Title: Quantum Matter Lecture

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Symmetry Protected Topological Phases (SPT)

Previously:
- SSB (if sym enforced, then long-range correlated states)
- Top. order (non-trivial w/o symmetry)
- SPT (short-range correlated, non-trivial when sym is enforced)
  - bulk physics relatively trivial
  - boundary non-trivial

1d "Cluster" state

$$H = -\sum_i Z_i X_{i+1} Z_{i+2}$$

Symmetry: $\mathbb{Z}_2 \times \mathbb{Z}_2$

$P_A = \prod_{i \in A} X_i \qquad P_B = \prod_{j \in B} X_j$

A B A B A B A B

ZXZ
$\quad ZXZ$
ZXZ
$\quad ZXZ$

all terms commute.

Ground state "stabilized" by $ZXZ = +1$

PBC:

A B A B A B A B

$Z X Z$
$\quad Z X Z$

$Z X Z$
$\quad Z X Z$

+2

all terms commute.

Ground state "stabilized" by $ZXZ = +1$

PBC: unique st.

Open boundary conditions:

$Z X Z$
$\quad Z X Z$
$\quad\quad Z X Z$
$\quad\quad\quad \ddots$
$\quad\quad\quad\quad Z X Z$

L qubits

$L-2$ constraints

$\Rightarrow 2^2 = 4$-fold degeneracy

Con

ng:

qubits

Constraints

$^2 = 4$-fold degeneracy

Can parameterize 4-fold subspace

$$P_A = \prod_{i \in A} X_i = \pm 1 \qquad P_B = \prod_{j \in B} X_j = \pm 1$$

local sym-breaking $H' = -\varepsilon Z_1 - \varepsilon Z_L$

favor $Z_1 = +1,\ Z_L = +1$

no local sym-preserving H' that can lift degeneracy

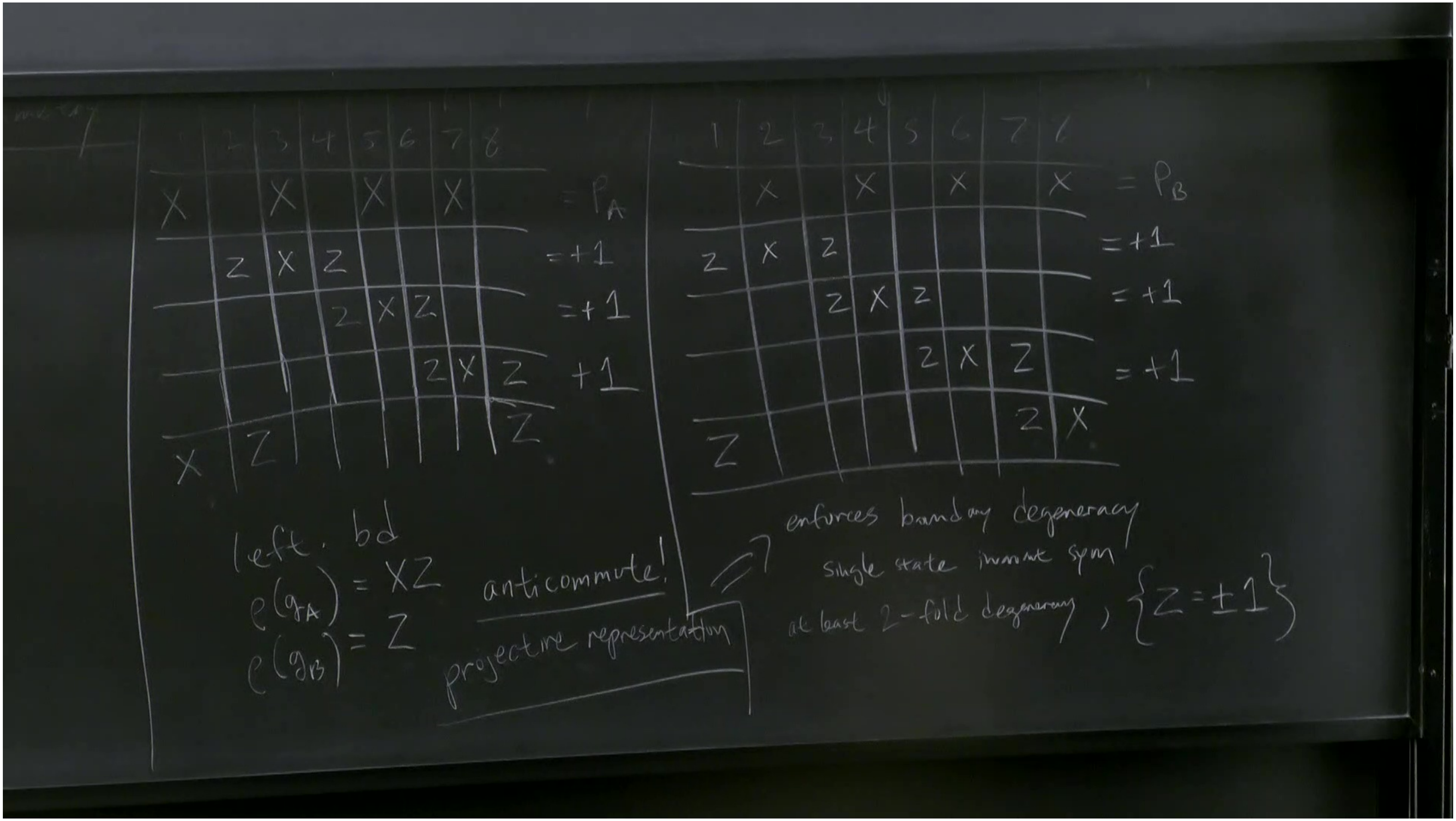
non-local symmetric way: $H' = -P_A - P_B$

