Title: Quantum Matter Lecture

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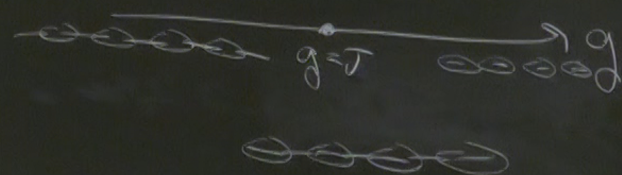
1d TFIM

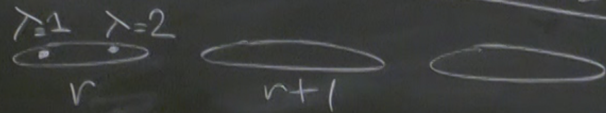
$$H = -J\sum_i Z_i Z_{i+1} - g\sum_i X_i$$

$$\gamma_{2i} = \Big(\prod_{j<i} X_j\Big) Z_i \quad , \quad \gamma_{2i+1} = \Big(\prod_{j<i} X_j\Big) Y_i$$

$$H = -J\sum_i i\,\gamma_{2i+1}\gamma_{2i+2} - g\sum_i i\,\gamma_{2i}\gamma_{2i+1}$$



$$Z_L Z_1 = (-P)(i\gamma_{2L+2}\gamma_1)$$



$$\gamma_{r\lambda} = \sum_k e^{ikr}\gamma_{k\lambda} \qquad \left(\gamma^{\dagger}_{k\lambda} = \gamma_{-k\lambda}\right)$$

$$\gamma_{k\lambda} = \left(\sum_r e^{-ikr}\gamma_{r\lambda}\right)$$

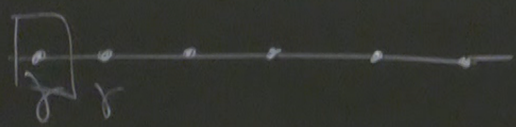
$$H = -\sum_r i\left(\sum_{kk'} g e^{i(k+k')r} - J e^{i(kr+k'(r+1))}\right)\gamma_{k1}\gamma_{k'2}$$

$$= -\sum_k i\left(g - Je^{-ik}\right)\gamma_{k1}\gamma_{-k2}$$

Each k-mode: $(\gamma_{k1}, \gamma_{-k2})$

$$H_k = \begin{pmatrix} 0 & -i(g - Je^{-ik}) \\ i(g - Je^{ik}) & 0 \end{pmatrix}$$

$$\pm E_k = \pm\left|i(g - Je^{-ik})\right| = \pm\sqrt{J^2 + g^2 - 2Jg\cos k}$$



$J = g$: gapless $k = 0$

Gap (and locality) control correlations, entanglement, stability of ground state

(Hastings 1008.5137)

