Title: Quantum Matter Lecture

Speakers: Yin-Chen He

Collection: Quantum Matter 2023/24

Date: March 28, 2024 - 10:15 AM

URL: https://pirsa.org/24030047

Fuzzy sphere regularization of 3D CFT

Recap: state-operator correspondence

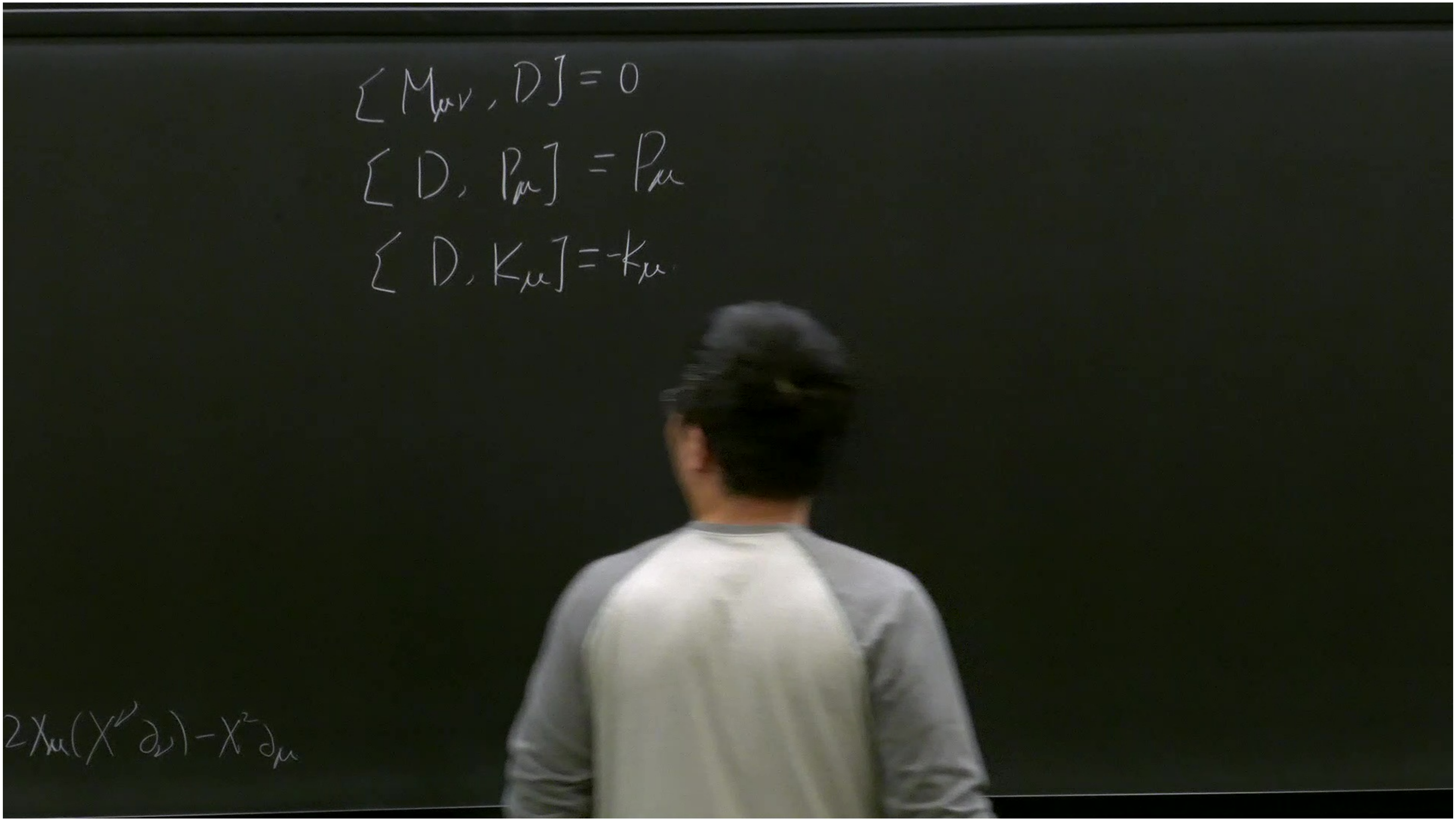
conformal generators $SO(d+1,1)$

1°. Translation: $P_\mu = \partial_\mu$

2°. Rotation: $M_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu$

3°. Dilatation: $D = x^\mu \partial_\mu$

4°. Special conformal transf. (SCT): $K_\mu = 2x_\mu (x^\nu \partial$


$[M_{\mu\nu}, D] = 0$
$[D, P_\mu] = P_\mu$
$[D, K_\mu] = -K_\mu$
$2x_\mu(x^\nu \partial_\nu) - x^2 \partial_\mu$

