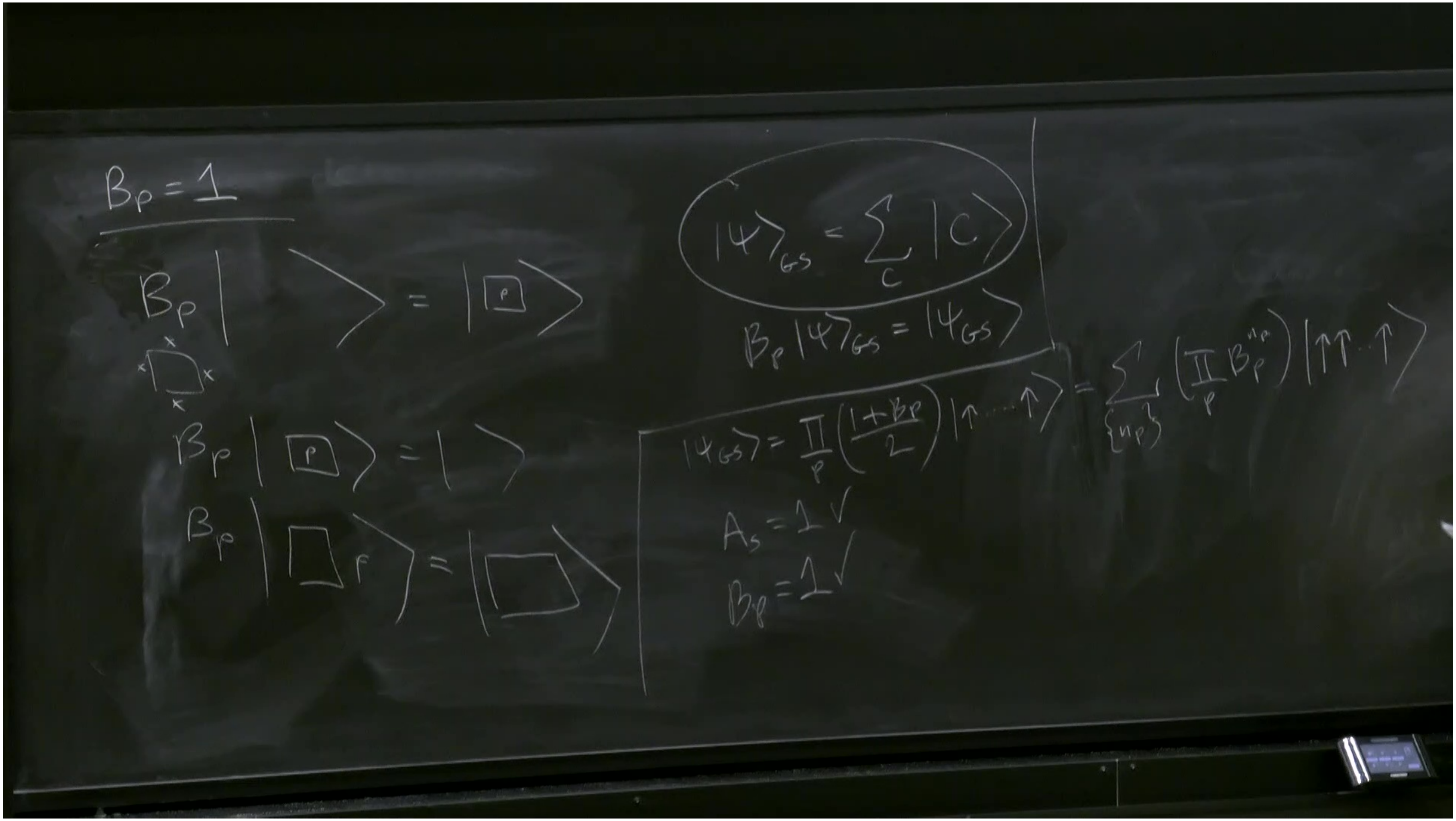


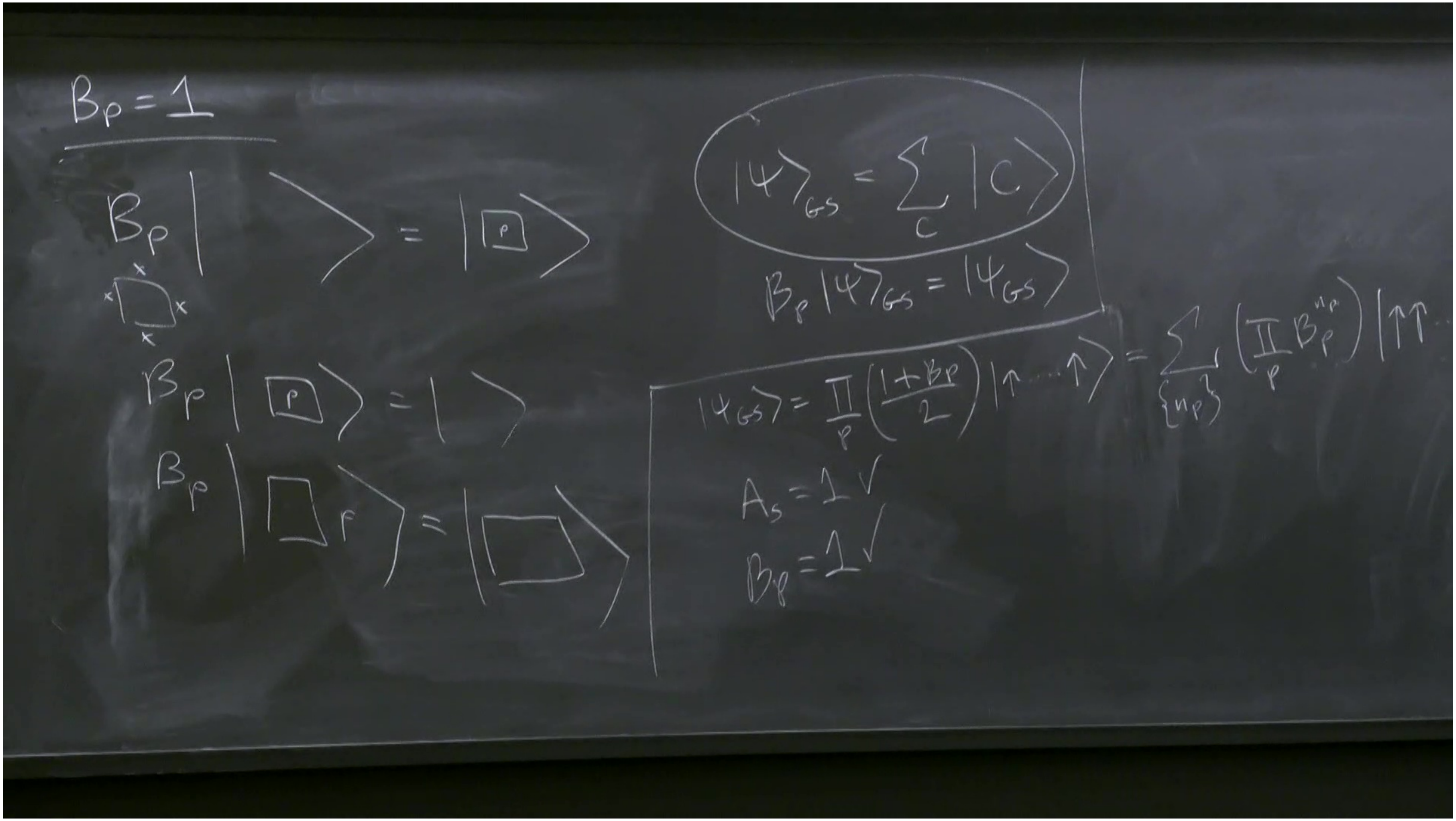
$$|\Psi\rangle = \sum_{\{s_i\}} \Psi(\{s_i\}) \, |\{s_i\}\rangle$$