Lieb-Robinson bound

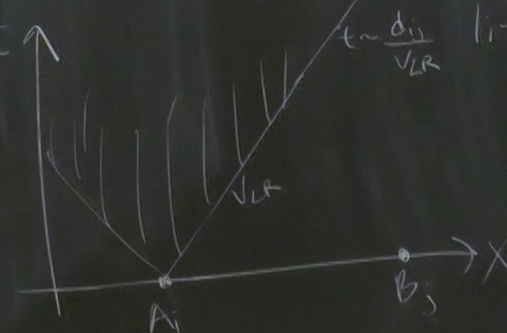
local Hamiltonian H

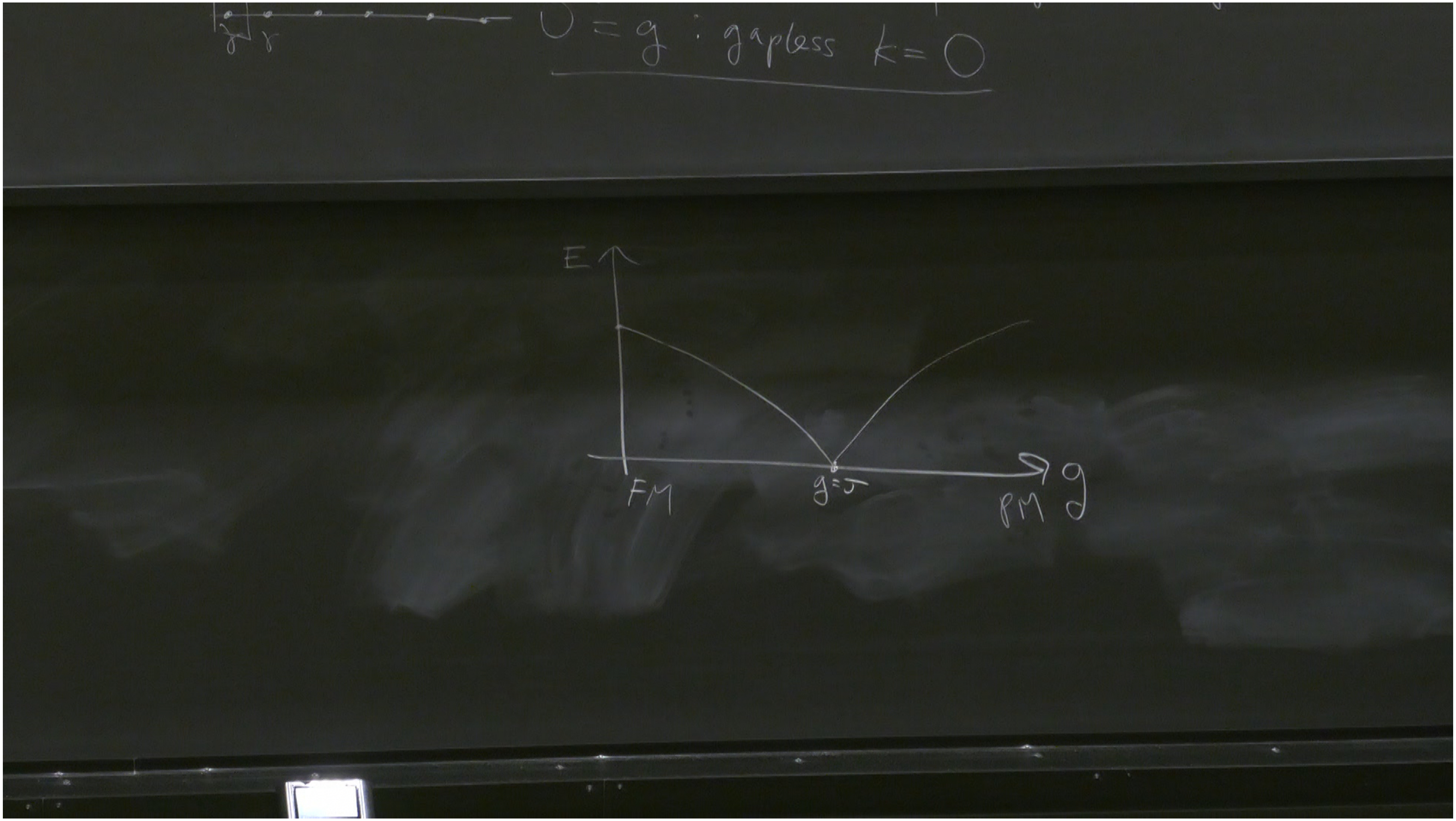
$$[A_i, B_j] = 0$$

$$A_i(t) \equiv e^{iHt} A_i e^{-iHt}$$

$$\| [A_i(t), B_j] \| \le c_1 \exp\left(-c_2 \left(d_{ij} - v_{LR} t\right)\right)$$

$$d_{ij} = |i-j|$$




= g : gapless k = 0
E
FM
g=J
PM
g

Local H w/ unique ground state $|\psi_0\rangle$ w/ gap Δ

① Correlations $\forall\, O_i, O_j$

$$\langle\psi_0|O_iO_j|\psi_0\rangle - \langle\psi_0|O_i|\psi_0\rangle\langle\psi_0|O_j|\psi_0\rangle \propto \exp(-d_{ij}/\xi)$$