Each boundary forms projective representation of symmetry

$\mathbb{Z}_2 \times \mathbb{Z}_2$

$g_A \quad g_B$

$$\rho(g_A)\rho(g_B) = \underline{c(g_A, g_B)}\,\rho(g_A g_B)$$


1 2 3 4 5 6 7 8
= P_A
= +1
= +1
+1
= P_B
= +1
= +1
= +1
left. bd
ρ(g_A) = XZ
ρ(g_B) = Z
anticommute!
projective representation
enforces boundary degeneracy
single state invariant sym
at least 2-fold degeneracy , {Z = ±1}

1d SPT has string / non-local order

PBC

$$Z \times Z$$
$$Z \times Z$$
$$Z \times Z$$
$$\cdots$$

$$= Z_\ell \underbrace{X \quad X \quad X \quad X}\, Z_r = +1 \text{ in g.s.}$$

$$\text{sym} = Z_2 \times Z_2$$

$$\boxed{S = \hat{O}_\ell \, \underline{P^{(g_A)}_{\text{trunc}}} \, \hat{O}_r}$$

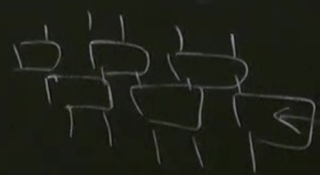
$$\langle S \rangle = O(1) \quad \text{indep. } |\ell - r|$$

charged under g_B

SPT <u>non-trivial</u> in presence of sym

different phases cannot be related

by finite depth local unitary (finite time H_{loc})
symmetric ... symmetric



each gate is symmetric

sym finite depth U_{sym}

$$[P_{trunc}, U_{sym}]$$

$$O_l \to U O_l U^\dagger$$

$$O_r \to U O_r U^\dagger$$

for trivial state

$$O_l \, P_{trunc} \, O_r$$

not charged

$$|SPT\rangle \neq U_{sym} |triv\rangle$$

$$= U_{non\text{-}sym} |triv\rangle$$

Controlled phase on 2-qubits A, B

$$CZ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

if $A = \downarrow$, applies Z to B

$$(CZ)_{12}\, X_2\, (CZ)_{12} = Z_1 X_2$$

$$(CZ)_{23}\, X_2\, (CZ)_{23} = X_2 Z_3$$

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CZ breaks $Z_2 \times Z_2$ sym

$$U = \prod_i CZ_{i,i+1}$$

takes $|+\rangle \longrightarrow$ cluster state