z-basis

$$s_i = \begin{cases} 1 & \uparrow \\ -1 & \downarrow \end{cases}$$


= [As, Bp]
As = Bp = 1
{si}) | {si}>
Z - basis
Si = 1, -1
As = 1 ⇒ {si} loop configuration
s1 s2 s3 s4 = 1
e.g.
|Ψ> = Σ Ψ(C) |C>
C
loops


$B_p = 1$
$|\psi\rangle_{GS} = \sum_c |c\rangle$
$B_p |\psi\rangle_{GS} = |\psi_{GS}\rangle$
$|\psi_{GS}\rangle = \prod_p \left(\frac{1+B_p}{2}\right) |\uparrow \cdots \uparrow\rangle = \sum_{\{n_p\}} \left(\prod_p B_p^{n_p}\right) |\uparrow\uparrow \cdots \uparrow\rangle$
$A_s = 1$ ✓
$B_p = 1$ ✓

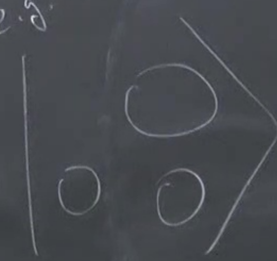

$B_p = 1$
$B_p | \; \rangle = | \boxed{p} \rangle$
$B_p | \boxed{p} \rangle = | \; \rangle$
$B_p | \; \square \; p \rangle = | \; \square \; \rangle$
$|\psi\rangle_{GS} = \sum_C |C\rangle$
$B_p |\psi\rangle_{GS} = |\psi_{GS}\rangle$
$|\psi_{GS}\rangle = \prod_p \left(\frac{1+B_p}{2}\right) |\uparrow \cdots \uparrow\rangle = \sum_{\{n_p\}} \left(\prod_p B_p^{n_p}\right) |\uparrow\uparrow \cdots$
$A_s = 1$ ✓
$B_p = 1$ ✓

$$|\psi\rangle_{GS} = \sum_{C} |C\rangle$$

$$B_p |\psi\rangle_{GS} = |\psi_{GS}\rangle$$

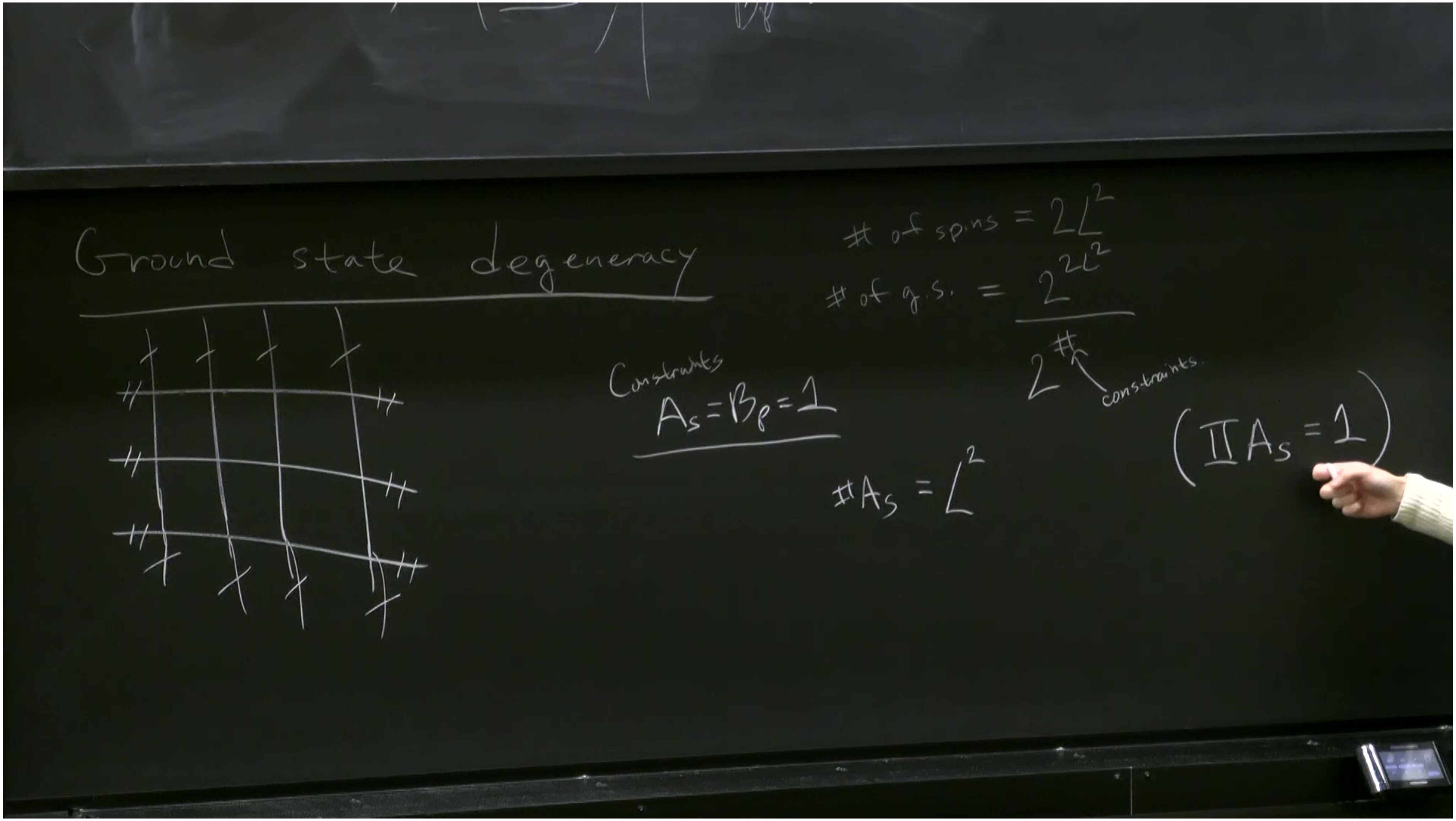
$$|\psi_{GS}\rangle = \prod_{p}\left(\frac{1+B_p}{2}\right)|\uparrow \cdots \uparrow\rangle = \sum_{\{n_p\}}\left(\prod_p B_p^{n_p}\right)|\uparrow\uparrow \cdots \uparrow\rangle = \sum_{\substack{C \\ \text{contractible}}} |C\rangle$$