$\xi = v_{LR}/\Delta$

$|\psi_{0a}\rangle$ ← g.s. degeneracy

$$P_0 = \sum_a |\psi_{0a}\rangle\langle\psi_{0a}|$$

$$\langle\psi_{0a}|O_iO_j|\psi_{0a}\rangle - \langle\psi_{0a}|O_iP_0O_j|\psi_{0a}\rangle \propto \exp(-d_{ij}/\xi)$$

w/ unique ground state $|\Psi_0\rangle$ w/ gap Δ

relations $\forall\, O_i O_j$

v_{LR}/Δ

$$\langle\Psi_0|O_iO_j|\Psi_0\rangle - \langle\Psi_0|O_i|\Psi_0\rangle\langle\Psi_0|O_j|\Psi_0\rangle \propto \exp(-d_{ij}/\xi)$$

$$P_0 = \sum_a |\Psi_{0a}\rangle\langle\Psi_{0a}|$$

$$\langle\Psi_{0a}|O_iO_j|\Psi_{0a}\rangle - \langle\Psi_{0a}|O_iP_0O_j|\Psi_{0a}\rangle \propto \exp(-d_{ij}/\xi)$$

$$\langle GHZ_+|Z_iZ_j|GHZ_+\rangle - \langle GHZ_+|Z_iP_0Z_j|GHZ_+\rangle = 0$$

$\propto \exp(-d_{ij}/\xi)$

$\{Z, P\} = 0$

$\langle Z_i Z_j \rangle - \langle Z_i \rangle\langle Z_j \rangle = 1$

local, sym O^{sym} $([O^{sym}, P] = 0)$

$\rangle \propto \exp(-d_{ij}/\xi)$

$P_0 Z_j |GHZ\rangle = 0$

$\langle GHZ | O_i^{sym} O_j^{sym} | GHZ \rangle - \langle GHZ | O_i^{sym} | GHZ \rangle \langle GHZ | O_j^{sym} | GHZ \rangle$

$\sim \exp(-d_{ij}/\xi)$

$\langle GHZ_+ | Z_i Z_j | GHZ_+ \rangle$

$|\psi_0\rangle$ ξ

② Entanglement

A

$$\underline{S_A \sim \xi |\partial A|} \quad \text{(area law)}$$

proven 1d

③ Stability of gap

Hastings-Wen quasi-adi

ment

$|\psi_0\rangle$ ξ

A

(area law)

