Title: Quantum Matter Lecture

Speakers: Yin-Chen He

Collection: Quantum Matter 2023/24

Date: March 11, 2024 - 2:00 PM

URL: https://pirsa.org/24030049

This week:

Dirac fermions

$\Downarrow$

Topological physics

1° fractional charge

2° Chern insulator

Chern number

Dirac fermions i

$(i\gamma_\mu \partial_\mu - m)\psi$

$\bar{\psi} = \psi^\dagger \gamma_0$,

4d spacetime,

3d spacetime,

Dirac fermions in QFT

$(i\gamma_\mu \cdot \partial_\mu - m)\psi = 0$

$\bar{\psi} = \psi^\dagger \gamma_0$, $\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$

4d spacetime, $\gamma_0, \gamma_1, \gamma_2, \gamma_3$, 4×4 matrix

3d spacetime, $\gamma_0, \gamma_1, \gamma_2$, 2×2 matrix, $\sigma^{x,y,z}$

2d Spacetime, γ_0, γ_1, two of $\sigma^{x,y,z}$

Translation

$$\psi_L(p) \to e^{i(\frac{\pi}{2}-p)}\psi_L(p) = i\, e^{-ip}\psi_L$$

$$\psi_R(p) \to -i\, e^{-ip}\psi_R$$

$$\psi \to i\sigma^z e^{-ip}\psi$$

$$\psi^\dagger \sigma^x \psi \longrightarrow \psi^\dagger e^{ip}(-i\sigma^z)\sigma^x(i\sigma^z)e^{-ip}\psi$$

$$= -\psi^\dagger \sigma^x \psi$$

$$\psi^\dagger \sigma^y \psi \longrightarrow -\psi^\dagger \sigma^y \psi$$

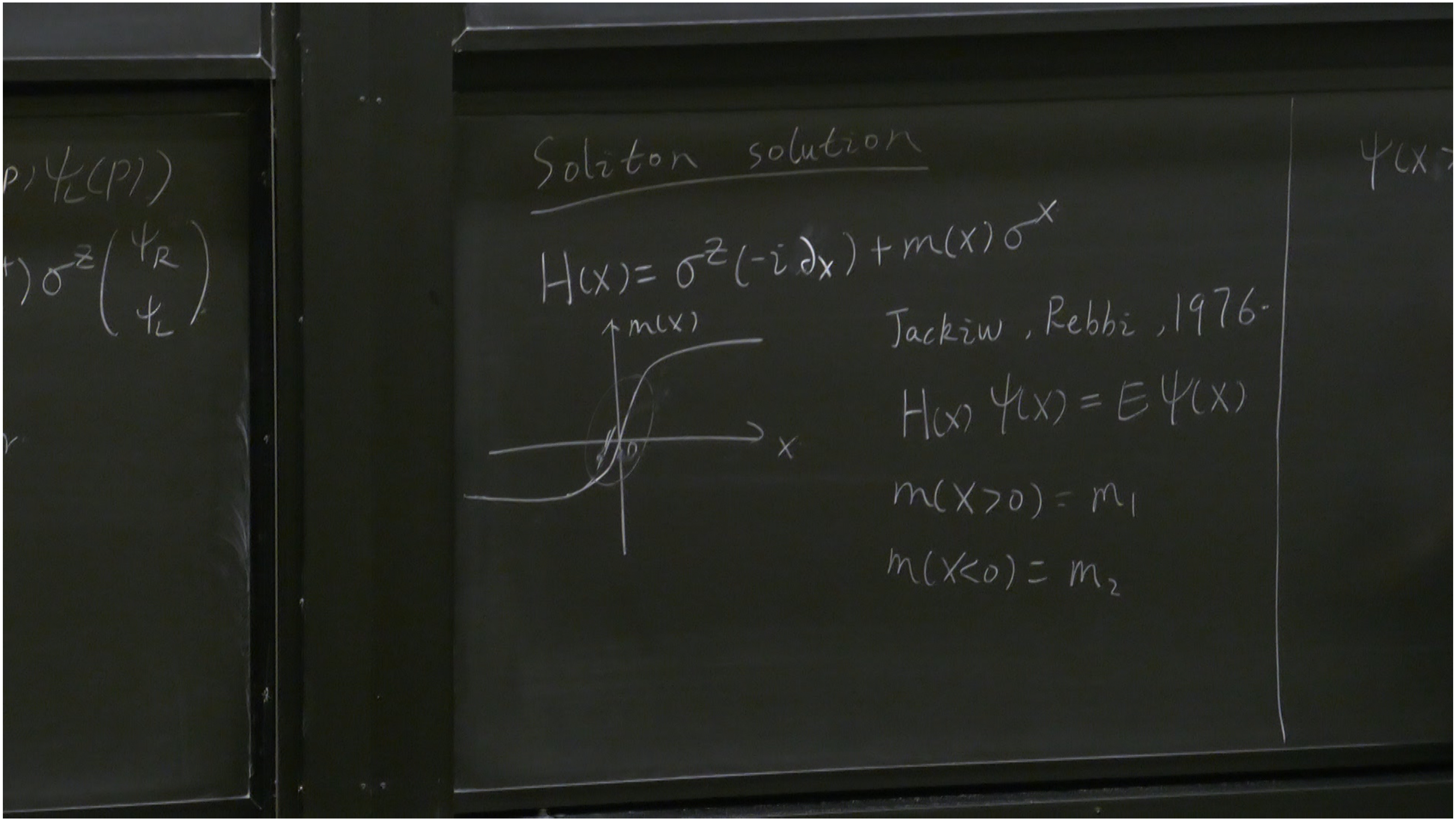
$$H' = -t\sum\left((-1)^j c_j^\dagger c_{j+1} + h.c.\right).$$