$[M_{\mu\nu}, D] = 0$

$[D, P_\mu] = P_\mu$

$[D, K_\mu] = -K_\mu$

Study a quantum Ham. H on S^{d-1}

$H|0\rangle = 0$,

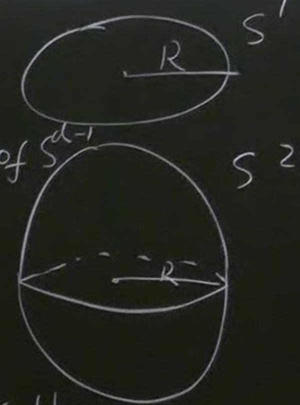
$H = D/R$, $M_{\mu\nu}$ is rotation of S^{d-1}

$[RH, P_\mu] = P_\mu$

$H|\phi\rangle = \frac{\Delta}{R}|\phi\rangle$, primary: $K_\mu|\phi\rangle = 0$

$H \underline{P_\mu|\phi\rangle} = \frac{\Delta+1}{R} P_\mu|\phi\rangle$

$H K_\mu|\phi\rangle = \frac{\Delta-1}{R} K_\mu|\phi\rangle$



ion of 3D CFT

tor correspondence

$SO(d+1,1)$

$P_\mu = \partial_\mu$

$M_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu$

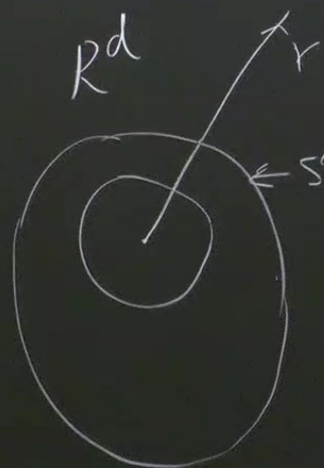
$D = x^\mu \partial_\mu$

formal transf. (SCT): $K_\mu = 2x_\mu (x^\nu \partial_\nu) - x^2 \partial_\mu$

$[M_{\mu\nu}, D] = 0$

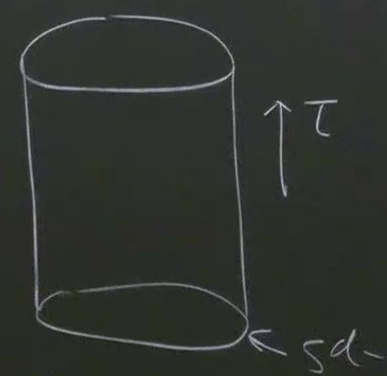
$[D, P_\mu] = P_\mu$