$$S_A \sim \xi |\partial A|$$

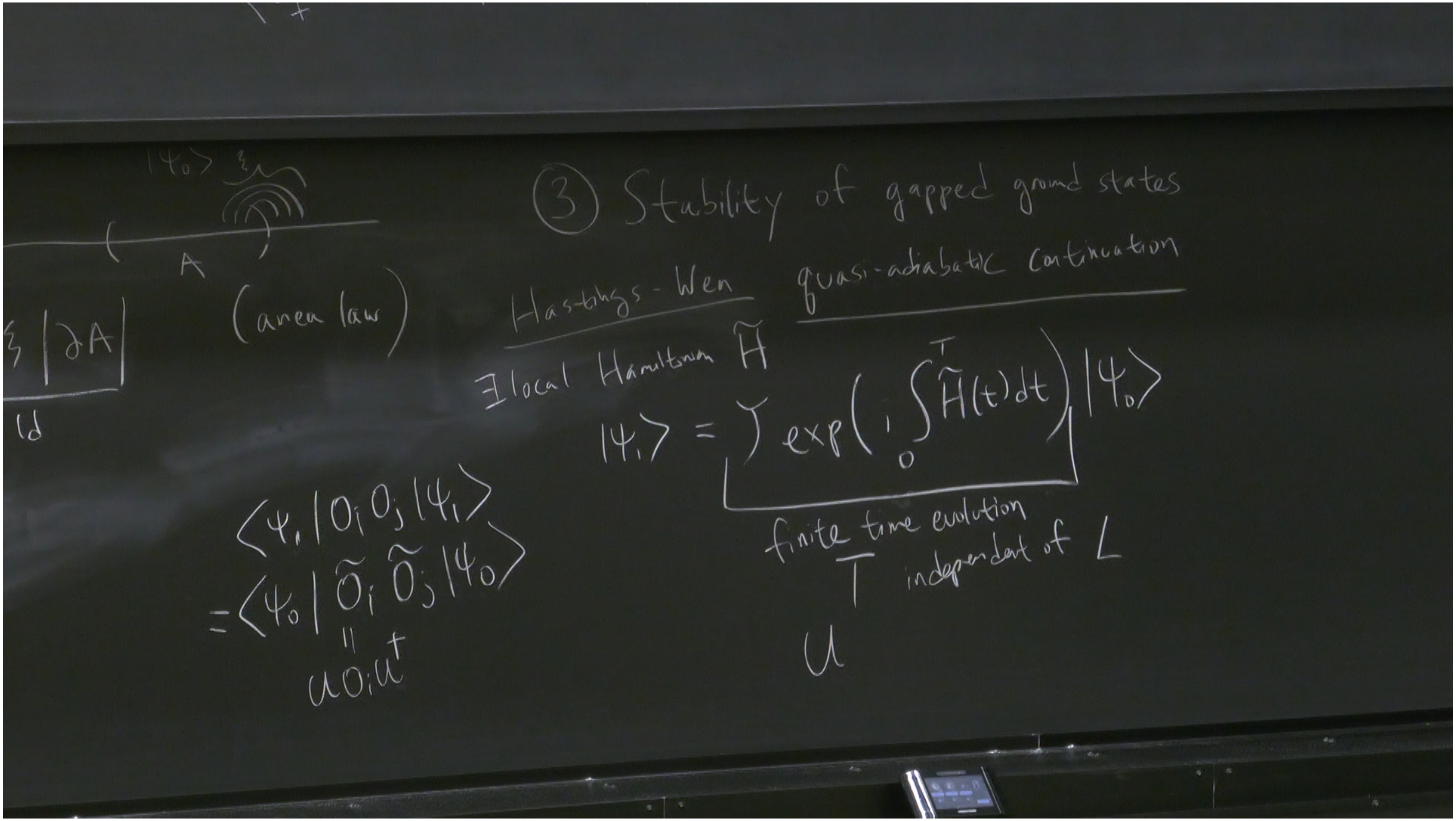
proven 1d

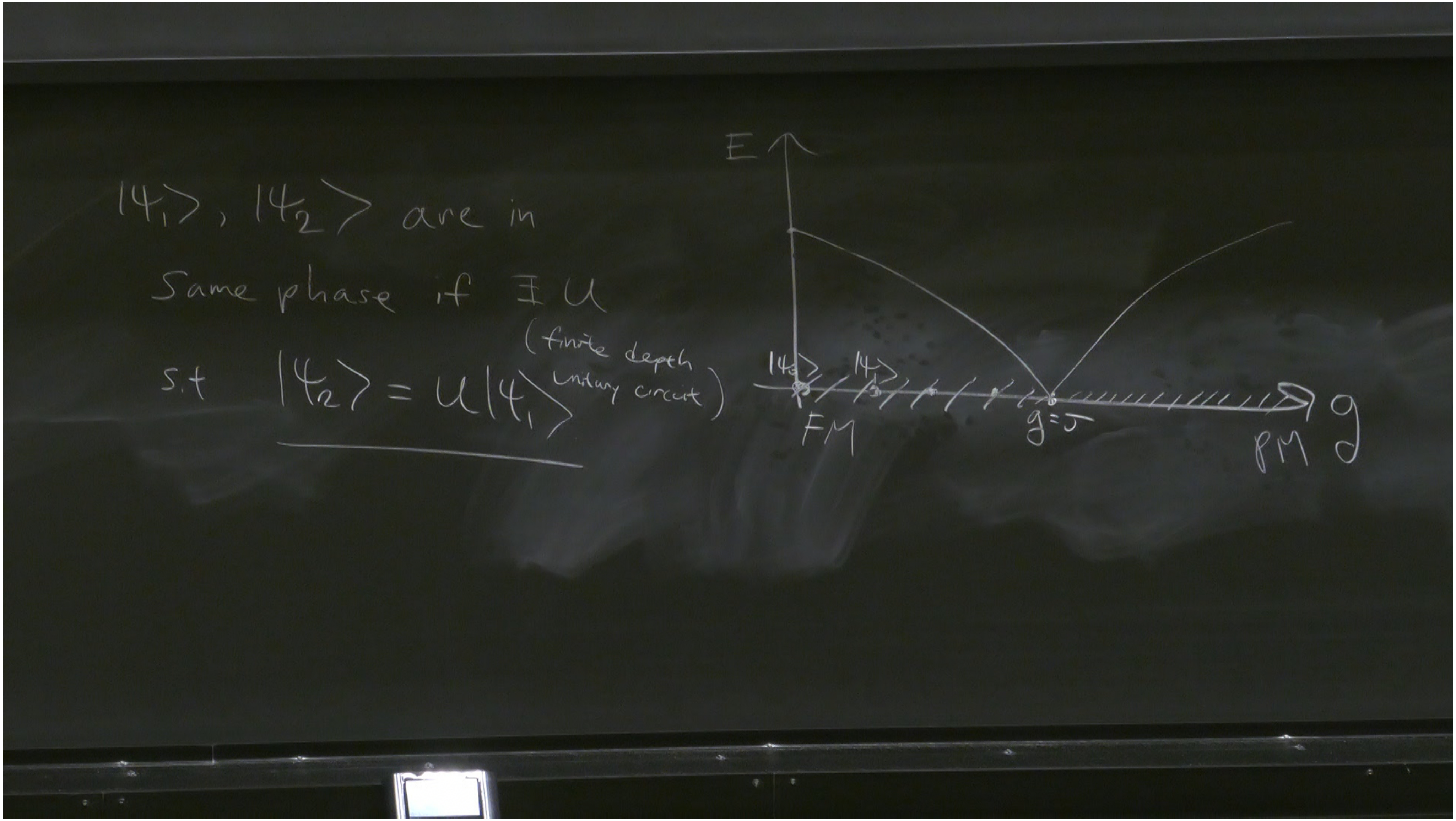
3 Stability of gapped ground states

Hastings-Wen quasi-adiabatic continuation

$\exists$ local Hamiltonian $\tilde{H}$

$$|\psi_1\rangle = \mathcal{T} \exp\left(i \int_0^T \tilde{H}(t)\, dt \right) |\psi_0\rangle$$


$|\Psi_0\rangle$ ξ
A
(area law)
$\xi|\partial A|$
ld
③ Stability of gapped ground states
Hastings-Wen quasi-adiabatic continuation
$\exists$ local Hamiltonian $\tilde{H}$
$|\Psi_1\rangle = \mathcal{T}\exp\left(i\int_0^T \tilde{H}(t)\,dt\right)|\Psi_0\rangle$
finite time evolution