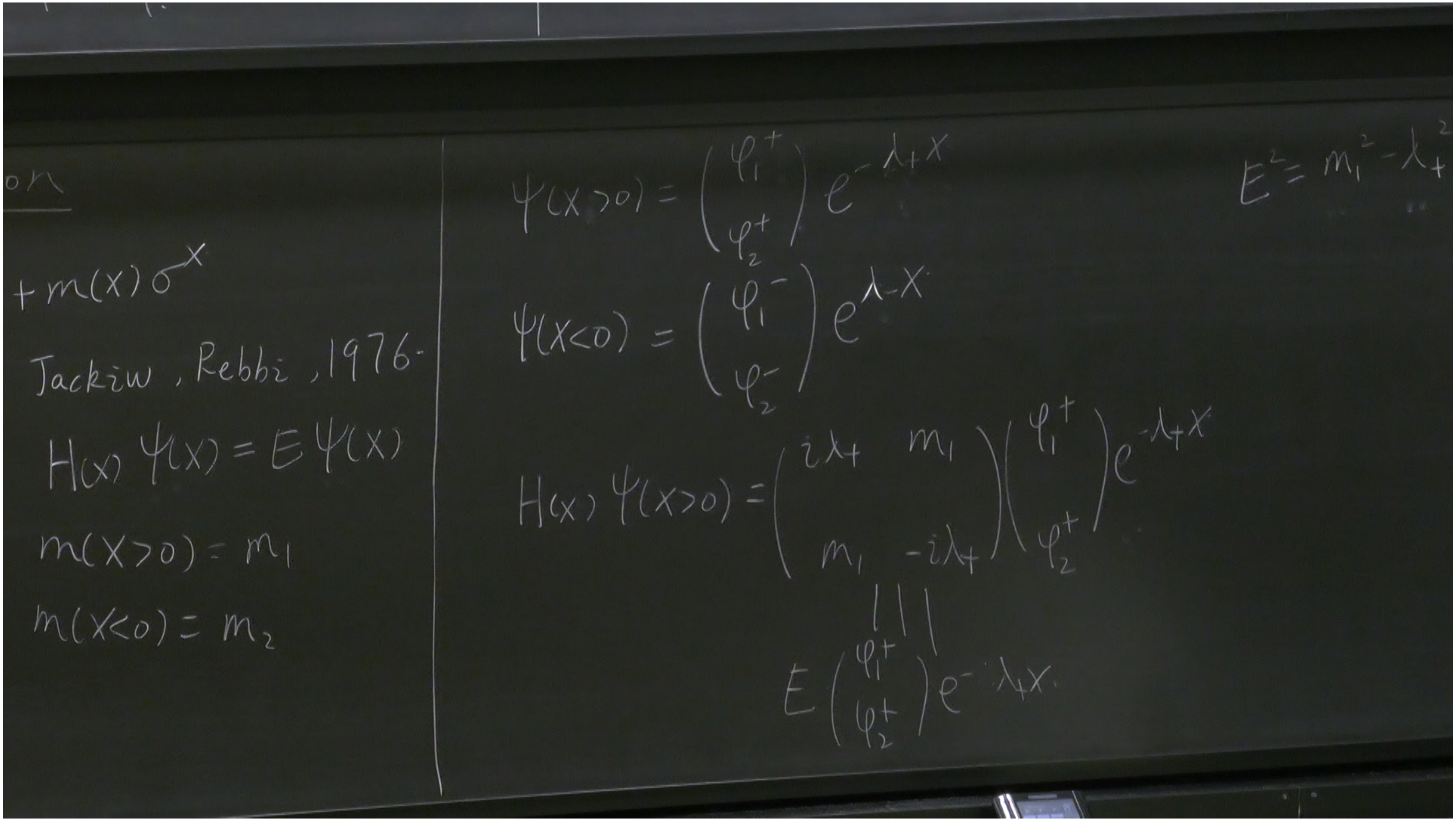
$$H' = -t\sum (-1)^j c_j^\dagger c_j .$$