$[D, K_\mu] = -K_\mu$

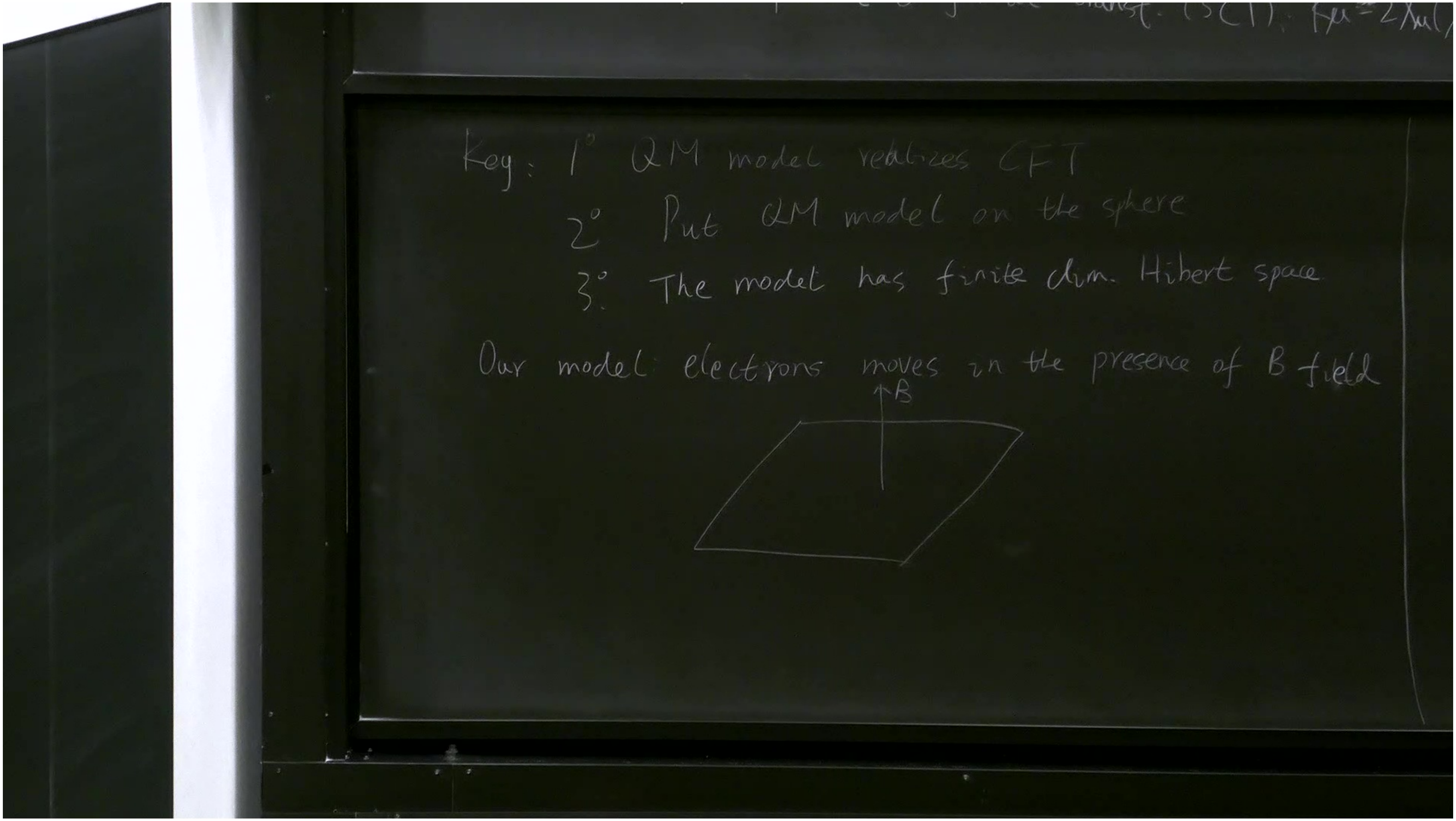


Weyl trans. $\longrightarrow$

$\tau = \log r$



Study a qu


Key: 1° QM model realizes CFT
2° Put QM model on the sphere
3° The model has finite dim. Hibert space
Our model: electrons moves in the presence of B field
B

Landau level, non-commutative geometry

$\vdots$

$n=2$

$n=1$

$n=0$

1°. Quantized energy $E_n = \frac{B}{M}\left(n+\frac{1}{2}\right)$

2°. Completely flat.

3°. Massive deg. at each level: $\frac{BA}{2\pi}$

$$\mathcal{L} = \frac{M}{2}\dot{\vec{X}}^2 - \dot{\vec{X}}\cdot\vec{A}, \quad A_i = -\frac{B}{2}\epsilon_{ij}X^j$$

$$\mathcal{L}_{eff} = -\dot{\vec{X}}\cdot\vec{A} = \frac{B}{2}\epsilon_{ij}\dot{X}^i X^j, \quad X^1 \sim \frac{p^2}{B}, \quad X^2 \sim \frac{p^1}{B}, \quad [X^1, p^1] \Rightarrow [$$