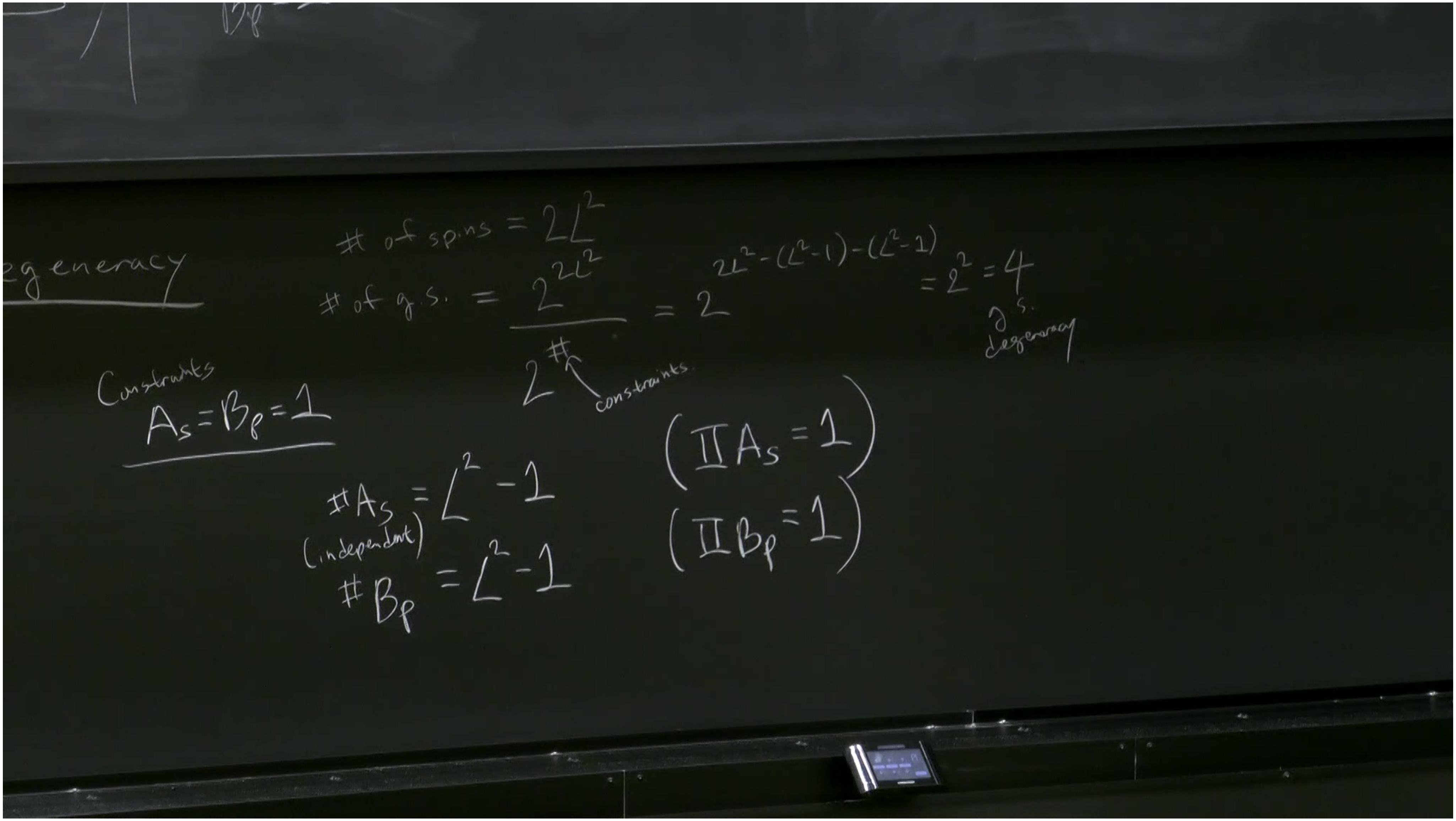
e.g.