Organic polymer, Su-Schrieffer-Heeger mode. 1979.

soliton excitation, fractional charge 1/2.

Atiyah-Singer index.


Soliton solution
$H(x) = \sigma^z(-i\partial_x) + m(x)\sigma^x$
Jackiw, Rebbi, 1976.
$m(x)$
x
$H(x)\psi(x) = E\psi(x)$
$m(x>0) = m_1$
$m(x<0) = m_2$


on
$+ m(x)\sigma^x$
Jackiw, Rebbi, 1976
$H(x)\psi(x) = E\psi(x)$
$m(x>0) = m_1$
$m(x<0) = m_2$
$\psi(x>0) = \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} e^{-\lambda_+ x}$
$\psi(x<0) = \begin{pmatrix} \varphi_1^- \\ \varphi_2^- \end{pmatrix} e^{\lambda_- x}$
$H(x)\psi(x>0) = \begin{pmatrix} i\lambda_+ & m_1 \\ m_1 & -i\lambda_+ \end{pmatrix} \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} e^{-\lambda_+ x}$
$|||$
$E \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} e^{-\lambda_+ x}$
$E^2 = m_1^2 - \lambda_+^2$

$$\begin{pmatrix}\varphi_1^+\\ \varphi_2^+\end{pmatrix} \sim \begin{pmatrix}\frac{E+i\sqrt{m_1^2-E^2}}{m_1}\\ 1\end{pmatrix}$$