geometry

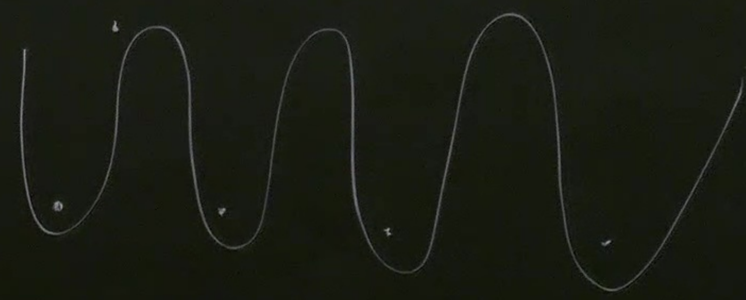
$$= \frac{B}{M}\left(n+\frac{1}{2}\right)$$

$\vdots$

$n=2$

$n=1$ } gap: $\frac{B}{M}$ >> interactions

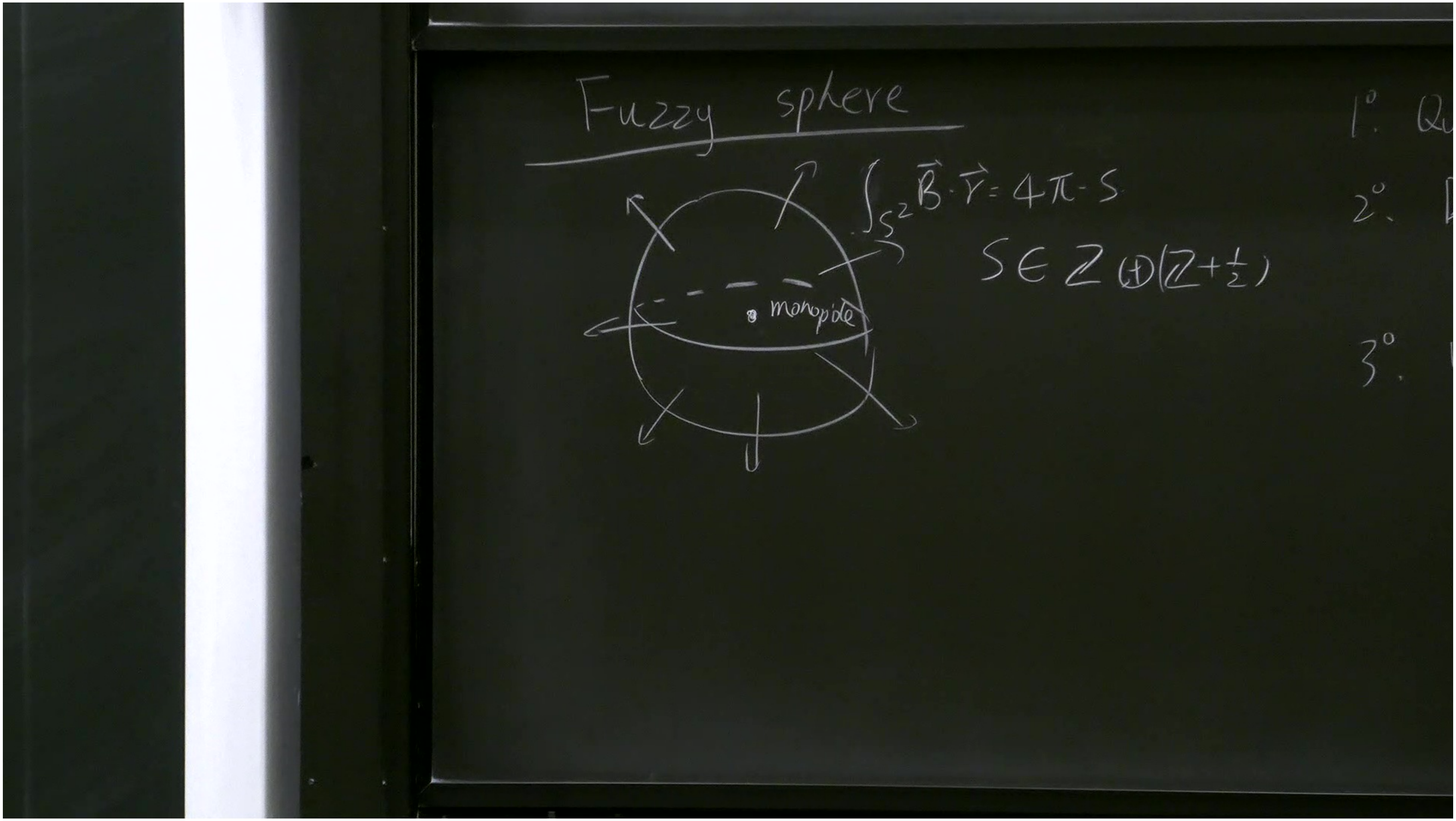
$n=0$

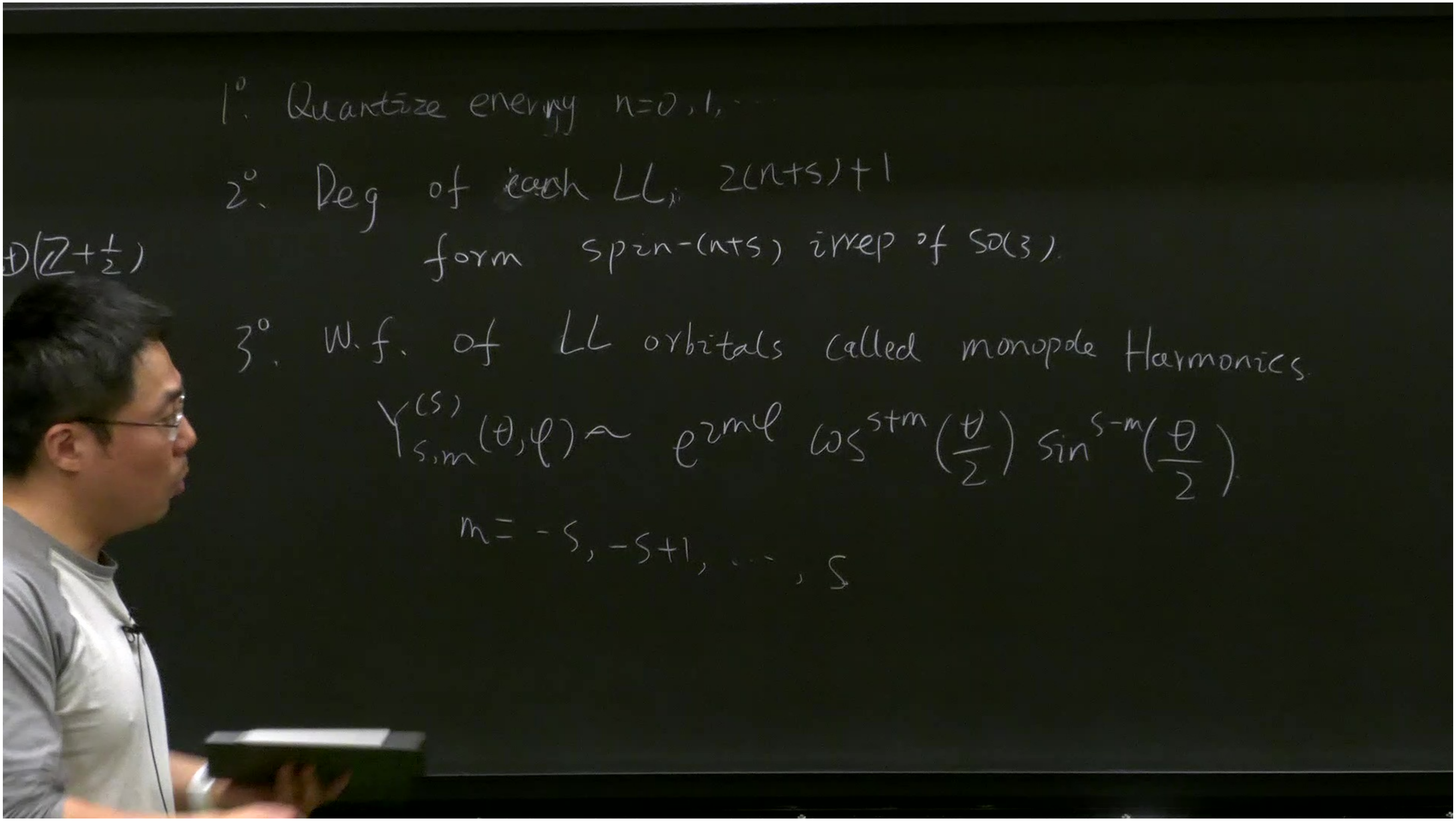


level: $\frac{BA}{2\pi}$

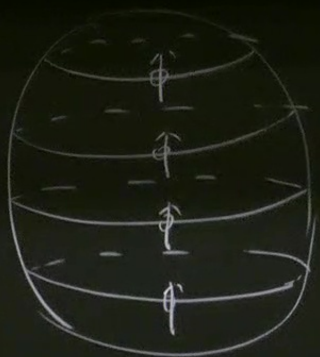
$$A_i = -\frac{B}{2}\epsilon_{ij}X^j$$

$$\ldots_{j}\,\dot{X}^i X^j, \qquad X^1 \sim \frac{p^2}{B}, \quad X^2 \sim \frac{p^1}{B}, \qquad [X^1, p^1] \Rightarrow [X^1, X^2] = \frac{i}{B}$$

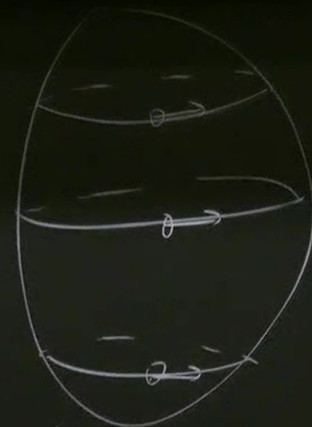

Fuzzy sphere
$\int_{S^2} \vec{B}\cdot\vec{r} = 4\pi\cdot S$
$S \in \mathbb{Z} \oplus (\mathbb{Z}+\frac{1}{2})$
monopole
1°.
2°.
3°.