T independent of L
U
$\langle\Psi_1|O_i O_j|\Psi_1\rangle$
$= \langle\Psi_0|\tilde{O}_i \tilde{O}_j|\Psi_0\rangle$
$U O_i U^\dagger$


|ψ₁>, |ψ₂> are in
Same phase if ∃ U
(finite depth unitary circuit)
s.t |ψ₂> = U|ψ₁>
E
|ψ₀>
|ψ₁>
FM
g=J
PM
g

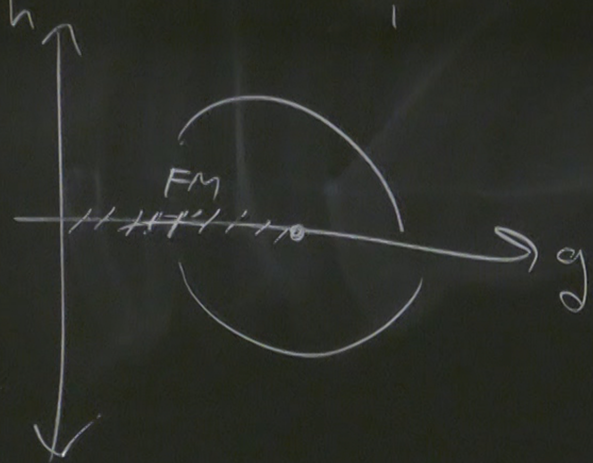
$$H' = H_{TFIM} - h\sum_i Z_i$$

$|\psi_1\rangle, |\psi_2\rangle$ are

Same phase if

s.t $|\psi_2\rangle = U|\psi$

$$H' = H_{TFIM} - h \sum_i Z_i$$



$|\psi_1\rangle, |\psi_2\rangle$ are in same phase if $\exists\, \mathcal{U}$ (finite depth unitary circuit)

s.t. $\underline{|\psi_2\rangle = \mathcal{U}|\psi_1\rangle}$