$$\frac{E}{m_1} = \frac{E}{m_2}$$

$1^\circ.\ m_1 = m_2$

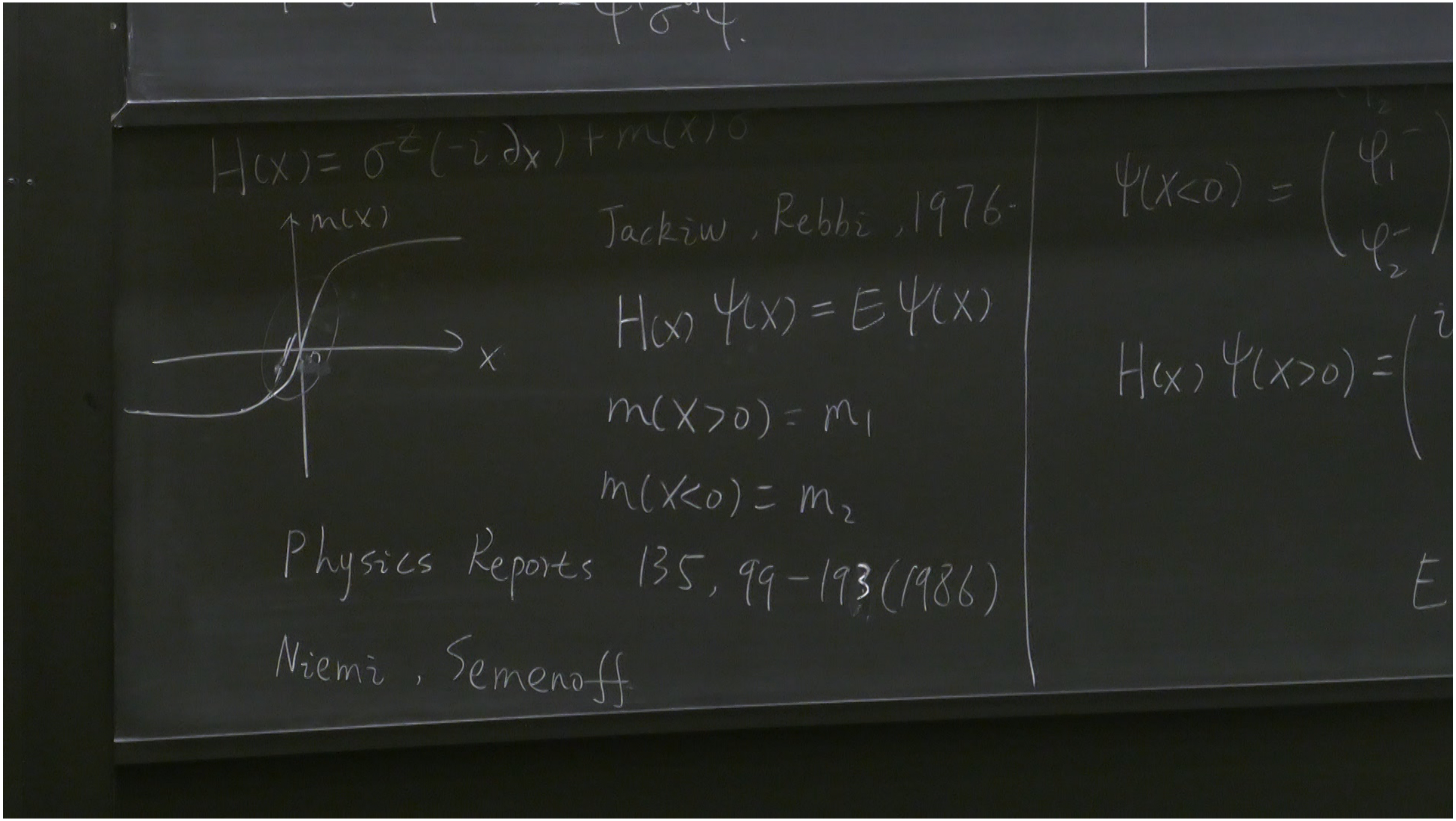
$\boxed{2^\circ.\ E=0,}$ $m_1 m_2 < 0$

$-\lambda_+ x$

$$E^2 = m_2^2 - \lambda_-^2, \quad \begin{pmatrix}\varphi_1^-\\ \varphi_2^-\end{pmatrix} = \begin{pmatrix}\frac{E+i\sqrt{m_2^2-E^2}}{m_2}\\ 1\end{pmatrix}$$

$$\begin{pmatrix} i\\ 1\end{pmatrix} e^{-m(x)\,x}$$

zero mode.


H(x) = σ^z(-i∂_x) + m(x)σ
Jackiw, Rebbi, 1976
m(x)
x
H(x)Ψ(x) = EΨ(x)
m(x>0) = m_1
m(x<0) = m_2
Physics Reports 135, 99-193 (1986)
Niemi, Semenoff
Ψ(x<0) = (φ_1^-, φ_2^-)
H(x)Ψ(x>0) =
E

$$\psi(x<0) = \begin{pmatrix} \varphi_1^- \\ \varphi_2^- \end{pmatrix} e^{\lambda_- x}$$

$$H(x)\,\psi(x>0) = \begin{pmatrix} i\lambda_+ & m_1 \\ m_1 & -i\lambda_+ \end{pmatrix} \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} e^{-\lambda_+ x}$$

$$\equiv E \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} e^{-\lambda_+ x}$$

$$\begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} \sim \begin{pmatrix} m_1 \\ 1 \end{pmatrix}$$

$$E^2 = m_2^2 - \lambda_-^2, \quad \begin{pmatrix} \varphi_1^- \\ \varphi_2^- \end{pmatrix} =$$

zero mode.

$$\begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix} \sim \begin{pmatrix} \frac{E + i\sqrt{m_1^2 - E^2}}{m_1} \\ 1 \end{pmatrix} \qquad \frac{E}{m_1} = \frac{E}{m_2}$$

$$1^\circ.\ \underline{m_1 = m_2}$$

$$) e^{-\lambda_+ x} \qquad E^2 = m_2^2 - \lambda_-^2, \begin{pmatrix} \varphi_1^- \\ \varphi_2^- \end{pmatrix} = \begin{pmatrix} \frac{E + i\sqrt{m_2^2 - E^2}}{m_2} \\ 1 \end{pmatrix} \qquad \boxed{2^\circ.\ E = 0,}\ m_1 m_2 < 0$$

$$\begin{pmatrix} i \\ 1 \end{pmatrix} e^{-m(x)\, x}$$

zero mode.