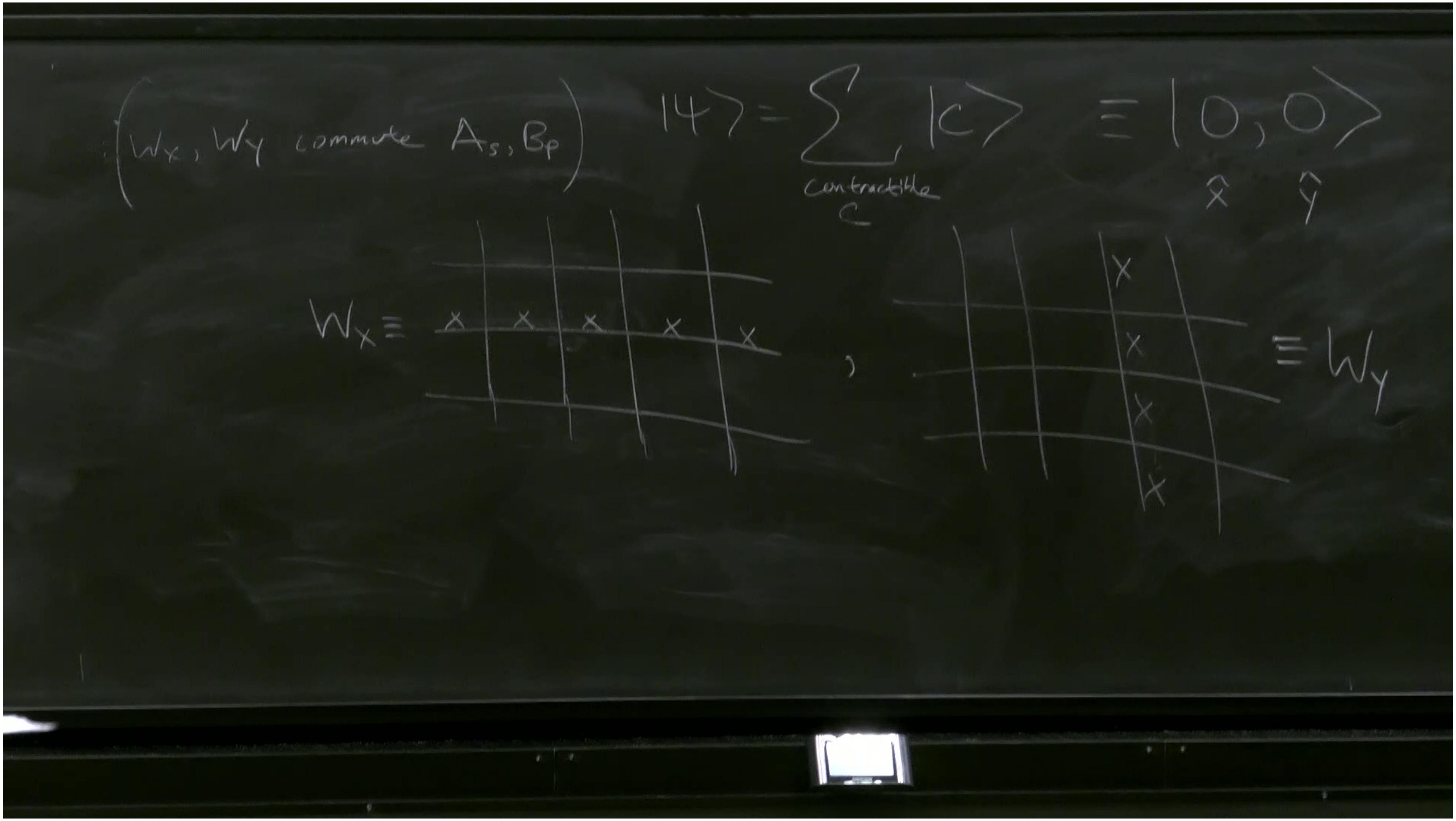


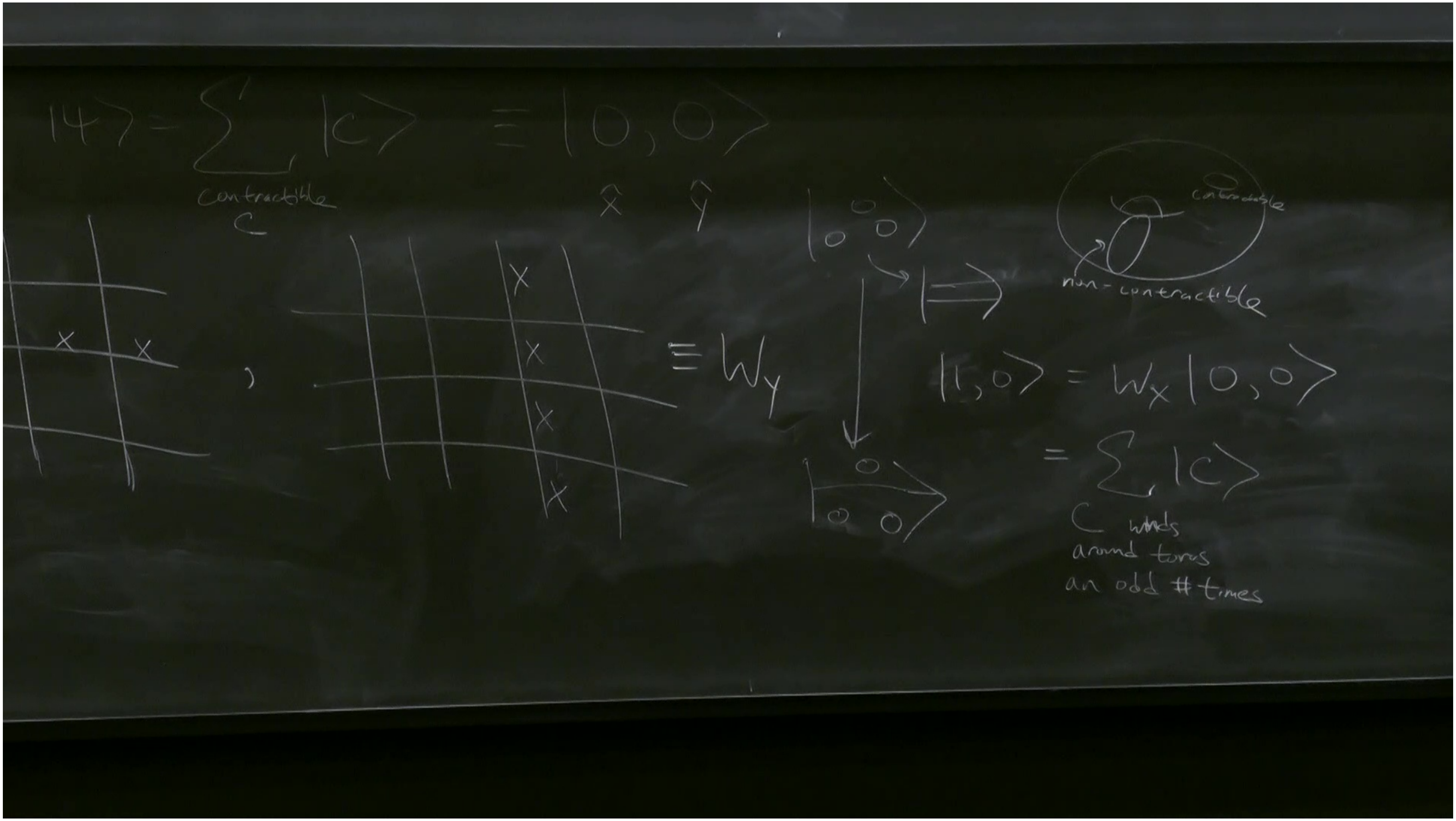
$A_s = 1$ ✓

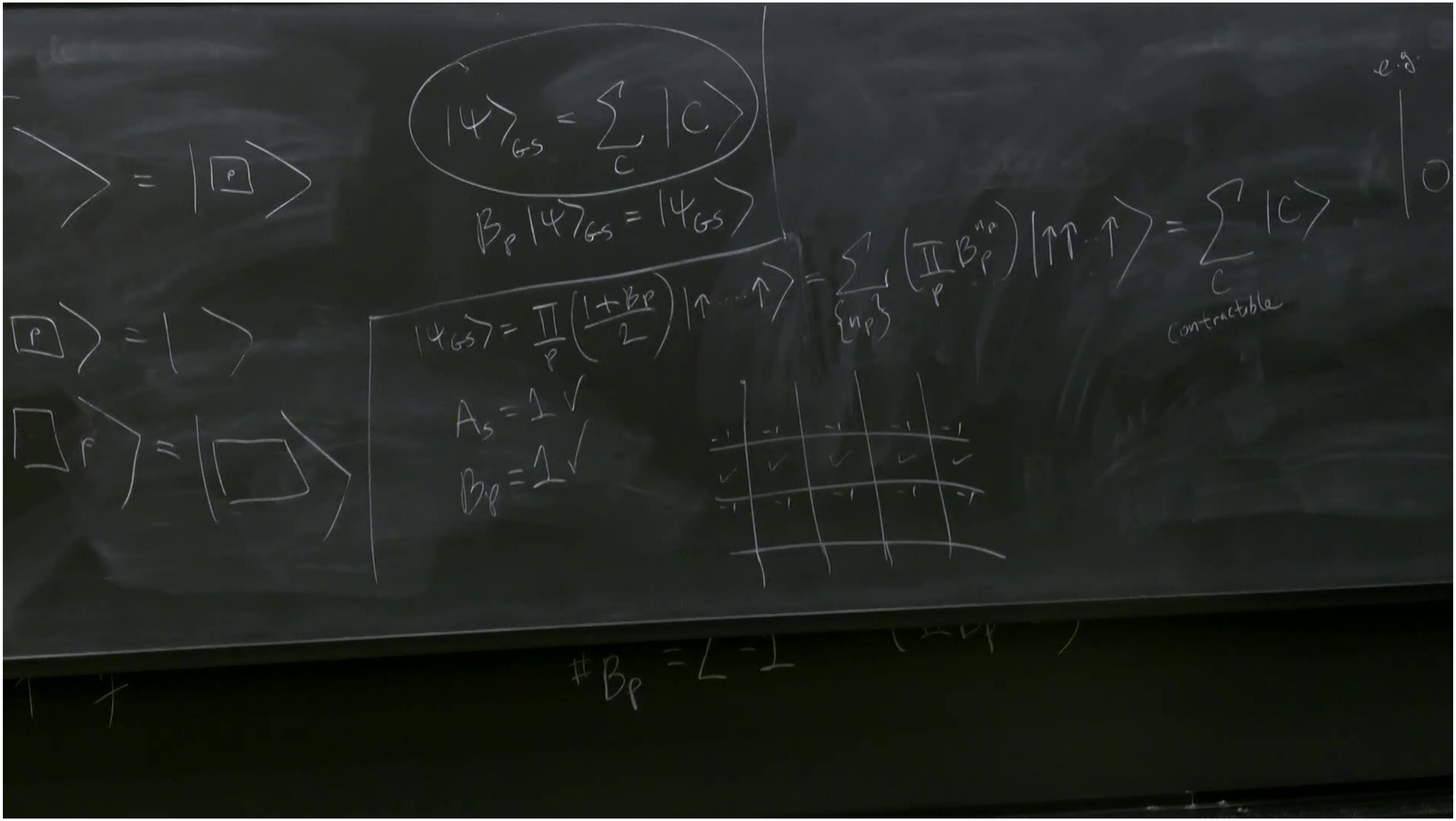
$B_p = 1$ ✓

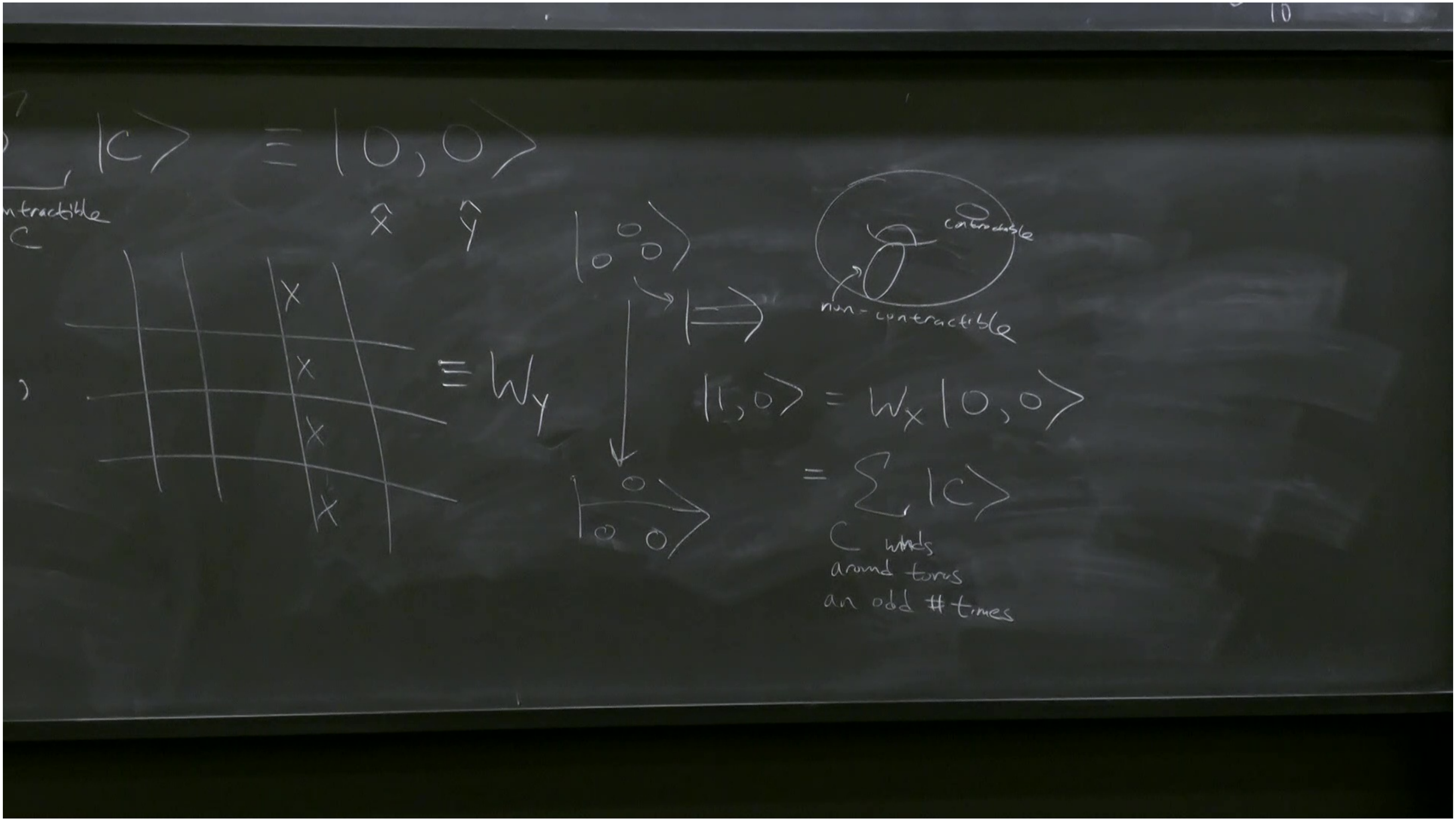

Ground state degeneracy
# of spins = 2L^2
# of g.s. = 2^{2L^2} / 2^{# constraints}
Constraints
A_s = B_p = 1
#A_s = L^2
(∏ A_s = 1)


egeneracy
# of spins = 2L^2
# of g.s. = 2^{2L^2} / 2^{#} = 2^{2L^2-(L^2-1)-(L^2-1)} = 2^2 = 4
constraints
g.s. degeneracy
Constraints
A_s = B_p = 1
#A_s = L^2 - 1
(independent)
#B_p = L^2 - 1
(∏A_s = 1)
(∏B_p = 1)


(Wx, Wy commute As, Bp)
|Ψ> = Σ |c> ≡ |0, 0>
contractible C
x̂ ŷ
Wx ≡
≡ Wy


|ψ> = Σ' |c> ≡ |0,0>
contractible C
x̂ ŷ
≡ W_y
contractible
non-contractible
|1,0> = W_x |0,0>
= Σ' |c>
C winds around torus an odd # times


|ψ⟩GS = Σ C |C⟩
Bp |ψ⟩GS = |ψGS⟩
|ψGS⟩ = Π p (1+Bp / 2) |↑ ... ↑⟩ = Σ {np} (Π p Bp^np) |↑↑ ... ↑⟩ = Σ C |C⟩
contractible
As = 1 ✓
Bp = 1 ✓
e.g.
#Bp