1°. Quantize energy $n=0,1,\cdots$
2°. Deg of each LL: $2(n+s)+1$
form spin-$(n+s)$ irrep of $SO(3)$.
3°. W.f. of LL orbitals called monopole Harmonics.
$Y^{(s)}_{s,m}(\theta,\varphi) \sim e^{im\varphi} \cos^{s+m}\left(\frac{\theta}{2}\right) \sin^{s-m}\left(\frac{\theta}{2}\right)$.
$m = -s, -s+1, \cdots, s$
$\oplus(\mathbb{Z}+\frac{1}{2})$

$c_\uparrow, c_\downarrow$



Ferromagnet



Paramagnet

$$H = \frac{1}{2Mr^2}\int d\vec{r}\; \psi^\dagger(\vec{r})(\partial_\mu + iA_\mu)\psi(\vec{r}) + H_{int},$$

$$\psi(\vec{r}) = \begin{pmatrix} \psi_\uparrow(r) \\ \psi_\downarrow(r) \end{pmatrix}$$

$$n^a = \psi^\dagger(\vec{r})\,\sigma^a\,\psi(\vec{r})$$

$$H_{int} = -\int d\vec{r}_a\, d\vec{r}_b\; \underline{U(\vec{r}_a - \vec{r}_b)}\; n^z(r_a)\, n^z(r_b) + h\int dr\; n^x(r)$$

e.g. $\delta(\vec{r}_a - \vec{r}_b)$, $\nabla^2\delta$, $\ldots$ $\nabla^{2n}\delta$

$$\psi_\sigma(\theta,\varphi) = \sum_{m=-s}^{s} c_{\sigma,m} Y^{(s)}_{s,m}(\theta,\varphi)$$

$$h \int \sin\theta \, d\theta \, d\varphi \sum_{m_1,m_2} (c^{+}_{\uparrow,m_1} \; c_{\downarrow,m_1}) \sigma^x \begin{pmatrix} c_{\uparrow,m_2} \\ c_{\downarrow,m_2} \end{pmatrix} \bar{Y}^{(s)}_{s,m_1}(\theta,\varphi) Y^{(s)}_{s,m_2}(\theta,\varphi)$$

$$\sim h \sum_m (c^{+}_{\uparrow,m} \; c_{\downarrow,m}) \sigma^x \begin{pmatrix} c_{\uparrow,m} \\ c_{\downarrow,m} \end{pmatrix}$$