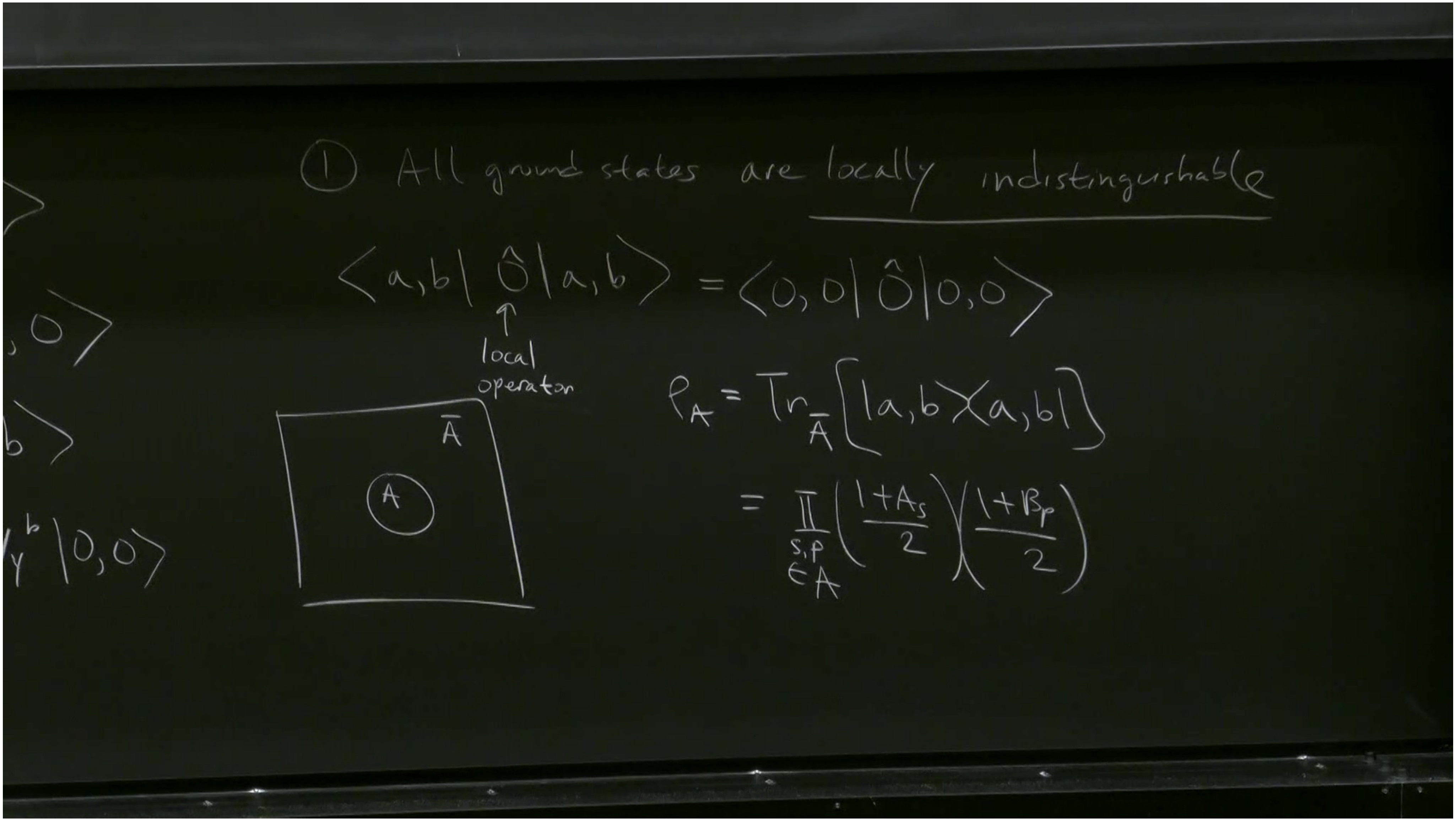

|c> ≡ |0,0>
ntractible
C
≡ W
contractible
non-contractible
|1,0> = W |0,0>
= ∑ |c>
C winds
around torus
an odd # times

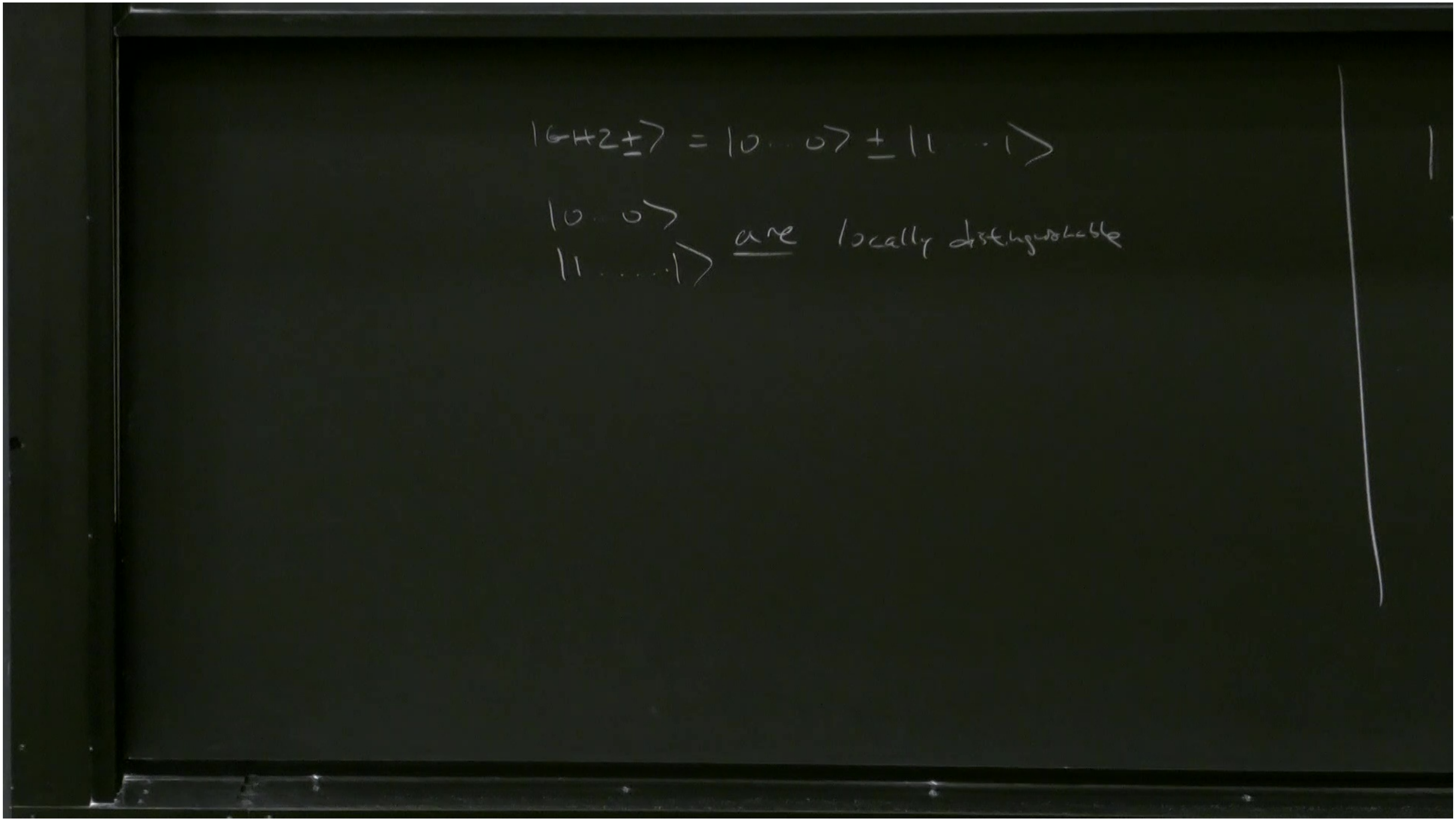
$$|0,1\rangle = W_y |0,0\rangle$$

$$|1,1\rangle = W_x W_y |0,0\rangle$$

$a = 0,1$
$b = 0,1$

$$|a,b\rangle = W_x^a W_y^b |0,0\rangle$$


① All ground states are locally indistinguishable
⟨a,b| Ô |a,b⟩ = ⟨0,0| Ô |0,0⟩
local operator
ρ_A = Tr_Ā [|a,b⟩⟨a,b|]
= ∏_{s,p ∈ A} ((1+A_s)/2) ((1+B_p)/2)
Ā
A


$|GHZ\pm\rangle = |0\cdots0\rangle \pm |1\cdots1\rangle$
$|0\cdots0\rangle$
$|1\cdots\cdots1\rangle$ are locally distinguishable

ground states are not "locally connectable"

$$\langle a,b|\hat{O}|a',b'\rangle \simeq 0 \qquad (a,b)\neq(a',b')$$

↑ local operator

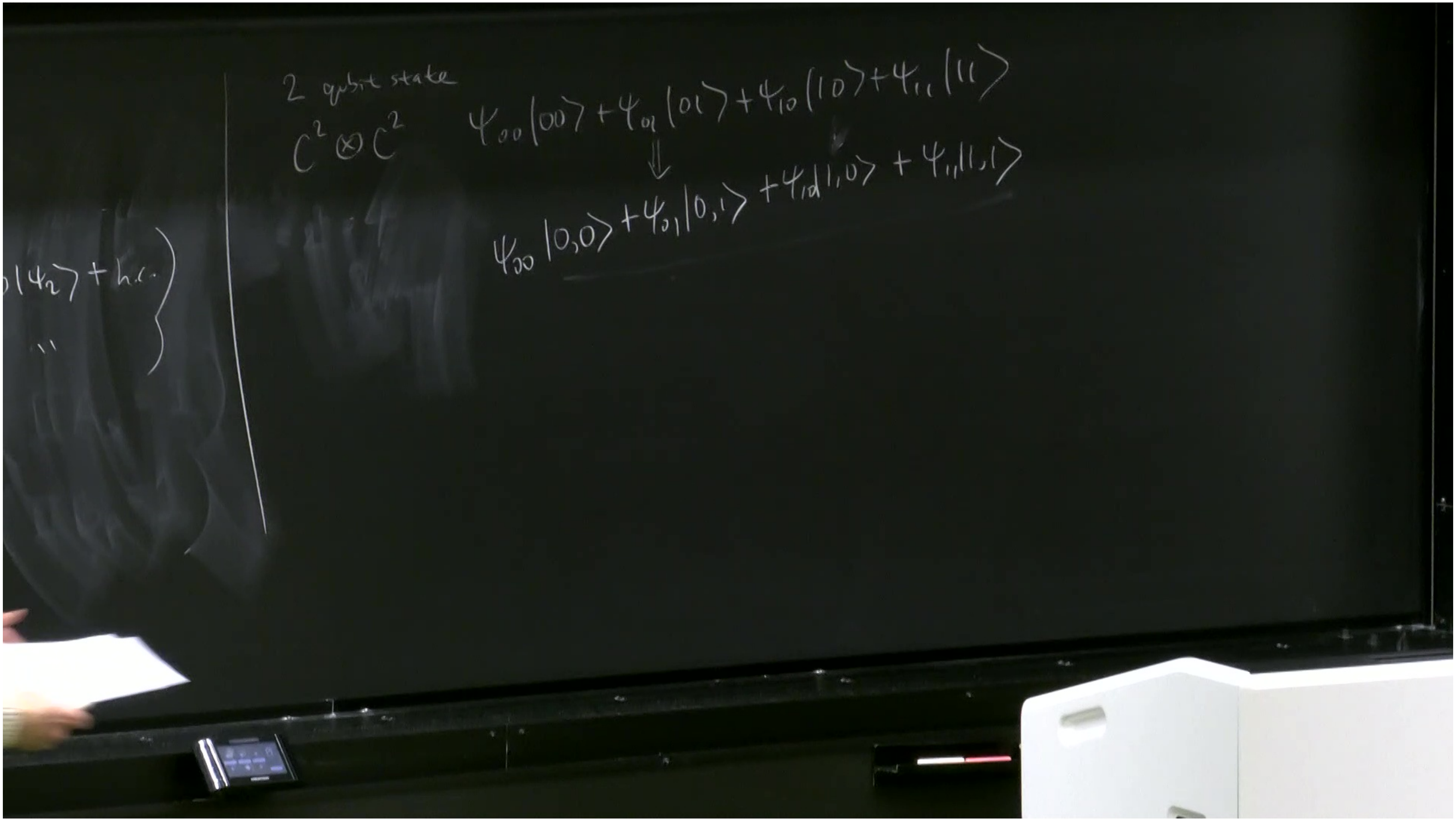
If $\hat{O}|\psi_1\rangle = |\psi_2\rangle$,

then $\hat{O}$ distinguish $|\psi_1\rangle+|\psi_2\rangle$

$|\psi_1\rangle-|\psi_2\rangle$

$$\langle O\rangle_1 + \langle O_2\rangle + \left(\langle\psi_1|O|\psi_2\rangle + h.c.\right)$$

$$'' \quad - \left(\quad '' \right)$$


2 qubit state
$\mathbb{C}^2 \otimes \mathbb{C}^2$
$\psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle$
$\Downarrow$
$\psi_{00}|0,0\rangle + \psi_{01}|0,1\rangle + \psi_{10}|1,0\rangle + \psi_{11}|1,1\rangle$